Moments of Meson PDFs from Lattice QCD

Status and Perspectives

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Why Meson PDFs …?

- Mesons are of course fundamental states in QCD

$\Rightarrow$ important to understand their structure

- we may learn about different aspects of QCD than from nucleons
- in particular: chiral symmetry (breaking)

It turns out

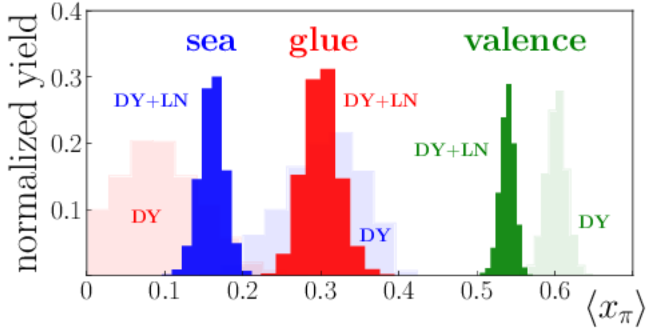
- there is surprisingly little known on meson PDFs

of course

- there is no meson target for experiments!

Strong Dependence on Data Set for Pion PDFs!

- recent Monte Carlo global QCD analysis of pion PDFs
  [Barry et al., PRL, 121, 152001 (2019)]
- Drell-Yan (DY) and Leading Neutron Electroproduction (LN) data
- DY: light shaded
  DY + LN: dark shaded



- analysis based on DY alone and DY + LN lead to significantly different pion momentum fractions
- more data and first principles theoretical analysis needed!

Opportunities in Upcoming Experiments

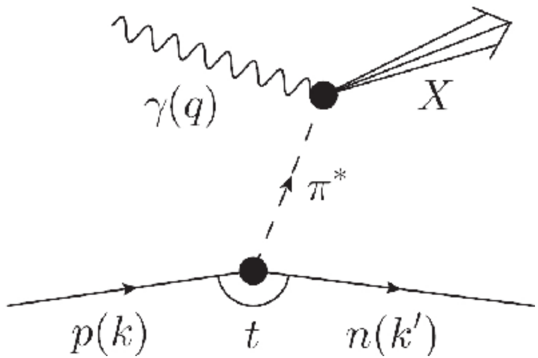
Pion PDFs will be further constrained by upcoming experiments

- COMPASS will provide new πA DY data
  wwwcompas.cern.ch/compass/future_physics/drellyan/

- Tagged DIS (TDIS) at JLAB pion structure through $ed \rightarrow eppX$
  [J. Annand et al., JLab experiment E12-15-006]

- a future electron-ion collider (eRHIC, JLEIC, LHCeC, …)

Sullivan Process relevant for PDFs:



[A. C. Aguilar et al., arXiv:1907.08218]

Beyond the Pion

- Pion is of fundamental interest as lightest (Goldstone) boson in QCD
  much lighter than proton due to **spontaneous chiral symmetry breaking**

- Kaon is still a Goldstone boson
  but with the heavier strange quark contributing to its mass
  "competition" between chiral symmetry breaking and quark mass

- ρ-meson has quark content identical to the pion
  but spin flipped
  not a Goldstone boson

$\Rightarrow$ we can hope to learn about spontaneous chiral symmetry breaking
and its interplay with quark masses and QCD mass generation

Parton Distribution Functions from Lattice QCD

Nowadays two complementary approaches:

- traditional: computing moments of PDFs
  [G. Martinelli and C. T. Sachrajda, Phys.Lett. B196, 184 (1987)]
  - relate moments of PDFs to matrix elements of local operators
  - only lowest few moments accessible
- novel: compute the x-dependence via quasi/pseudo/…-PDFs
  [X. Ji, Phys. Rev. Lett. 110, 262002 (2013), …]
  - direct access to PDFs
  - several extrapolations and systematics need to be understood

here we focus on the traditional approach

Moments of PDFs from Lattice QCD

- Mellin moments of PDFs

$$\langle x^n \rangle_q = \int_0^1 x^n \left(f_q(x) + (-1)^{n+1} f_{\bar{q}}(x) \right) dx$$

- through operator product expansion at leading twist related to matrix elements of twist two operators $\mathcal{O}_q^{\mu\nu\ldots\rho}$

$$\langle H | \bar{q} \gamma_{\{\mu} \overleftrightarrow{D}_\nu \cdots \overleftrightarrow{D}_{\rho\}} q | H \rangle - \text{traces}\,.$$

- note that lattice has access to
  - valence + sea contribution for n odd (e.g. $\langle x \rangle$)
  - valence only contribution for n even (e.g. $\langle x^2 \rangle$)

Distinguishing Valence and Sea contributions

- identify connected contribution to matrix element with valence $\langle x \rangle_v$ and disconnected contribution with sea $\langle x \rangle_s$?
- strictly speaking not possible!
- however, the following sum rule must hold

$$\langle x \rangle_u^{\mathrm{conn}} + \langle x \rangle_d^{\mathrm{conn}} + \sum_q \langle x \rangle_q^{\mathrm{disc}} = \langle x \rangle_v + N_f \langle x \rangle_s = 1 - \langle x \rangle_G$$

  for the renormalised moments

- as long as disconnected contributions are omitted, comparison to phenomenology is approximate only

Mixing under Renormalisation

- main limitation in lattice calculation of moments:
  operator mixing
- in addition: higher moments require more non-zero components of meson momentum
- with the right choice of operators mixing can be avoided up to $\langle x^3 \rangle$
  [Capitani and Rossi, Nucl.Phys. B433 (1995); Beccarini et al., Nucl.Phys. B456 (1995)]
- e.g. for $\langle x \rangle$

$$\mathcal{O}_{44} = \bar{q}\left[\gamma_0 \overleftrightarrow{D}_0 - \frac{1}{3}\sum \gamma_k \overleftrightarrow{D}_k\right] q$$

does not mix and requires zero total momentum only

Mixing under Renormalisation

- for $\langle x^2 \rangle$ there is no mixing with

$$\mathcal{O}_q^{012} = \bar{q}\gamma_{\{0}\overleftrightarrow{D}_1\overleftrightarrow{D}_{2\}}q$$

  with two non-zero spatial components of momentum

- and $\langle x^3 \rangle$ a rank four operator does not mix

$$\mathcal{O}_q^{0123} = \bar{q}\gamma_{\{0}\overleftrightarrow{D}_1\overleftrightarrow{D}_2\overleftrightarrow{D}_{3\}}q$$

  with three non-zero spatial components of momentum

- for rank five operators mixing is unavoidable

Reconstruction of PDFs from First Few Moments?

- reconstruction of full x-dependence requires **infinitely** many moments
- in particular for large x higher moments are required
- one may use parametrizations like in global fits and information about the heavy quark limit
  [Detmold, Melnitchouk, Thomas, Eur.Phys.J.direct 3 (2001); Phys.Rev. D68 (2003)]
- alternative: compare moments directly
  - moments extracted from experimental data rely on parametrizations

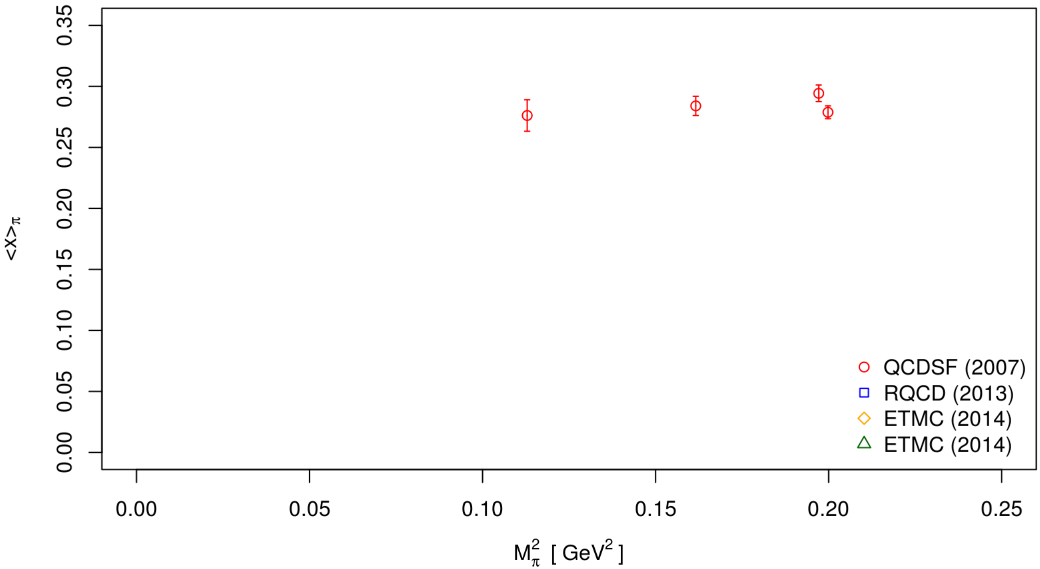
$\Rightarrow$ **a PDF computation solely from first principles difficult**

- maybe best to combine lattice and experiment in global analysis!

Moments of Pion PDFs from LQCD

- pion is studied best using lattice techniques
- quenched data from 2005 or older
  [Best et al., Phys.Rev.D56 (1997); ZeRo Collaboration, Eur. Phys. J. C40, 69 (2005); S. Capitani et al., Phys. Lett. B639, 520 (2006)]
- since 2007 $N_f = 2$ or $N_f = 2 + 1 + 1$ standard
  QCDSF (2007), Best et al., $N_f = 2$ Wilson clover
  RQCD (2013), Bali et al., PoS LATTICE2013 (2014) 447, $N_f = 2$ Wilson clover
  ETMC (2014), CU, unpublished, $N_f = 2$ twisted mass
  ETMC (2014), Abdel-Rehim, CU et al., Phys.Rev. D92 (2015), $N_f = 2$ twisted clover
  ETMC (2019), Oehm, CU et al., Phys.Rev. D99 (2019), $N_f = 2 + 1 + 1$ twisted mass
- results for $\langle x \rangle$, $\langle x^2 \rangle$ and $\langle x^3 \rangle$ (sometimes with mixing)
- fermionic disconnected contributions omitted throughout
- gluon contribution to $\langle x \rangle$ only quenched
  [Meyer, Negele, Phys.Rev. D77 (2008)]

Pion $\langle x \rangle$ from $N_f = 2$ LQCD



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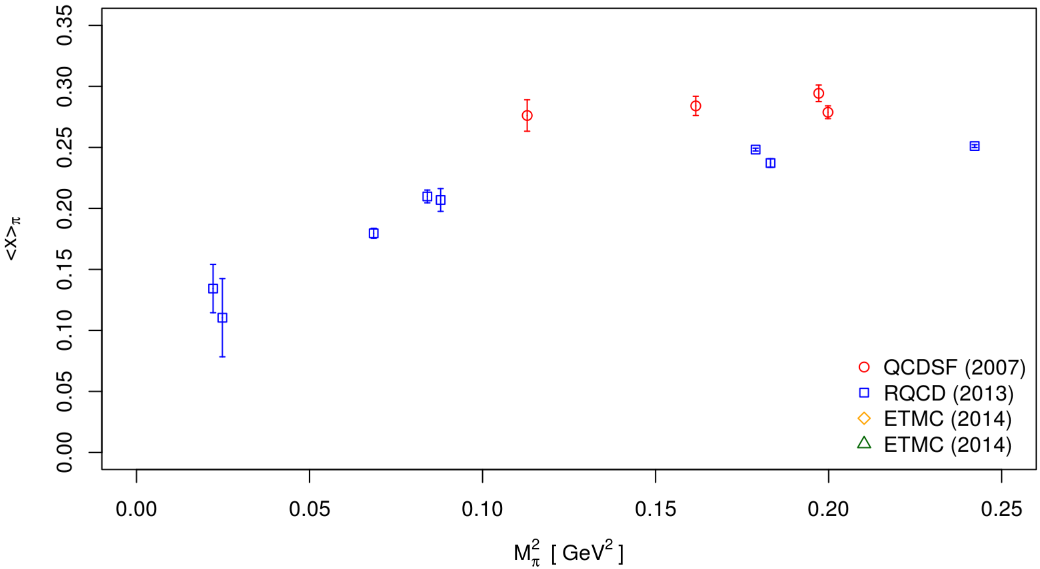

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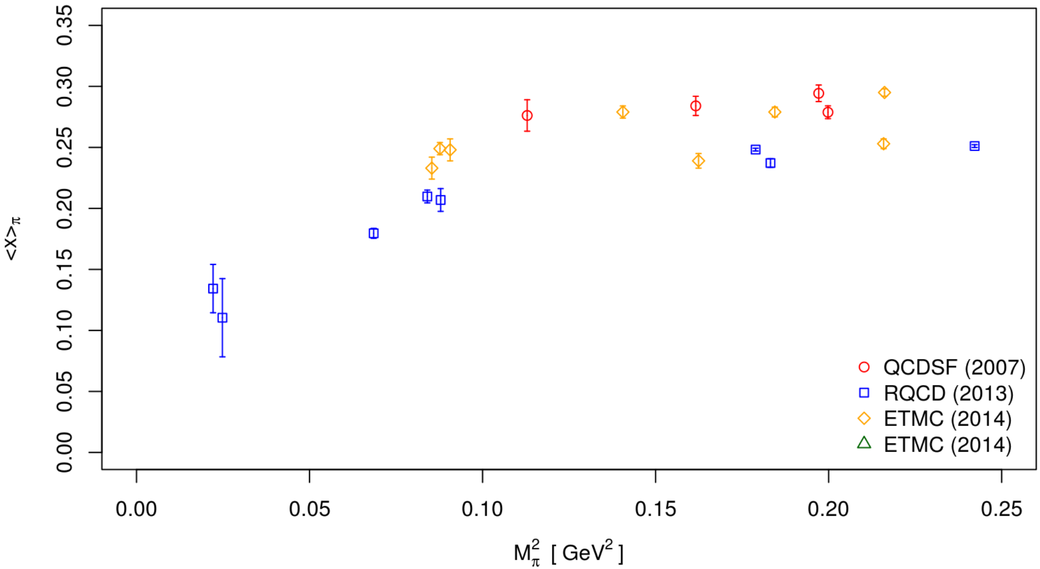

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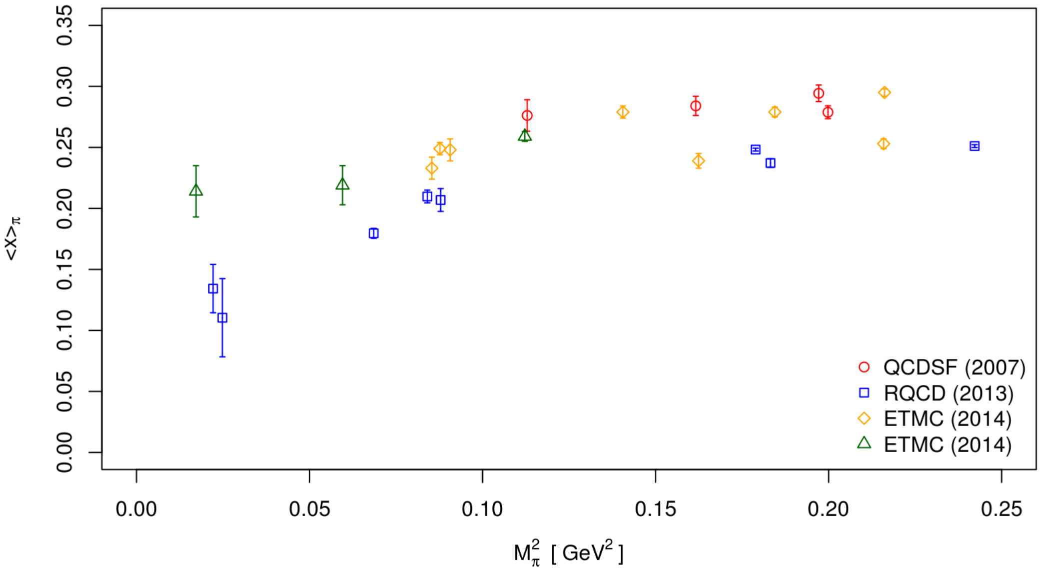
Pion $\langle x \rangle$ from $N_f = 2$ LQCD



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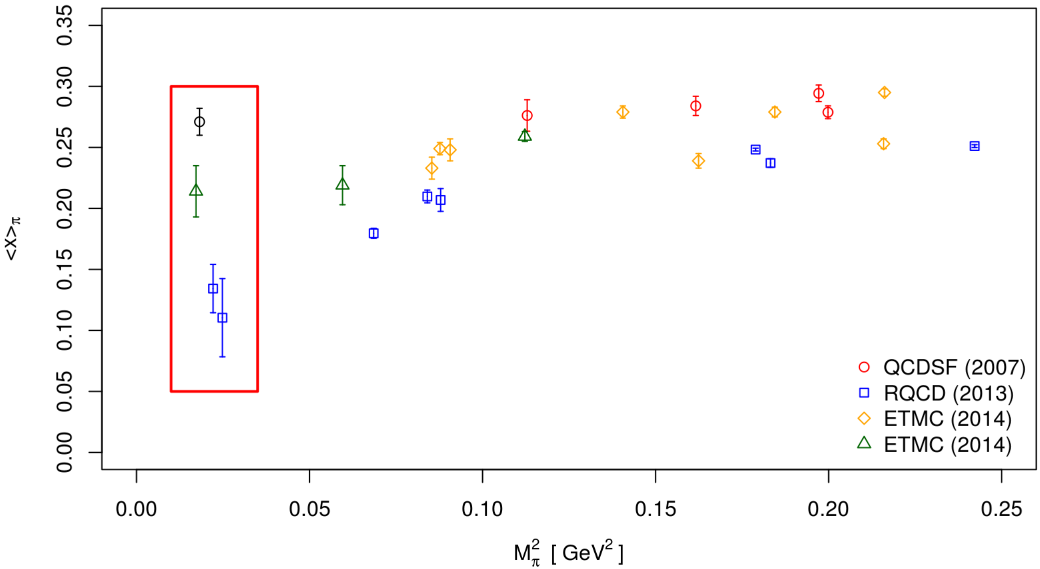

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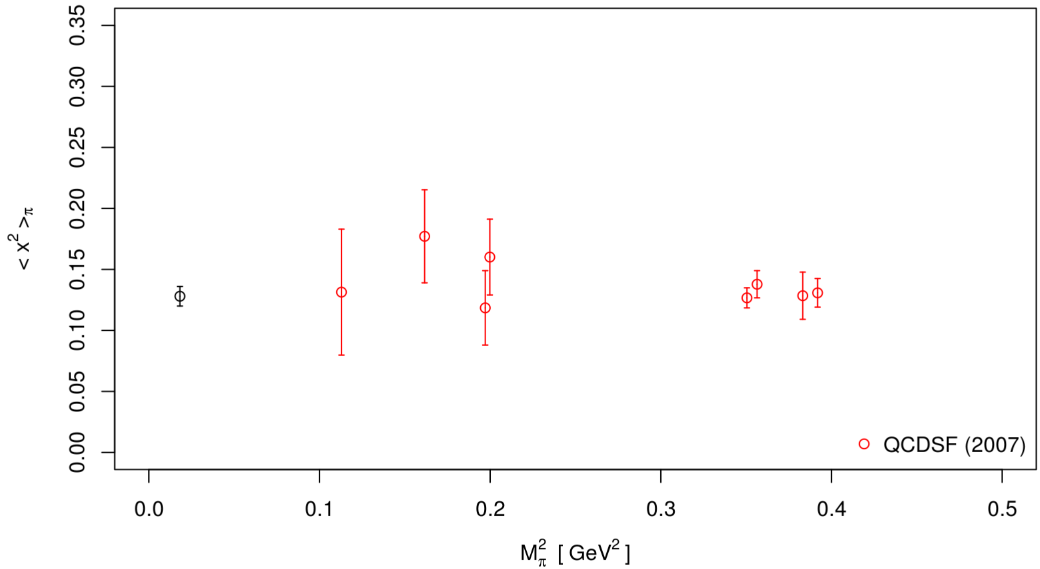
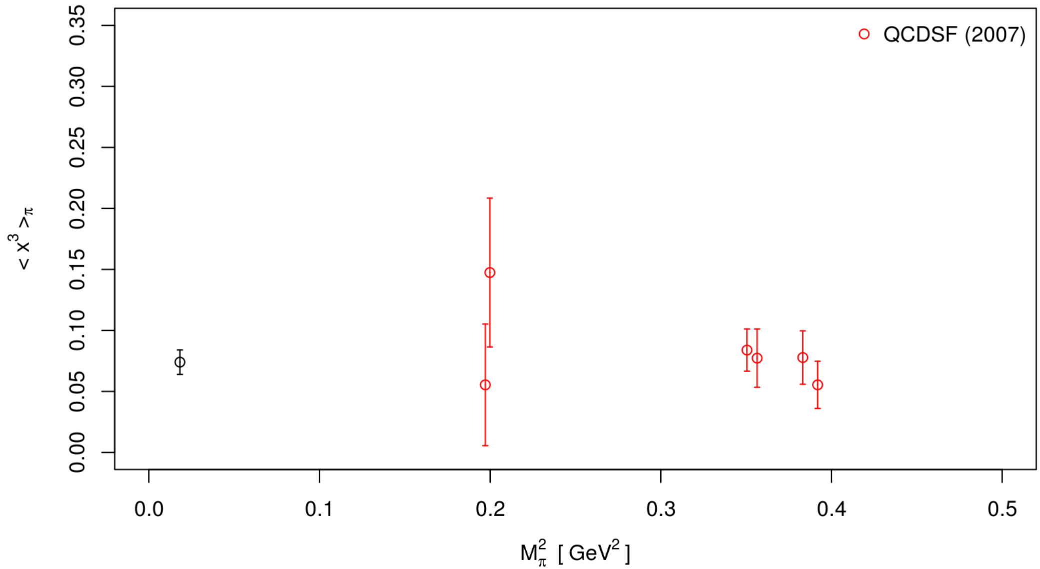
Pion $\langle x^2 \rangle$ from $N_f = 2$ LQCD

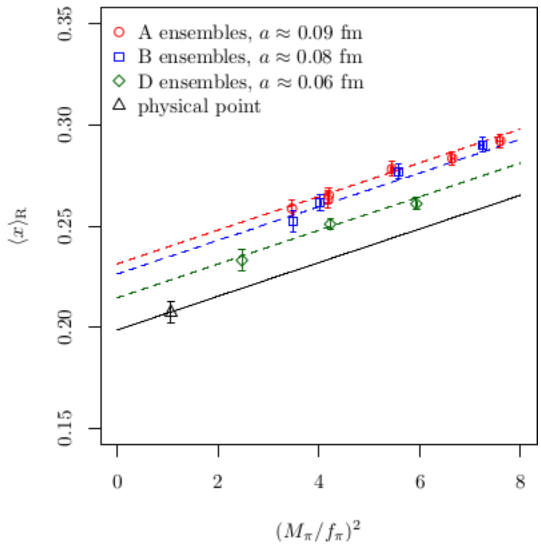


Pion $\langle x^3 \rangle$ from $N_f = 2$ LQCD



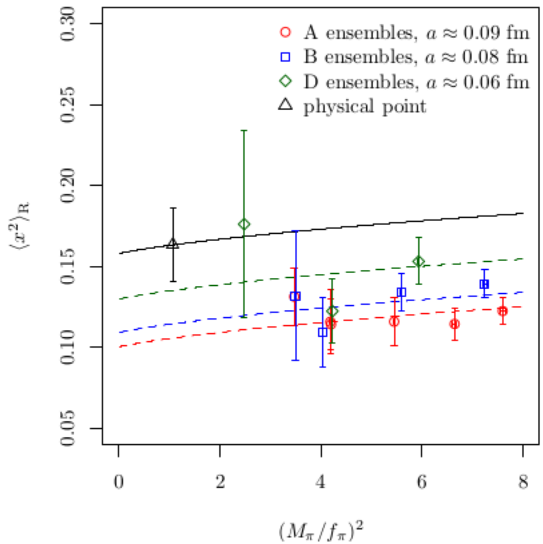
Pion $\langle x \rangle$ from $N_f = 2 + 1 + 1$ LQCD

- ETMC results with $N_f = 2 + 1 + 1$
  [Oehm, CU et al., Phys.Rev. D99 (2019)]
- three lattice spacings
- chiral extrapolation
- at 2 GeV in the $\overline{\mathrm{MS}}$ scheme
- good agreement with ETMC $N_f = 2$
  [Abdel-Rehim et al., Phys.Rev. D92 (2015)]
- sizable pion mass and lattice spacing dependence



Pion $\langle x^2 \rangle$ from $N_f = 2 + 1 + 1$ LQCD

- ETMC results with $N_f = 2 + 1 + 1$
  [Oehm, CU et al., Phys.Rev. D99 (2019)]
- three lattice spacings
- chiral extrapolation
- at 2 GeV in the $\overline{\text{MS}}$ scheme
- large errors, M_π and a dependence hardly resolvable
- good agreement to QCDSF (2007)



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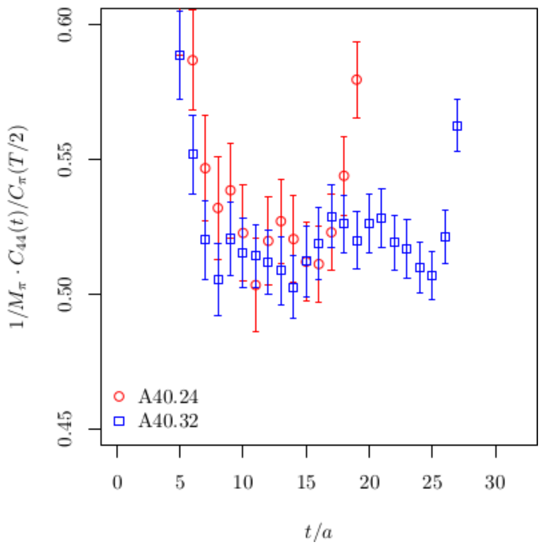
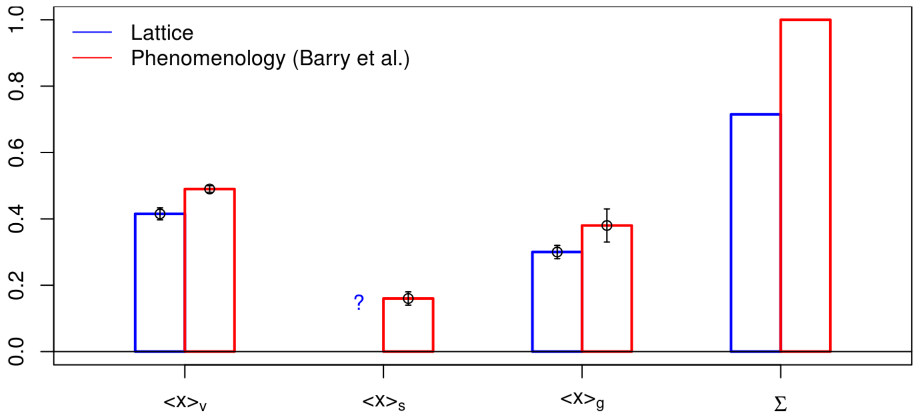
Signal Quality and Finite Volume Effects

- example bare 3pt function
- finite volume effects major issue for nucleon structure
- not so for pion moments
- different volumes $L = 24, 32$ compatible within errors
  [Oehm, CU et al., Phys.Rev. D99 (2019)]
- increasing noise with increasing n

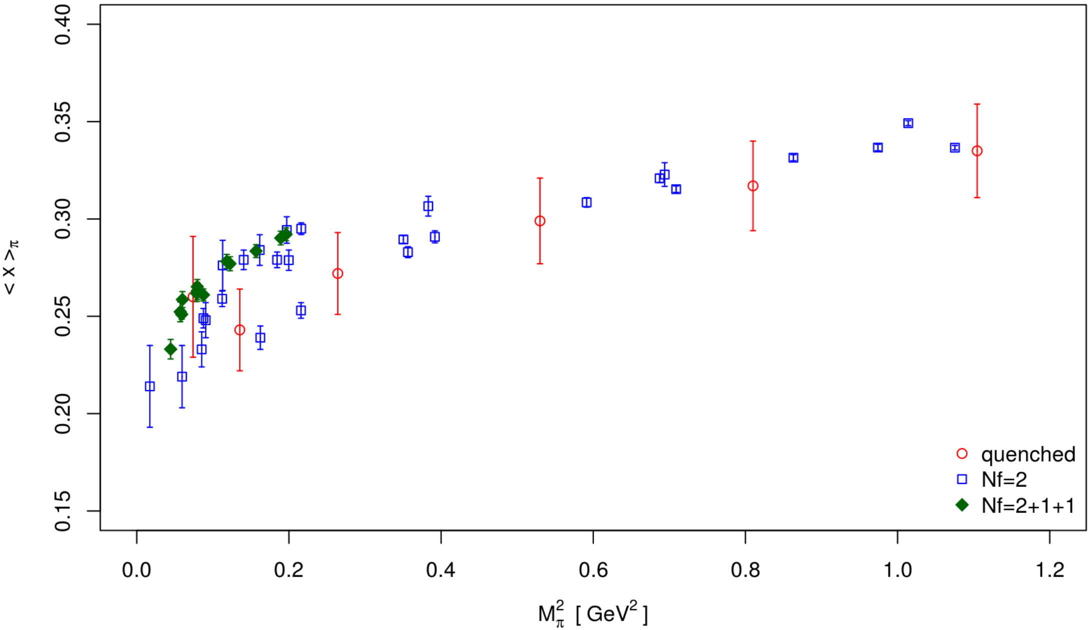


Comparison with Phenomenology for $\langle x \rangle$



- lattice gluon from quenched study by Meyer and Negele
  [Meyer, Negele, Phys.Rev. D77 (2008)]
- other lattice results from $N_f = 2 + 1 + 1$
  [Oehm, CU et al., Phys.Rev. D99 (2019)]

From Quenched to $N_f = 2 + 1 + 1$



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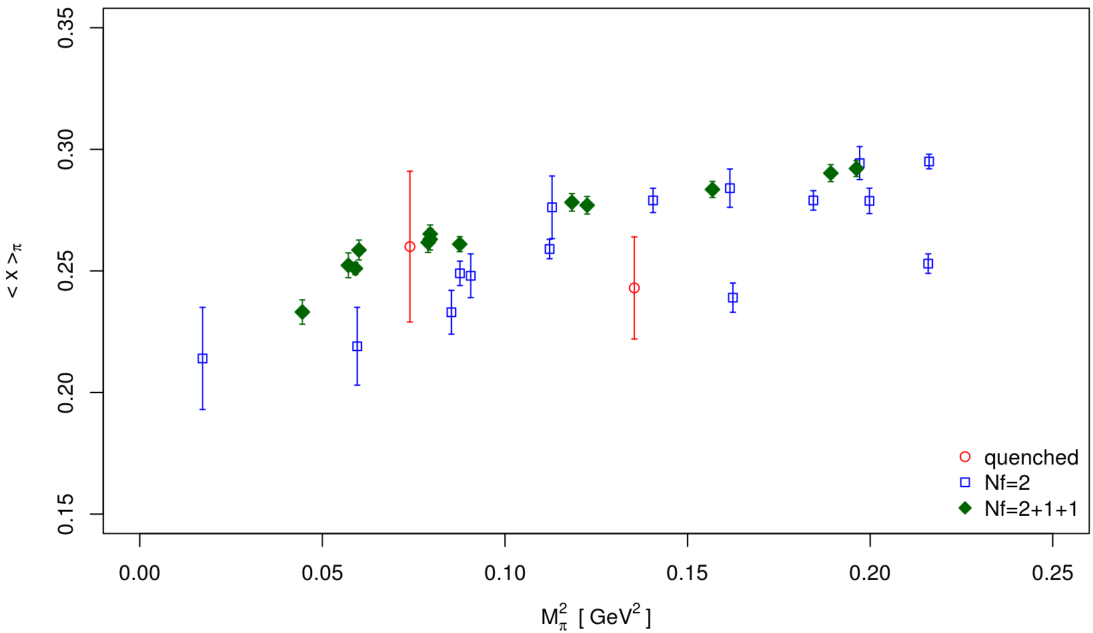
From Quenched to $N_f = 2 + 1 + 1$



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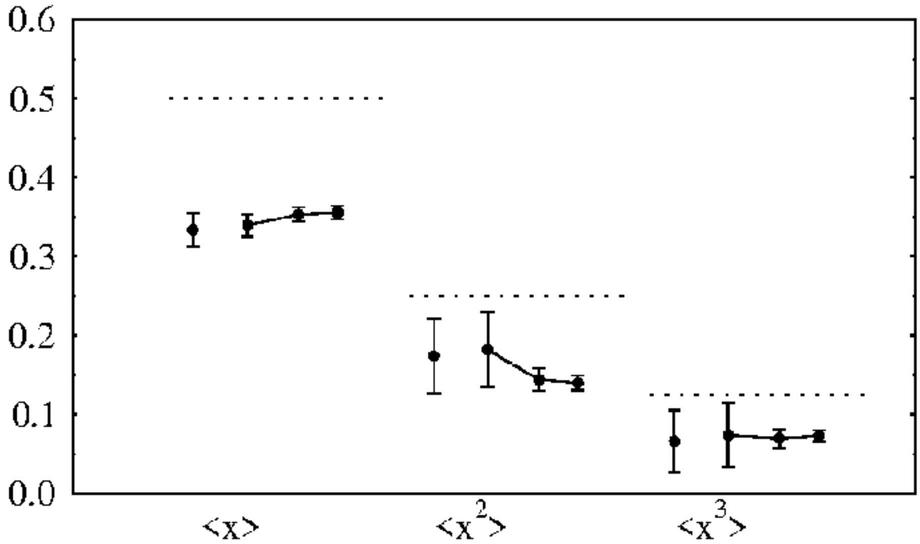
Moments of the ρ-Meson

- only one quenched LQCD pioneering study of moments for the ρ-meson
  [Best et al., Phys.Rev.D56 (1997)]
- single Lattice spacing and three heavy pion mass values
- authors compute moments of unpolarised and polarised structure functions

$\Rightarrow$ access to the spin decomposition

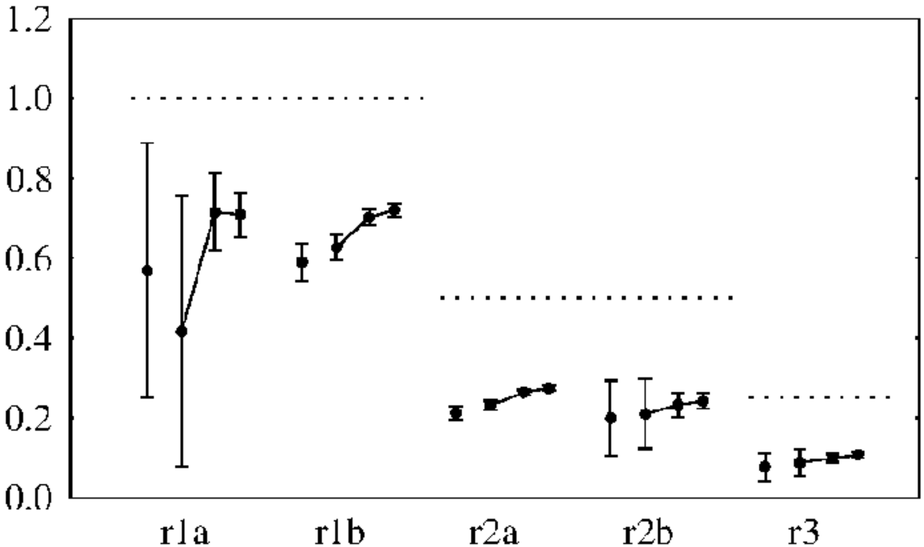
- ρ-**meson stable** due to quenching and large pion masses
- in a dynamical calculation with decaying ρ probably a treatment along Lellouch-Lüscher and Briceño-Hansen-Walker-Loud needed
  [Lellouch, Lüscher, Commun.Math.Phys. 219 (2001); Briceño, Hansen, Walker-Loud, Phys.Rev. D91 (2015)]

Unpolarised PDFs of the ρ-Meson



[Best et al., Phys.Rev.D56 (1997)]

Polarised PDFs of the ρ-Meson



[Best et al., Phys.Rev.D56 (1997)

Moments of the ρ-Meson

their conclusions:

- moments of unpolarised PDFs **very similar** to the pion ones

$\Rightarrow$ $$f^{\pi} \approx f^{\rho}$$

- about 70% of the total spin carried by valence quarks
  interestingly, similar to the nucleon
  [Alexandrou et al., Phys.Rev.Lett. 119 (2017)]
- the lowest moment of difference in parton distributions of an $m = 0$ and $m = 1$ target surprisingly large
  particular to spin-1 states

$\Rightarrow$ substantial orbital angular momentum of the valence quarks?

Perspectives

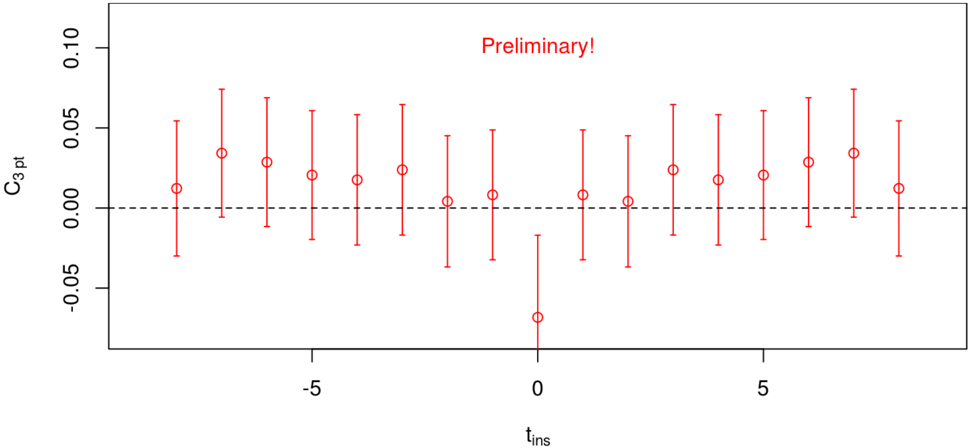
Pion moments:

- computations of the fermionic disconnected contribution to $\langle x \rangle$ on their way
- investigation with $N_f = 2 + 1 + 1$ directly at the physical pion mass
  to finally check the chiral extrapolation
- $\langle x^3 \rangle$ with chiral and continuum extrapolations
- comparison to quasi/pseudo-PDF results

Other mesons:

- moments for the kaon are being computed
- the ρ must be treated as a resonance

Disconnected Contribution to $\langle x \rangle_\pi$



- preliminary result for the disconnected contribution
  [ETMC, M. Petschlies, B. Kostrzewa, in progress]
- normalisation still missing, data quite noisy

Summary

- new experimental effort to unravel pion structure
- tension in phenomenological analyses

$\Rightarrow$ LQCD can help to clarify

- $\langle x^n \rangle_\pi$ with $n = 1, 2, 3$ available
  however, so far without fermionic disconnected contributions
- remaining uncertainty in the chiral extrapolation
- very interesting, but old results for the ρ meson

I would like to thank F. Steffens for very helpful discussions and comments