Renormalization of nonlocal quasi-PDF operators

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PDFs from lattice

- Quasi-PDF/LaMET
  - [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']
- (Light quark) current correlation functions
  - [Braun and Müller, EPJC 08']
- Lattice cross sections
  - [Ma and Qiu, 14' & PRL 17']
- Ioffe-time /pseudo-distribution
  - [Radyushkin, PRD 17']
- More approaches
  - [Liu and Dong, PRL 94']
  - [Detmold and Lin, PRD 06']
  - [Davoudi and Savage, PRD 12']
  - [Chambers et al., PRL 17']

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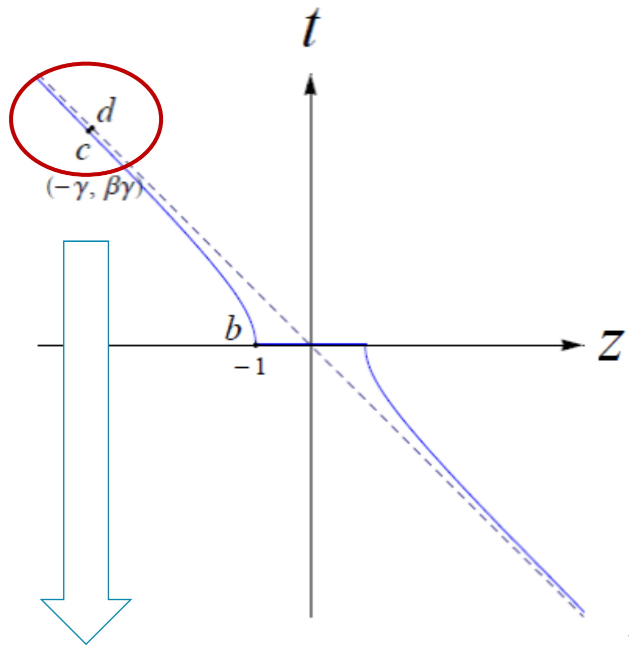
Large momentum is required to approach light cone physics, access information on higher moments, or reach large Ioffe-time

PDFs from LaMET



- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']
  - Appropriately chosen $\tilde{q}$ can be calculated on the Euclidean lattice, e.g.

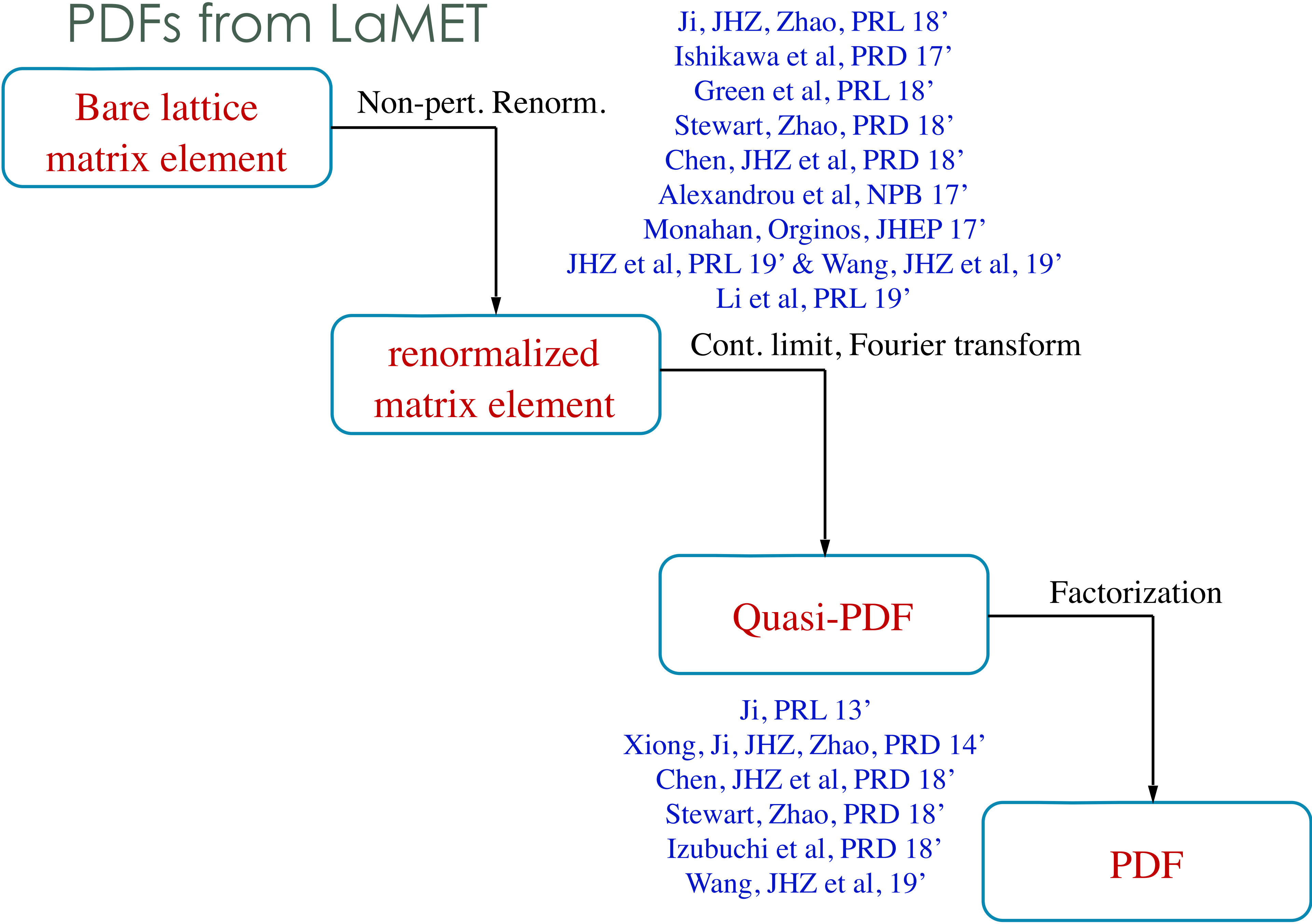
$$\tilde{q}(x, P_z, \tilde{\mu}) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{ixzP_z} \langle P | \bar{\psi}(z) \Gamma \mathscr{P} \exp^{-ig\int_0^z dz' A_z(z')} \psi(0) | P \rangle$$

  - A finite but large P_z already offers a good approximation, where (leading) frame-dependence can be removed through a factorization formula

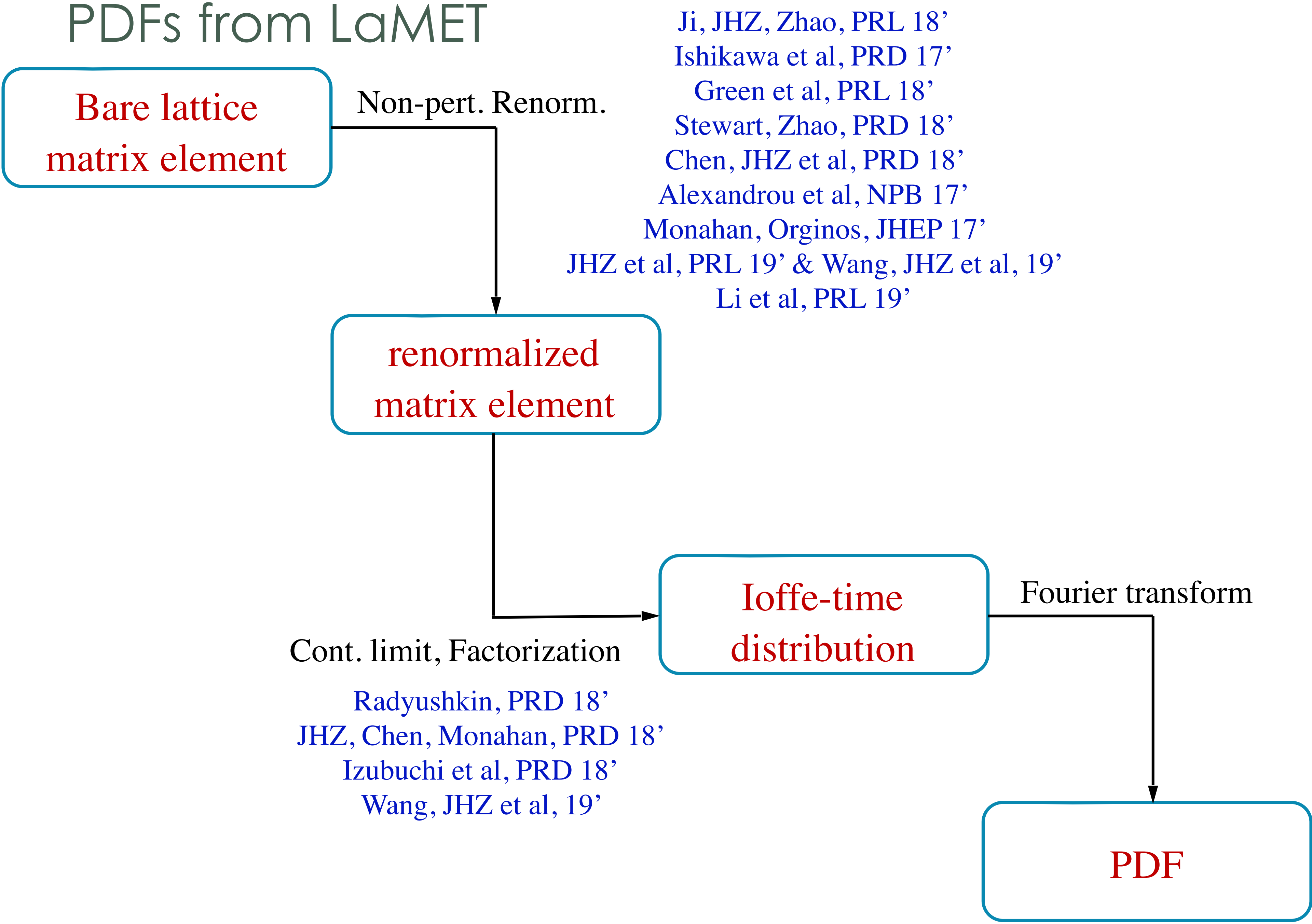
$$\tilde{q}(x, P_z, p_z^R, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{p_z^R}{\mu}\right) q(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2(1-x)P_z^2}\right)$$

[Braun, Vladimirov JHZ, PRD 19']

PDFs from LaMET



PDFs from LaMET



Quark quasi-PDFs

- Multiplicatively renormalized
- Auxiliary field approach [Dorn, Fortsch. Phys. 86', Ji, JHZ, Zhao, PRL 18', Green, Jansen and Steffens, PRL 18']
  - Spacelike Wilson line replaced by two-point function of auxiliary heavy quark field

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \overline{Q}(x) in \cdot DQ(x)$$

  - Nonlocal quark bilinear operator becomes product of two local currents

$$O(x,y) = \overline{\psi}(x)\Gamma L(x,y)\psi(y) \quad \longrightarrow \quad O(x,y) = \overline{\psi}(x)\Gamma Q(x)\overline{Q}(y)\psi(y)$$

  - Integrating out the auxiliary field (taking into account the potential mass term generated by radiative corrections)

$$O_R = Z_{\bar{j}}^{-1} Z_{j}^{-1} e^{\delta \bar{m}|z_2 - z_1|} \overline{\psi}(z_2)\Gamma L(z_2, z_1)\psi(z_1)$$

- Feynman diagrammatic approach [Ishikawa, Ma, Qiu, Yoshida, PRD 17']
  - All power divergences come from self energy of Wilson lines, and can be removed by an effective mass renormalization

Quark quasi-PDFs

- Nonlocal quasi-PDF operators at different z do not mix under renormalization. Two ways to perform renormalization:
  - Calculate the endpoint renormaliztion factors and the Wilson line mass counterterm nonperturbatively
  - Calculate the renormalization factors as a whole for each z (e.g. RI/MOM [Stewart, Zhao, PRD 18']])

$$\tilde{h}(z, P_z, a^{-1}) = \frac{1}{2P^0}\langle P|O_{\gamma^t}(z)|P\rangle$$

$$O_\Gamma(z) = \bar{\psi}(z)\Gamma U(z,0)\psi(0)$$

$$U(z,0) = P\exp\left(-ig\int_0^z dz' A_z(z')\right)$$

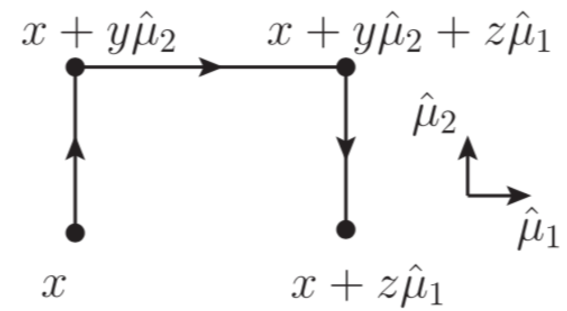
$$\tilde{h}_R(z, P_z, p_z^R, \mu_R) = Z^{-1}(z, p_z^R, a^{-1}, \mu_R)\tilde{h}(z, P_z, a^{-1})\Big|_{a\to 0}$$

$$Z(z, p_z^R, a^{-1}, \mu_R) = \frac{\sum_s \langle p, s|O_{\gamma^t}(z)|p, s\rangle}{\sum_s \langle p, s|O_{\gamma^t}(z)|p, s\rangle_{\text{tree}}}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

Quark quasi-PDFs

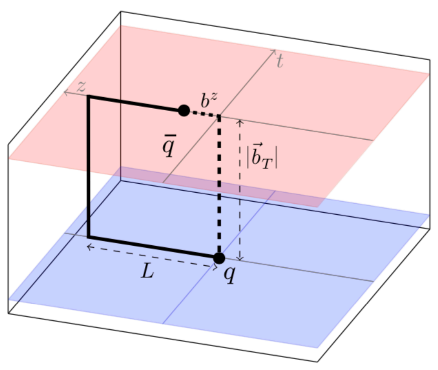
- Extension to operators with staple-shaped Wilson lines, required for TMD studies on lattice
  - Perturbative calculation at one-loop [Constantinou et al, PRD 19']

$$\mathcal{O}_\Gamma \equiv \bar{\psi}(x)\Gamma W(x, x+y\hat{\mu}_2, x+y\hat{\mu}_2+z\hat{\mu}_1, x+z\hat{\mu}_1) \times \psi(x+z\hat{\mu}_1),$$



  - Argued based on auxiliary field approach [Ebert, Stewart, Zhao, 19']

$$\begin{aligned}\mathcal{O}_0^\Gamma(b^\mu, a, L) &\equiv \bar{\psi}_0(b^\mu) W_{\hat{z}} \frac{\Gamma}{2} W_T W_{\hat{z}}^\dagger \psi_0(0) \\ &= Z_{q,\mathrm{wf}}\, e^{\delta m(L+|L-b^z|+b_T)} \Big(\bar{\psi}(b^\mu) W_{\hat{z}} \frac{\Gamma}{2} W_T W_{\hat{z}}^\dagger \psi(0)\Big)_R \\ &\equiv \tilde{Z}_B \Big(\bar{\psi}(b^\mu) W_{\hat{z}} \frac{\Gamma}{2} W_T W_{\hat{z}}^\dagger \psi(0)\Big)_R .\end{aligned}$$



Gluon quasi-PDFs

- Gluon PDF (unpol.) [Collins, Soper, NPB 82']

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ixP^+\xi^-} \langle P|F_a^{+i}(\xi^-)\mathcal{W}(\xi^-,0)F_a^{+i}(0)|P\rangle$$

- Naively expected gluon quasi-PDF operators ($\{\mu,\nu\} = \{t,z\}$)

$$O_g^{\mu\nu}(z,0) = F^{\mu\alpha}(z)\mathcal{W}(z,0)F_\alpha^{\ \nu}(0)$$

  - They mix in general with other operators under renormalization

- Appropriate choices can be multiplicatively renormalized
  - Auxiliary field approach [Wang, Zhao, JHEP 18', JHZ, Ji, Schaefer, Wang, Zhao, PRL 19']
    - Non-local gluon quasi-PDF operator ⟶ product of two local composite operators
  - Feynman diagrammatic approach [Li, Ma, Qiu, PRL 19']
    - All components of $F^{\mu\nu}(z)W(z,0)F^{\rho\sigma}(0)$ renormalize multiplicatively
    - Choosing appropriate combinations also gives multiplicatively renormalizable gluon quasi-PDF operators

Gluon quasi-PDFs

- The non-local gluon quasi-PDF operator can be replaced by a product of two local composite operators

$$\mathcal{O}_g^{(3)}(z_2, z_1) = J_1^{ti}(z_2)\overline{J}_{1,i}{}^{z}(z_1)$$

$$J_1^{ti}(z_2) = F_a^{ti}(z_2)\mathcal{Q}_a(z_2), \quad \overline{J}_{1,i}{}^{z}(z_1) = \overline{\mathcal{Q}}_b(z_1)F_{b,i}{}^{z}(z_1)$$

- Local operator mixing [Joglekar, Lee, Annals Phys. 76', Collins, 11']
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - For $J_1^{\mu\nu}$, the operators allowed to mix are [Dorn et al, Annalen Phys. 83']

$$J_2^{\mu\nu} = n_\rho(F_a^{\mu\rho}n^\nu - F_a^{\nu\rho}n^\mu)\mathcal{Q}_a/n^2,$$

$$J_3^{\mu\nu} = (-in^\mu A_a^\nu + in^\nu A_a^\mu)((in\cdot D - m)\mathcal{Q})_a/n^2,$$

  - General mixing pattern

$$\begin{pmatrix} J_{1,R}^{\mu\nu} \\ J_{2,R}^{\mu\nu} \\ J_{3,R}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ 0 & 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{\mu\nu} \\ J_2^{\mu\nu} \\ J_3^{\mu\nu} \end{pmatrix},$$

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$$J_2^{\mu\nu} = n_\rho(F_a^{\mu\rho}n^\nu - F_a^{\nu\rho}n^\mu)\mathcal{Q}_a/n^2,$$

$$J_3^{\mu\nu} = (-in^\mu A_a^\nu + in^\nu A_a^\mu)((in\cdot D - m)\mathcal{Q})_a/n^2,$$

  - Renormalization constants are not all independent

$$\begin{pmatrix} J_{1,R}^{z\mu} \\ J_{3,R}^{z\mu} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{z\mu} \\ J_3^{z\mu} \end{pmatrix}, \qquad J_{1,R}^{ti} = Z_{11}J_1^{ti}, \quad J_{1,R}^{ij} = Z_{11}J_1^{ij}$$

  - Different components renormalize differently due to Lorentz symmetry breaking

Gluon quasi-PDFs

- We can identify building blocks that can be used to construct multiplicatively renormalizable gluon quasi-PDFs, e.g.

$$\mathcal{O}_R^1(z_2, z_1) \equiv J_{1,R}^{ti}(z_2)\overline{J}_{1,R}^{ti}(z_1)$$

- After integrating out the auxiliary field

$$O_R^1(z_2, z_1) = (F^{ti}(z_2)L(z_2, z_1)F^{ti}(z_1))_R = Z_{11}^2 e^{\overline{\delta m}|z_2 - z_1|} F^{ti}(z_2)L(z_2, z_1)F^{ti}(z_1)$$

- Four such operators have been identified [JHZ, Ji, Schaefer, Wang, Zhao, PRL 19']

$$O_g^{(1)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ t}(0), \quad O_g^{(2)}(z,0) \equiv F^{zi}(z)\mathcal{W}(z,0)F_i^{\ z}(0),$$
$$O_g^{(3)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ z}(0), \quad O_g^{(4)}(z,0) \equiv F^{z\mu}(z)\mathcal{W}(z,0)F_\mu^{\ z}(0),$$

- Using their multiplicative renormalizability, we can renormalize them in the RI/MOM scheme [Wang, JHZ et al, PRD 19']

- Inserting gluon (quark) quasi-PDF operators into a quark (gluon) state yields finite mixing

Factorization

- Coordinate space [Wang, JHZ et al, PRD 19']

$$\tilde{h}_{q_i,R}(z,P^z,\mu) = \int_{-1}^{1} du\, \mathcal{C}_{q_iq_j}(u,\mu^2z^2)h_{q_j}(u\nu,\mu) + \int_{-1}^{1} du\, \nu\mathcal{C}_{qg}(u,\mu^2z^2)h_g(u\nu,\mu)$$

$$\tilde{h}_{g,R}(z,P^z,\mu) = \int_{-1}^{1} du\, \mathcal{C}_{gg}(u,\mu^2z^2)h_g(u\nu,\mu) + \int_{-1}^{1} du \frac{\mathcal{C}_{gq}(u,\mu^2z^2)}{\nu}h_{q_i}(u\nu,\mu)$$

- Momentum space

$$\tilde{f}^{(n)}_{g/H}(x,P^z,p^R_z,\mu_R) = \int_{-1}^{1}\frac{dy}{|y|}\Big[C_{gg}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{g/H}(y,\mu) + C_{gq}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{q_j/H}(y,\mu)\Big] + \mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),$$

$$\tilde{f}_{q_i/H}(x,P^z,p^R_z,\mu_R) = \int_{-1}^{1}\frac{dy}{|y|}\Big[C_{q_iq_j}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{q_j/H}(y,\mu) + C_{qg}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{g/H}(y,\mu)\Big] + \mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big), \tag{2.53}$$

- Perturbative matching coefficients have been available at one-loop

Polarized gluon PDF

- For

$$\Delta f_{g/H}(x,\mu) = i\epsilon_{\perp ij}\int \frac{d\xi^-}{2\pi x P^+} e^{-i\xi^- xP^+}\langle P|F^{+i}(\xi^- n_+)\mathcal{W}(\xi^- n_+,0;L_{n_+})F^{j+}(0)|P\rangle$$

- We have identified three multiplicatively renormalizable quasi-PDF operators [JHZ, Ji, Schaefer, Wang, Zhao, PRL 19']

$$\Delta O_g^1(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{tj}(z_1),$$
$$\Delta O_g^2(z,0) = i\epsilon_{\perp,ij}F^{zi}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$
$$\Delta O_g^3(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$

- Renormalization and factorization are similar to the unpolarized case [Wang, JHZ et al, PRD 19']

- Perturbative matching coefficients also available at one-loop

Summary

- Rapid progress has been achieved in the past few years on computations of x-dependence of hadron structure from lattice QCD
- Applications to nucleon PDFs have yielded encouraging results, but so far only to isovector quark combinations which do not mix with gluons
- There also exist exploratory studies on GPDs, TMDs (Collins-Soper kernel, soft function)
- Theory inputs ready for gluon PDF and flavor-singlet quark PDF
- Efficiency of different approaches in approaching lightcone physics can be tested with similar lattice setup
    - Synergy between lattice effort towards a better understanding of nucleon structure?