QCD at the high-energy frontier

T. Lappi

University of Jyväskylä, Finland

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Outline

Setup:

- High energy limit of Quantum Chromodynamics: gluon saturation
- CGC, Wilson line, classical color field
- Dilute-dense processes: orders in α_s and $\ln \sqrt{s}$

Details:

- High energy evolution at NLO
- Inclusive Deep inelastic scattering, at NLO
- Exclusive processes, UPC
- Forward rapidity in proton-nucleus at NLO

This talk intended as a broad overview,

But occasionally go into detail to demonstrate what is going on.

Gluon saturation

IMF, parton model perspective

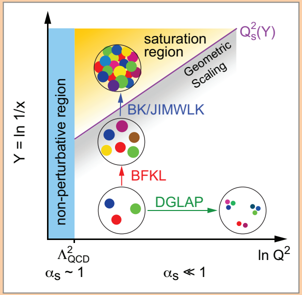
- Evolution with Q^2 or x: cascade of gluons
- Small x: phase space density of gluons large
  $\Longrightarrow$ nonlinear interactions, depending on
  - Size of one gluon $\sim 1/Q^2$
  - Transverse space available
  - Coupling

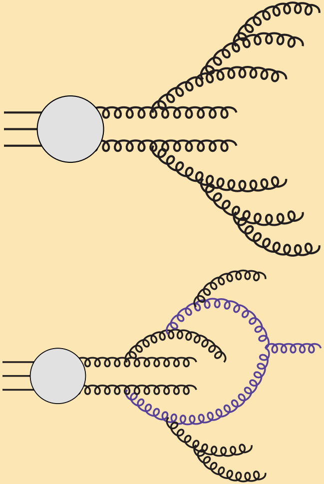
Gluon mergings matter when

$$\pi R_p^2 \sim \alpha_s x G(x, Q_s^2)/Q_s^2$$

▼

"Phase diagram":

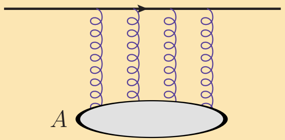




But don't calculate like this!

Eikonal scattering off target of glue

Instead of counting gluons, look at scattering amplitudes

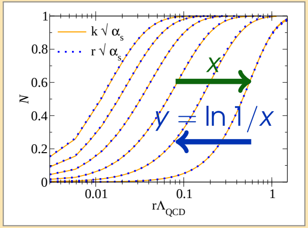


- Dilute probe through target color field
- At high energy interaction is eikonal, relevant degree of freedom is **Wilson line** (= scattering amplitude of colored parton)

$$V = \mathbb{P}\exp\left\{-ig\int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_c) \qquad \text{coordinate space!}$$

- Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_c} \mathrm{Tr}\, V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

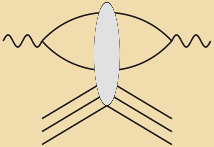


- Gluon TMD's (WW, dipole) FT's of $\mathcal{N}(r) \sim \alpha_s [xG] r^2$
- **Saturation** = unitarity requirement for amplitude (built in as group theory constraint for $\mathrm{SU}(N_c)$)
- $1/Q_s$ is Wilson line $\perp$ **correlation length**

Power counting for dilute-dense processes

Dilute-dense process at LO

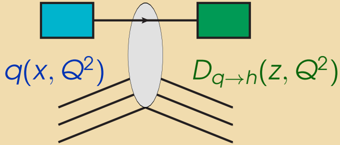
DIS



- $\gamma^* \to q\bar{q}$ dipole
  interacts with target color field
- Total cross section
  $2\times$Im-part of amplitude
- Exclusive & inclusive processes

"Dipole model": Nikolaev, Zakharov 1991
Fits to HERA data: e.g. Golec-Biernat, Wüsthoff 1998

Forward hadrons



- High x q/g from probe:
  collinear pdf
- $|$quark amplitude$|^2 \sim$ dipole
- Indep. fragmentation

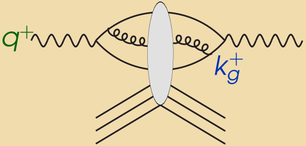
"Hybrid formalism" Dumitru, Jalilian-Marian 2002

Universality: both involve same dipole amplitude $\mathcal{N} = 1 - S$

Dilute-dense process at LL

Add one **soft** gluon: large logarithm of energy, i.e. $1/x$

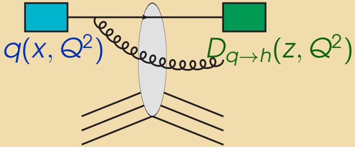
DIS



- Soft gluon: large logarithm

$$\alpha_s \int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \alpha_s \ln \frac{1}{x_{Bj}}$$

Forward hadrons



- Soft gluon $k^+ \to 0$: same large $\ln 1/x$
- Collinear gluon $k_T \to 0$:
  also DGLAP evolution of pdf, FF

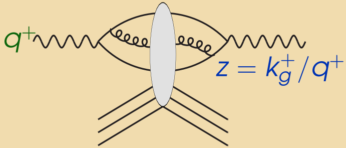
Dumitru et al 2005

Absorb large $\ln 1/x$ into renormalization of Wilson line:
JIMWLK equation, or **BK equation** for dipole Balitsky 1995, Kovchegov 1999

Dilute-dense process at NLO

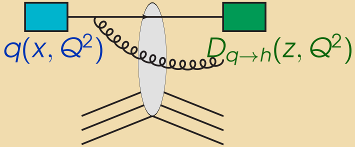
Add one gluon, but **not** necessarily soft

DIS



- DIS impact factor
  Balitsky & Chirilli 2010, Beuf 2017

Forward hadrons



- NLO single inclusive
  Chirilli et al 2011

- Leading small-k^+ gluon already in BK-evolved target
- Need to **subtract** leading log from cross section, (high energy) **factorization**

Schematically $$\sigma_{NLO} = \int dz \Big[\overbrace{\sigma(z) - \sigma(z=0)}^{\sigma_{\text{sub}}} + \overbrace{\sigma(z=0)}^{\text{absorb in BK}} \Big] \quad z = \frac{k_g^+}{q^+}$$

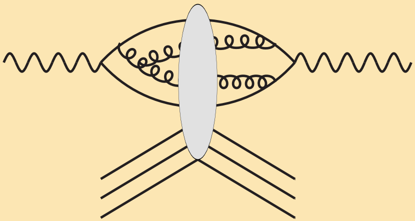
NLO to NLL

NLO evolution equation:

- Consider NNLO DIS
- Extract leading soft logarithm
- Lengthy calculation:

  Balitsky & Chirilli 2007

  $\Longrightarrow$ NLO BK/JIMWLK equation
- But additional resummations needed for practical phenomenology



(+ many diagrams at same order)

- $\alpha_s^2 \ln^2(1/x)$: two iterations of LO BK
- $\alpha_s^2 \ln 1/x$: NLO BK
- α_s^2: part of NNLO impact factor (not calculated)

Summary: power counting & state of the art

$$\sigma \sim \overbrace{\mathcal{O}(1)}^{\text{LO}} + \overbrace{\mathcal{O}(\alpha_s \ln 1/x)}^{\text{LL}} + \overbrace{\mathcal{O}(\alpha_s)}^{\text{NLO}} + \overbrace{\mathcal{O}(\alpha_s^2 \ln 1/x)}^{\text{NLL}}$$

Calculated at NLO/NLL

- JIMWLK/BK evolution Balitsky, Chirilli 2008, Grabovsky, Lublinsky, Mulian 2012
- Total DIS cross section $m_q = 0$ Balitsky, Chirilli 2010, Beuf 2011-2017
- Single inclusive particles in fwd rapidity hh-collisions Chirilli, Xiao, Yuan + others 2011 –
- Diffractive dijets in DIS Boussarie et al 2014
- Exclusive light vector mesons (with PDA's) Boussarie et al 2016

So far only LO/LL, but NLO/NLL under way:

- Forward rapidity dijets in pA Partial results: Mulian & Iancu, Ayala et al
- Diffractive structure functions
- Total DIS cross section with massive quarks
- Exclusive quarkonium in DIS/UPC (with NRQCD)

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Calculated at NLO/NLL

- **1:** JIMWLK/BK evolution Balitsky, Chirilli 2008, Grabovsky, Lublinsky, Mulian 2012
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- Diffractive structure functions
- Total DIS cross section with massive quarks
- **3:** Exclusive quarkonium in DIS/UPC (with NRQCD)

The rest of this talk: **1, 2, 3, 4**

BK evolution at NLO

The NLO BK equation

as derived by Balitsky and Chirilli, 2007

Equation: $y = \ln 1/x$-dependence from

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \boldsymbol{K_1} \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \boldsymbol{K_f} \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} \boldsymbol{K_2} \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$

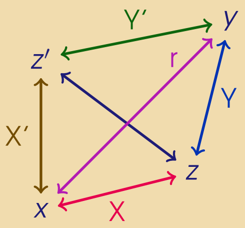
Notations & details

- $S(x-y) \equiv (1/N_c) \langle \mathrm{Tr}\, V^\dagger(x) V(y) \rangle$
- $\otimes = \int d^2z$ or $\int d^2z\, d^2z'$
- Here large N_c & mean field:

$$\langle \mathrm{Tr}\, V^\dagger V \,\mathrm{Tr}\, V^\dagger V \rangle \to \langle \mathrm{Tr}\, V^\dagger V \rangle \langle \mathrm{Tr}\, V^\dagger V \rangle$$

(This gives BK, instead of JIMWLK)

Coordinates



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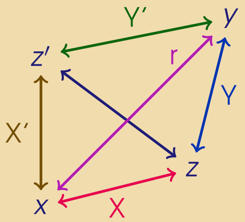

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

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- Leading order

Kernels

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$$\left.\left. + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

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- Leading order
- Running coupling (Terms with β function coefficient)

Kernels

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- Leading order
- Running coupling (Terms with β function coefficient)
- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)

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- Leading order
- Running coupling (Terms with β function coefficient)
- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)
- Nonconformal double log $\Longrightarrow$ blows up for $r = 0$

Resummations

Following Iancu et al 2015

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \boldsymbol{K_1} \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \boldsymbol{K_f} \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} \boldsymbol{K_2} \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$

Rapidity local resummation procedure:

- β-function terms in K_1 into running coupling: K_{Bal}
- Double transverse logarithms in K_1 into $K_{\mathrm{DLA}} \sim J_1(\ln r^2)/\ln r^2$.
- Single logs in K_2 into $K_{\mathrm{STL}} \sim r^{\alpha_s A_1}$ with DGLAP anomalous dimension A_1
- Subtract double counting K_{sub}, include rest of NLO K_1^{fin} $\Longrightarrow$ Mäntysaari, T.L. 2016 :

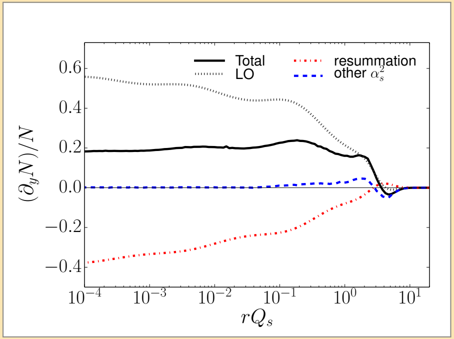
$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \left[K_{DLA} K_{STL} K_{Bal} - K_{\mathrm{sub}} + K_1^{\mathrm{fin}} \right] \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)] + N_F\text{-part}$$

- $\exists$ Also alternative cumbersome but better defined kinematical constraint Beuf 2014, implementation Albacete 2015

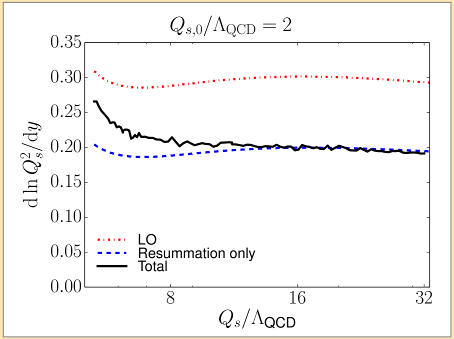
NLL evolution in $Y \equiv \ln k^+$ with resummation

Mäntysaari, T.L. 2016

Evolution speed vs r



Evolution speed of Q_s



- Resummations essential to get stable results

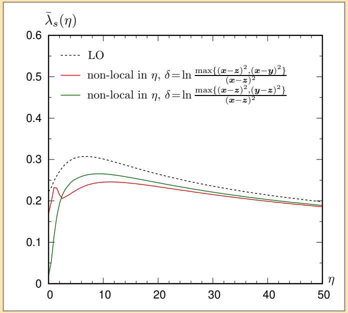
  $\Longrightarrow$ good HERA fit with "resummation only" Iancu et al 2015

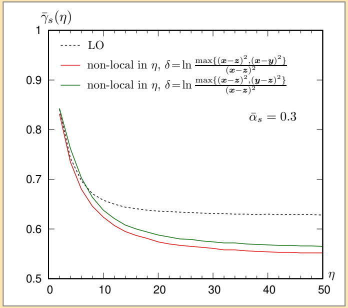
NLL evolution in $\eta \equiv \ln 1/k^-$ with resummation

Ducloué et al 2019

- Change evolution variable from $Y = \ln k^+ \sim \ln W^2$ to $\eta = Y - \ln(Q_0^2 r^2) \sim \ln 1/x_{Bj}$
- More stable with respect to scale choice in resummations
- Initial value problem better defined
- Evolution speed, anomalous dimension (only) slightly lower than leading order

$\Longrightarrow$ This is good for phenomenology



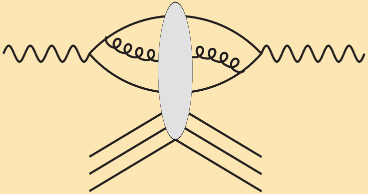


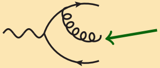
DIS at NLO

Inclusive DIS

Balitsky, Chirilli 2010, Beuf 2011-2017

- At NLO need
  - Real: $q\bar{q}g$ state in dipole
  - Virtual: 1-loop corrections to the $\gamma^* q\bar{q}$-vertex
- Divergences cancel between real and virtual
- Massive quarks: in progress Beuf, T.L., Paatelainen
- Soft log factorized into BK evolution of target rest is NLO "γ^* impact factor"





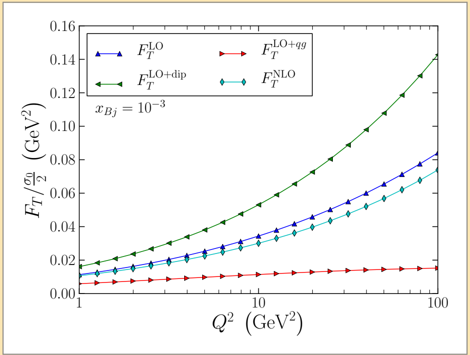
$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_\mathrm{F} \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{\mathrm{d}z_2}{z_2} \left[\mathcal{K}_{L,T}^{\mathrm{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\mathrm{NLO}}(0, X(z_2)) \right].$$

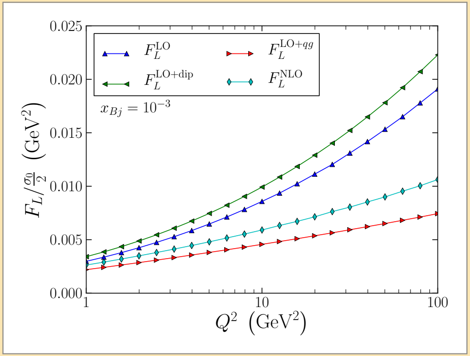
Note on factorization

- Target rapidity scale must be $X(z_2)$, depends on integration variable z_2
- Naive/"k_T-factorization"/CXY subtraction with $X(z_2) = x_{Bj}$ is unstable.

1st numerical implementation: general features

Ducloué, Hänninen, T.L., Zhu 2017



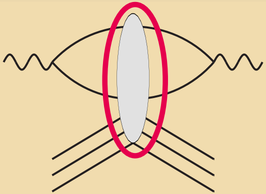


- NLO corrections of reasonable magnitude,
  after major cancellation between different terms
- Factorization procedure (still) somewhat naive, not good at large Q^2
- Starting point for comparison with experimental data

Exclusive DIS & UPC's

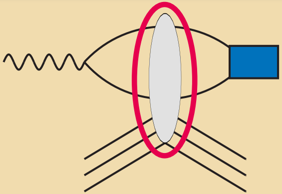
Exclusive processes in dipole picture

Total cross section



$\sigma_{\text{tot}} \sim$ forward elastic amplitude

Diffractive DIS



Exclusive $\sim |\text{same amplitude}|^2$

Same QCD-evolved amplitude describes both

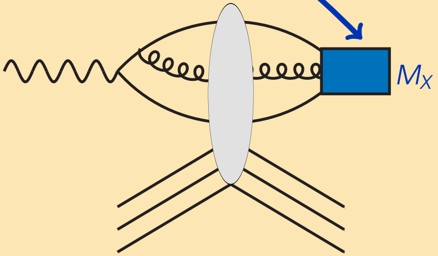
Unified description is a major advantage of the dipole picture

In hard scattering limit $\mathcal{N} \sim xg(x, Q^2)$

$\Longrightarrow$ often quoted formula $\frac{d\sigma^{\gamma^* H \to VH}}{dt} = \frac{16\pi^3 \alpha_s^2 \Gamma_{ee}}{3\alpha_{\text{em}} M_V^5} \left[xg(x, Q^2)\right]^2$

Exclusive DIS at NLO

- Different final states
  jets, vector mesons, . . .



Known at NLO

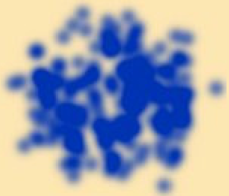
- Dijets Boussarie et al 2014
- Exclusive light vector mesons
  PDA for meson Boussarie et al 2016

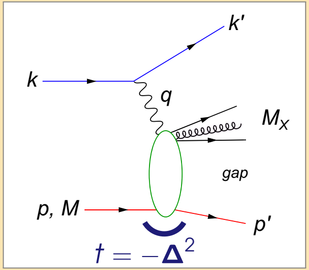
NLO calculations in progress

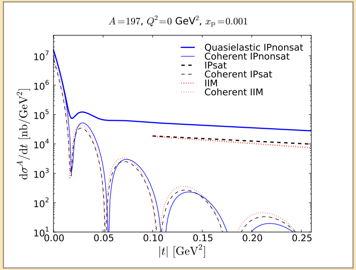
- Diffractive structure functions: fixed M_X
- Quarkonium, with NRQCD for meson
  Heavy quarks are important:
  allow $Q^2 = 0$ in weak coupling

How to measure transverse geometry of gluons

Diffractive DIS gives Fourier transform of gluon distribution







$$\mathcal{N}(\mathbf{\Delta}) = \int d^2\mathbf{b} e^{i\mathbf{b}\cdot\mathbf{\Delta}} \mathcal{N}(\mathbf{b})$$

Coherent target intact; measure **average** gluon distribution

$$-t \sim \frac{1}{R_A^2} \sim 0.01\text{GeV}^2 \text{ (nucleus)}$$

Incoherent target breaks without color exchange: **fluctuations**

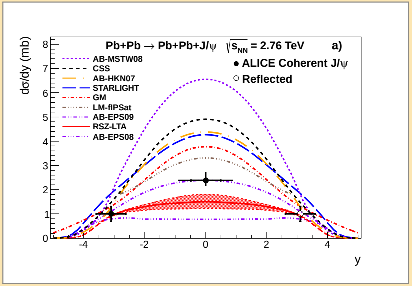
$$-t \sim \frac{1}{R_p^2} \sim 1\text{GeV}^2 \text{ (nucleus} \to \text{nucleons)}$$

(Both very impotant for QGP physics)

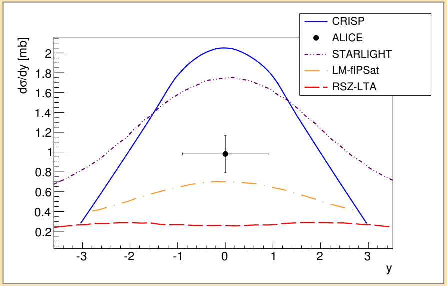
UPC results from LHC

- To understand b distribution need average **and** fluctuations:
  coherent **and** incoherent
- This is equally true for $\gamma^{(*)}A$ and $\gamma^{(*)}p$

Highest energy data so far: ultraperipheral collisions at LHC:



$\gamma A \to J/\Psi + A$ Eur. Phys. J. C **73** (2013) 2617



ALICE $\gamma A \to J/\Psi + A^*$

Plot from Phys. Rev. C **92** (2015) 064903

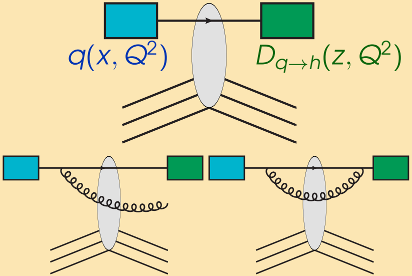
- $Q^2 = 0 \implies J/\Psi$ only at one scale: heavy quark mass.
- Q_s is p_T-scale: to study CGC dynamics need Q^2-dependence: EIC

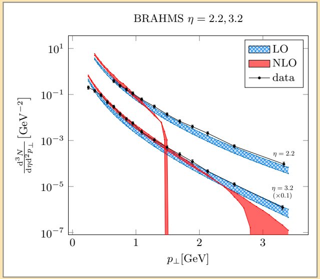
Forward particle production

Particle production in forward pA

Particle production in forward pA:
"hybrid formalism"

- Quark/gluon from collinear pdf (large-x)
- LO: deflected by target field
- NLO: 1-loop virtual and radiative corrections
- 1-loop factorization formulae
  Chirilli, Xiao, Yuan 2011
  - Soft divergence: target BK
  - Collinear: DGLAP for pdf, FF
  - Rest: "hard function"
- 1st result Stasto et al 2013 : NLO cross section negative at large p_T.
- This now understood as a problem with the "naive" factorization procedure: exactly as for DIS





Conclusions

QCD at the HE frontier, via CGC effective theory:

- Resummation of large logs of energy into JIMWKL/BK evolution
- Access to gluon saturation
- Inclusive and exclusive processes in consistent framework
  - Small-x physics at EIC
  - Dilute-dense processes at LHC
  - Intial conditions of heavy ion collisions.
- Moving to NLO
  - Loop calculations done for many processes
  - In $1/x$-evolution requires resummations

    & factorization needs to be consistent with these

    $\Longrightarrow$ challenges in implementation still being worked out