Lattice QCD Results on Moments of Nucleon Parton Distribution Functions and Generalized Form Factors

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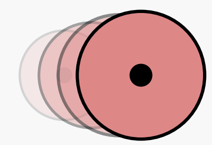
Outline

Moments of PDFs

- Lower moments of PDFs from the lattice
  - Axial and tensor charge
  - Intrinsic quark spin contribution to nucleon spin
- Moments of PDFs
  - Momentum fraction
  - Helicity and transversity
- Momentum dependence
  - Generalized form factors

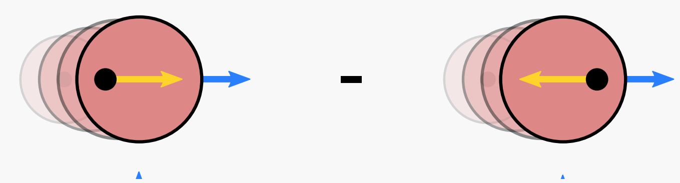
Unpolarised

$$\mathcal{O}_V^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



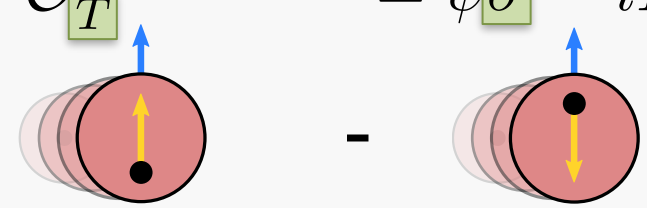
Helicity

$$\mathcal{O}_A^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



Transverse

$$\mathcal{O}_T^{\nu\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



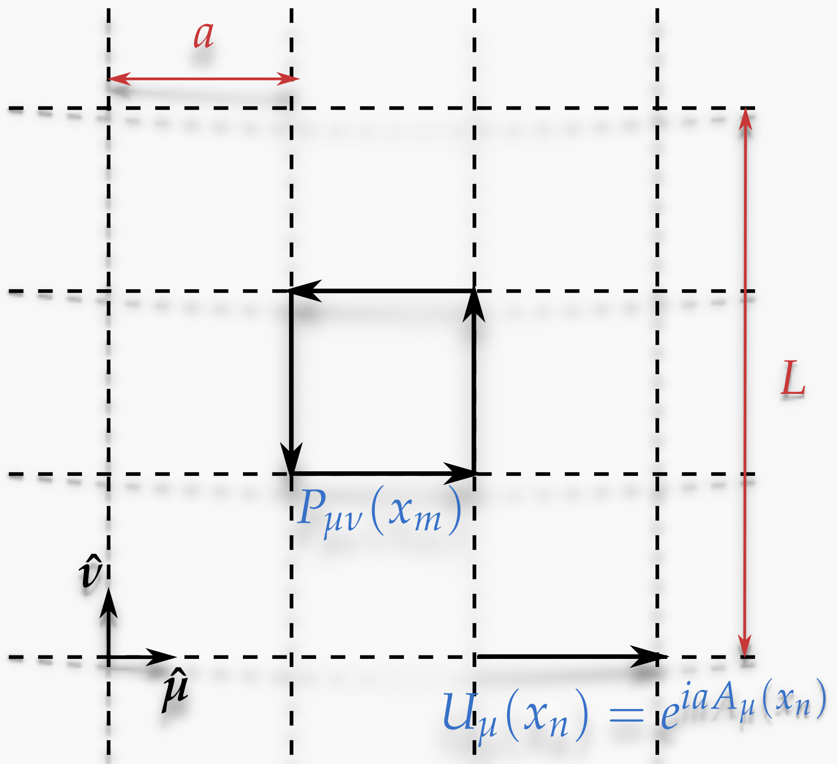
Lattice QCD — ab initio simulation of QCD

Numerical solution of Quantum Chromodynamics

- Direct simulation, starting from the QCD Lagrangian

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D[U] \mathcal{O}[M^{-1}] e^{-S_g[U] + \ln \det M}$$

- Path integral formulation allows for Markov Chain Monte-Carlo simulation
- Integrate-out fermion fields: Need inverse: ***M*-1**
- Sample representative sets of gluon fields ***U***



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Lattice QCD — ab initio simulation of QCD

Freedom in choice of:

- quark masses (heavier is cheaper)
- lattice spacing a (larger is cheaper)
- lattice volume $L^3 \times T$ (smaller is cheaper)

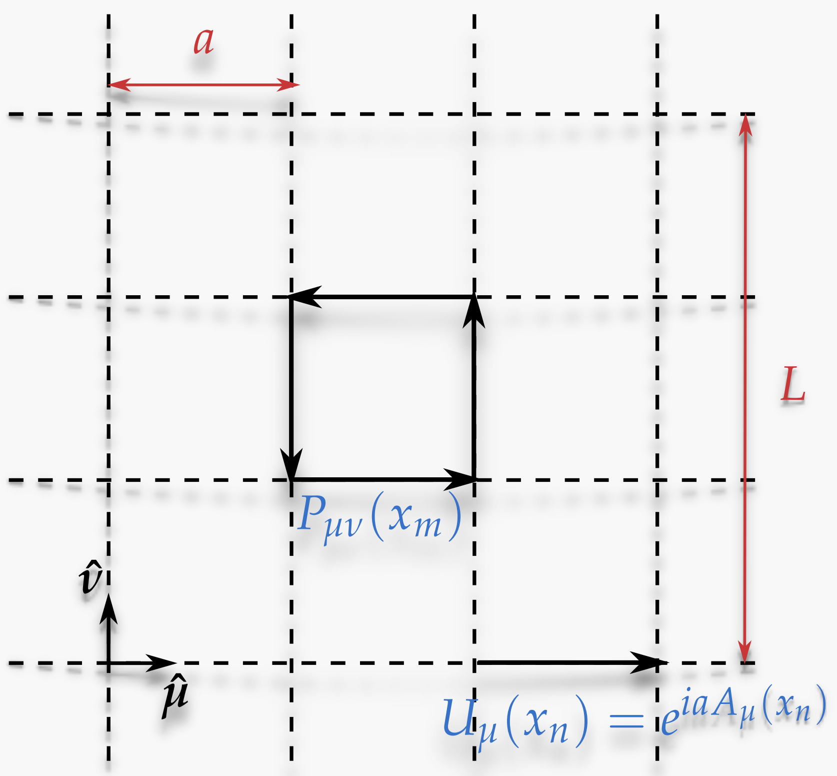
Choice of discretisation scheme

e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall

Trade — offs and advantages for each differ

Eventually, all schemes must agree:

- At the continuum limit: $a \rightarrow 0$
- At infinite volume limit $L \rightarrow \infty$
- At physical quark mass



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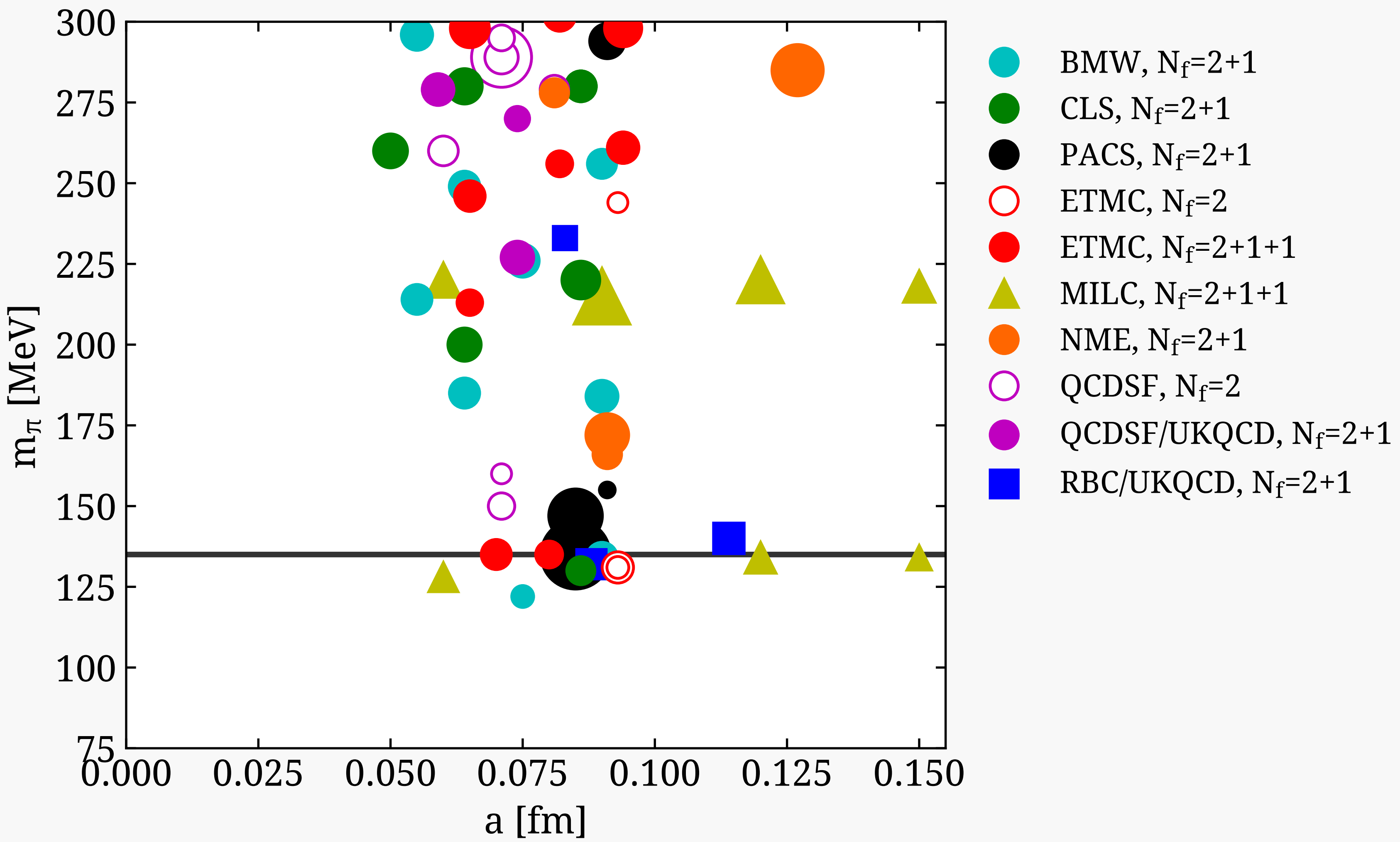
Simulation landscape



Selected lattice simulation points used for hadron structure

- Multiple collaborations simulating at physical pion mass
- Size of points indicates $m_\pi L$

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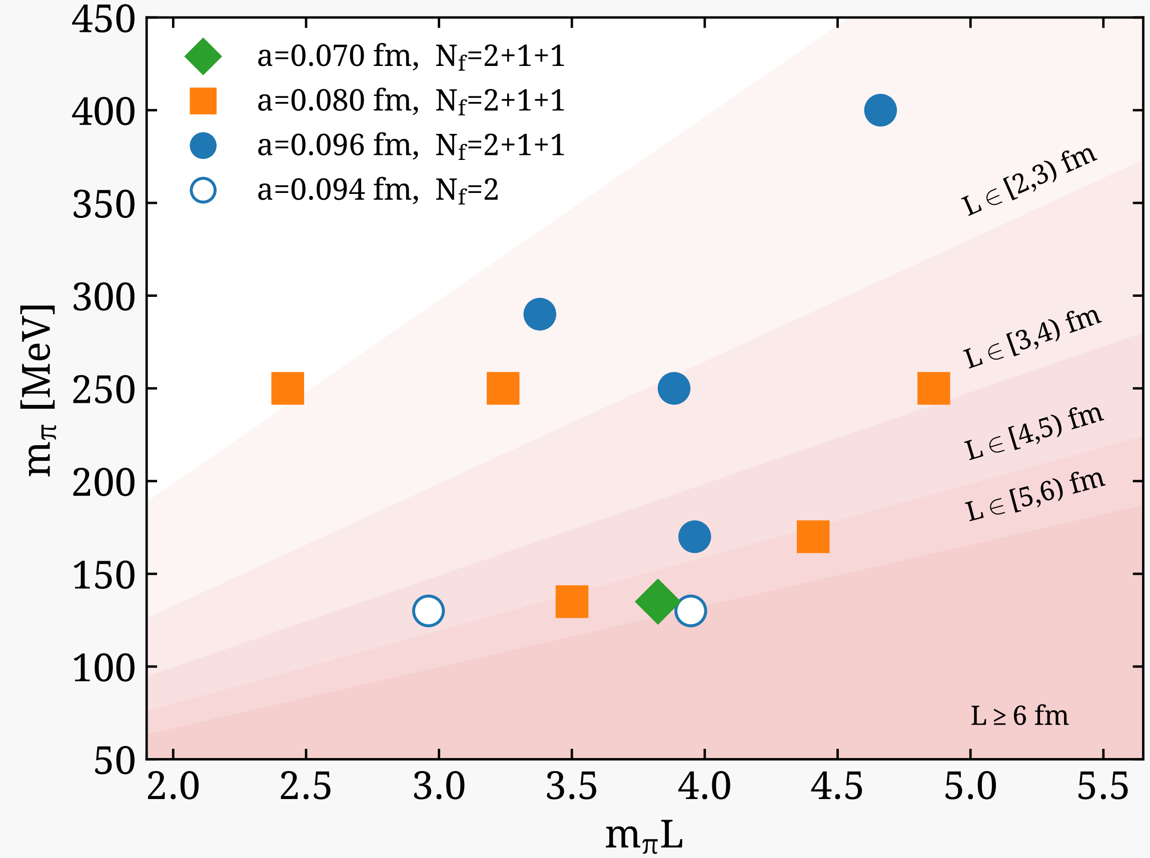
Twisted Mass QCD

$$S^{\ell}_{tm} = \sum_x \bar{\chi}_\ell(x) \left[D_W(U) + \frac{i}{4} c_{\rm sw} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m + i\mu_\ell \tau^3 \gamma^5 \right] \chi_\ell(x)$$

Formulation particularly attractive for nucleon structure

- Tune to "maximal twist": tune Wilson mass (m) to its critical value ($m_{\rm crit}$)
- $\mathcal{O}(a)$ improved operators without requiring further operator improvement
- Several ensembles at physical quark mass
- Generation of additional ensembles ongoing



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Extended Twisted Mass Collaboration

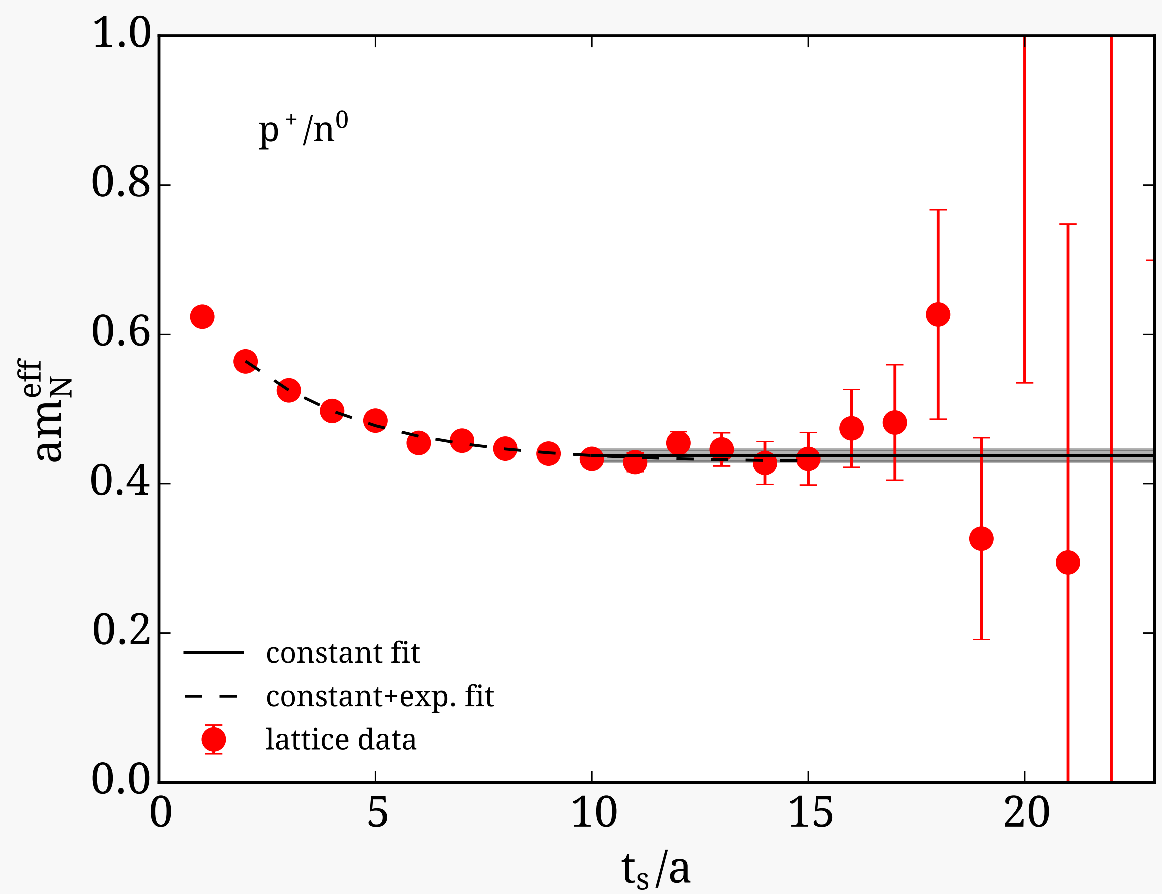
Nucleon structure on the lattice

Two-point correlation functions

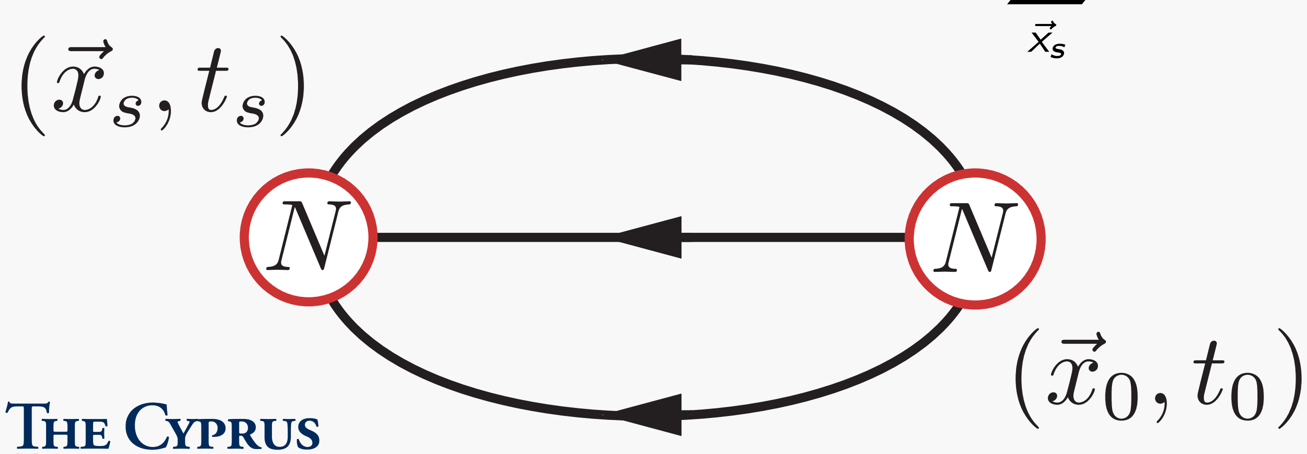
- **Statistical error:** $N^{-½}$ with Monte Carlo samples
- Correlation functions exponentially decay with time-separation

Systematic uncertainties

- Extrapolations: a, L, m_π
- Contamination from higher energy states



$$\sum_{\vec{x}_s} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(x_s) | \chi_N^{\alpha}(0) \rangle = c_0 e^{-E_0 t_s} + c_1 e^{-E_1 t_s} + \ldots$$



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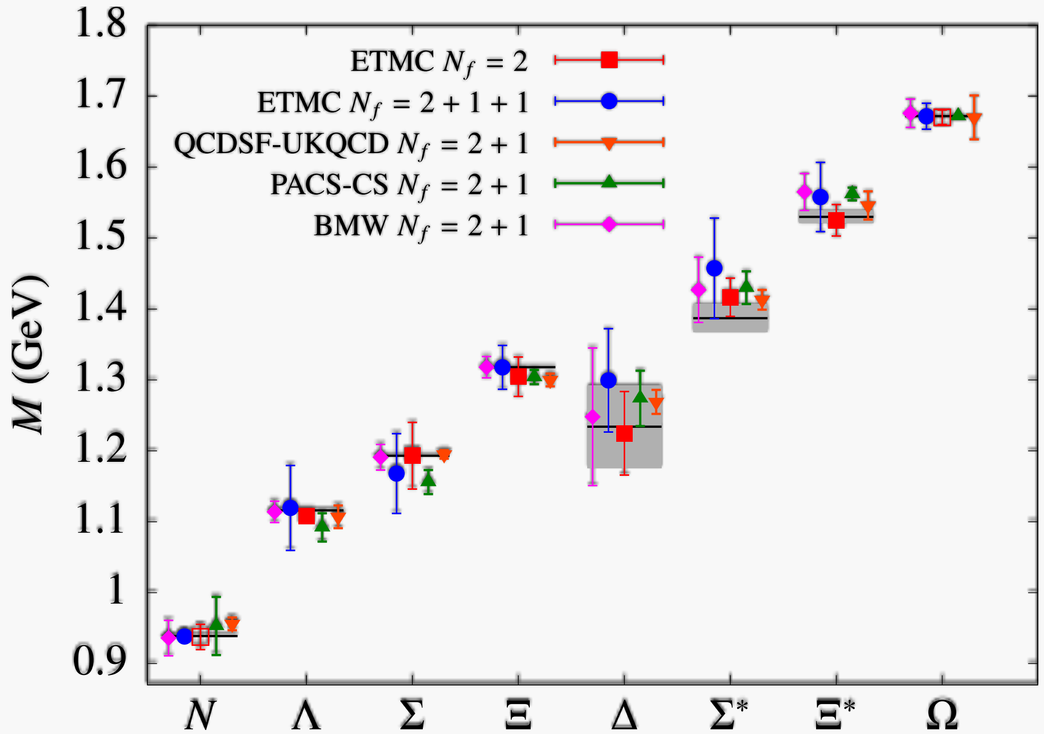
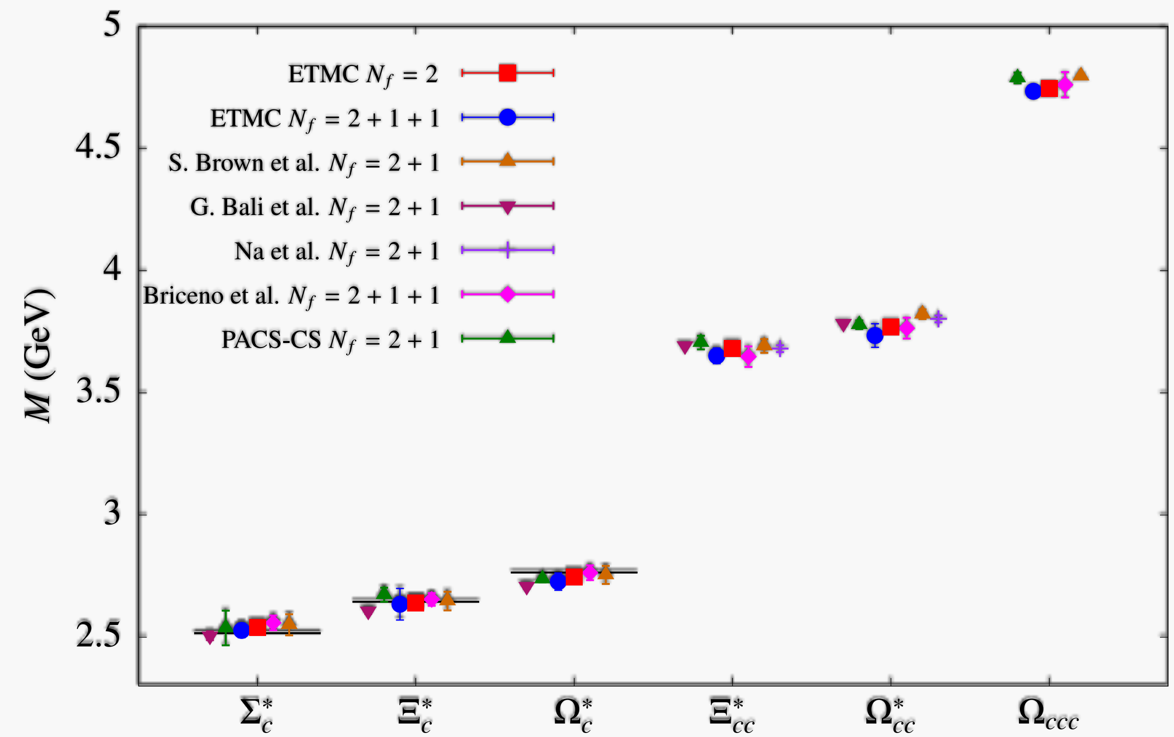
Baryon spectrum





Summary plots from arXiv:1704.02647

Reproduction of light baryon masses

- Agreement between lattice discretisations
- Reproduction of experiment

Prediction of yet to be observed baryons

- Confidence through agreement between lattice schemes

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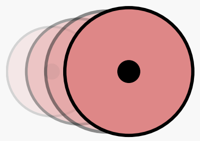
Nucleon structure on the lattice

On the lattice, moments of parton distribution functions are readily accessible as matrix elements of local operators

Unpolarised

$$\mathcal{O}_{V}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

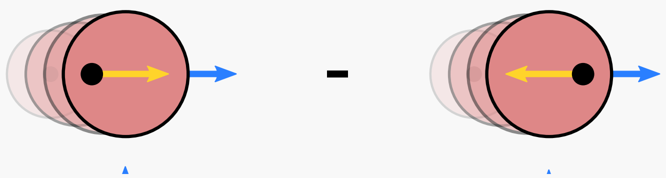
$$\langle 1\rangle_{u-d} = g_V\,,\quad \langle x\rangle_{u-d}\,,\quad \ldots$$



Helicity

$$\mathcal{O}_{A}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

$$\langle 1\rangle_{\Delta u-\Delta d} = g_A\,,\quad \langle x\rangle_{\Delta u-\Delta d}\,,\quad \ldots$$



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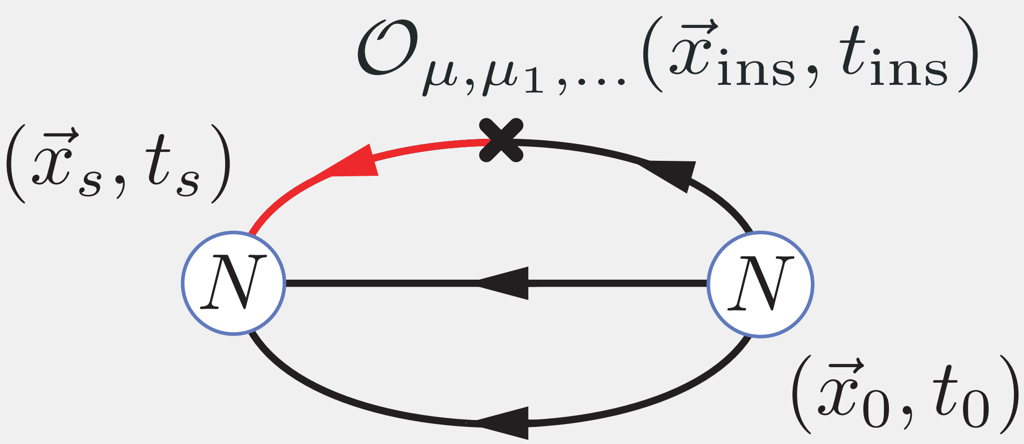
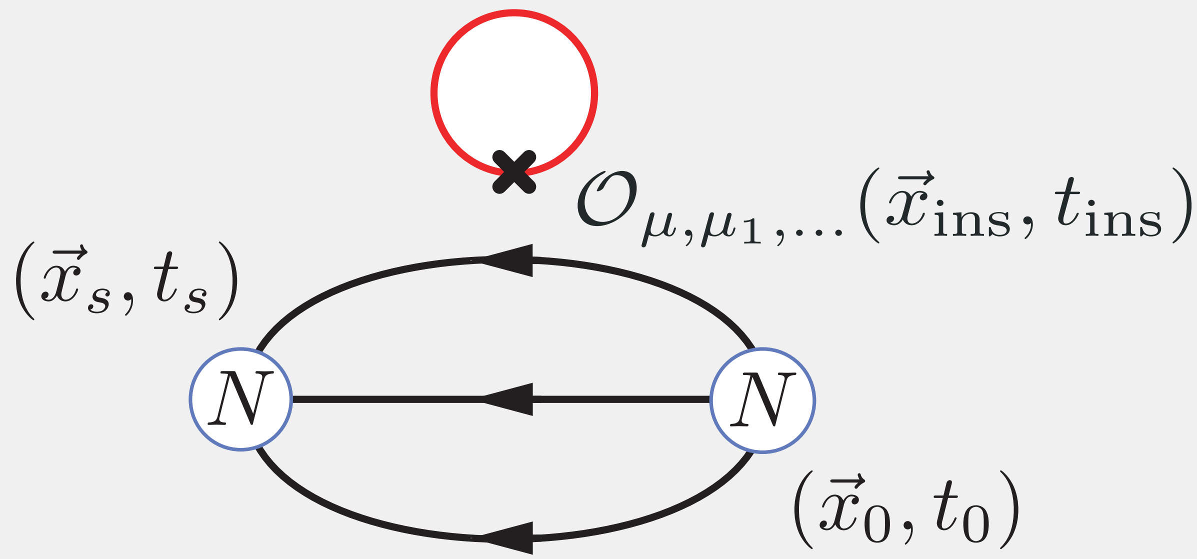
$$\langle 1\rangle_{\delta u-\delta d} = g_T\,,\quad \langle x\rangle_{\delta u-\delta d}\,,\quad \ldots$$

Lattice evaluation of matrix elements

$$G_\mu(\Gamma;\vec{q};t_s,t_{\rm ins}) = \sum_{\vec{x}_s\vec{x}_{\rm ins}} e^{-i\vec{q}.\vec{x}_{\rm ins}}\Gamma^{\alpha\beta}\langle\bar{\chi}^\beta_N(\vec{x}_s;t_s)|\mathcal{O}^\mu(\vec{x}_{\rm ins};t_{\rm ins})|\chi^\alpha_N(\vec{0};0)\rangle$$





Analyses for identifying excited state contributions

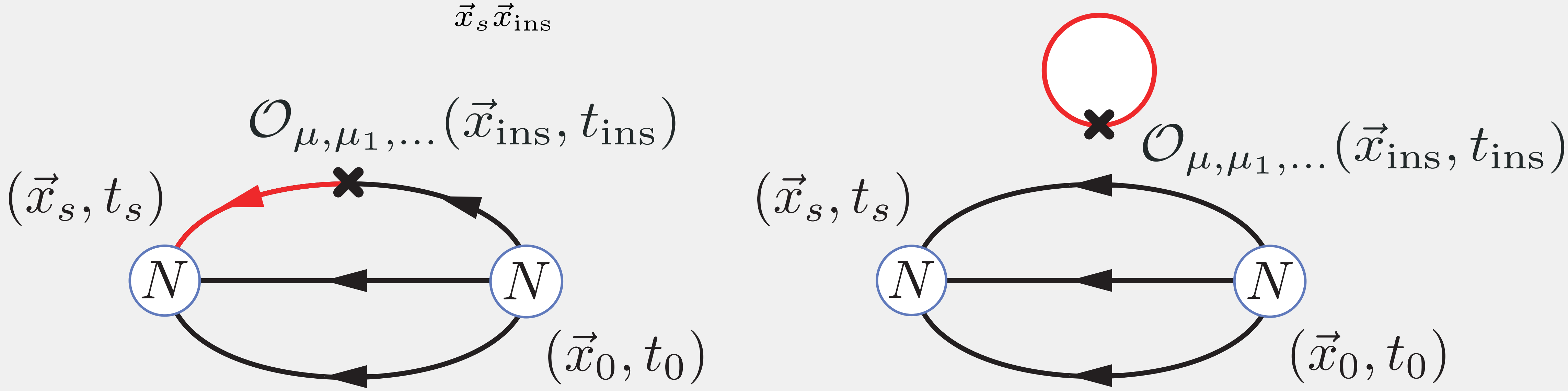
- **Plateau**:

$$R(t_s,t_{\rm ins},t_0) \xrightarrow[t_{\rm ins}-t_0\to\infty]{t_s-t_{\rm ins}\to\infty} \mathcal{M}[1+\mathcal{O}(e^{-\Delta(t_{\rm ins}-t_0)},e^{-\Delta'(t_s-t_{\rm ins})})]$$

fit to constant w.r.t. $t_{\rm ins}$ for multiple values of t_s

Lattice evaluation of matrix elements

$$G_\mu(\Gamma;\vec{q};t_s,t_{\rm ins}) = \sum_{\vec{x}_s \vec{x}_{\rm ins}} e^{-i\vec{q}.\vec{x}_{\rm ins}} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^\beta(\vec{x}_s;t_s)|\mathcal{O}^\mu(\vec{x}_{\rm ins};t_{\rm ins})|\chi_N^\alpha(\vec{0};0)\rangle$$



Analyses for identifying excited state contributions

- **Two-state fit**: Fit, two- and three-point simultaneously, including first excited state
- **Sum over $t_{\rm ins}$**:

$$\sum_{t_{\rm ins}} R(t_s,t_{\rm ins},t_0) \xrightarrow{t_s-t_0\to\infty} \text{Const.} + \mathcal{M}(t_s-t_0) + \mathcal{O}(t_s e^{-\Delta t_s})$$

fit to linear form, matrix element is the slope.

Lattice evaluation of matrix elements

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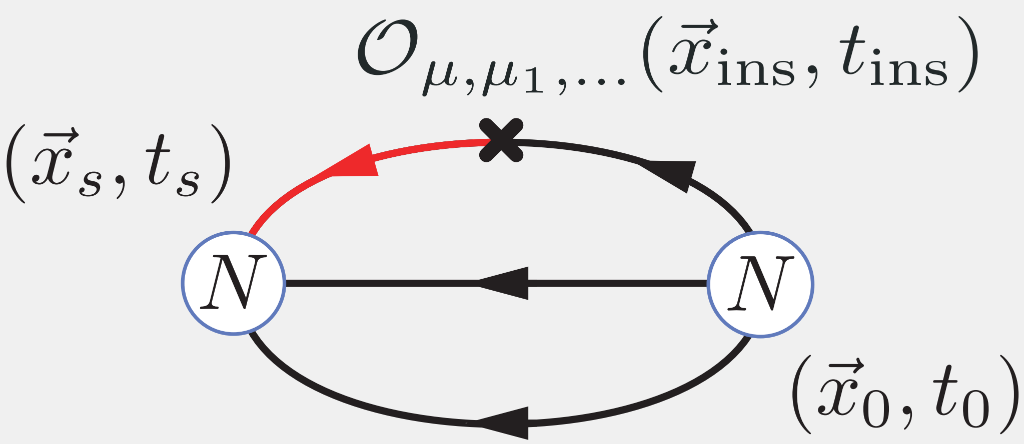

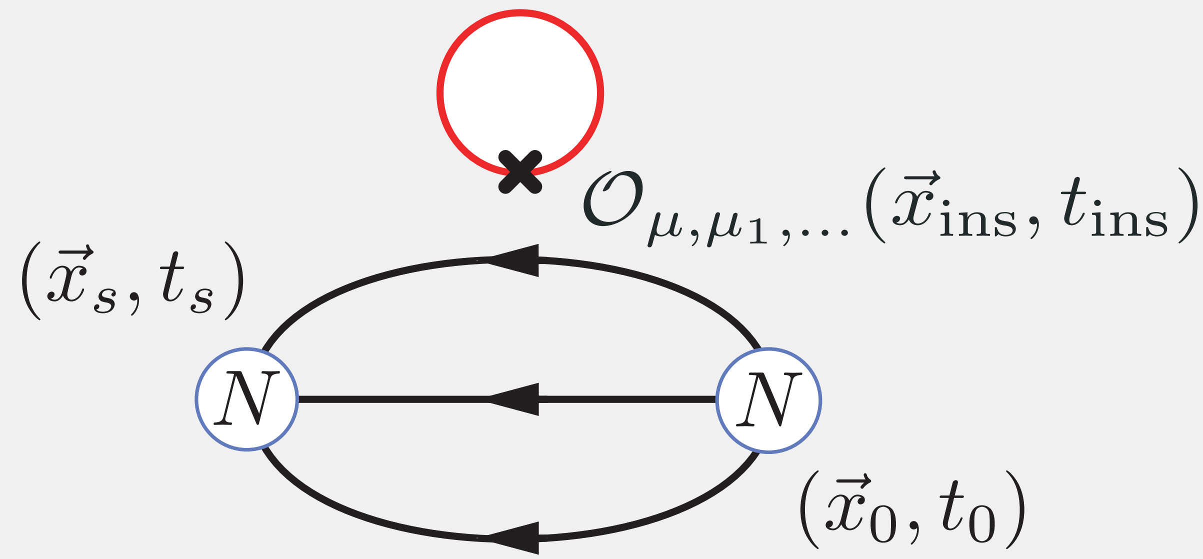

Ana... ontributions

- T... simultaneously, including first excited state
- Su...

... $\text{Const.} + \mathcal{M}(t_s - t_0) + \mathcal{O}(t_s e^{-\Delta t_s})$

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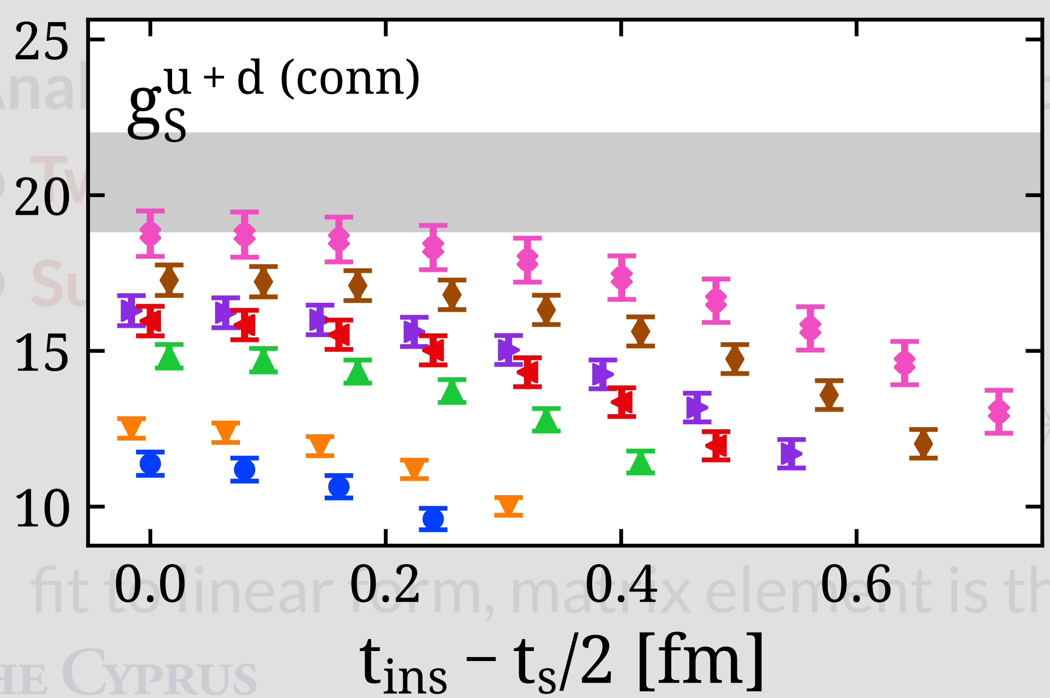

Connected, ~12,000 statistics

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Lattice evaluation of matrix elements

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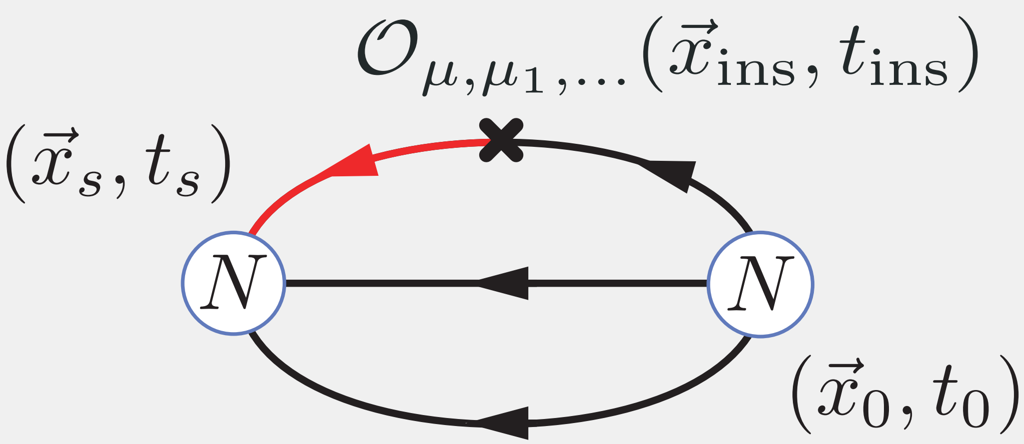

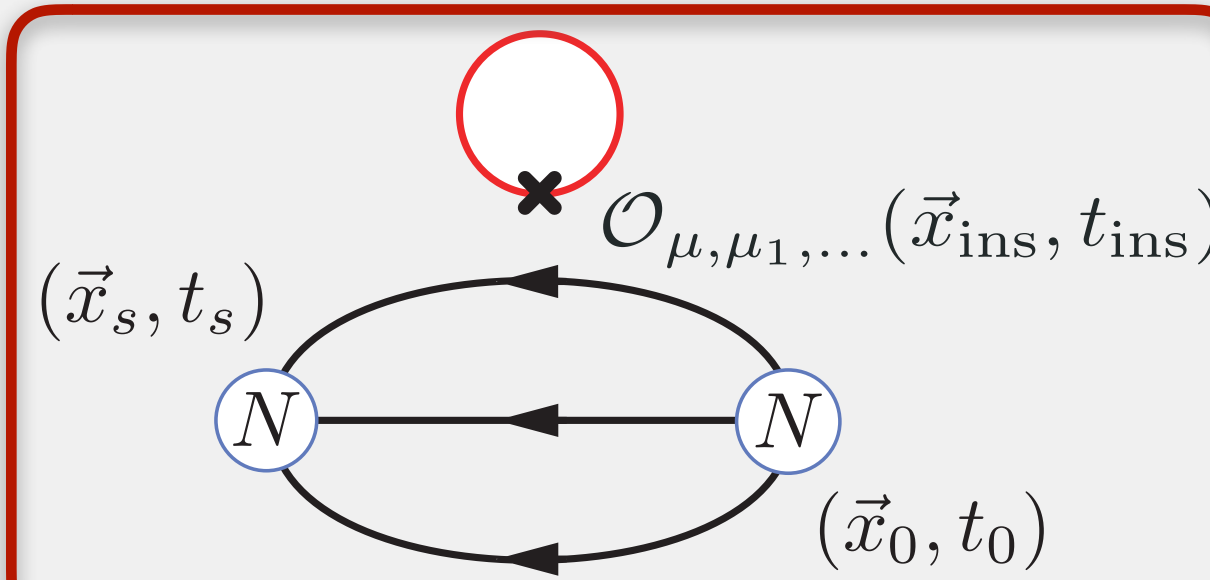

"Disconnected" contributions – estimate stochastically

Ana... ...ntributions

- ... simultaneously, including first excited state
- ...

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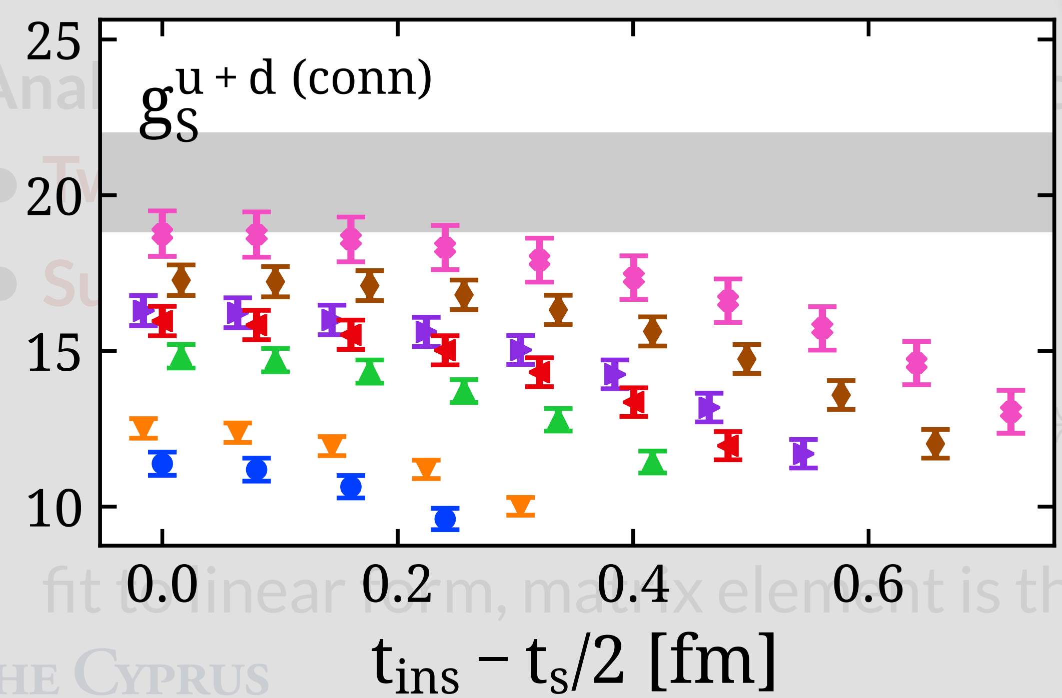
fit to linear form, matrix element is the slope.



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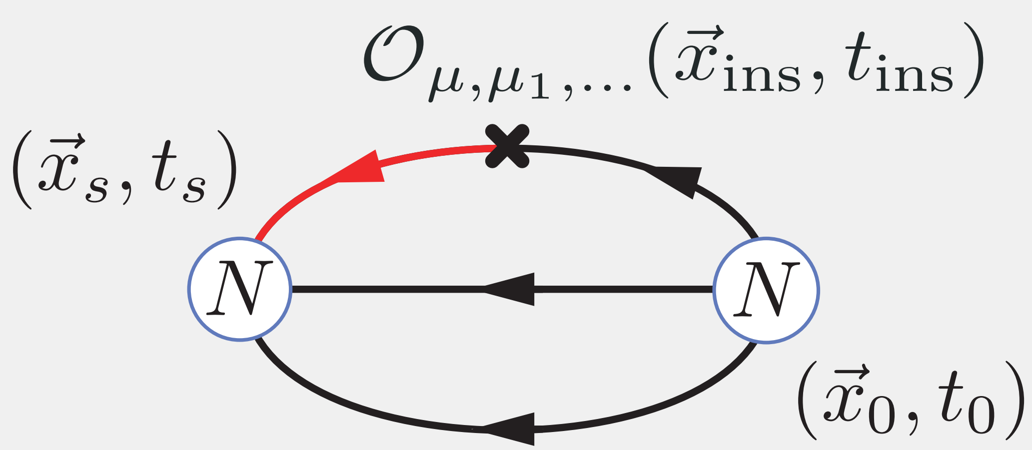

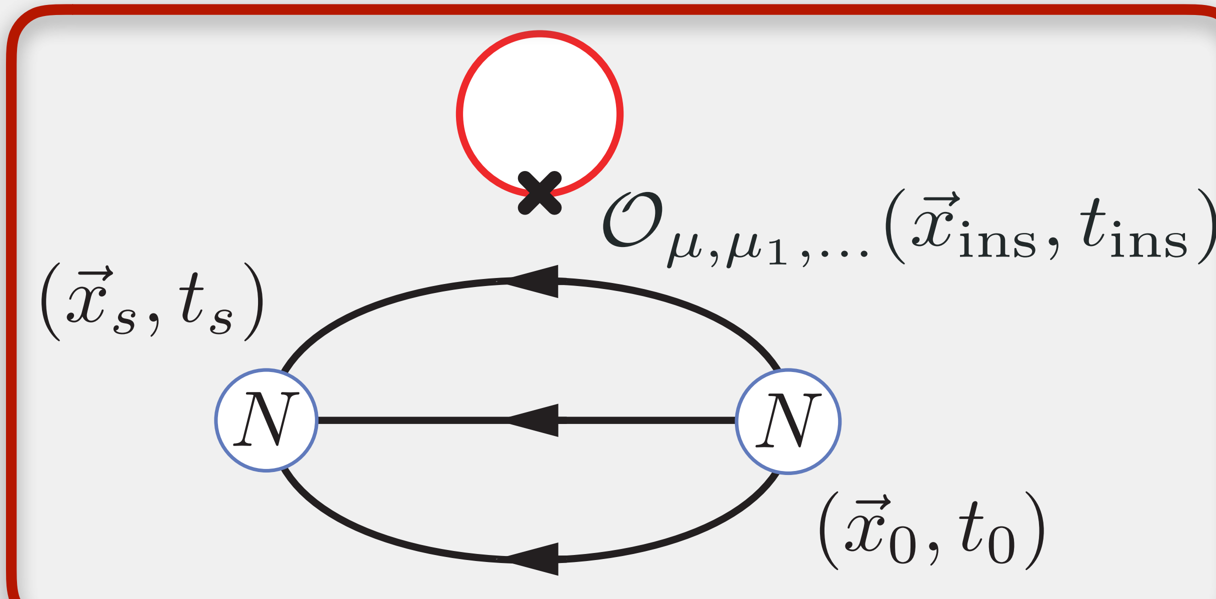

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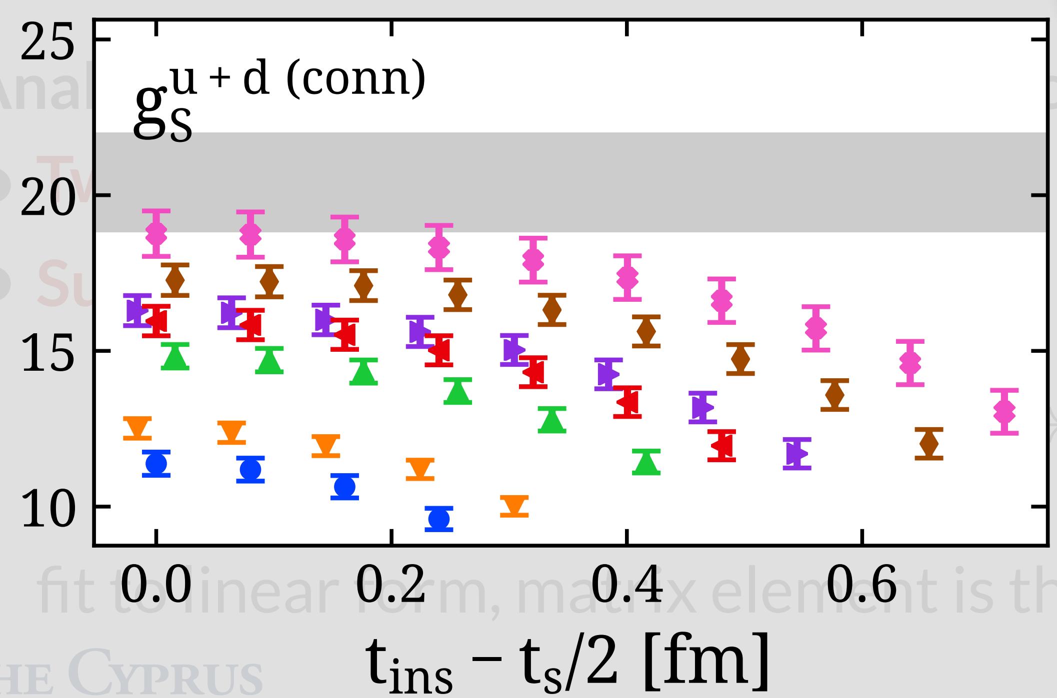

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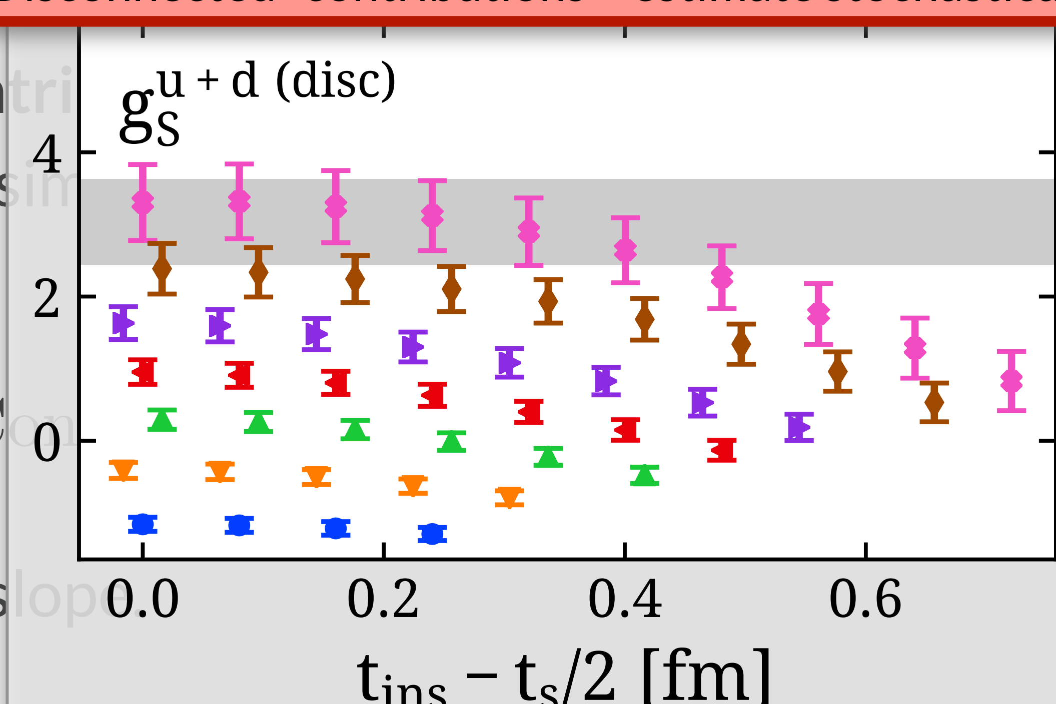

Disconnected, ~198,000 statistics

fit to linear form, matrix element is the slope

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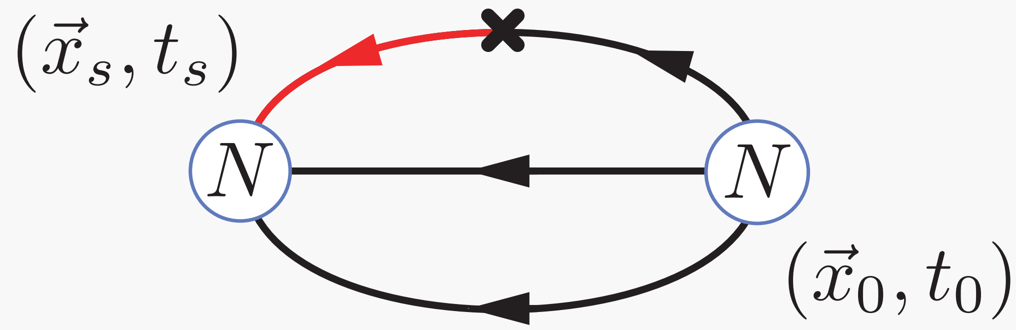
Axial Matrix Elements

Axial charge:

- Well known from β-decay
- Readily accessible on the lattice
- Benchmark quantity in lattice QCD

$$\mathcal{O}^A = \bar{u}\gamma_5\gamma_k u - \bar{d}\gamma_5\gamma_k d$$



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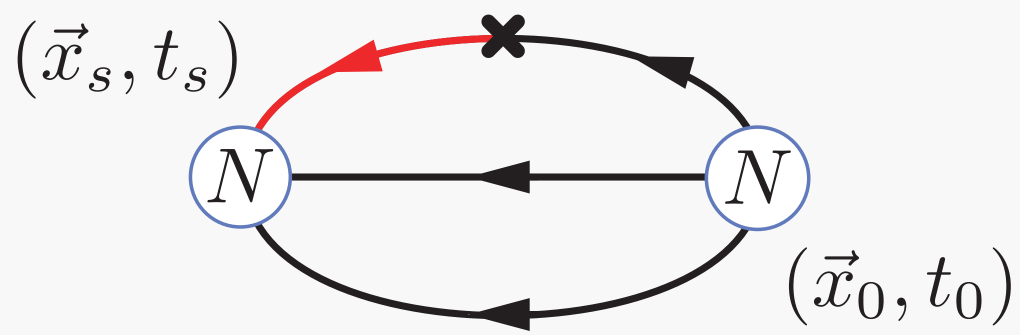

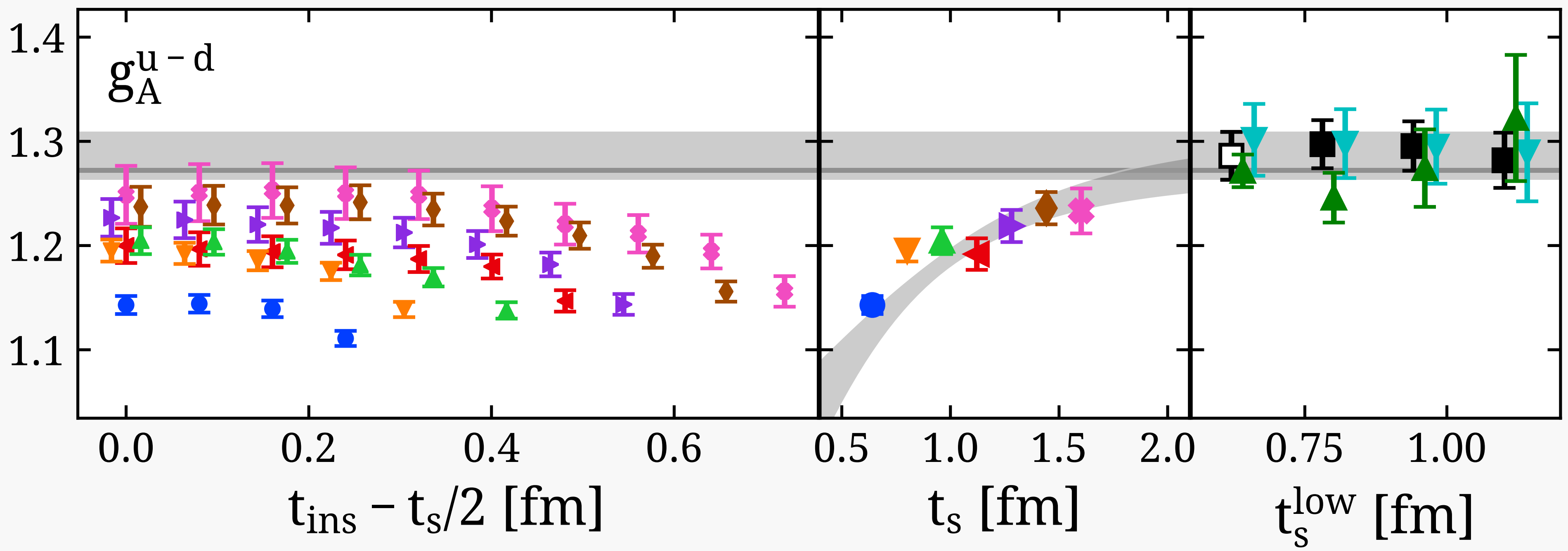

Treatment of excited states

Asymptotic value consistent between methods

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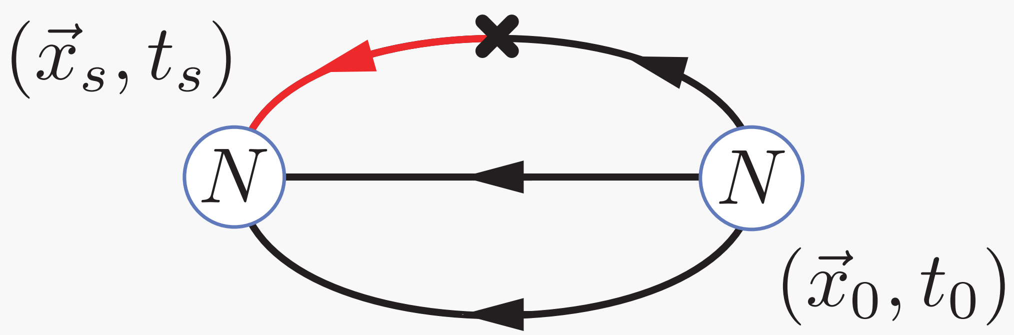

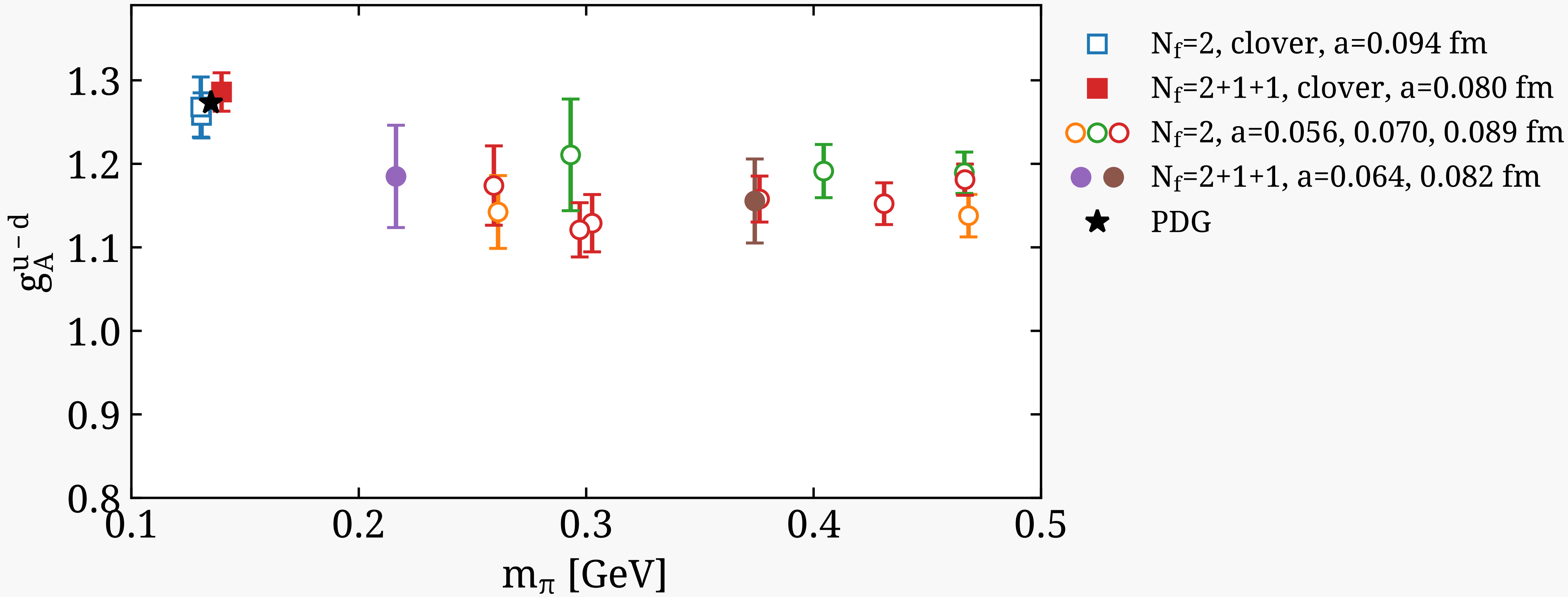

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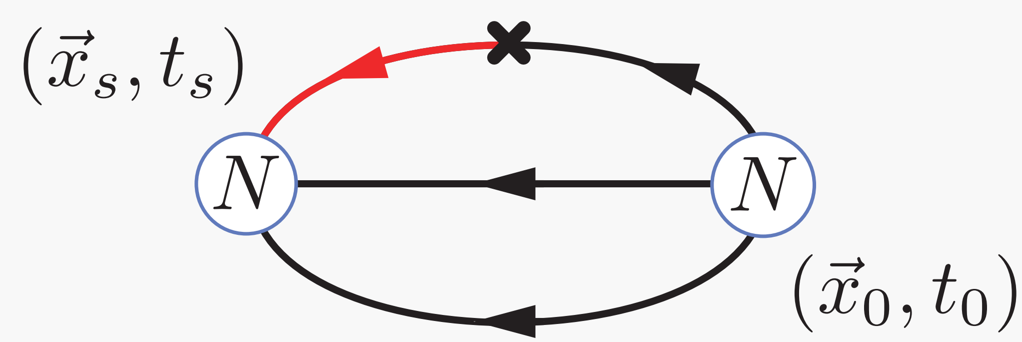

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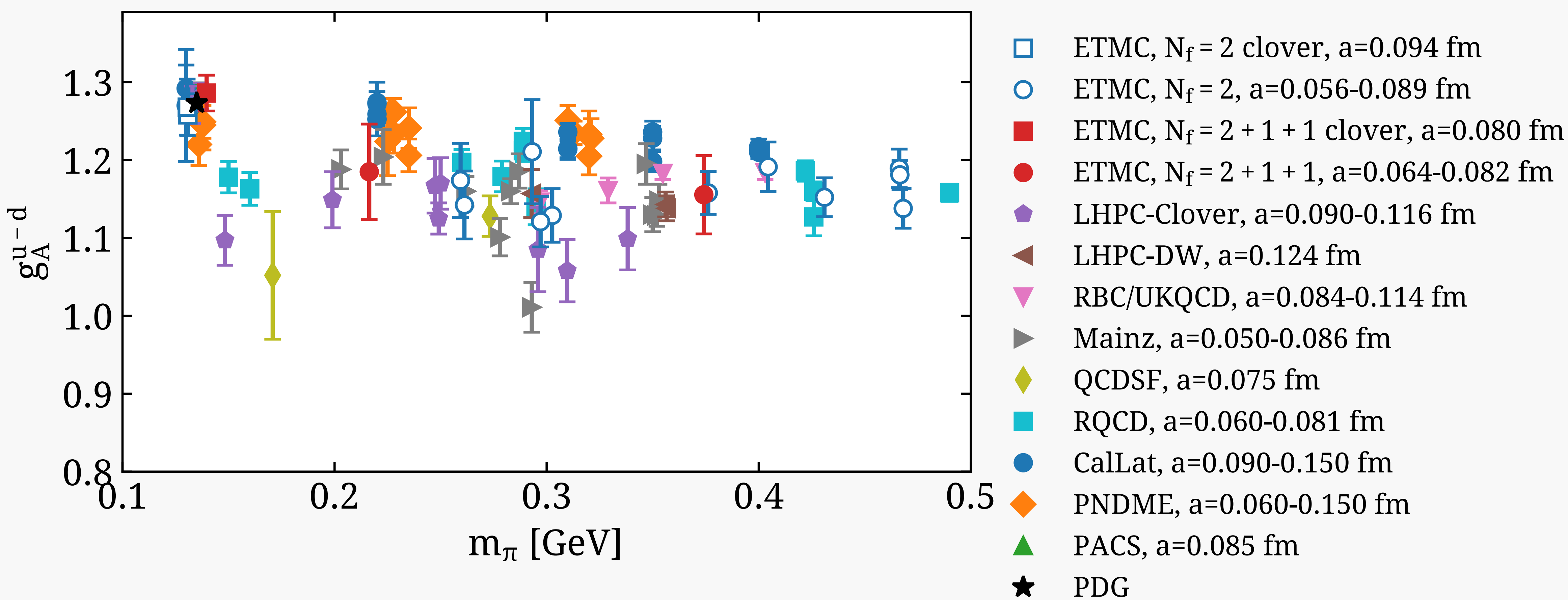

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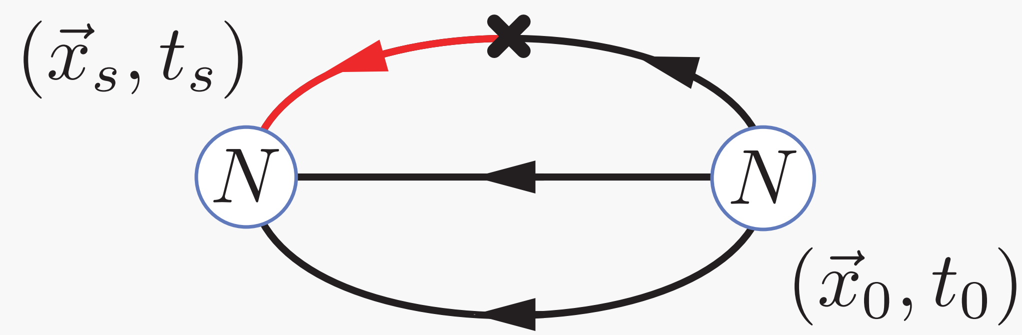

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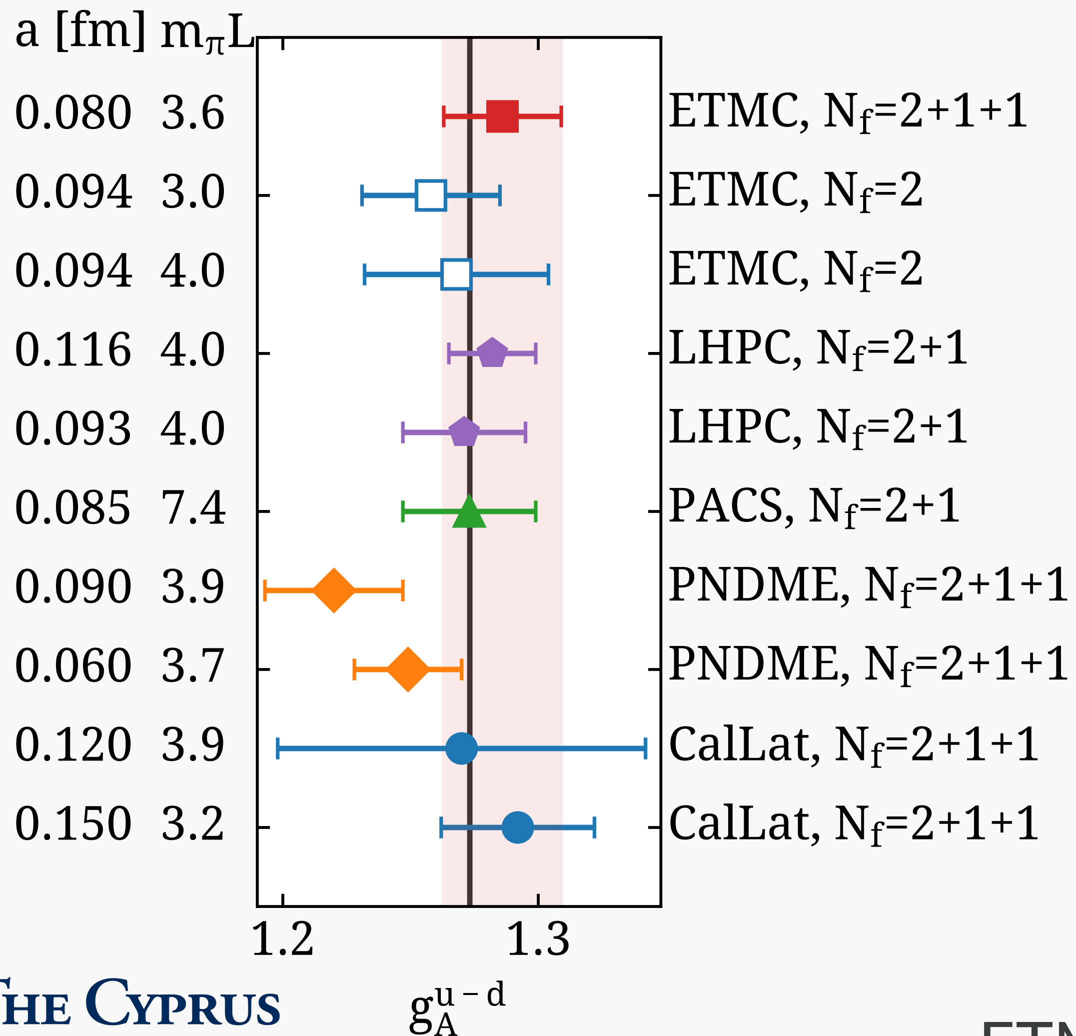


Lattice status

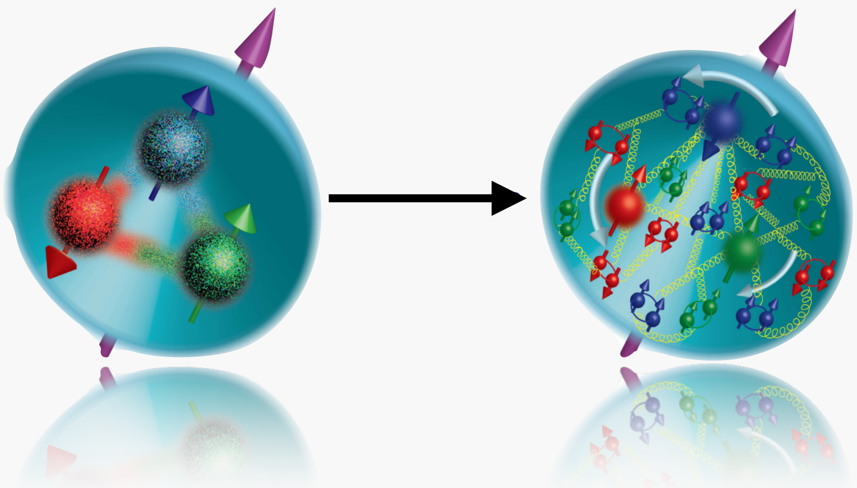
- Range of lattice actions
- Range of volumes and lattice spacings
- Agreement with experiment:
  - Physical point simulations
  - Careful analysis of ground state dominance

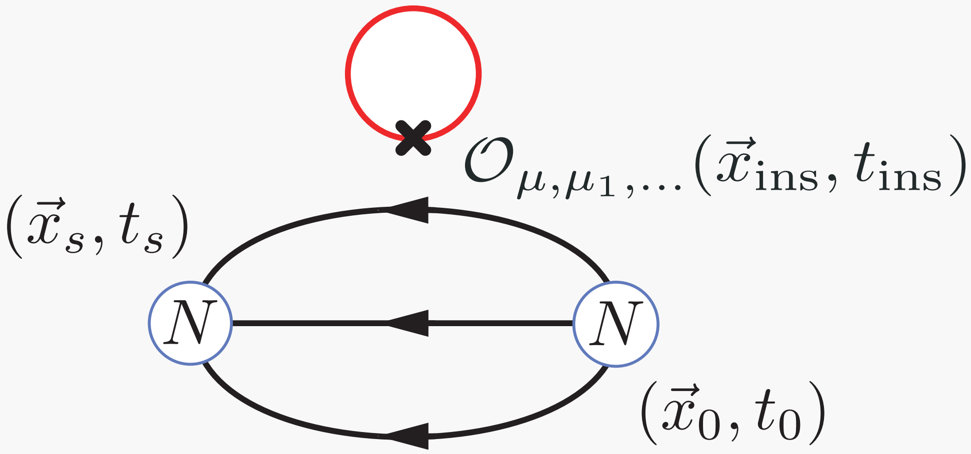
ETMC, arXiv:1909.00485. Poster by S. Bacchio

Nucleon spin

Quark intrinsic spin contributions to nucleon spin

$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{q=u,d,s,\ldots} g_A^q$$





Quark intrinsic spin contributions to nucleon spin

- Need linear combination of isovector (u-d) and isoscalar (u+d) contributions for individual up- and down-quarks
- Strange quark contribution is sea-quark contribution only (disconnected diagrams)
- Need O(10) - O(100) times more statistics

Nucleon spin

Quark intrinsic spin contributions to nucleon spin

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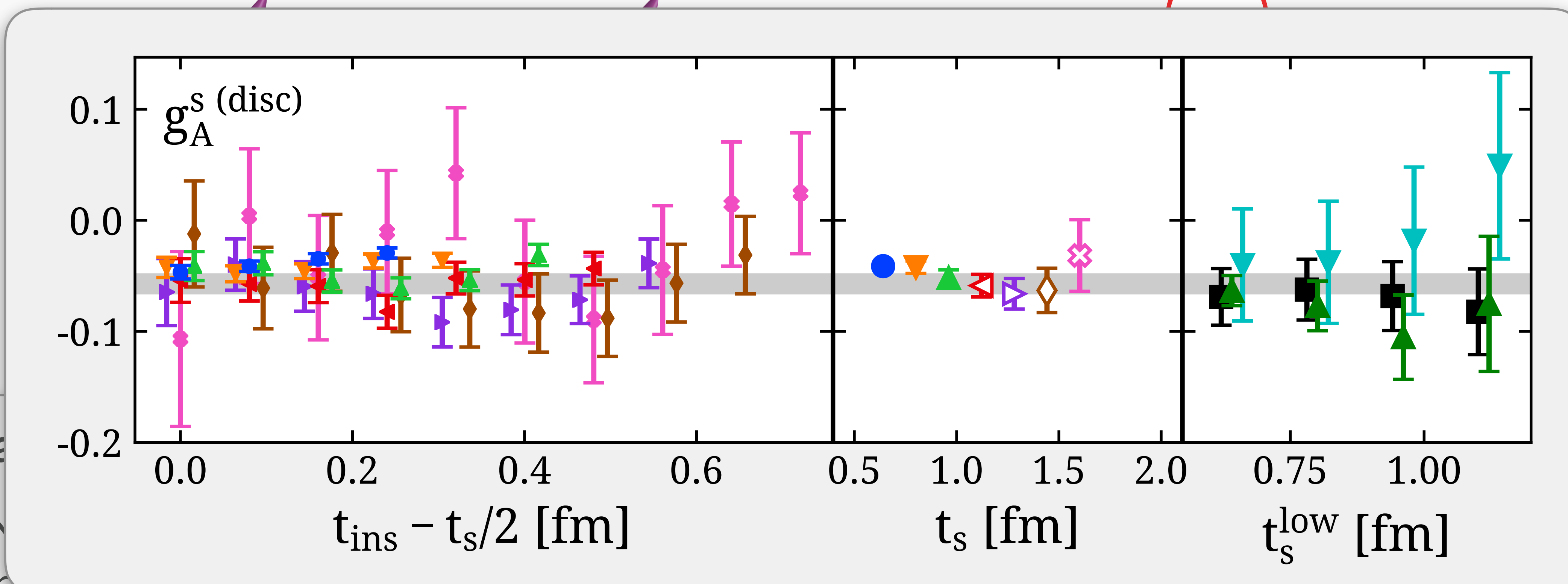

Qua

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  in…
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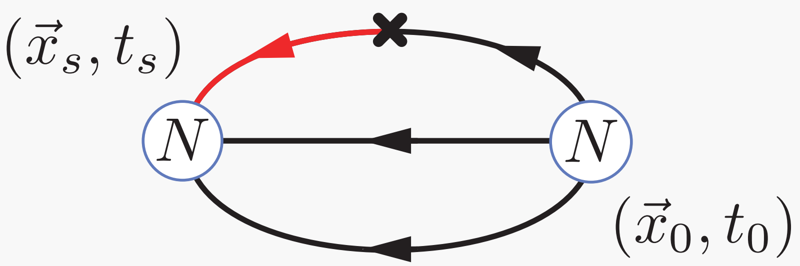
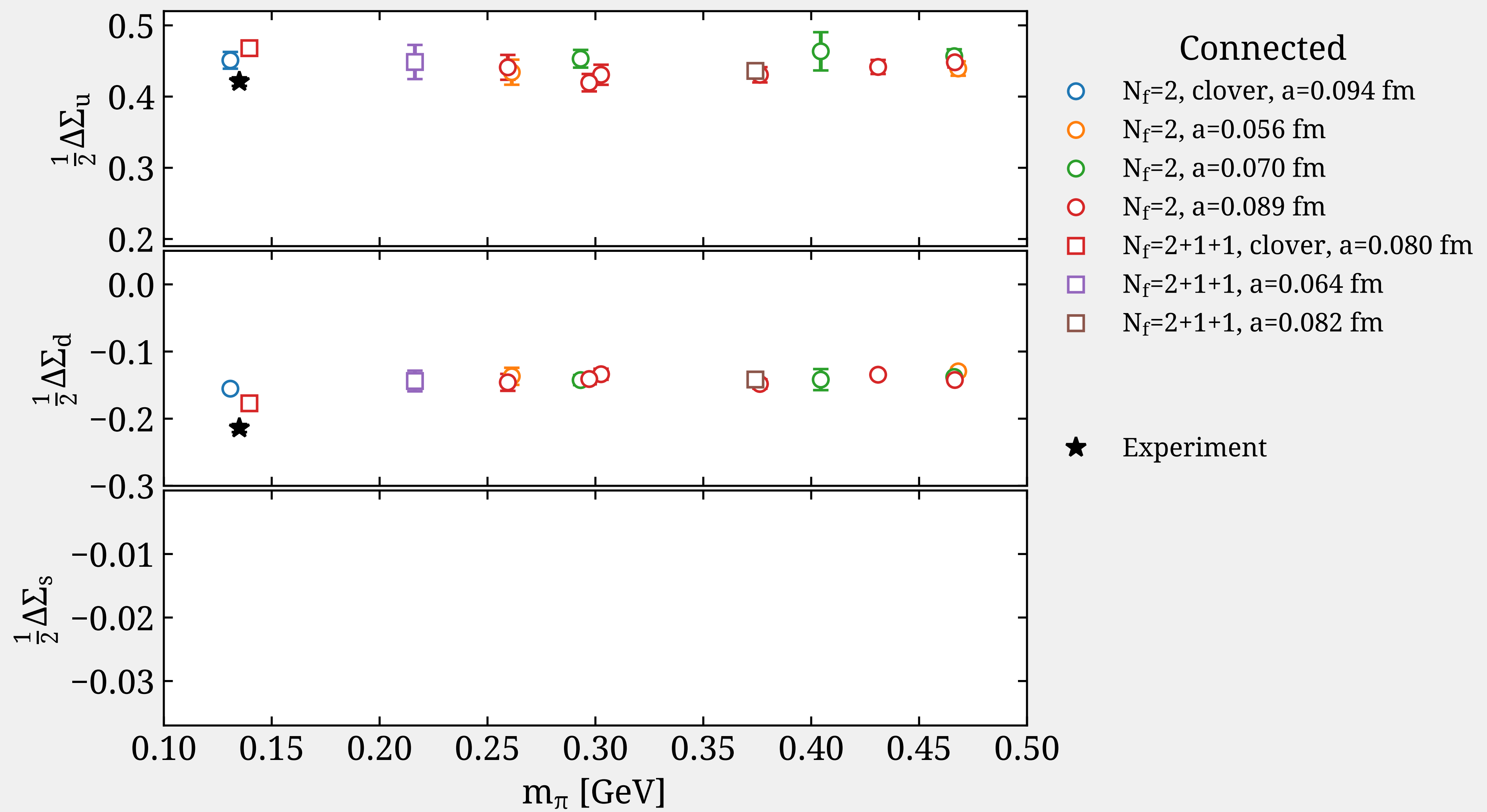
ETMC, arXiv:1909.00485. Poster by S. Bacchio

Quark intrinsic spin contributions

Quark intrinsic spin contributions to nucleon spin

- Mild cut-off effects
- Strange and down-quark contributions negative





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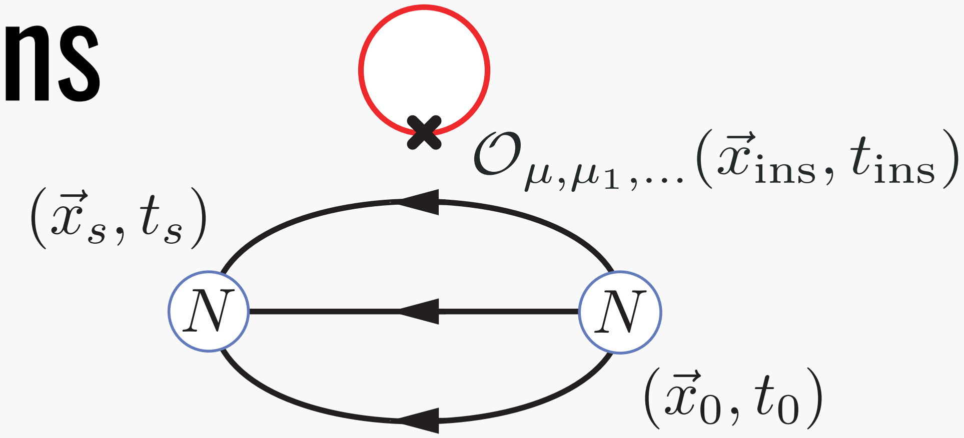
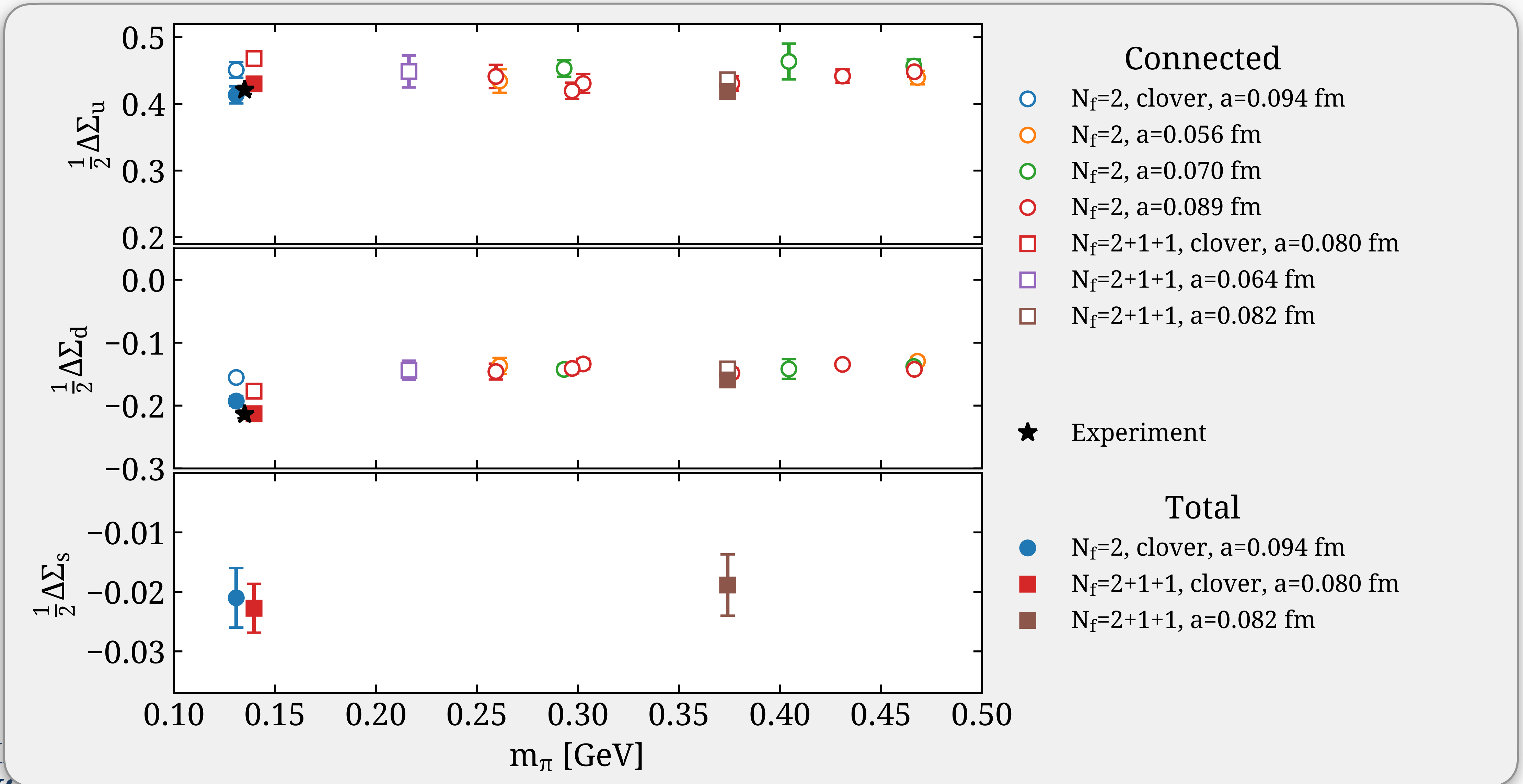
Quark intrinsic spin contributions



Quark intrinsic spin contributions to nucleon spin

- Mild cut-off effects
- Strange and down-quark contributions negative
- Disconnected contributions: agreement with experiment



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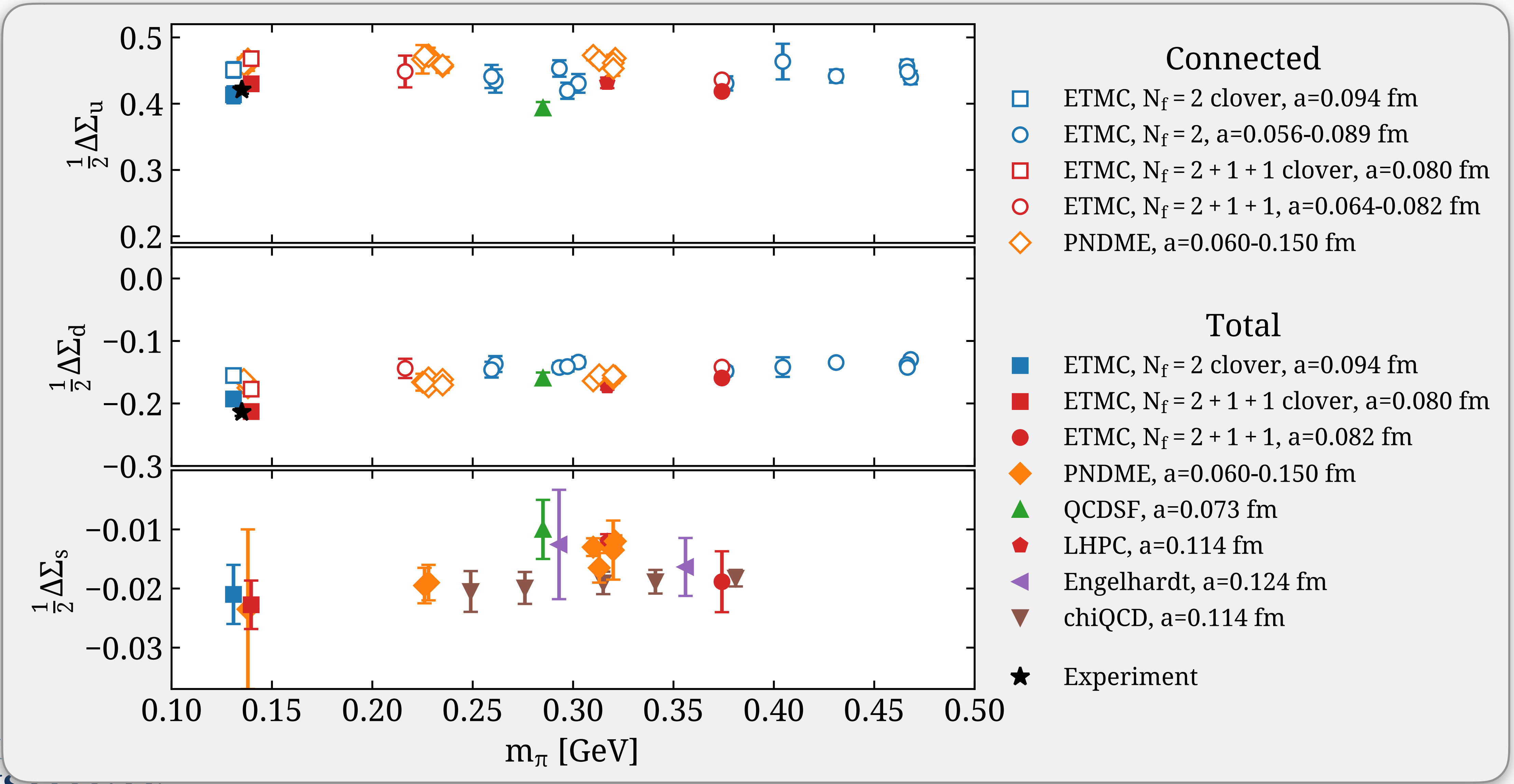
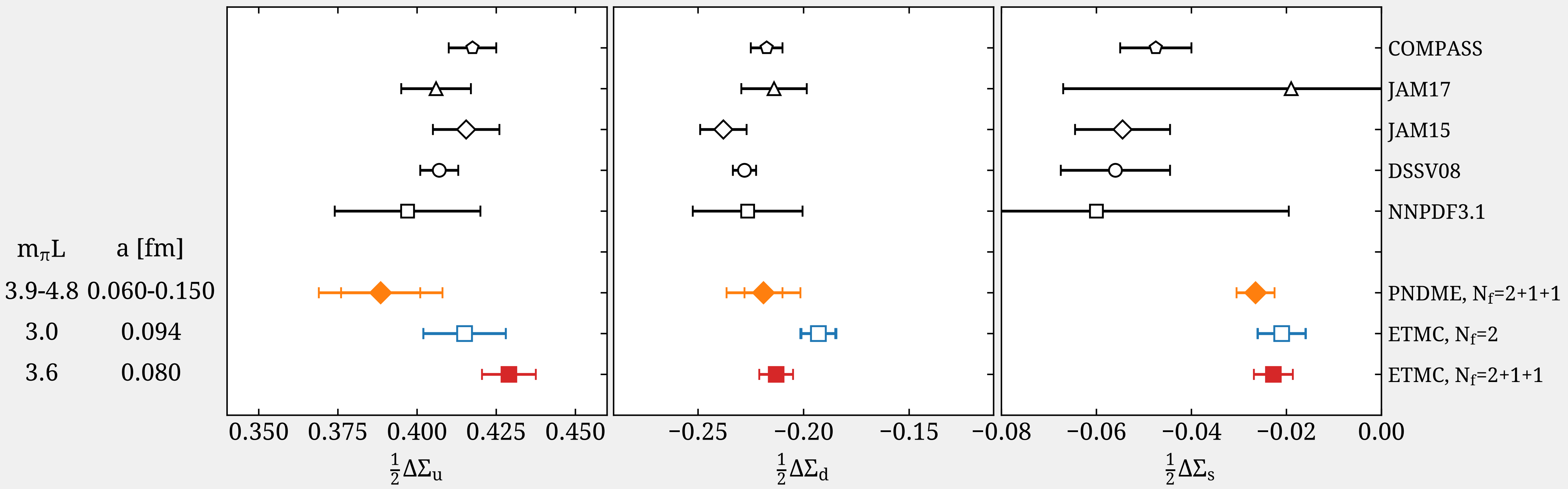


Quark intrinsic spin contributions

Quark intrinsic spin contributions to nucleon spin

- Individual up-, down-, and strange-quark intrinsic spin contributions to nucleon spin
- Lattice comparison to experiment



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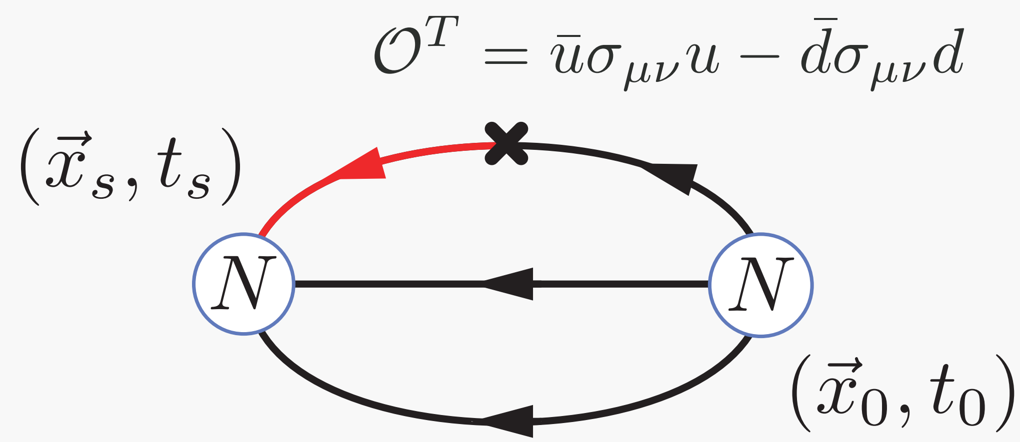
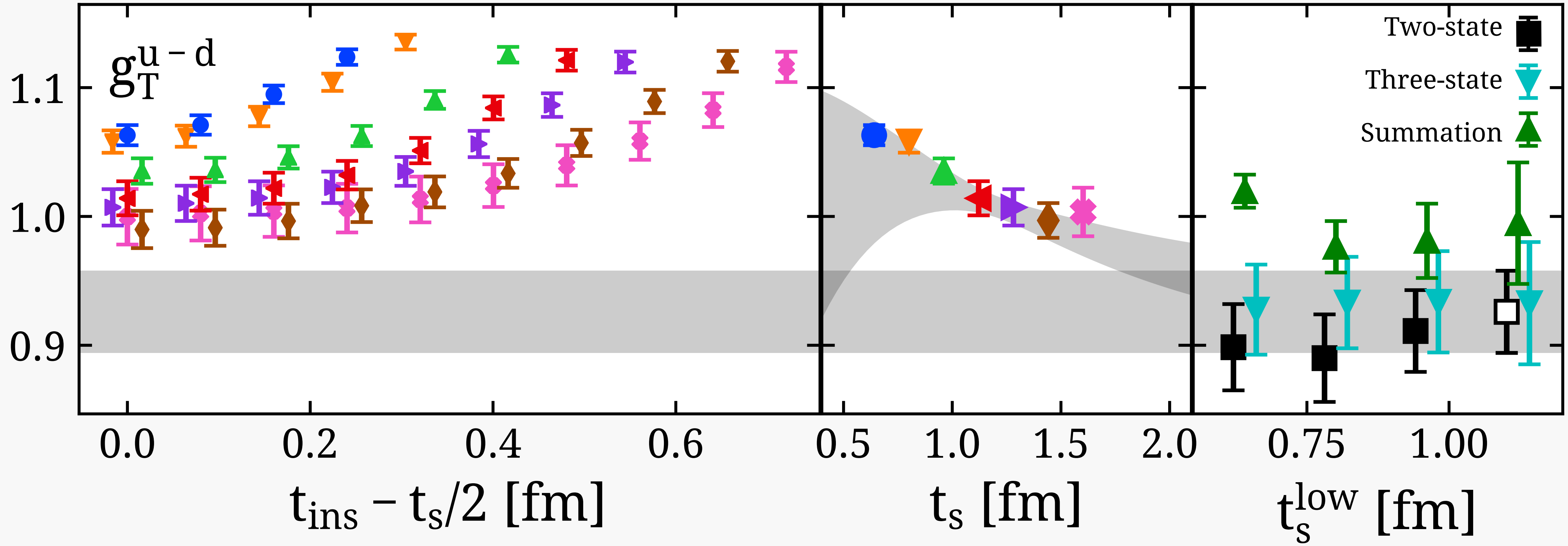
ETMC, arXiv:1909.00485. Poster by S. Bacchio

Tensor charge

Isovector tensor charge

- Slower convergence to ground state compared to g_A

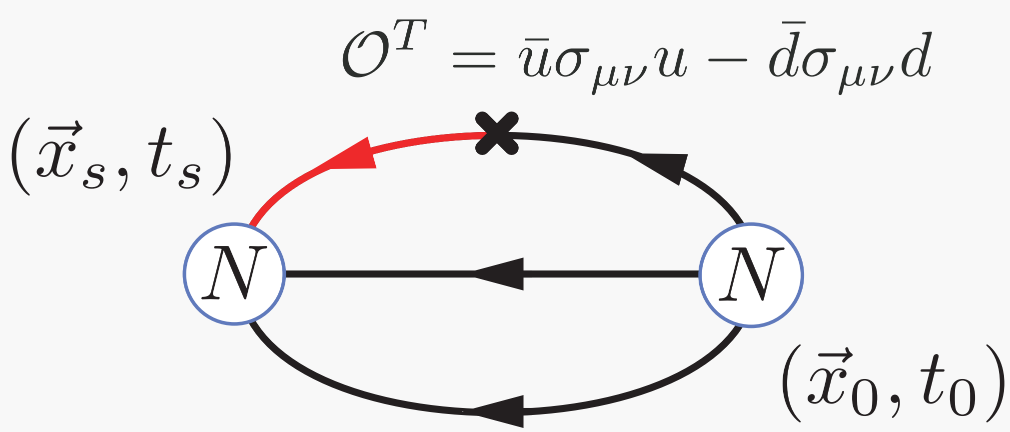


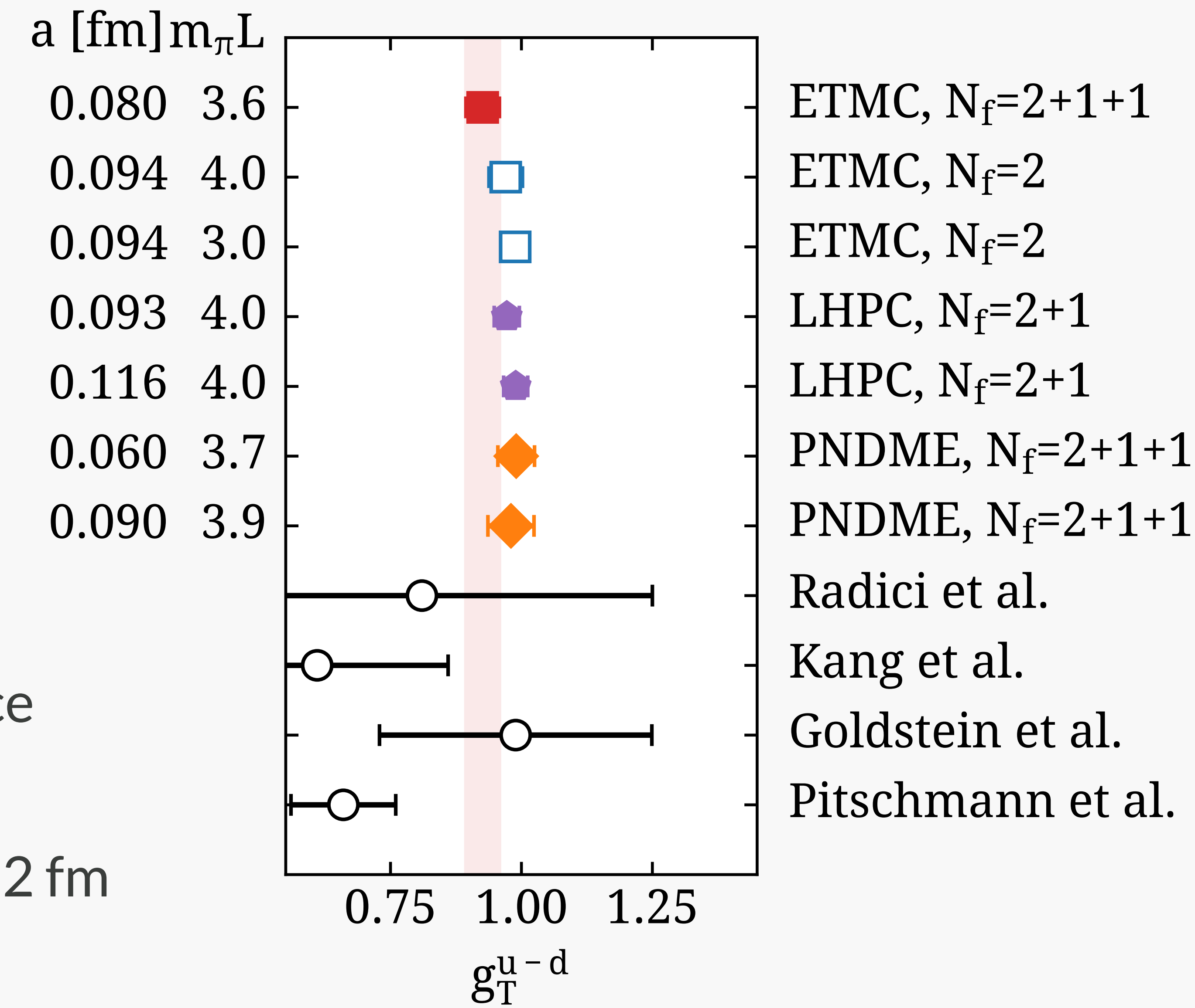


ETMC, arXiv:1909.00485. Poster by S. Bacchio

Tensor charge

Isovector tensor charge



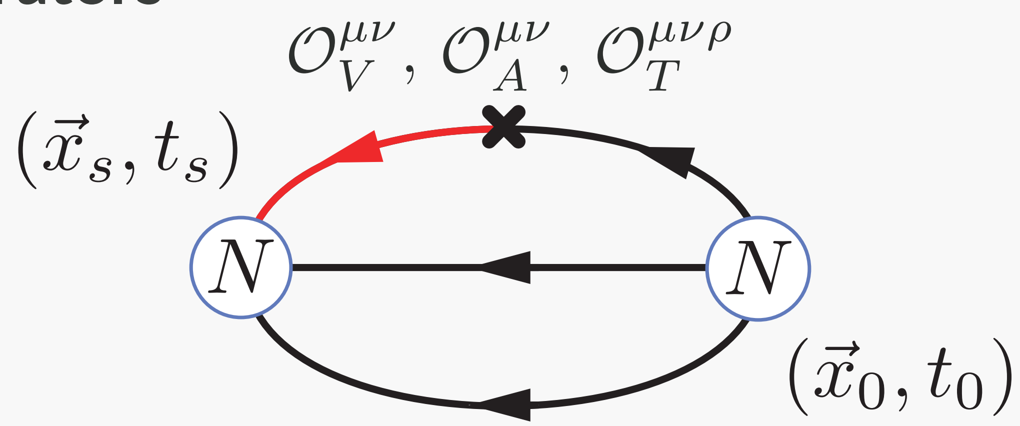


- Precise results from multiple lattice
- Volumes between L=4.4 to 5.8 fm
- Lattice spacing spanning 0.06 - 0.12 fm

ETMC, arXiv:1909.00485. Poster by S. Bacchio

Moments of PDFs

Isovector matrix element of 1st-derivative operators



- Unpolarized (momentum fraction)

$$\mathcal{O}_V^{\mu\nu} = \bar{\psi}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi, \; \Pi_V^{44}(\Gamma_0) = -\frac{3m_N}{4}\langle x\rangle_{u-d}$$

- Helicity

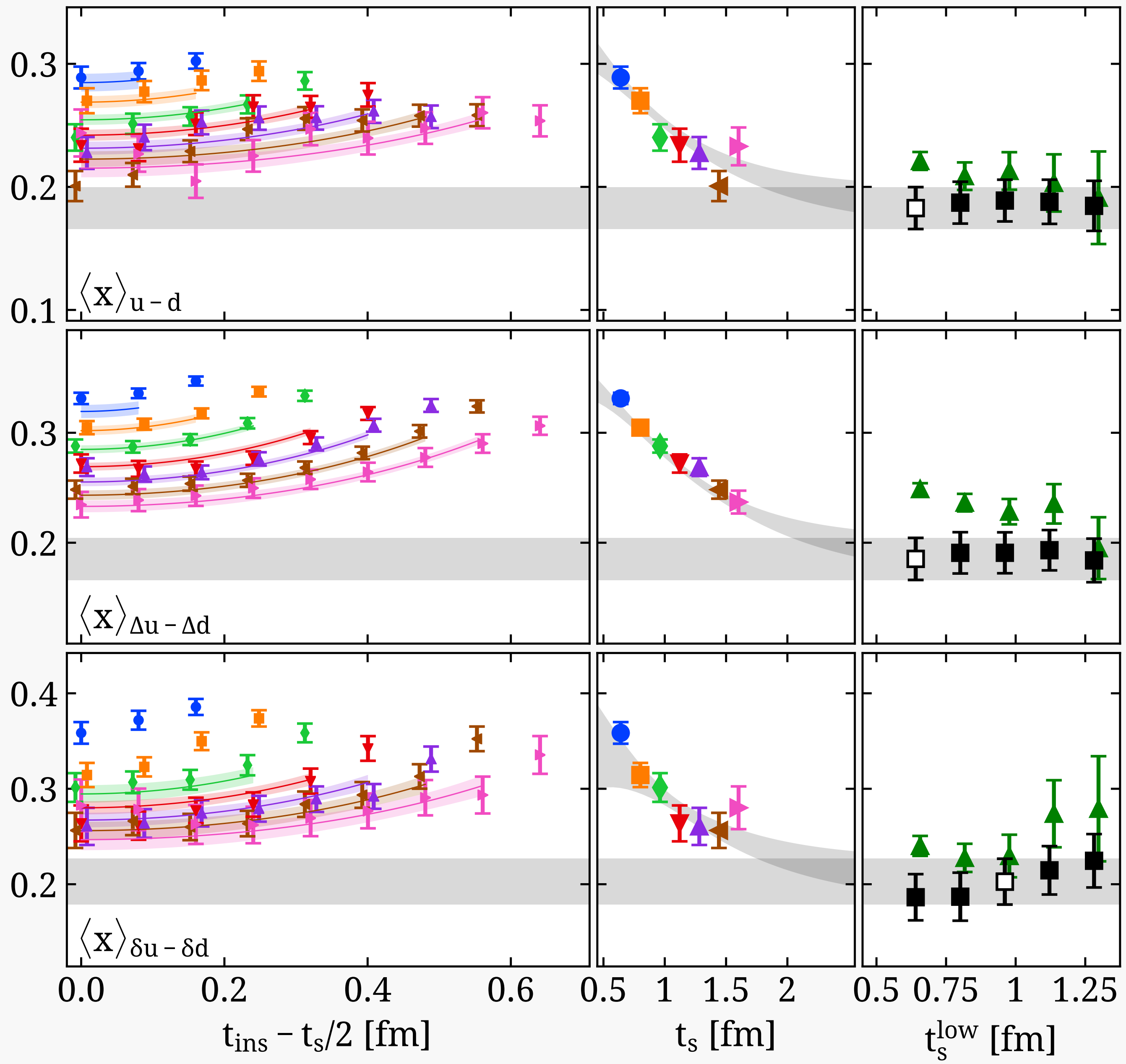
$$\mathcal{O}_A^{\mu\nu} = \bar{\psi}\gamma_5\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi, \; \Pi_A^{j4}(\Gamma_k) = -\frac{im_N}{2}\delta_{jk}\langle x\rangle_{\Delta u-\Delta d}$$

- Transversity

$$\mathcal{O}_T^{\mu\nu\rho} = \bar{\psi}\sigma^{[\mu\{\nu]}\overleftrightarrow{D}^{\rho\}}\psi, \; \Pi_T^{\mu\nu\rho}(\Gamma_k) = i\epsilon_{\mu\nu\rho k}\frac{m_N}{8}(2\delta_{4\rho} - \delta_{4\mu} - \delta_{4\nu})\langle x\rangle_{\delta u-\delta d}$$

Moments of PDFs

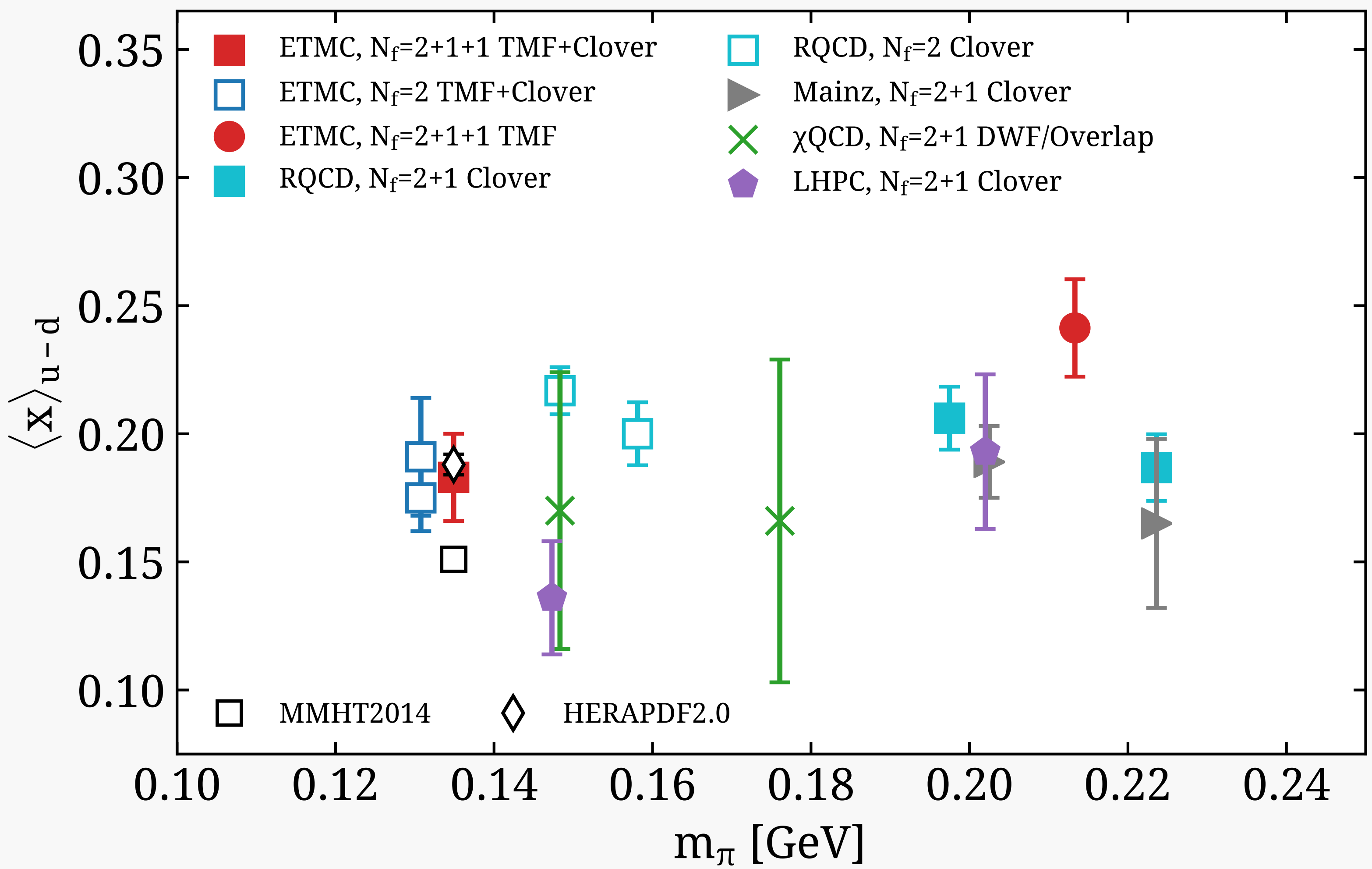
Isovector matrix element of 1st-derivative operators



ETMC, arXiv:1908.10706

Moments of PDFs

Isovector matrix element of 1st-derivative operators



- New results directly at the physical point
- Physical point results within phenomenological spread

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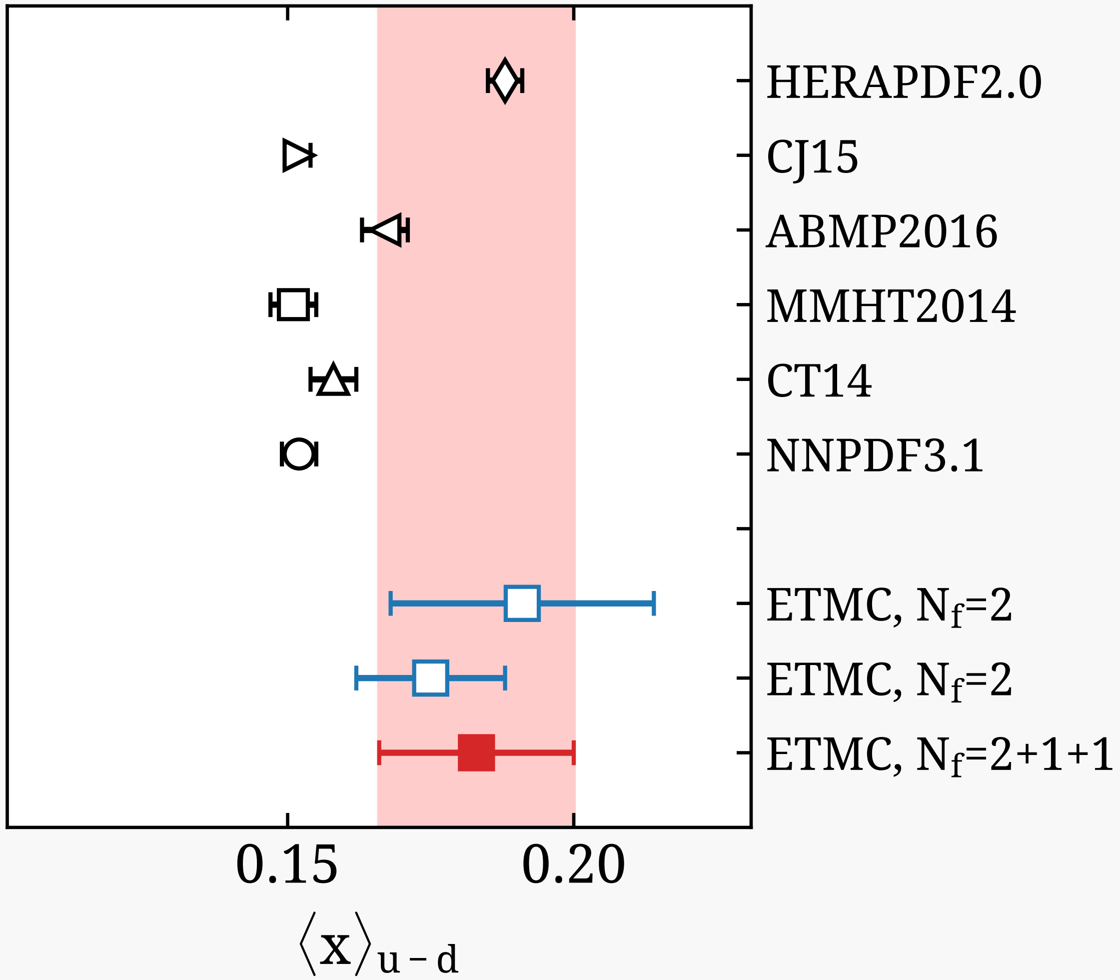
ETMC, arXiv:1908.10706

Moments of PDFs

Isovector matrix element of 1st-derivative operators



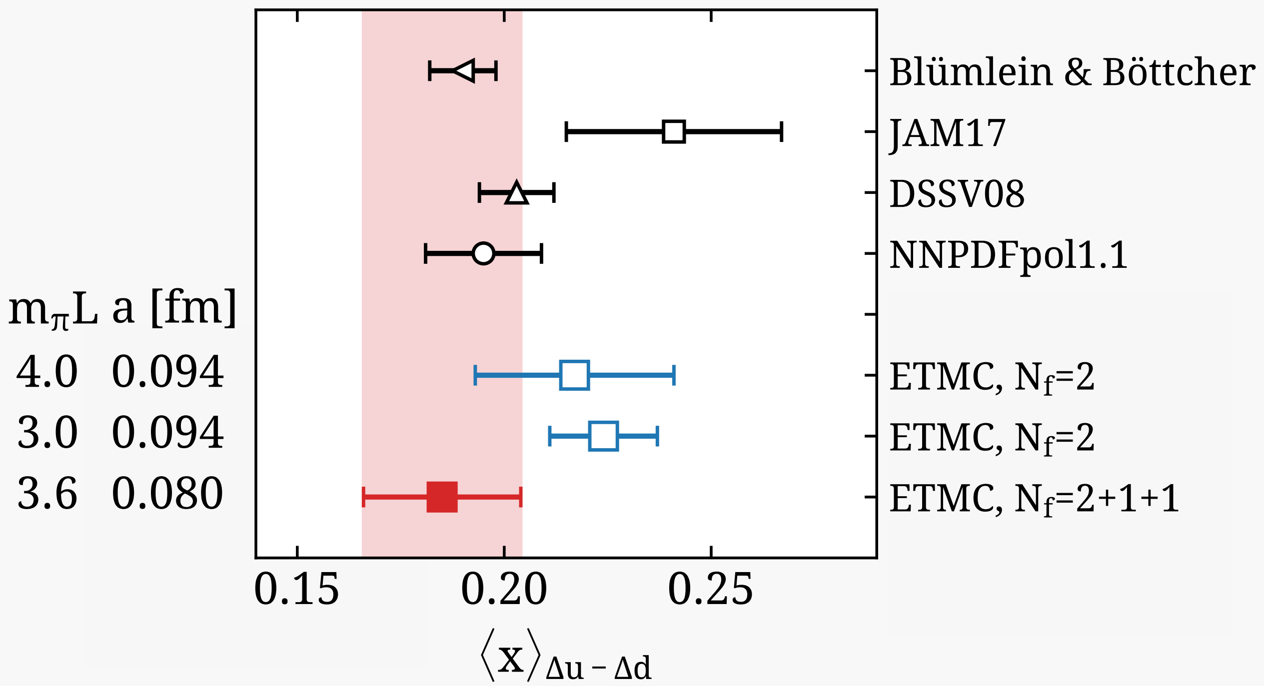
$m_\pi L$	a [fm]
4.0	0.094
3.0	0.094
3.6	0.080

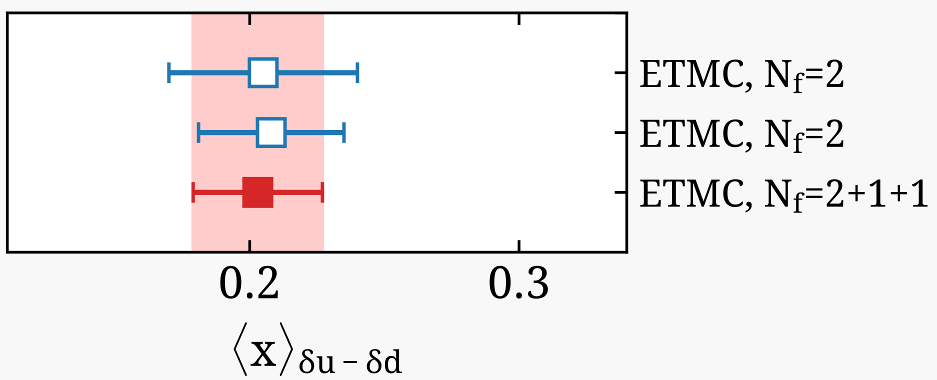
- Check of volume effects
- Slightly higher value compared to most phenomenological extractions

ETMC, arXiv:1908.10706

Moments of PDFs

Isovector matrix element of 1st-derivative operators





- Not shown: One other lattice study at 150 MeV: RQCD, arXiv:1812.08256, PRD
  - Extracted from single separation ~1.1 fm
  - Consistent with our plateau at similar separation

ETMC, arXiv:1908.10706

Nucleon Generalized Form-Factors

Matrix element:

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{V,A}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[...\Big]u_N(p,s)$$

Three vector and two axial GFFs:

$$\text{Vector}: A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}$$

$$\text{Axial}: \tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5$$

Ji spin sum: $J^q = \frac{1}{2}[A^q_{20}(0) + B^q_{20}(0)]$

$A^{u-d}_{20}(0) = \langle x\rangle_{u-d}$: directly calculated at Q^2=0

$B^{u-d}_{20}(0)$: need to model $B^{u-d}_{20}(Q^2)$ and take $Q^2 \longrightarrow 0$

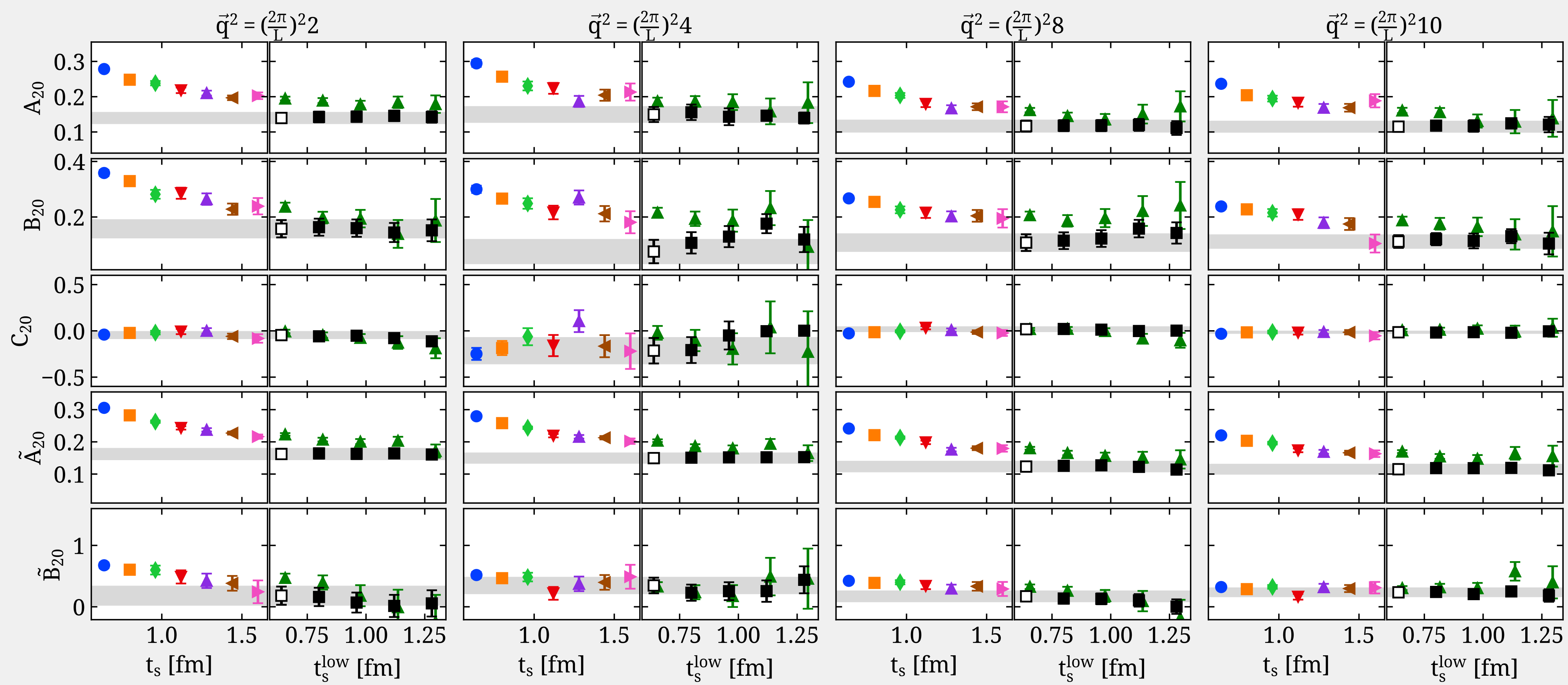
Nucleon Generalized Form-Factors

Solve via SVD of matrix of kinematics:

$$\Pi^{\mu\nu}(\Gamma;\vec{q}) = \mathcal{G}^{\mu\nu}(\Gamma;\vec{q})F(Q^2)$$

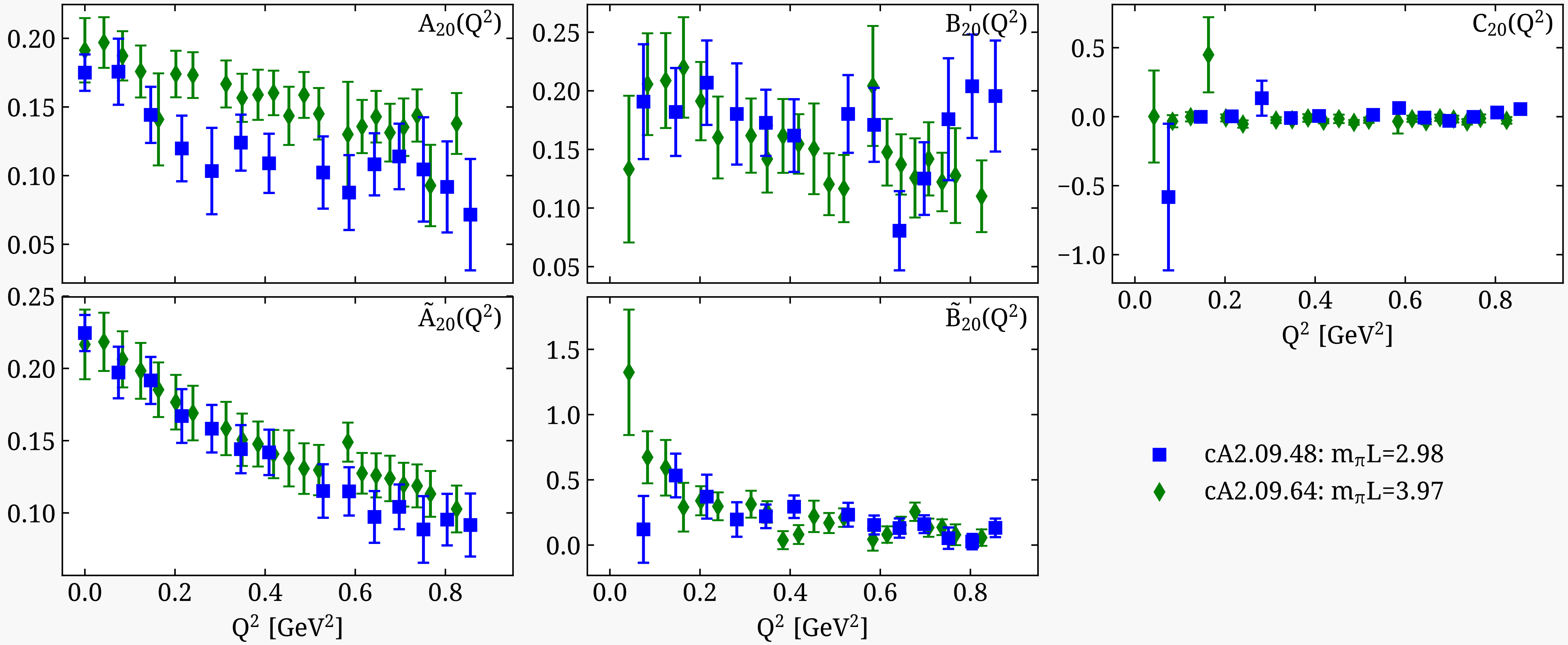
$\boldsymbol{\Pi}$ lattice measurements, $\mathcal{G}$ kinematics, $\boldsymbol{F}$ vector of GFFs

Excited state analysis as charges but for each Q^2 separately



ETMC, arXiv:1908.10706

Nucleon Generalized Form-Factors



GFFs finite volume effect study using N_f=2 twisted mass

- No detectable effects from the init volume between $m_\pi L \simeq 3$ and 4

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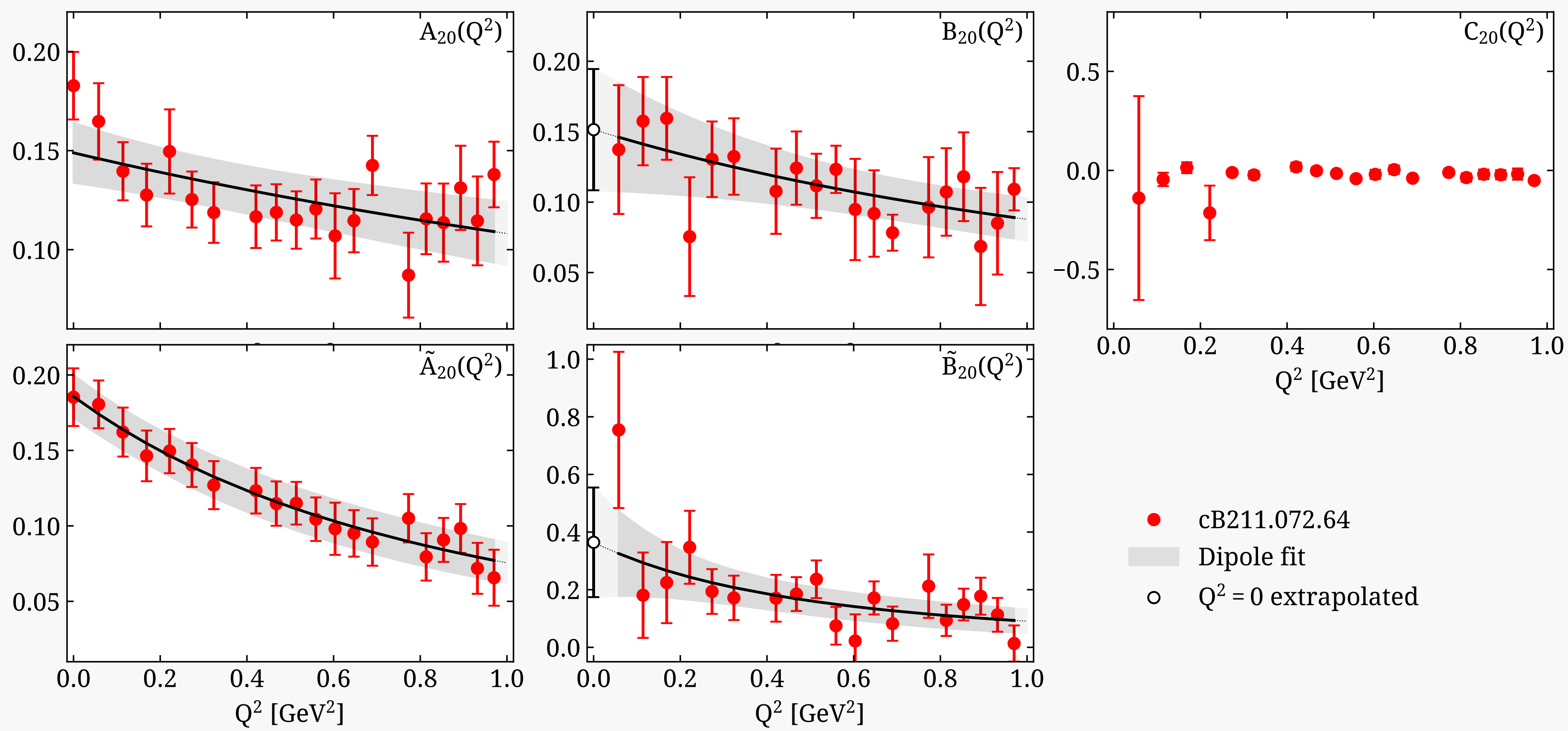
ETMC, arXiv:1908.10706

Nucleon Generalized Form-Factors



N_f=2+1+1 at physical pion mass

- Dipole fits model well B_{20} and $\tilde{A}_{20}$
- Tripole fits also model well B_{20} and B_{20}

$$J^{u-d} = \frac{1}{2}[A_{20}^{u-d}(0) + B_{20}^{u-d}(0)] = 0.167(24)(04)$$

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ETMC, arXiv:1908.10706

Summary

- Nucleon moments of PDFs
  - Local matrix elements of nucleon
  - Physical pion masses available by multiple collaborations
  - Disconnected contributions for individual $u,\ d,\ s,\ c$ contributions
  - Analyses focusing on ensuring ground-state dominance
- Gluonic contributions also available (not covered here)
- Results available at physical quark masses:
  - Nucleon charges and moments of PDFs comparable to experiment
  - Flavor separation thanks to techniques for disconnected contributions
  - Momentum-dependence of moments (FFs and GFFs)
- Ongoing:
  - Finer lattices and larger volumes

Contributors

ETM Collaboration

Extended Twisted Mass Collaboration

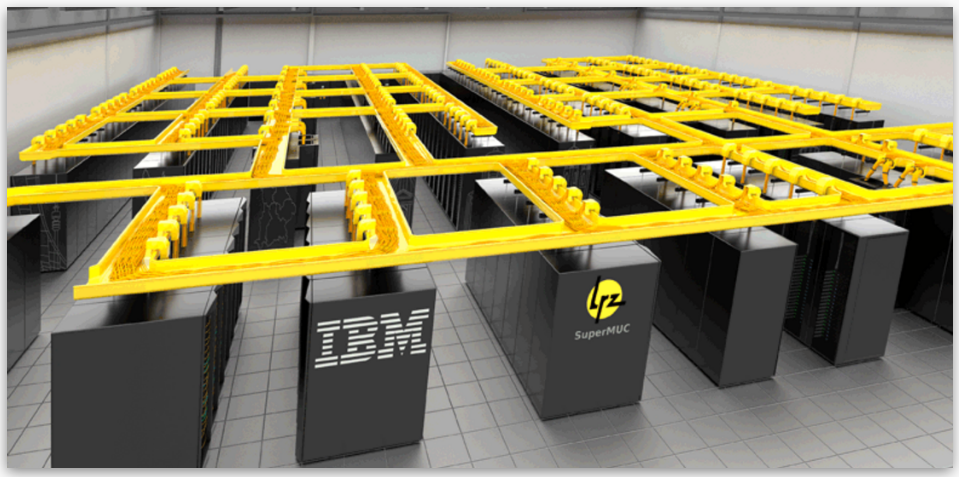
Nucleon Structure

C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, C. Lauer, A. Scapellato, F. Steffens, A. Vaquero

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Jena, Münster), Italy (Rome I, II, III), Netherlands (Groningen), Poland (Poznan), Switzerland (Bern), UK (Liverpool), US (Temple, PA)

Acknowledgements

Gauge Field generation:
SuperMUC @ LRZ



- tmLQCD: `github.com/etmc/tmLQCD`
- DDalphaAMG adapted to twisted mass: `github.com/sbacchio/DDalphaAMG`

Propagators & Contractions:
Piz Daint @ CSCS

- QUDA Multi-Grid
- Custom contraction code: `github.com/ETMC-QUDA/quda-QKXTM-Multigrid-PlugIn`

NextQCD, EXCELLENCE/1918/0129

2016-2020 RESTART SEARCH

Κυπριακή Δημοκρατία

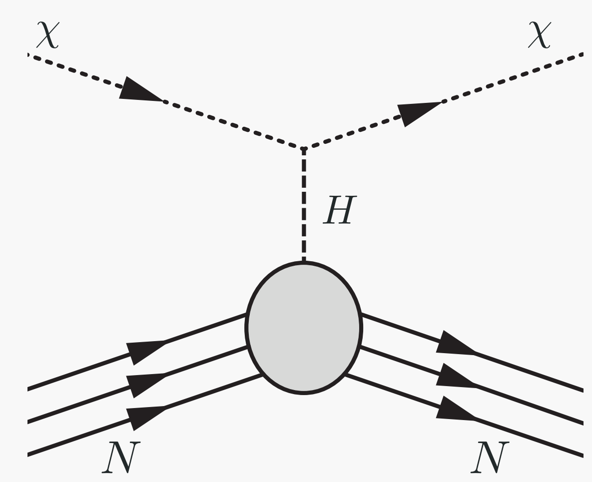
Διαρθρωτικά Ταμεία της Ευρωπαϊκής Ένωσης στην Κύπρο

Ευρωπαϊκή Ένωση
Ευρωπαϊκά Διαρθρωτικά και Επενδυτικά Ταμεία

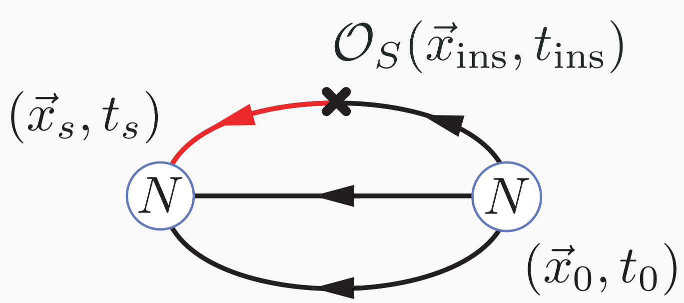
Backup

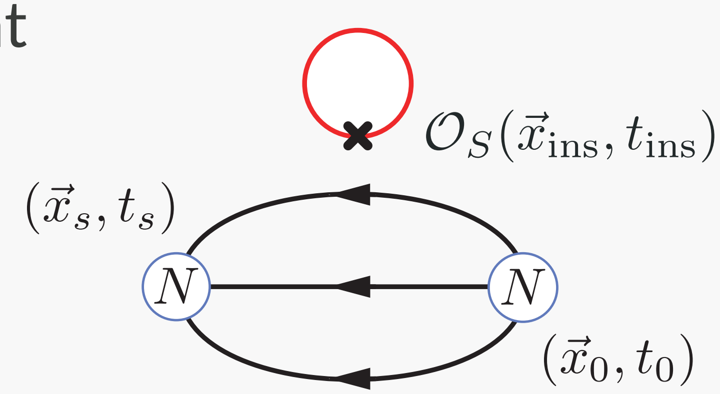
Nucleon σ-terms

- Pion nucleon σ-term: $\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$
- Strange σ-term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
- Scattering cross sections of scalars with nucleon (e.g. neutralino through Higgs)



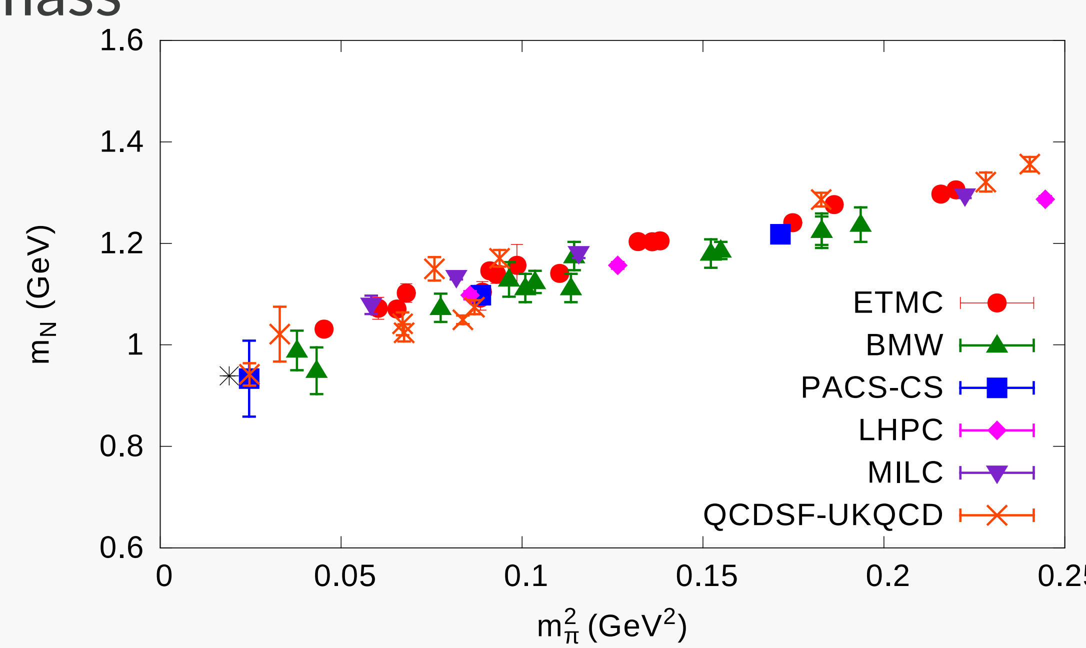
Direct: through matrix element





Feynman - Hellmann: dependence on quark mass

- Requires modelling of m_q dependence
- Weak m_s dependence



Poster by W. Soeldner, spectrum and σ-terms from QCDSF

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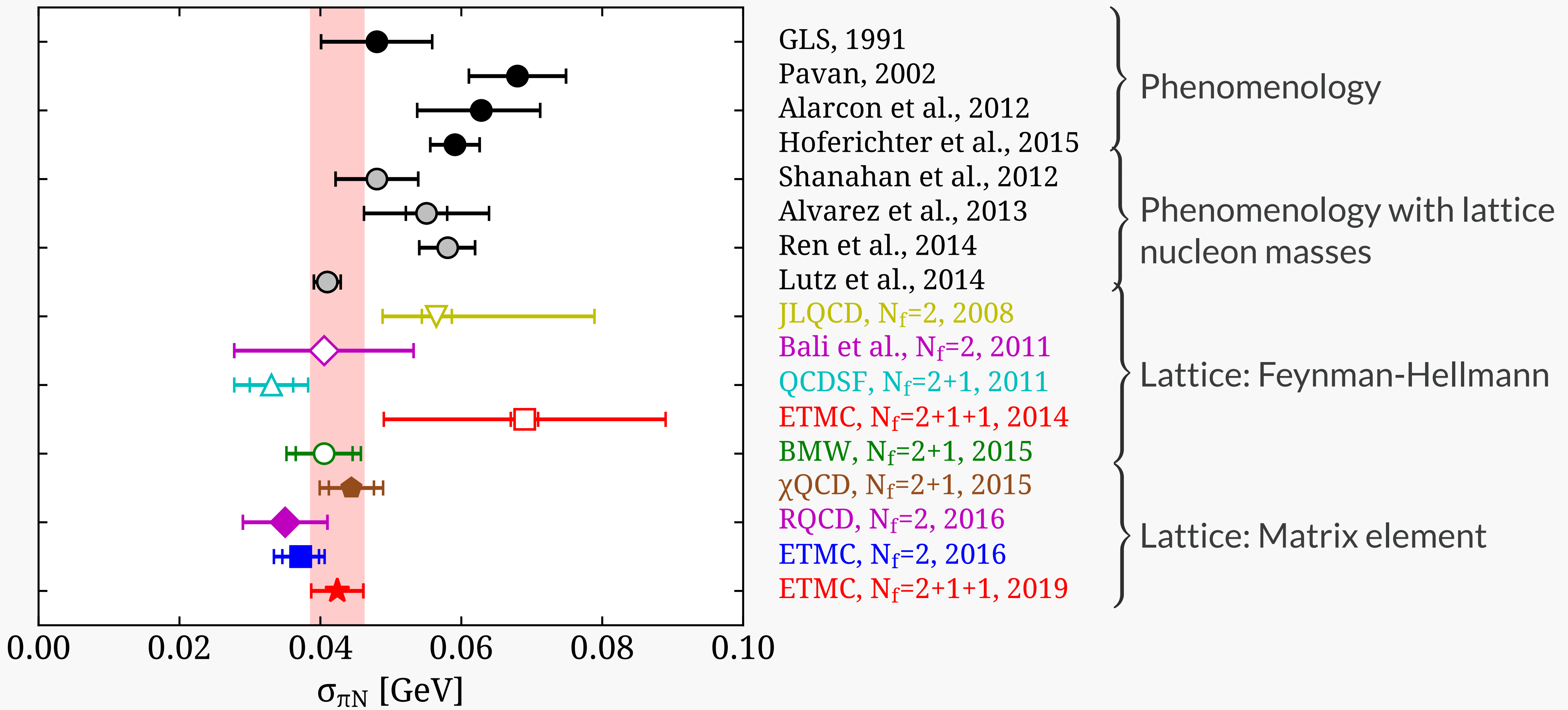
Nucleon σ-terms

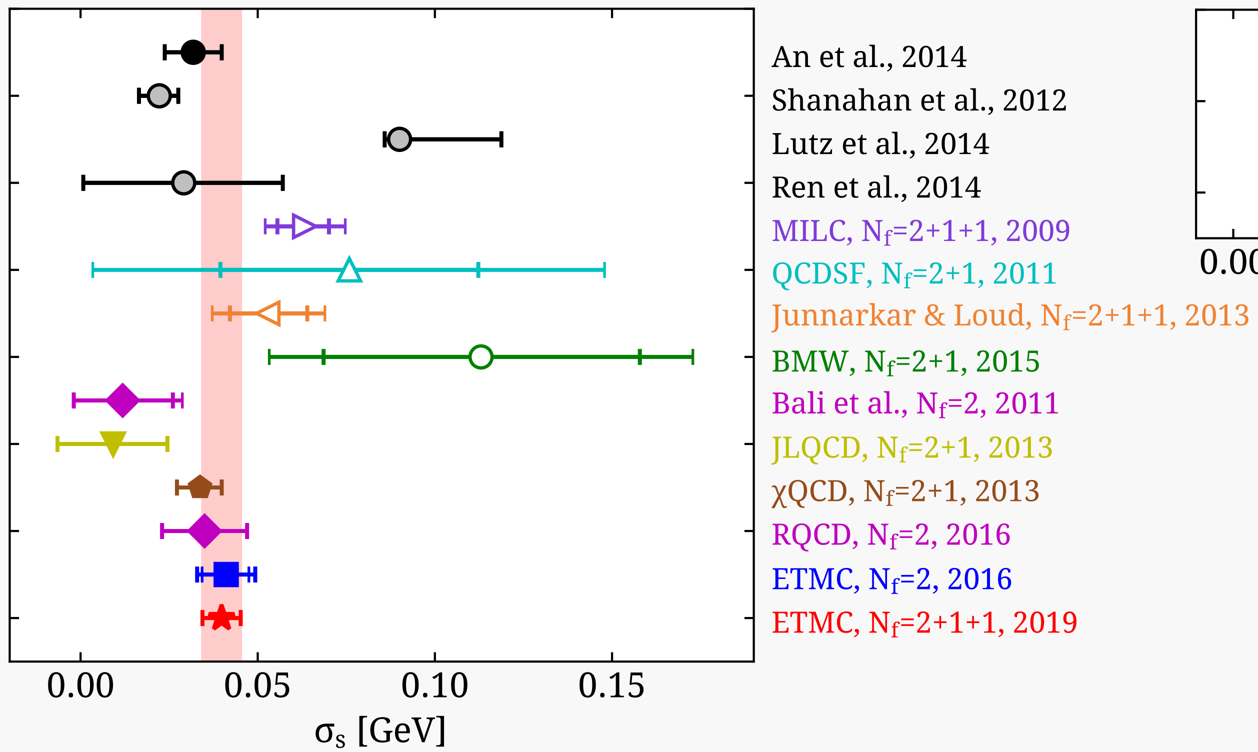
Light σ-term $\sigma_{\pi N}$

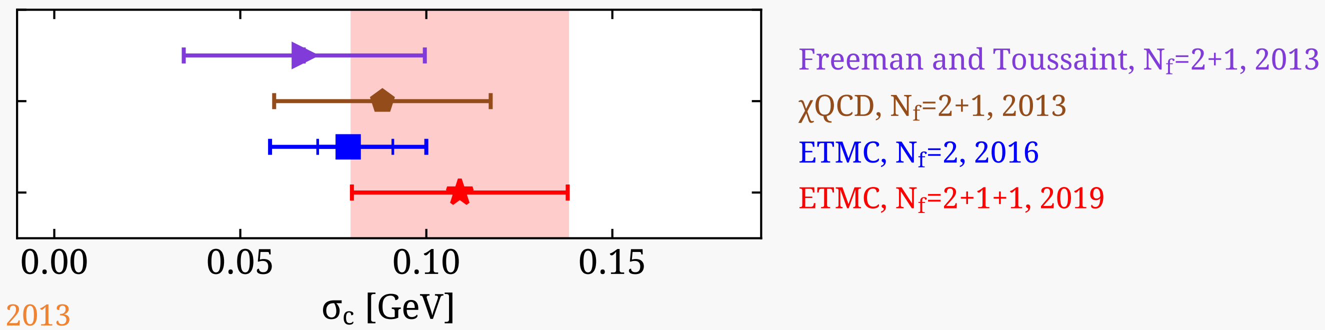


- Phenomenology suggests larger value for $\sigma_{\pi N}$
- Proposal to compute π-N scattering lengths on the lattice

Nucleon σ-terms

Good signal also for strange and charm σ-terms





- General consistency between lattice results and experimental determinations

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Nucleon Electromagnetic Form-Factors

Matrix element:

$$\langle N(p',s')|j^\mu|N(p,s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\bar{u}(p',s')\mathcal{O}^\mu u(p,s)$$

$$\mathcal{O}^\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N}F_2(q^2), \quad q = p' - p$$

Sachs form-factors:

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2}F_2(q^2)$$

Isovector & Isoscalar combinations:

$$j_\mu^v = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d, \; j_\mu^s = \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$$

$$F^p - F^n = F^u - F^d$$

$$F^p + F^n = \frac{1}{3}(F^u + F^d)$$

Assuming flavour isospin symmetry

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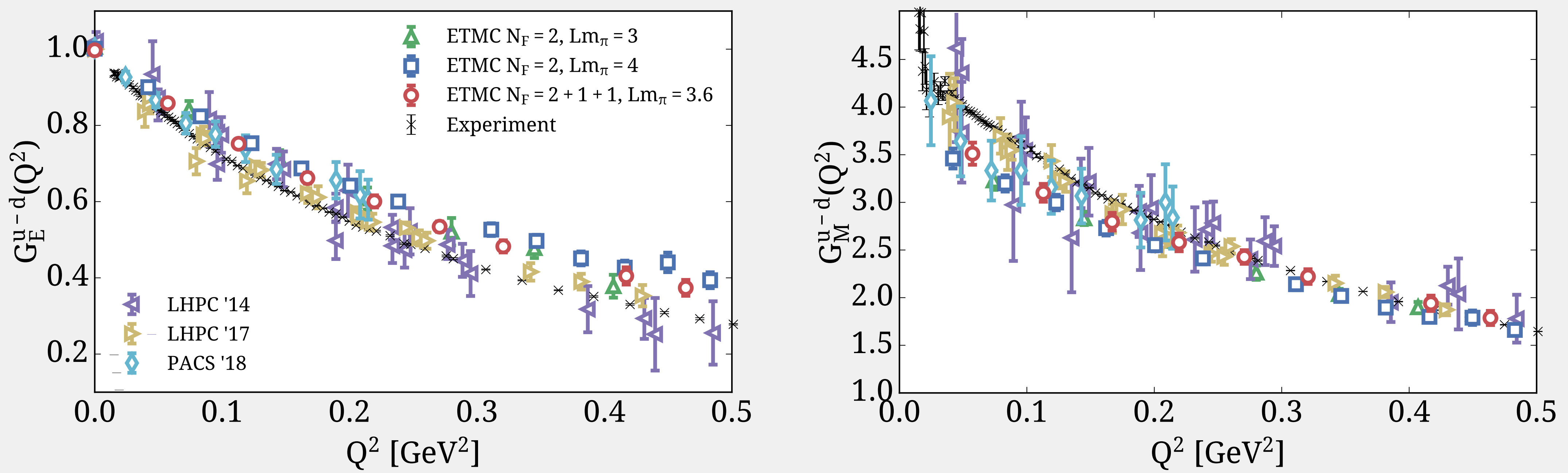
Nucleon Electromagnetic Form-Factors



Isovector EM form-factors

- Multiple groups with physical pion mass
- G_E: slight tension at high Q^2
- G_M: slight tension at small Q^2

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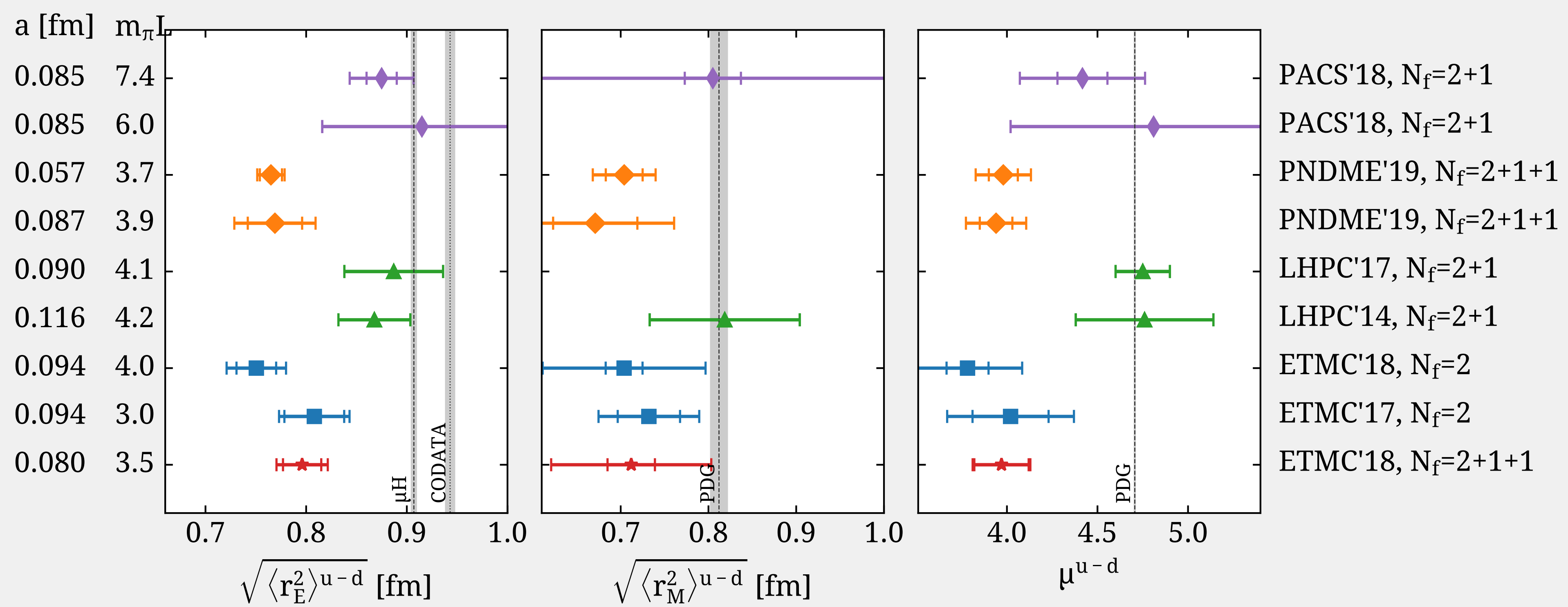
arXiv:1812.10311 PRD

Nucleon Electromagnetic Form-Factors



Isovector EM form-factors

- Estimation of all systematics still ongoing
- Continuum and volume extrapolations still needed

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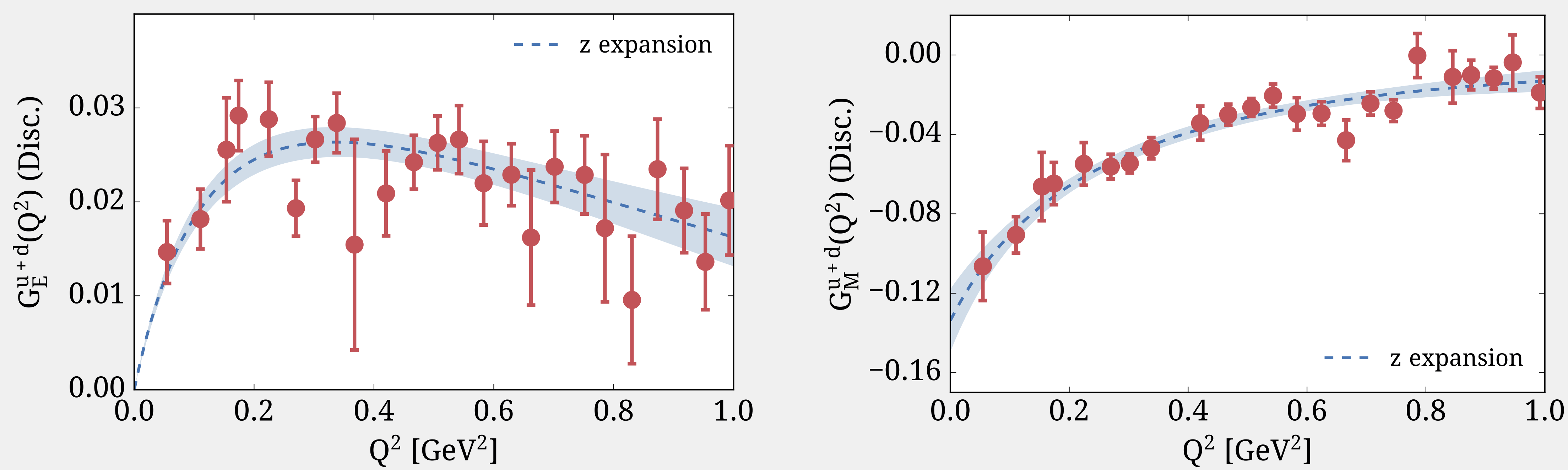
arXiv:1812.10311 PRD

Nucleon Electromagnetic Form-Factors

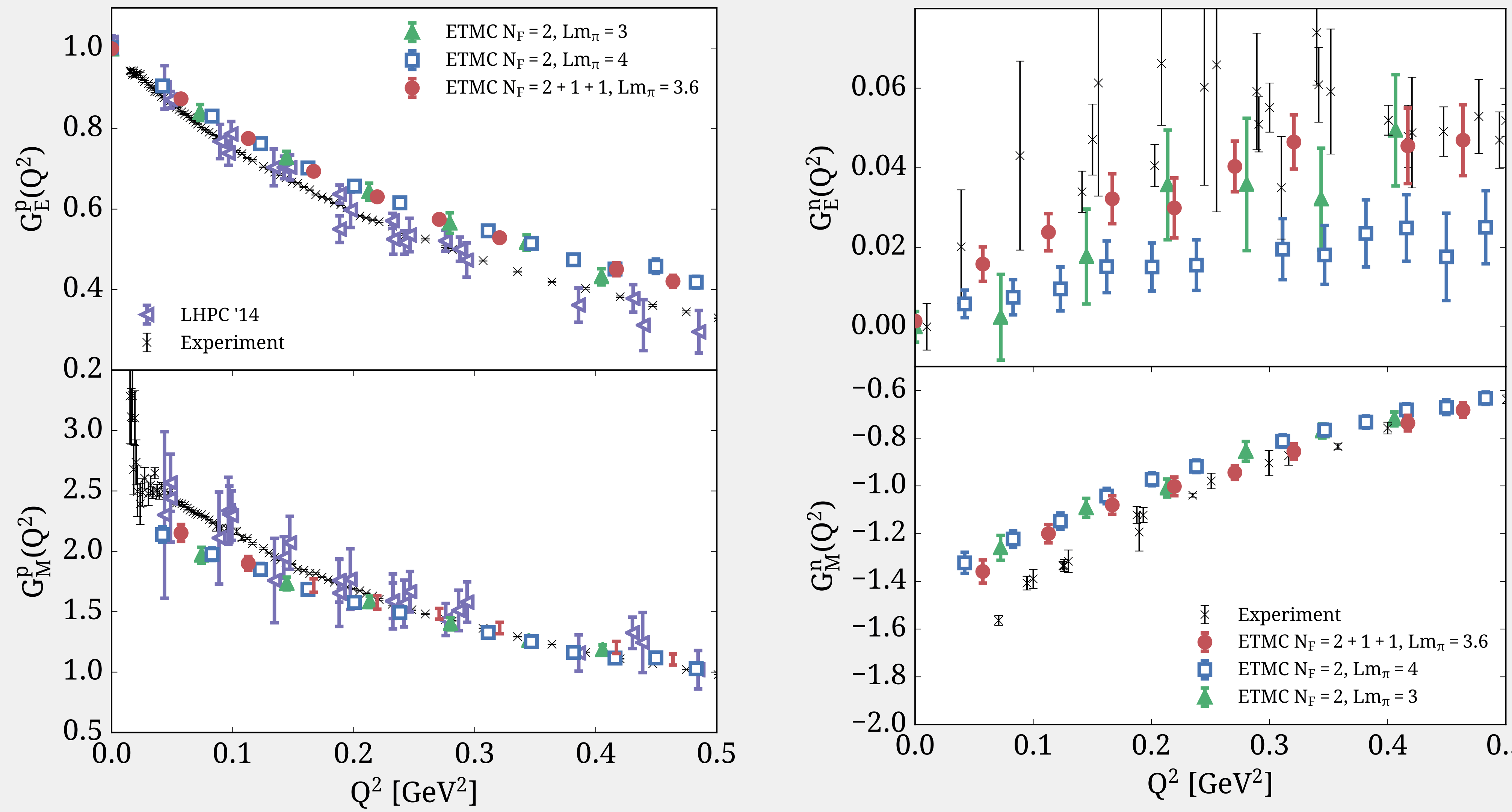


Proton/neutron separated: need disconnected diagrams

- Good signal thanks to hierarchical probing and use of boosted frames
- Small in magnitude (few percent level)

arXiv:1812.10311 PRD

Nucleon Electromagnetic Form-Factors



Proton/neutron separated EM form-factors

- Including disconnected; non-negligible effect for neutron G_E
- G_E: slight tension at high Q^2
- G_M: slight tension at small Q^2

arXiv:1812.10311 PRD

Axial Form Factors

★ **Axial and induced pseudo-scalar**

$$\langle N(p',s')|A^3_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}(p',s')[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q_\mu\gamma_5}{2m_N}G_p(q^2)]\frac{1}{2}u_N(p,s)$$

with:

$A^3_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x)$, and $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

- Known to less accuracy experimentally (e.g. compared to electromagnetic)
  - Via elastic scattering: $\nu_\mu + n \to \mu^- + p$
  - Via charged pion electroproduction
- Required in neutrino oscillation experiments. Traditionally modelled with a dipole form: $G_A(Q^2) = \frac{g_A}{(1+\frac{Q^2}{M_A^2})^2}$ to extract the "axial radius": $\langle r_A^2\rangle = \frac{12}{M_A^2}$

- From ν-scattering: M_A=1.026(21) GeV
- MiniBooNE (2002): M_A=1.35(17) GeV
- Recent re-analysis using z-expansion (2016): M_A=1.01(24) GeV

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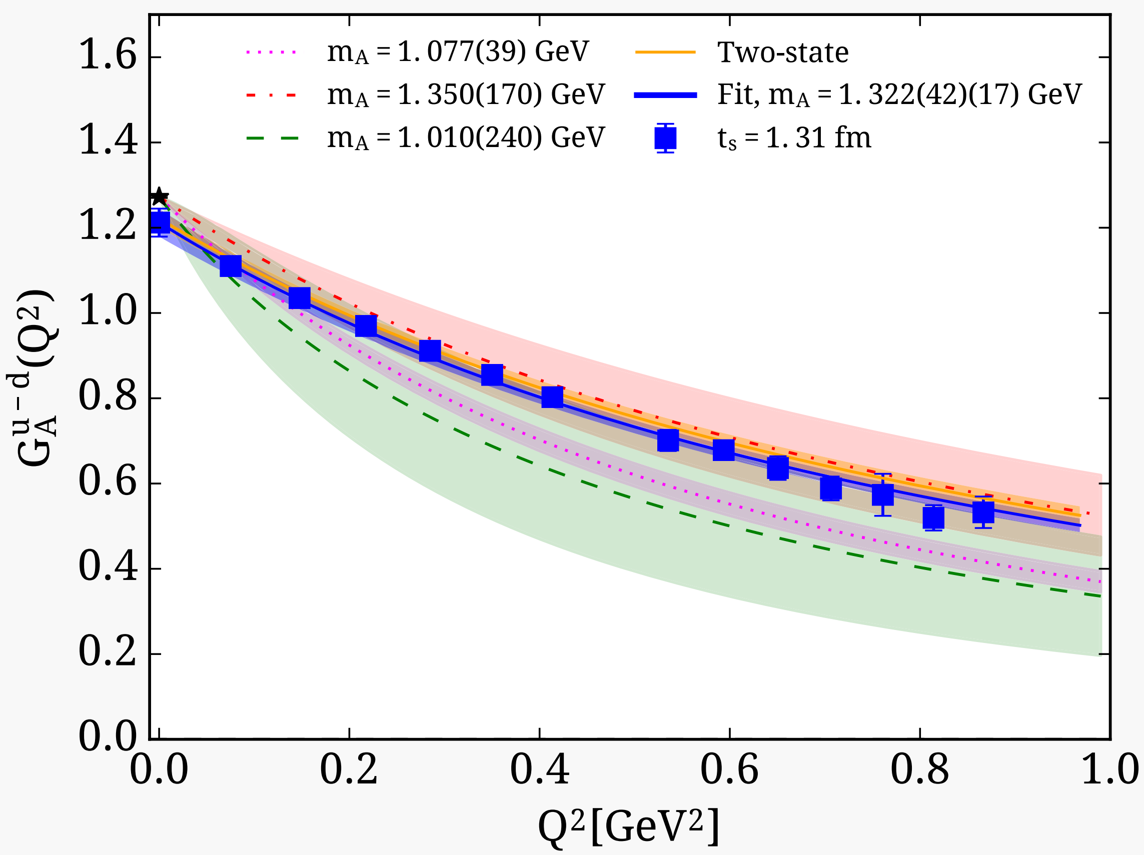
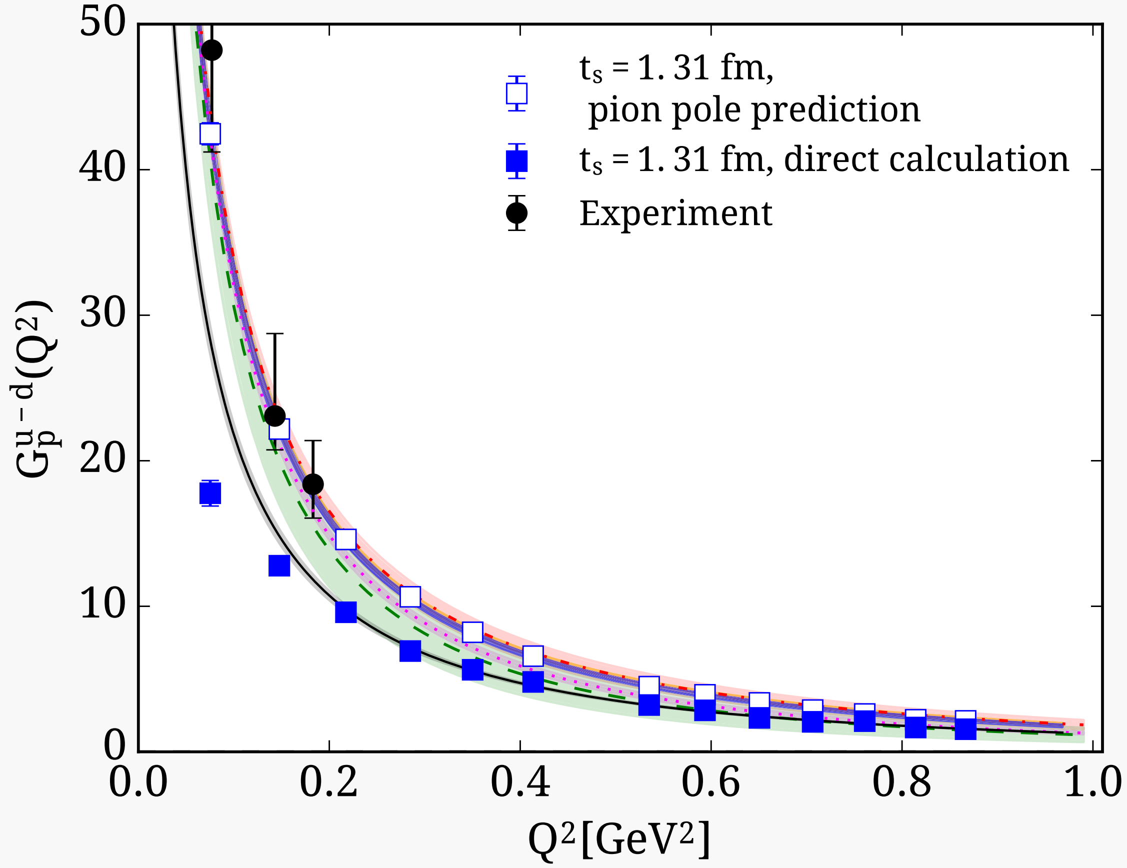
Axial Form Factors





- Axial and induced pseudo scalar from one ensemble with m_π=130 MeV
- Axial form-factor within bands of experimental determinations
- Induced pseudo scalar does not follow the pion pole prediction

$$G_p(Q^2) = G_A(Q^2)\frac{C}{Q^2 + m_\pi^2}$$

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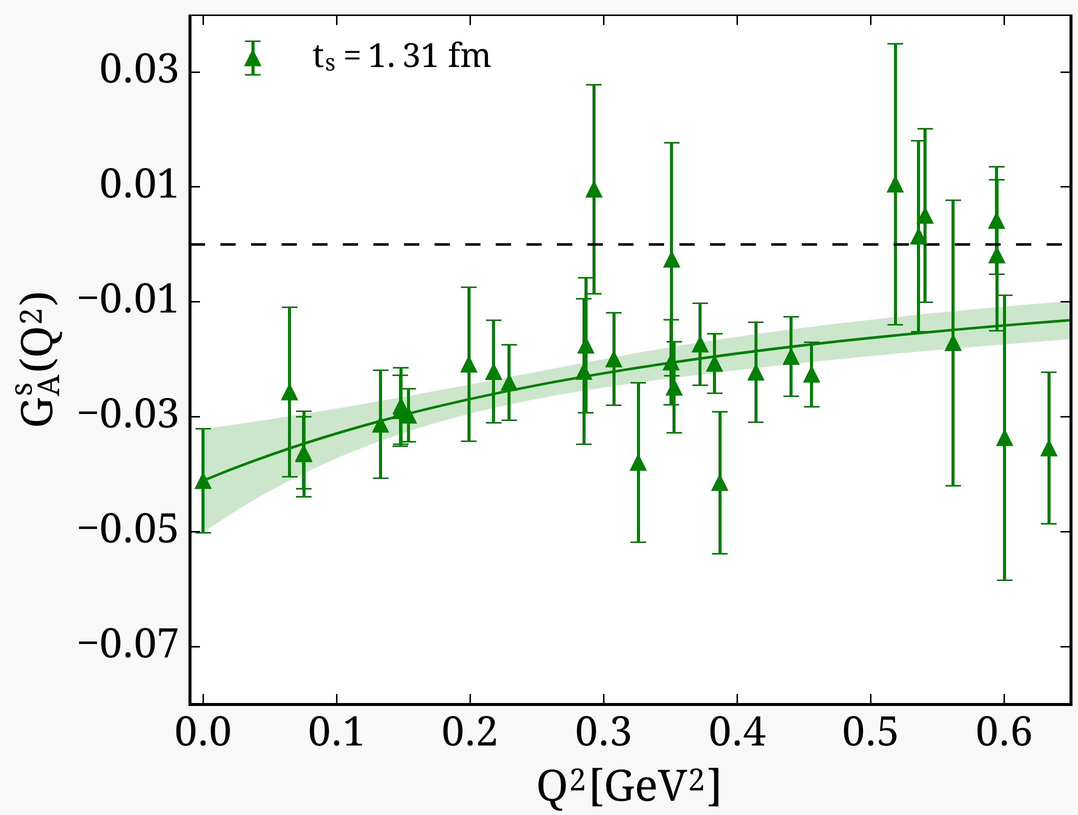
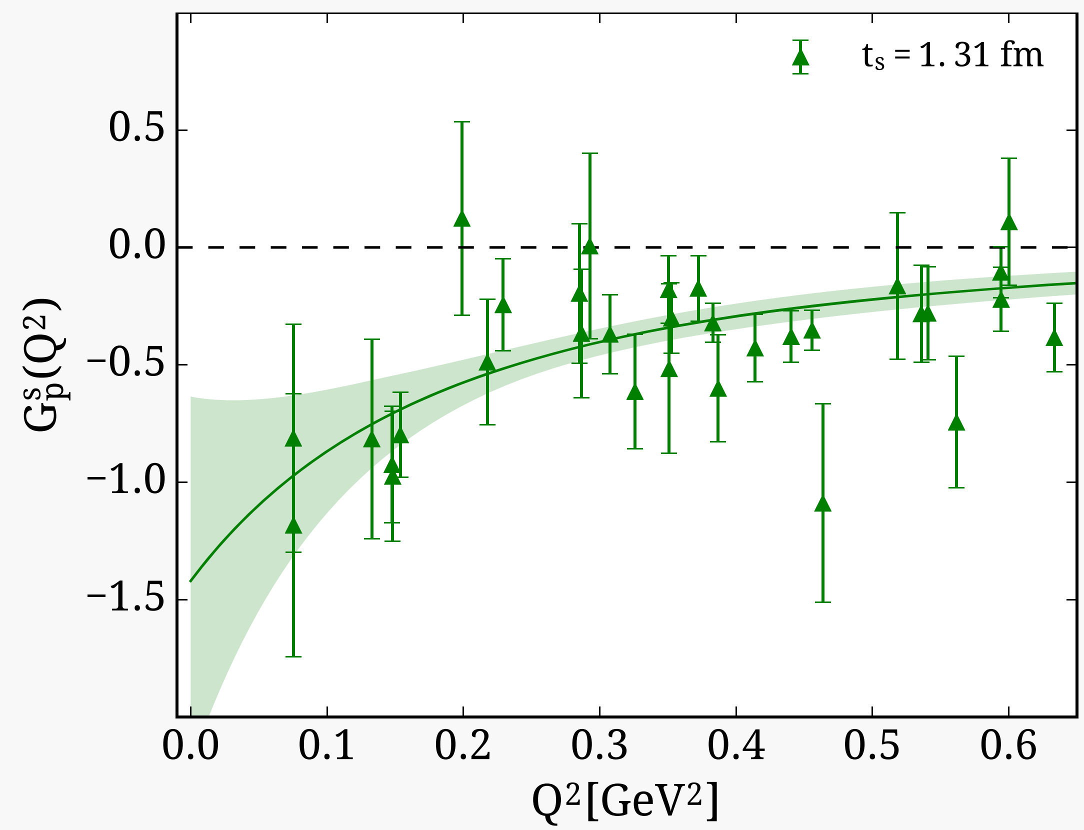
arXiv:1705.03399, PRD

Axial Form Factors





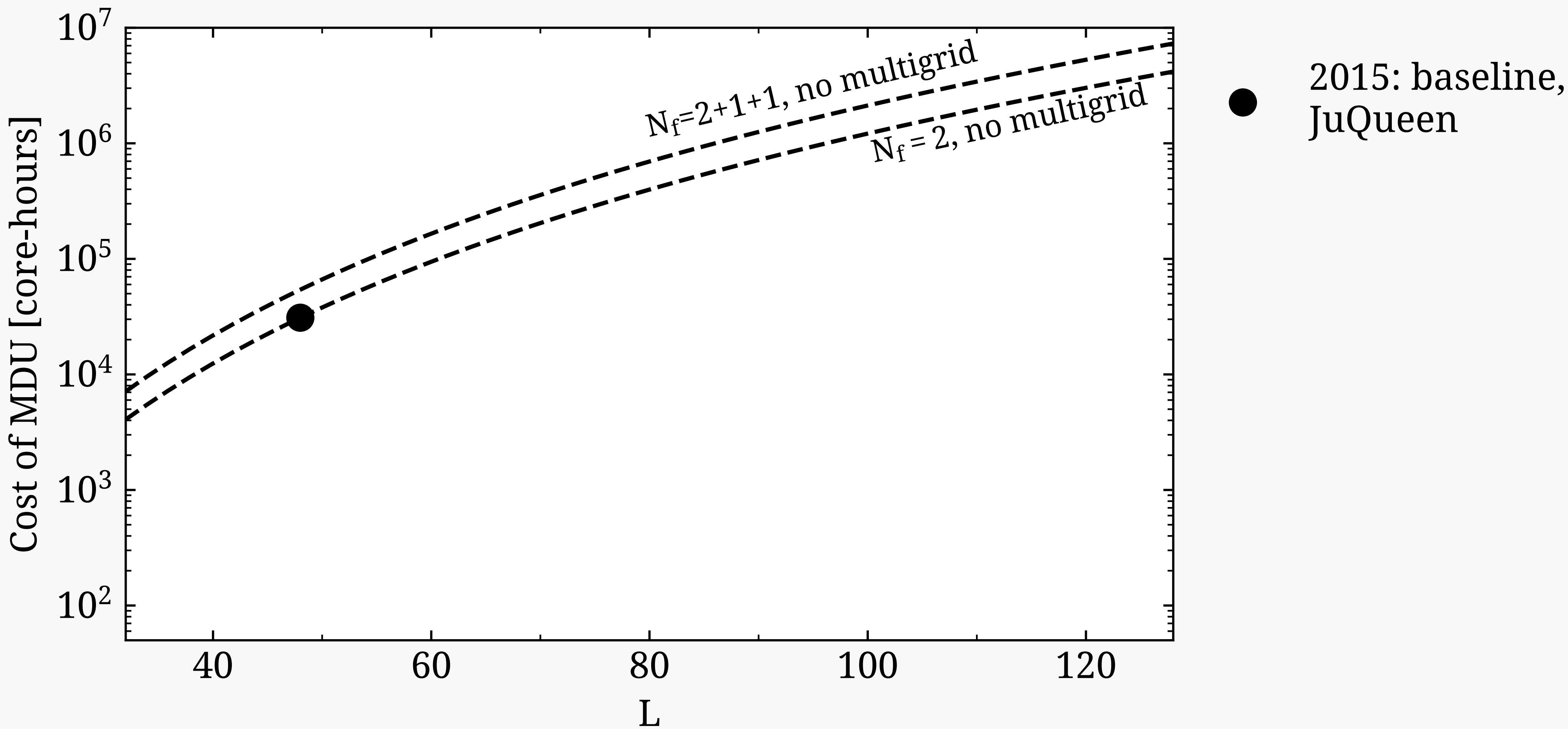
- Strange components from disconnected contributions
- Negative and non-zero signal

arXiv:1705.03399, PRD

Lattice QCD — Large-scale simulations

A combination of

- Use of advanced computers
- Development of new algorithms



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Lattice QCD — Large-scale simulations

A combination of

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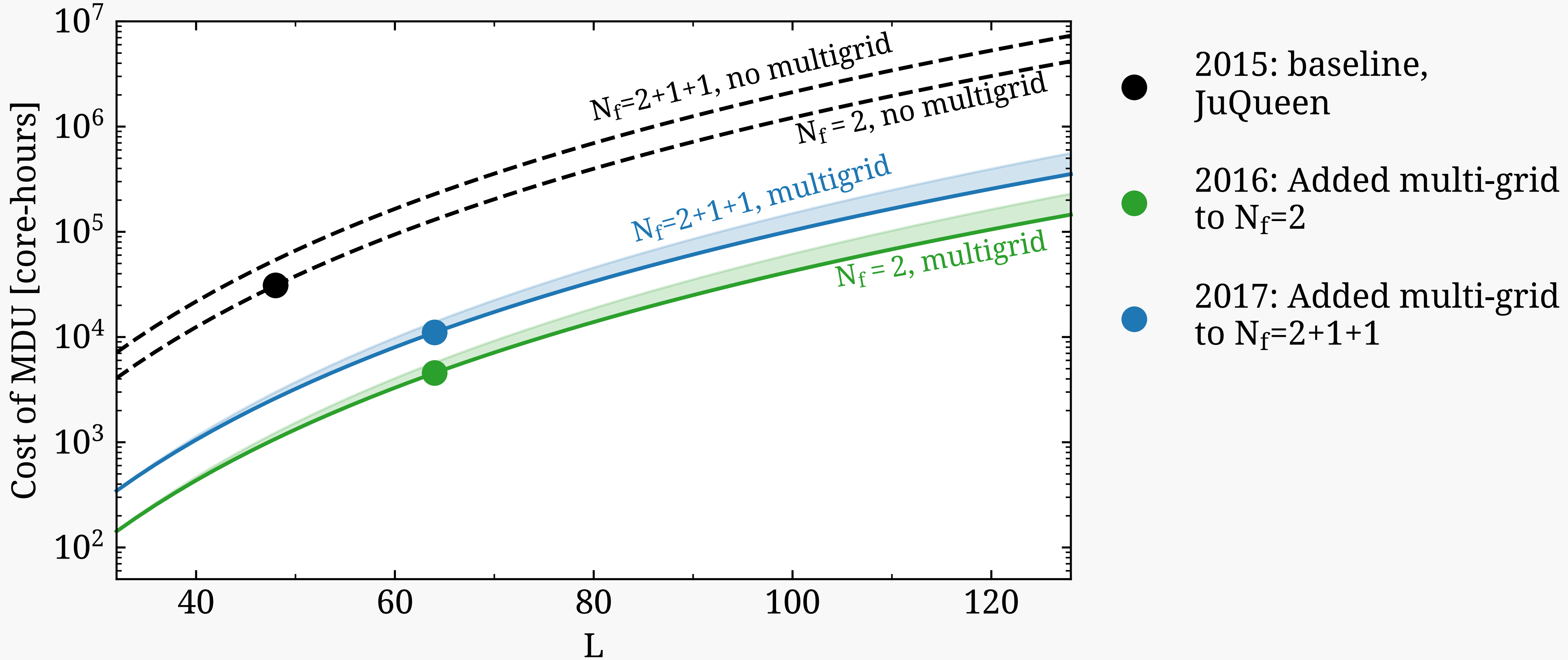
- Development of new algorithms



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Lattice QCD — Large-scale simulations

A combination of

- Use of advanced computers
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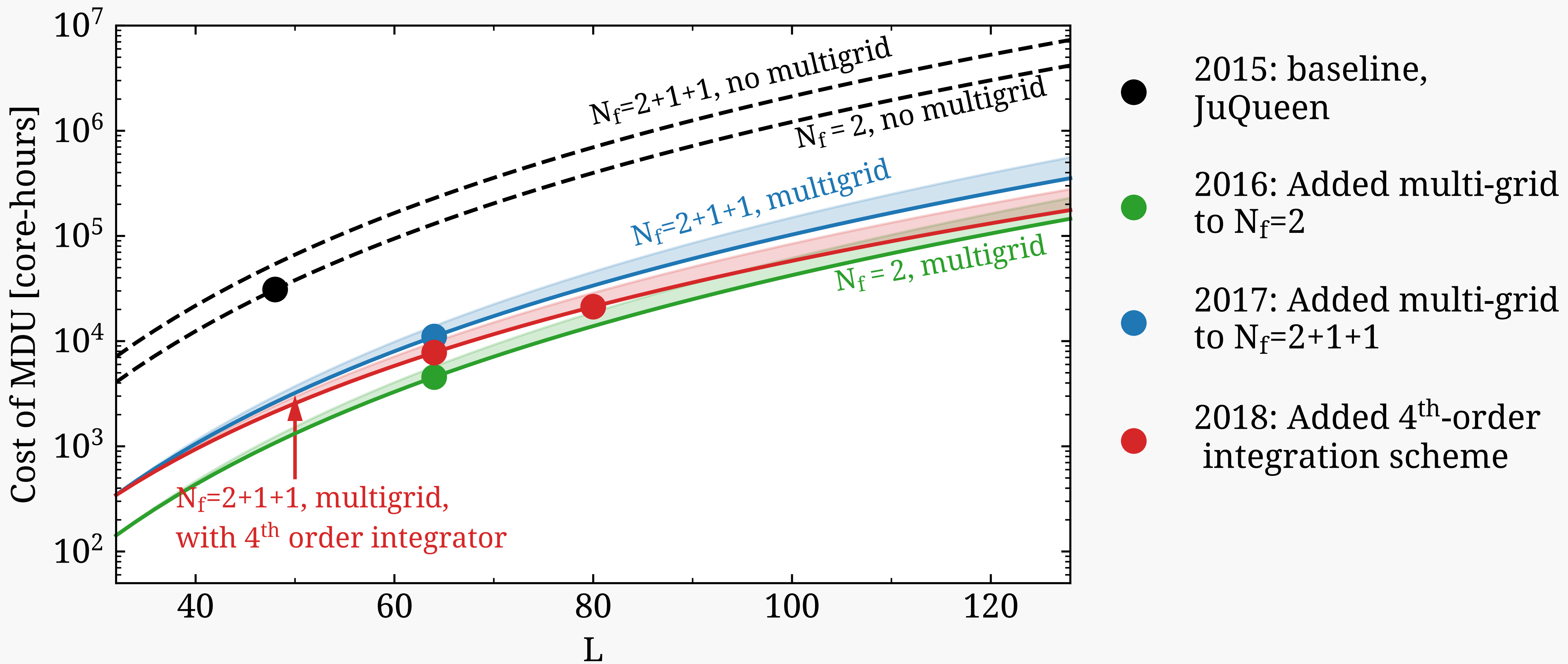

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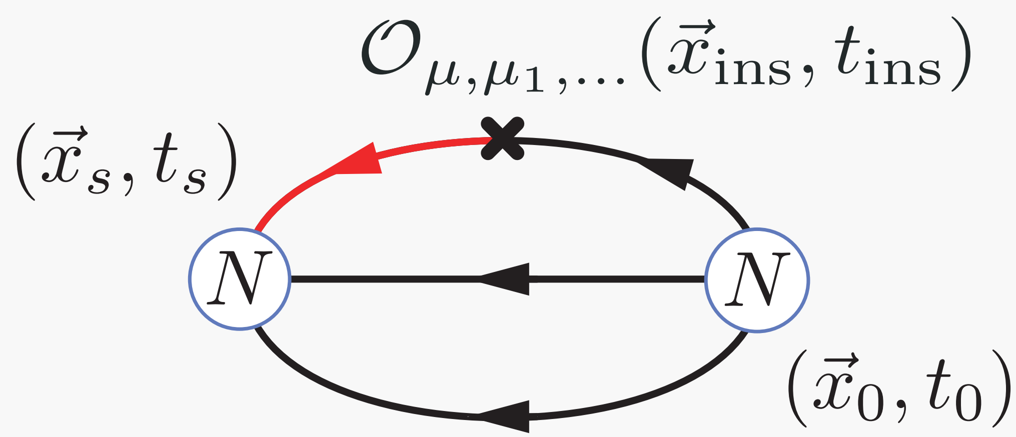
Setup



Connected:

- Sequential inversion through the sink
- Fix: Sink-source separation (t_s), nucleon polarisation (Γ), sink momentum p'

t_s/a	$N_{\rm src} \times N_{\rm conf}$		
	cB211.072.64	cA2.09.48	cA2.09.64
8	1×750	–	–
10	2×750	16×578	–
12	4×750	16×578	16×333
14	6×750	16×578	16×515
16	16×750	88×530*	32×515
18	48×750	88×725*	–
20	64×750	–	–
2-point	264×750	100×2153	32×515

* Only for Γ_0

- Four polarisations Γ_0, Γ_k
- Sink momentum fixed to $\vec{p}' = \vec{0}$
- More statistics for two-point function from disconnected calculation

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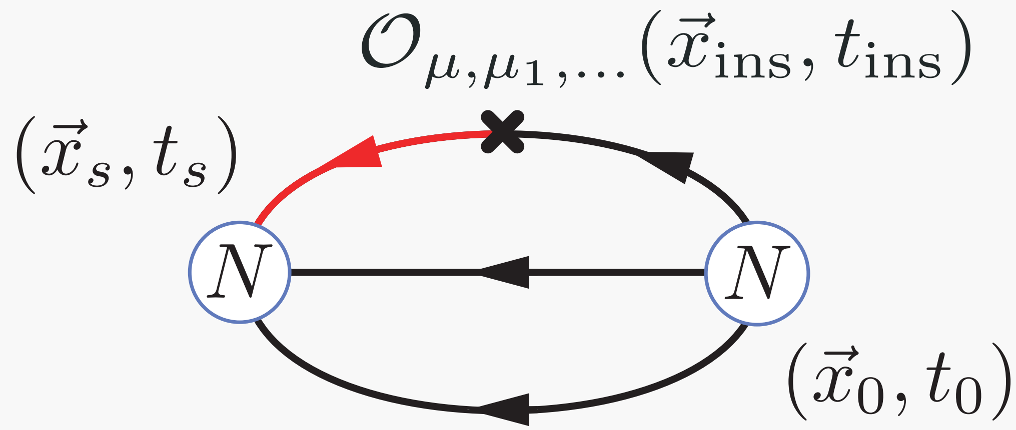
Setup



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t_s/a	$N_{\text{src}} \times N_{\text{conf}}$ cB211.072.64	cA2.09.48	cA2.09.64
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18	48×750	88×725*	–
20	64×750 ×4	–	–
2-point	264×750	100×2153	32×515

* Only for Γ_0

- Four polarisations Γ_0, Γ_k
- Sink momentum fixed to $\vec{p}\,' = \vec{0}$
- More statistics for two-point function from disconnected calculation
- Increased statistics with increasing t_s

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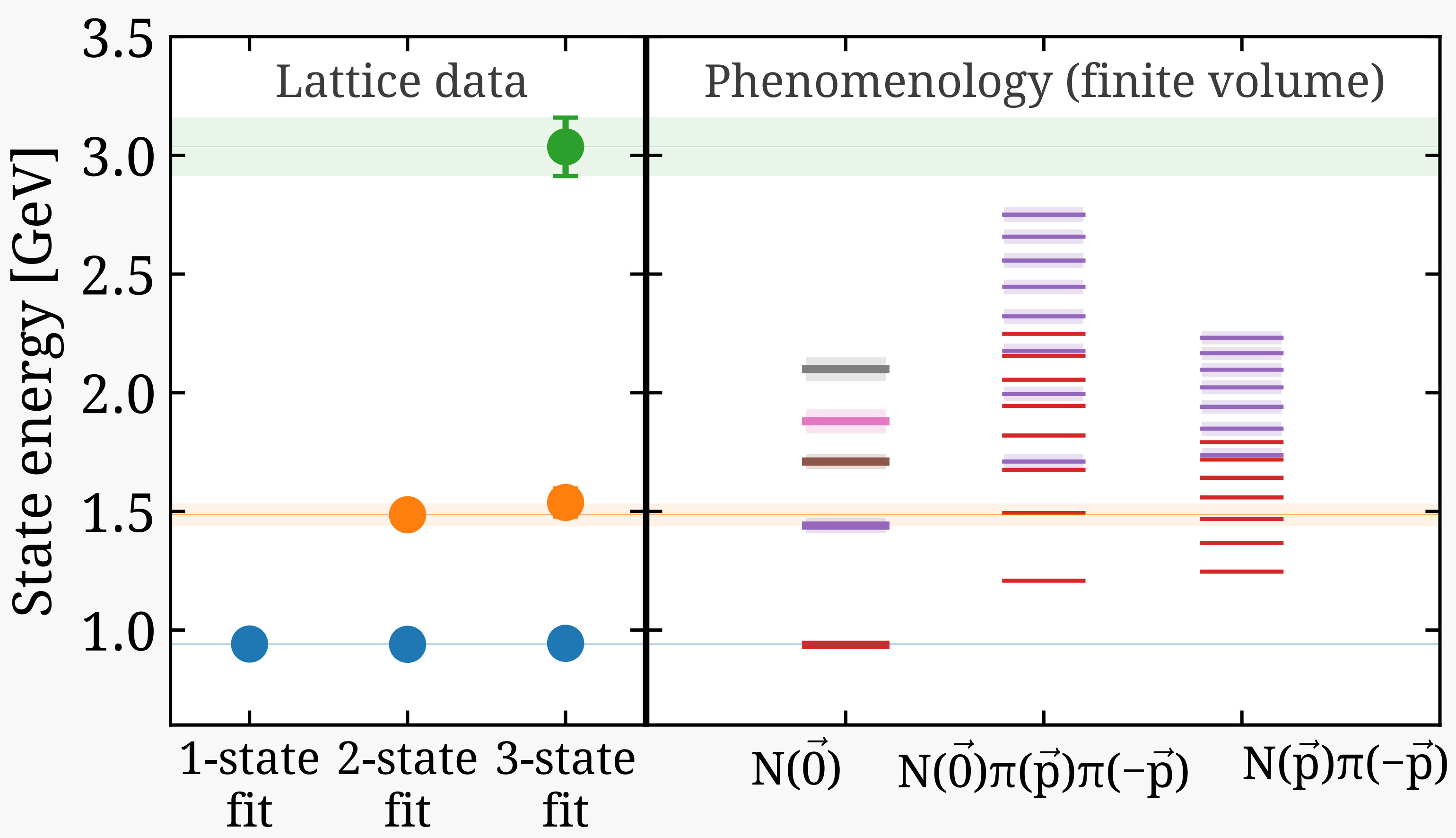
Treatment of excited states

- Analysis of nucleon two-point correlation function



Excited state consistent between 2- and 3-state fit

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