Extraction of pseudo-PDFs *or why Ioffe-Time Distributions are our friend*

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Outline

- LaMET, quasi-PDFs, pseudo-PDFs and Good Lattice Cross Sections
- Pion as a theatre for PDFs - pPDFs and GLCS
- pPDFs in Nucleon
- Summary

Introduction

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}\psi(0) \mid P\rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

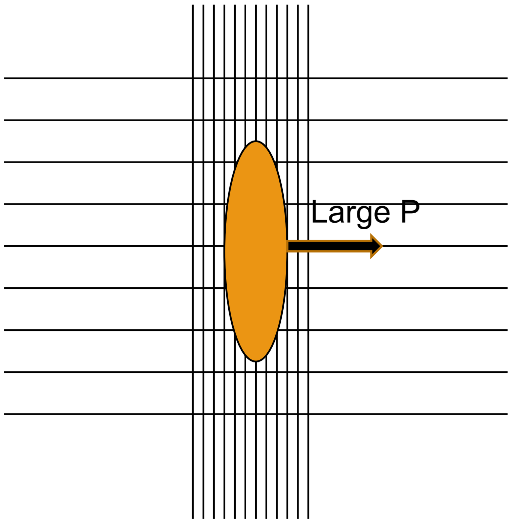
$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\ldots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\ldots P_{\mu_n}a^{(n)}$$

- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

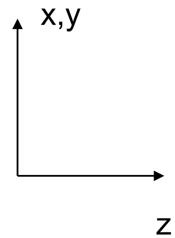
Moment Methods

- Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
- Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

Solution….



Large-Momentum Effective Theory (LaMET)



"Equal time" correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

Pseudo-PDFs

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $z = (0, z_-, 0_T)$

Ioffe-Time Distribution

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Lattice "building blocks" that of quasi-PDF approach.

$\Downarrow$ *Lorentz covariant*

$$\mathcal{M}(\nu, z^2) = \int_{-1}^{1} dx\, e^{i\nu x} \mathcal{P}(x, z^2)$$ ⟵ *pseudo-PDF*

$$\Downarrow$$

$$f(x) = \mathcal{P}(x,0) \underset{z_3^2 \to 0}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, -z_3^2)$$

pPDFs - II

To deal with UV divergences, introduce reduced distribution $\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu,\mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$$\Downarrow$$

Need data for all ν, *or additional physics input*

Inverse problem

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Moments

J Karpie, K Orginos, S Zafeiropoulos, arXiv:1807.10933

- Can obviate need for inverse through computation of moments

$$\mathfrak{M}(\nu, z^2) = \sum_{n=0}^{\infty} i^n \frac{\nu^n}{n!} a_n(\mu^2) K_n(\mu^2 z^2) + \mathcal{O}(z^2)$$

Mellin moments of PDF

Matching coefficient

"Good Lattice Cross Sections"

$$\sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P \rangle$$

Ma and Qiu, Phys. Rev. Lett. 120 022003

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

Calculated in LQCD

Parton Distribution function

Calculated in perturbation theory ("process dependent")

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Encompasses qPDF/pPDF

Gauge-Invariant Currents

$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0)$$

$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0)$$

Flavor-changing

+ analogous gluon operators

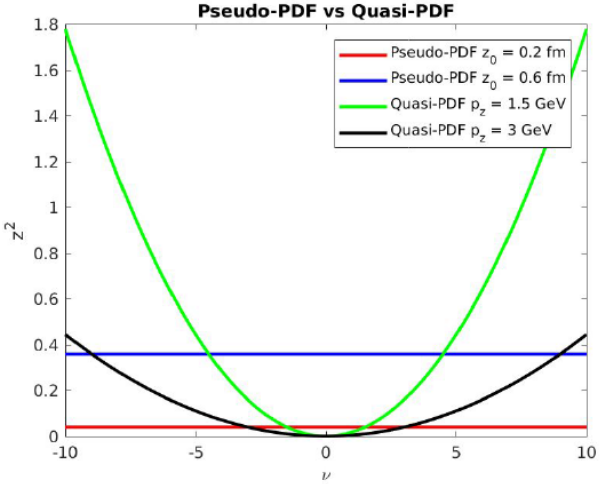
Quasi- vs Pseudo- vs GLCS

– All integrals of Ioffe-Time Distribution Function
– Should yield same PDF after matching and systematic controls

Quasi-PDF

$$\mathcal{Q}(x, p_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x}\mathcal{M}(\nu, -\nu^2/p_3^2)$$

$$\mathcal{P}(x, -z_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x}\mathcal{M}(\nu, -z_3^2)$$



Pseudo-PDF and GLCS

For pPDF + GLCS, z sets short-distance scale.

$$z << \frac{1}{\Lambda_{\text{QCD}}}$$

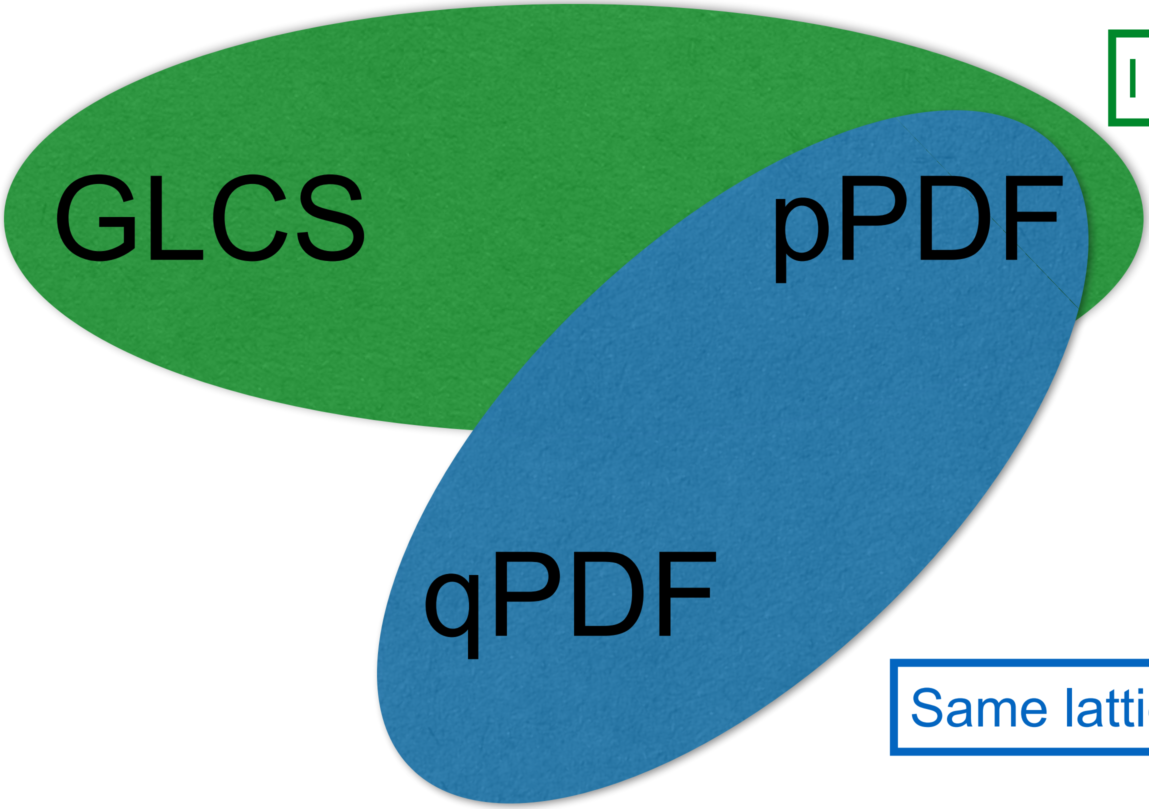
N.B. All approaches require large momentum - but for pPDF and GLCS to ensure range in Ioffe time to solve *inverse problem*.

$$P \longrightarrow \sqrt{s}$$ *Collision energy*

$$z \longrightarrow \frac{1}{Q}$$ *Hard Probe*

Analogous matching to light-cone PDFs

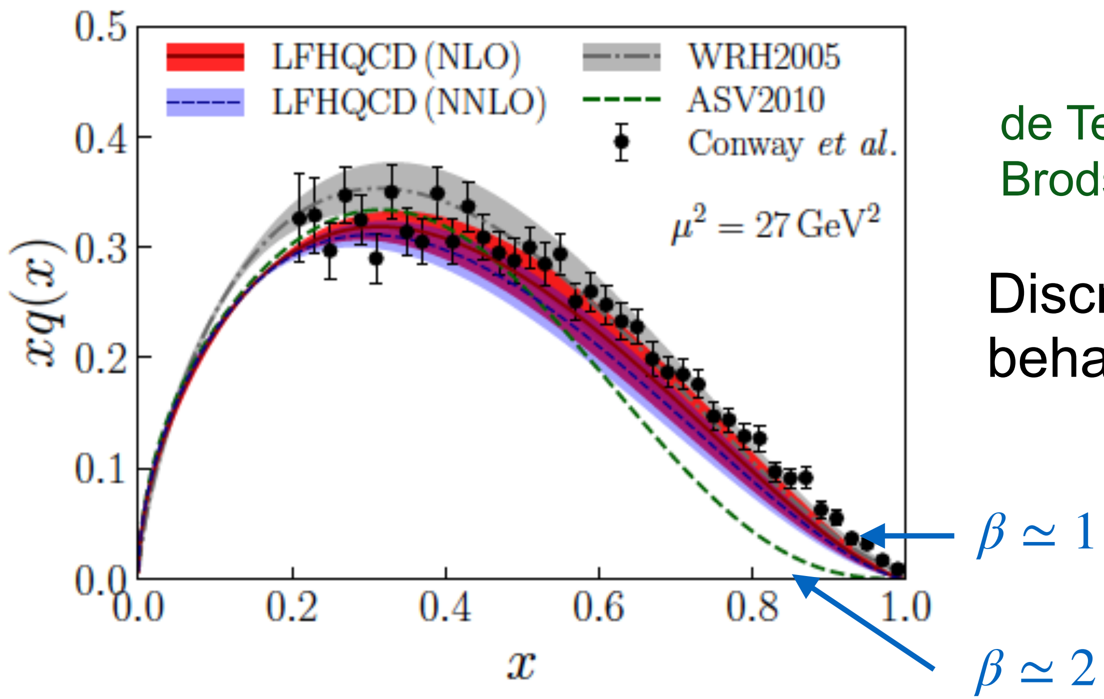
I will discuss these



Same lattice building blocks

Pion Valence PDF

- u distribution of FNAL E615 to leading order
- C12-15-006 at Hall A will look at structure of pion
- C12-15-006A at Hall A will look at structure of Kaon
- No free pion target



de Teramond, liu, Sufian, Dosch, Brodsky, Deur, PRL (2018)

Discrepancy in large-x behavior of pion distribution

Why the Pion?

- Pion less computationally demanding that nucleon?
  - *Larger signal-to-noise ratio*

$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^\dagger(0,0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}} \to e^{-E(\vec{p})t}$$

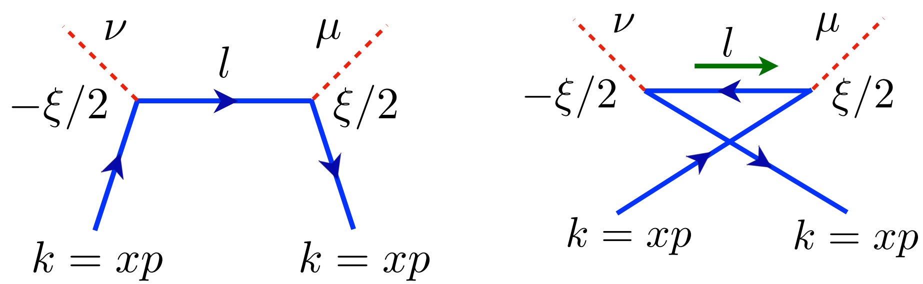
$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \to \begin{cases} e^{-m_\pi t} & \text{Mesons} \\ e^{-(3m_\pi/2)t} & \text{Baryons} \end{cases}$$

- Important constraint on systematic uncertainty is understanding operator renormalization
  - *Operator renormalization "independent" of external states*
- Several different calculations using the different approaches
  - *Lattice cross-section approach straightforward for mesons, challenging for baryons*

Good Lattice Cross Section

$$\sigma_{ij}^{\mu\nu}(\xi, p) = \langle \pi(p) \mid \mathcal{O}_{ij}^{\mu\nu}(\xi) \mid \pi(p)\rangle$$
$$= \xi^4 \langle \pi(p) \mid \mathcal{J}_i^{\mu}(\xi/2)\mathcal{J}^{\nu} + j(-\xi/2) \mid \pi(p)\rangle$$

Calculate K at tree-level between quark states



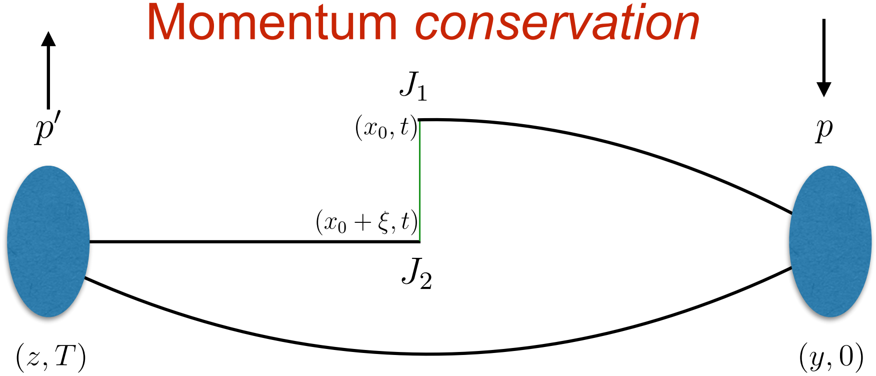
Process, i.e. current, dependent $\frac{1}{2}[\sigma_{V,A}^{\mu\nu}(\xi,p) + \sigma_{A,V}^{\mu\nu}(\xi,p)]$

$$\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\nu, \xi^2) + (p^\mu \xi^\nu - \xi^\mu p^\nu)T_2(\nu, \xi^2)$$

Sequential-Source Approach

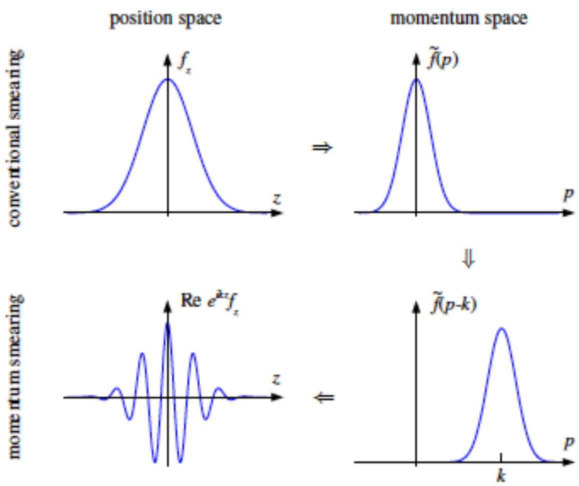
Momentum *conservation*

Momentum projection



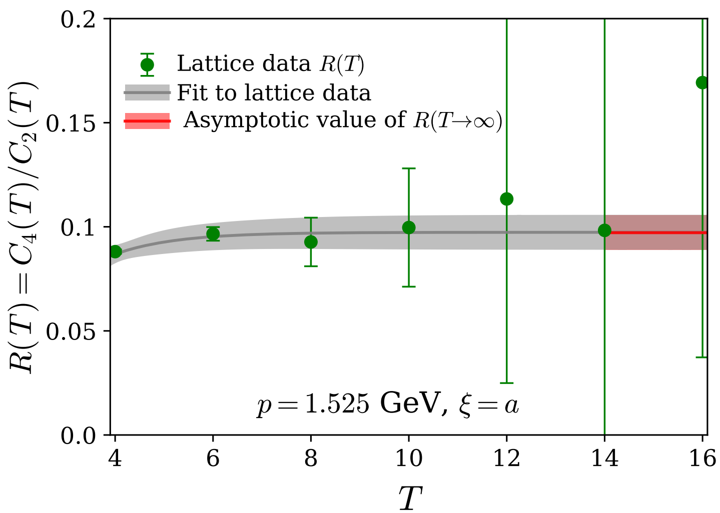
Momentum projection

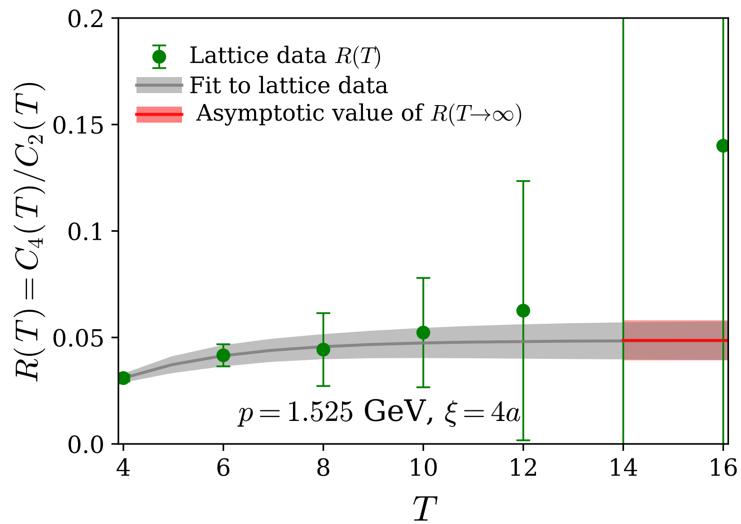
Challenges of Higher Momenta



Boosted interpolating operators

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

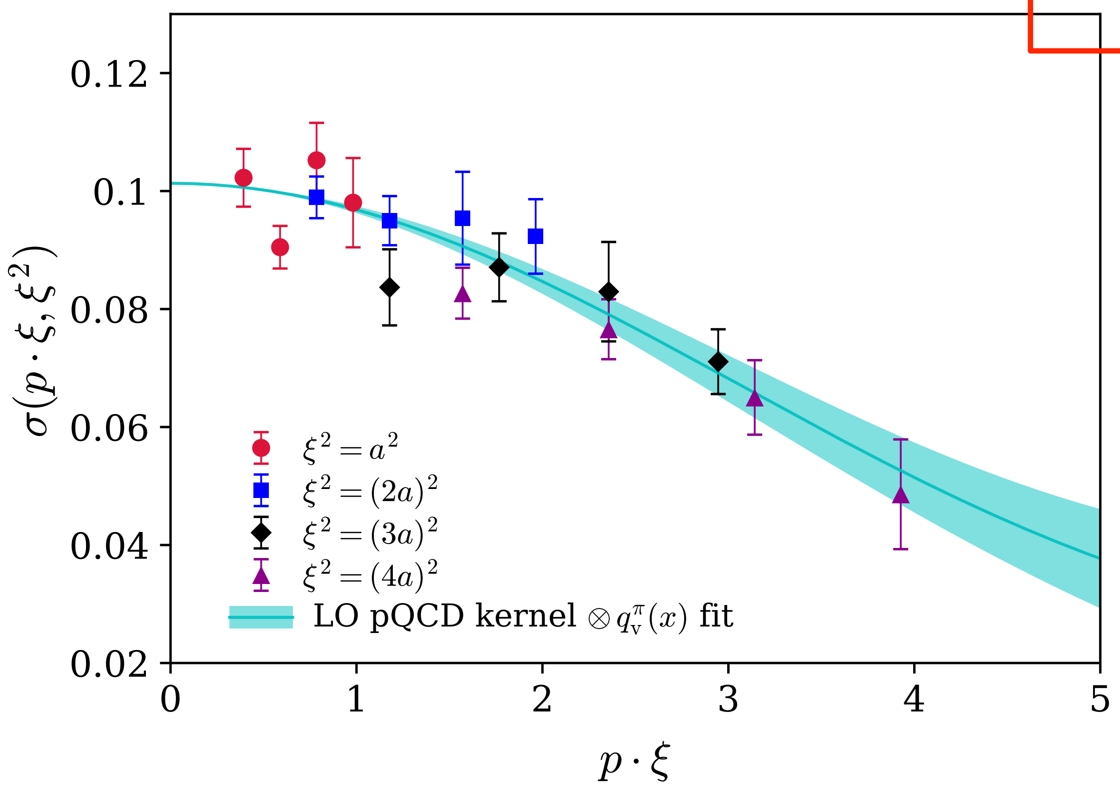




Good Lattice Cross Section

2 + 1 Clover-fermion action

– Lattice spacing a ~ 0.127 fm
– Pion mass 415 MeV
– 32^3 x 96 lattice, 490 Configs



Inverse problem: extract PDF

"Inverse Problem" - ill-posed inverse Fourier transform.

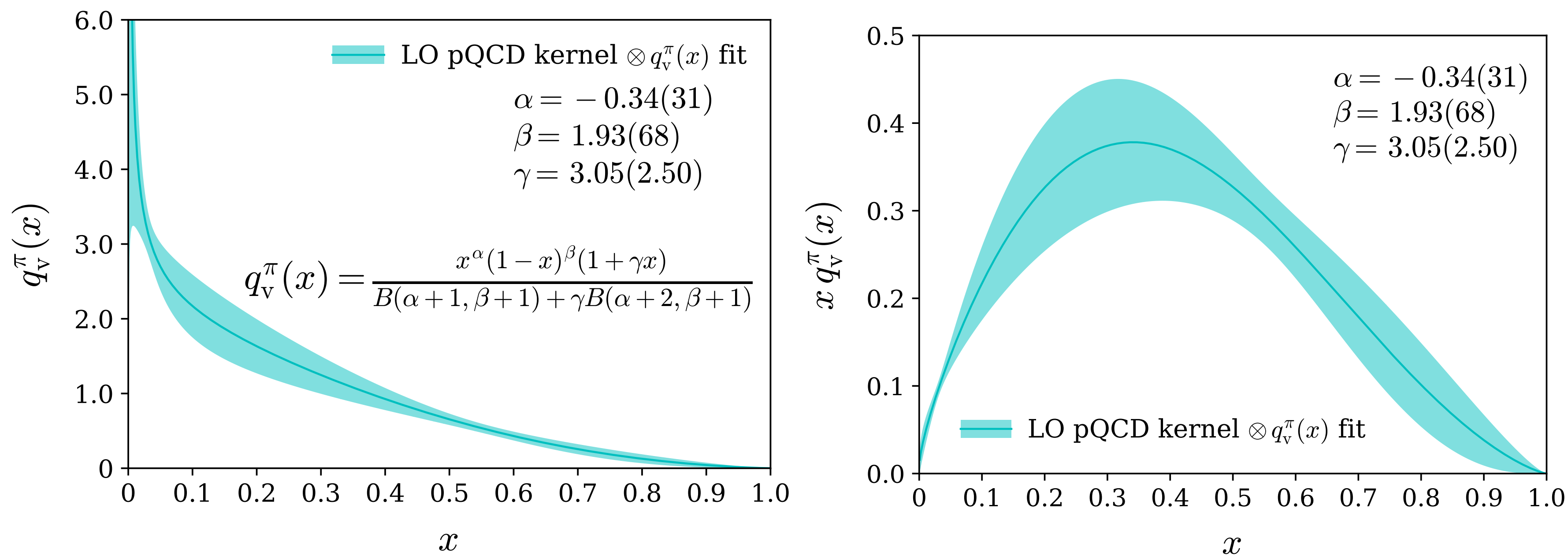
$$\sigma_n(\nu,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) K_n^a(x\nu,\xi^2,x^2P^2,\mu^2) + \mathcal{O}(\xi^2\Lambda^2_{\rm QCD})$$

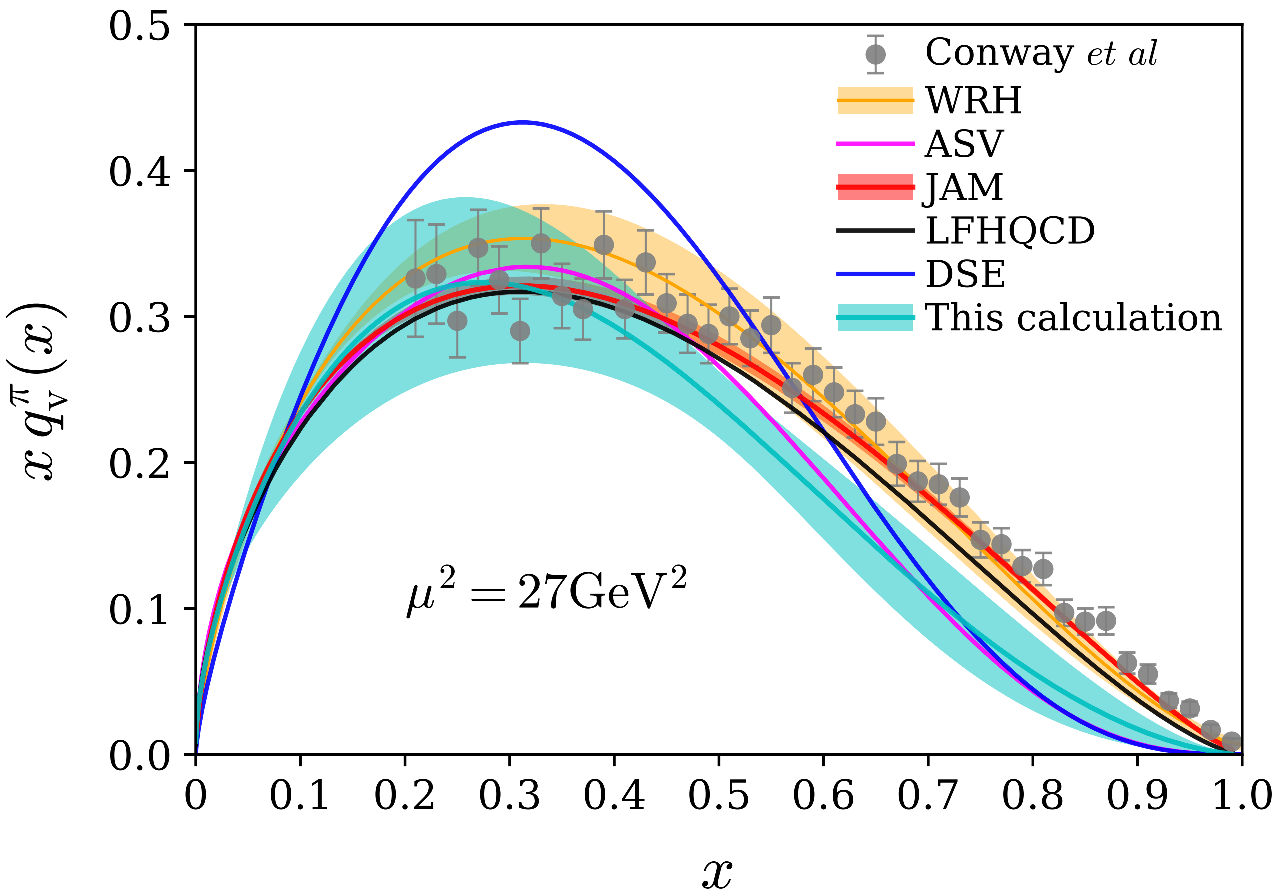
Calculate on Lattice

Calculate in PQCD

Extract PDF?

Similar challenge to global fitting community!





LO Calculation: “guess” scale

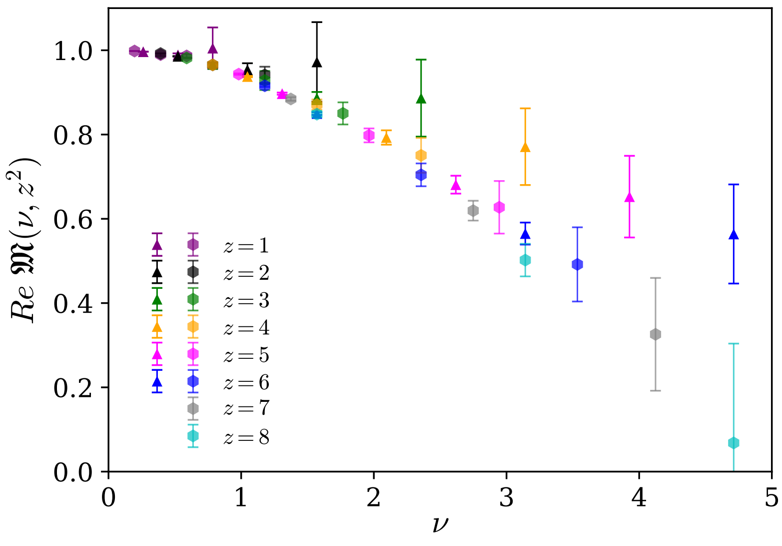
Systematic study at final lattice spacing NLO kernel: coming soon

Pseudo-PDF Approach

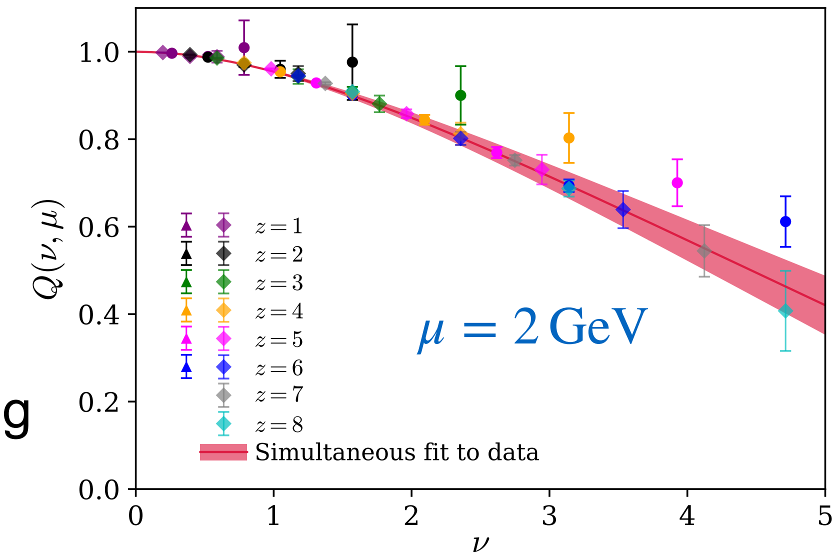
ID	a (fm)	m_π (MeV)	β	am_l	am_s	$L^3 \times T$	N_{cfg}
a127m415	0.127(2)	415(23)	6.1	-0.280	-0.245	$24^3 \times 64$	2147
a127m415L	0.127(2)	415(23)	6.1	-0.280	-0.245	$32^3 \times 96$	2560

See Savvas Zeiferopoulos, Thursday PM

← Same ensemble as GLCS

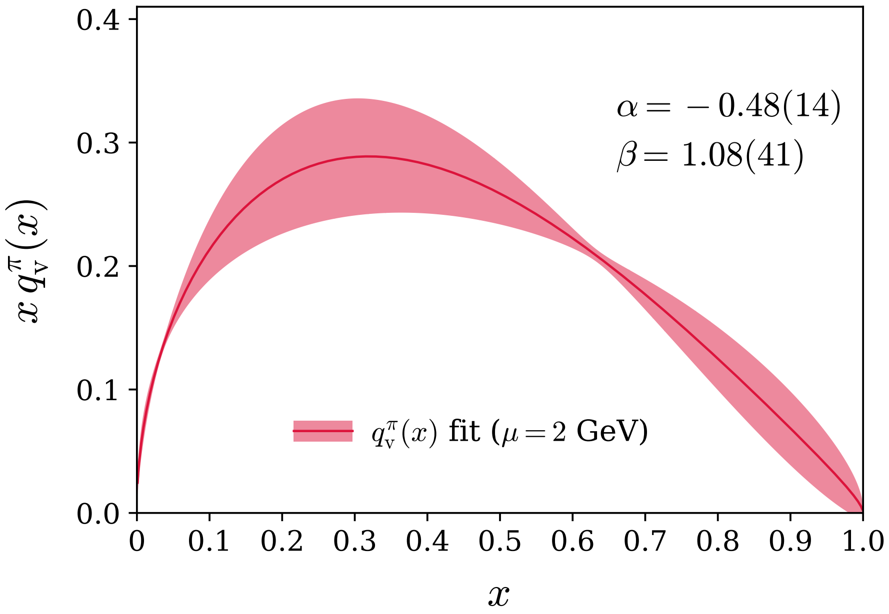


⟹ After Matching

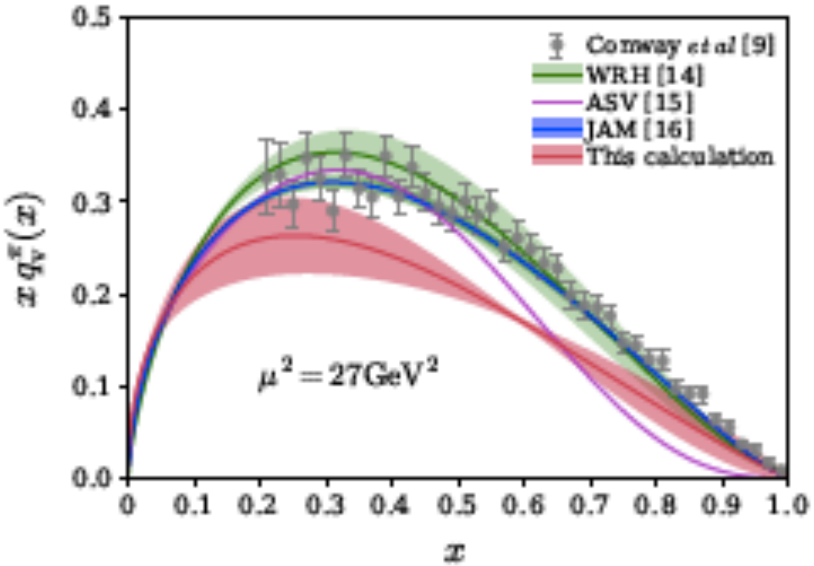


B.Joó, J.Karpie, K.Orginos, A.Radyushkin, D.Richards, R.Sufian, S.Zafeiropoulos, arXiv:1909.08517

Pion pPDF - II

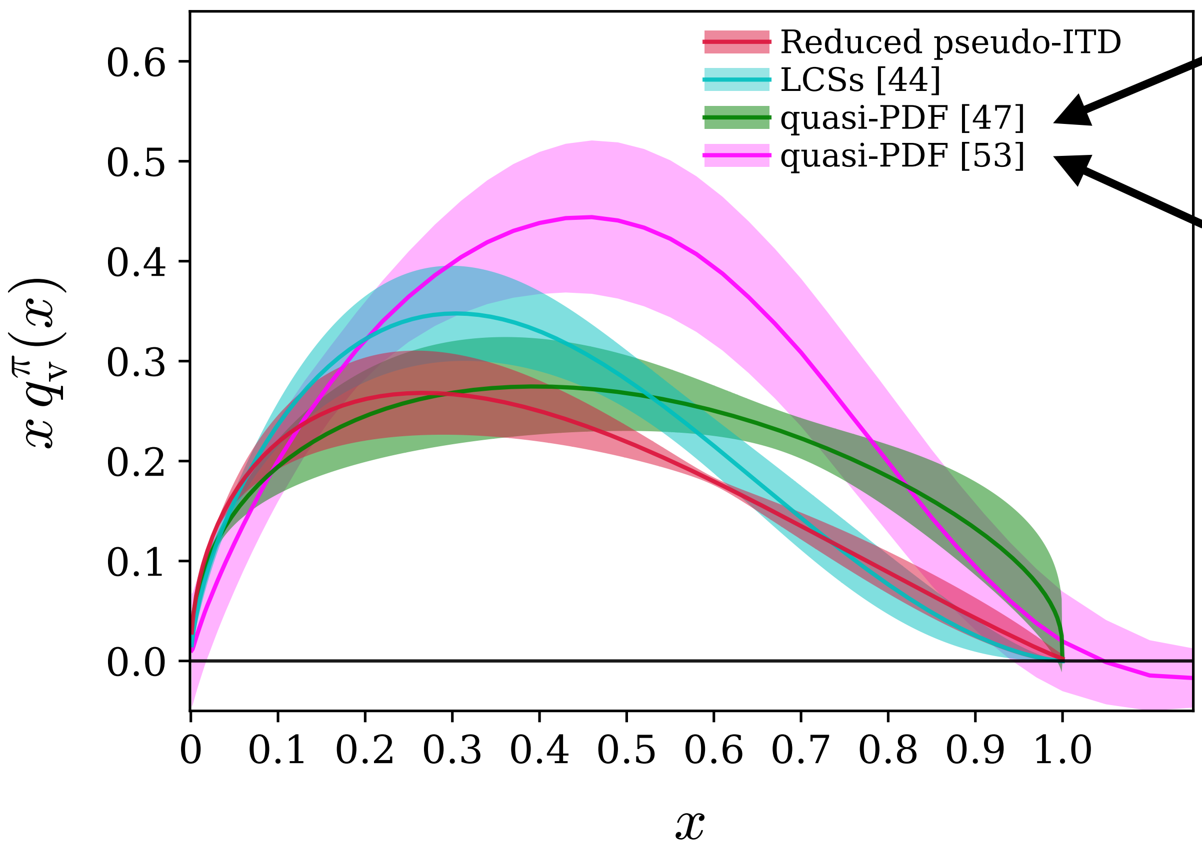


$\alpha = -0.48(14)$
$\beta = 1.08(41)$
$q_v^\pi(x)$ fit ($\mu = 2$ GeV)
$x\, q_v^\pi(x)$
x



Conway et al [9]
WRH [14]
ASV [15]
JAM [16]
This calculation
$\mu^2 = 27\text{GeV}^2$
$x\, q_v^\pi(x)$
x

Pion pPDF - III



T.Izubuchi *et al.*, Phys. Rev. 100, 034516

J-H Zhang *et al.*, Phys. Rev. D **100**, 034505

Nucleon pseudo-PDF

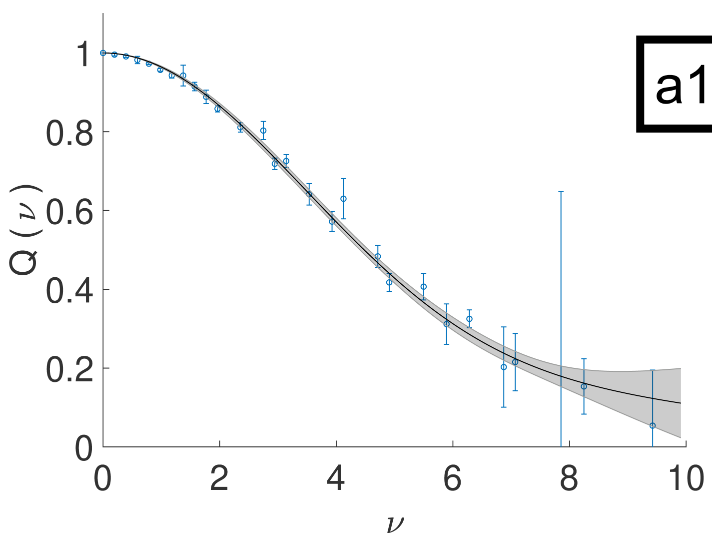
See Savvas Zeiferopoulos, Thursday PM

Ground-breaking quenched calculation: K. Orginos et al., PRD96 (2017), 094503

B.Joo et al., arXiv:1908.09771

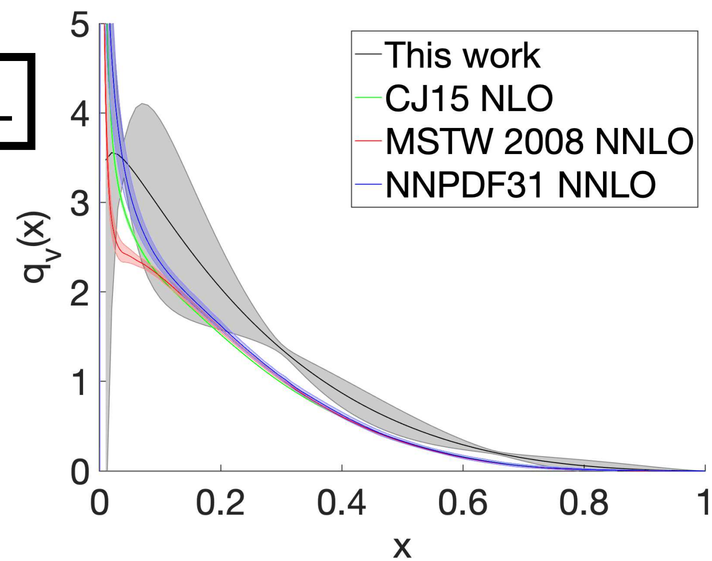
ID	a(fm)	M_π(MeV)	β	c_{SW}	am_l	am_s	$L^3 \times T$	N_{cfg}
$a127m415$	0.127(2)	415(23)	6.1	1.24930971	-0.2800	-0.2450	$24^3 \times 64$	2147
$a127m415L$	0.127(2)	415(23)	6.1	1.24930971	-0.2800	-0.2450	$32^3 \times 96$	2560
$a094m390$	0.094(1)	390(71)	6.3	1.20536588	-0.2350	-0.2050	$32^3 \times 64$	417

← Finer lattice spacing

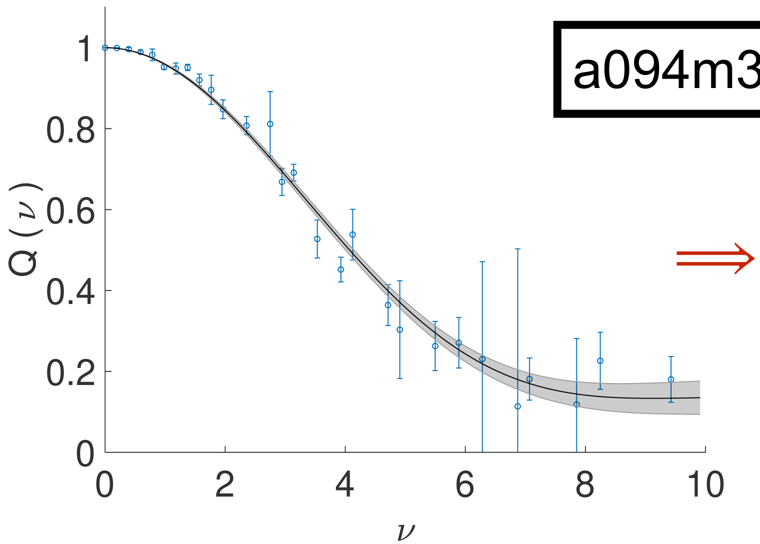


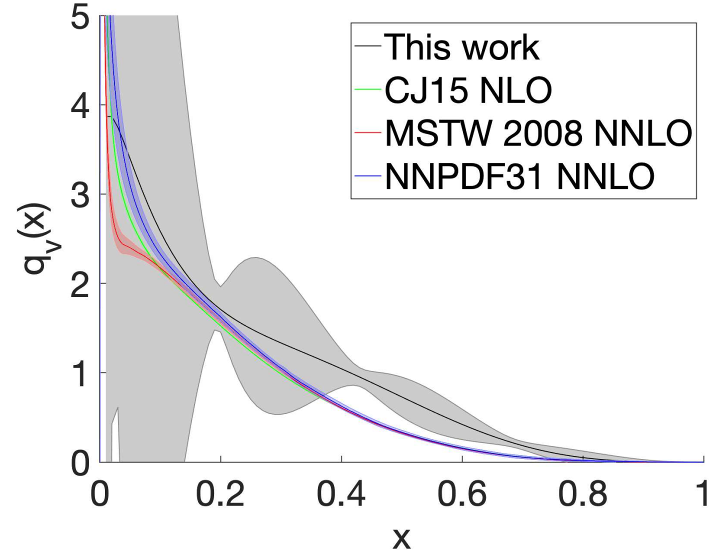
a127m415L

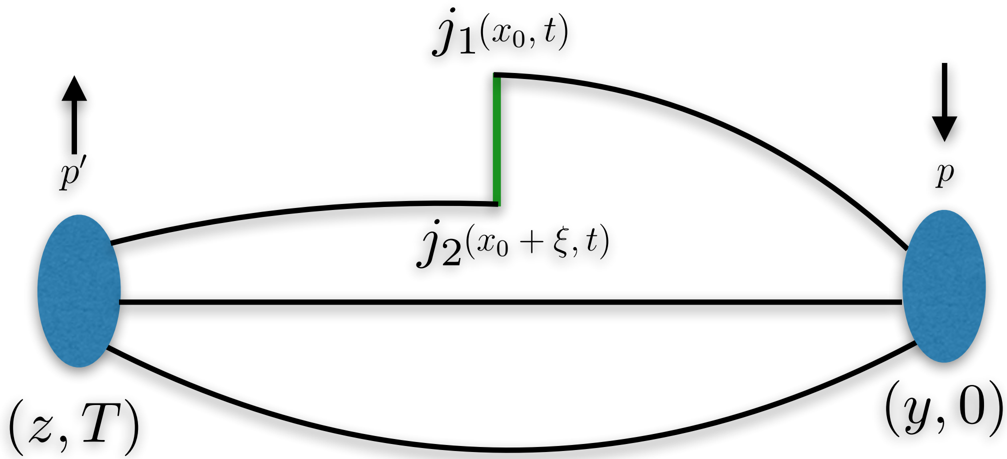
⟹



Nucleon GLCS?







Does not admit sequential source method!

Summary

- Revolution in the study of x-dependent measures of hadron structure
  - Impact global fitting community? Unclear for valence PDFs BUT
  - First-Principles calculation
- Pseudo-PDF/GLCS approach has a well-defined short-distance scale: factorize short-distance physics from perturbative scale.
- Solution of inverse problem: common to all attempts to extract PDFs. Appeal to global fitting community
- Systematic study of pion PDF using GLCS approach is in preparation; NLO perturbative kernel. Q. Ma
- To control systematics
  - fine lattices - to ensure in perturbative regime
  - large momenta - to provide range in Ioffe time

Extension to 3D imaging through GPDs and TMDs, see Michael Engelhardt - *opportunity to predict and constrain experiment*

Moments of Pion PDF