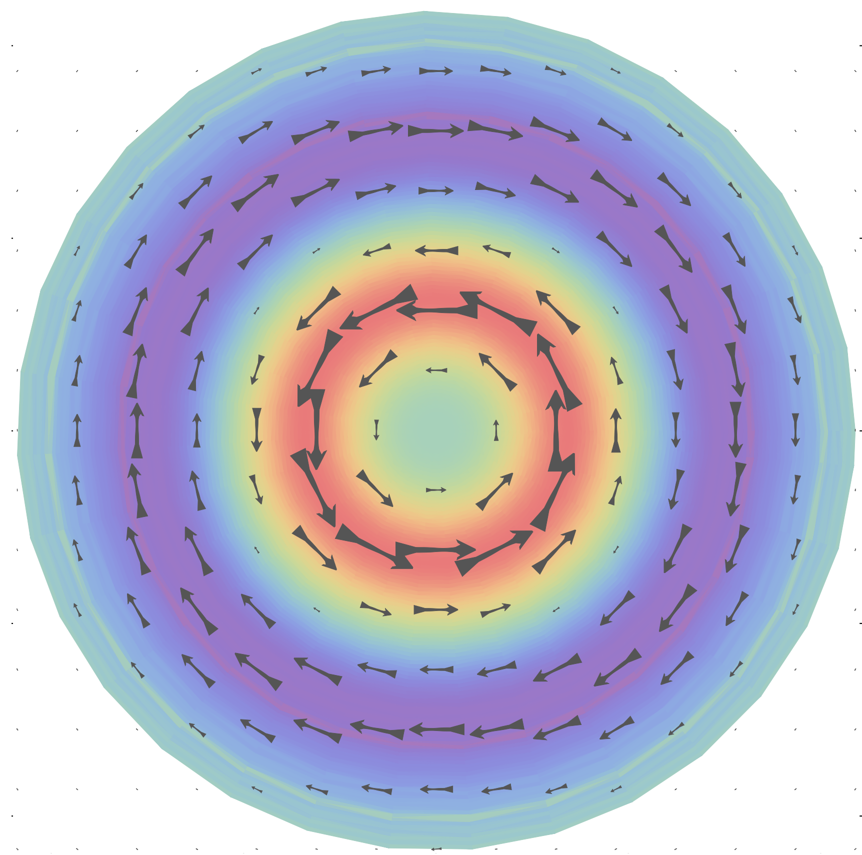
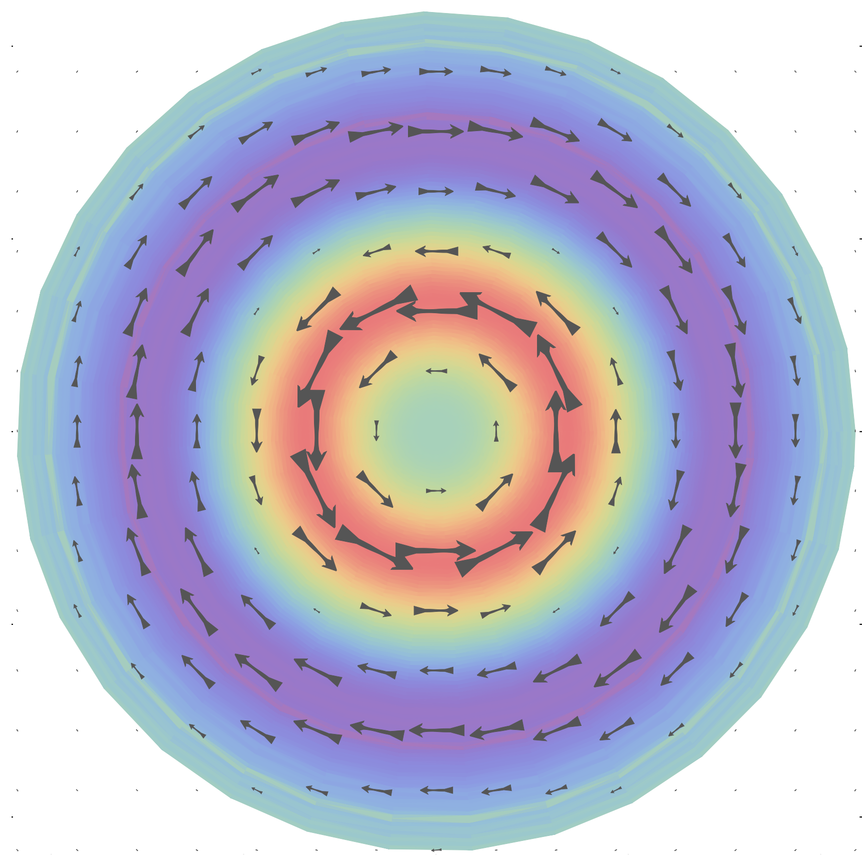
Proton pressure distribution from LQCD



Phiala Shanahan

Massachusetts Institute of Technology

Proton pressure distribution
(and the gluon structure of nucleons and nuclei)
from LQCD



Phiala Shanahan

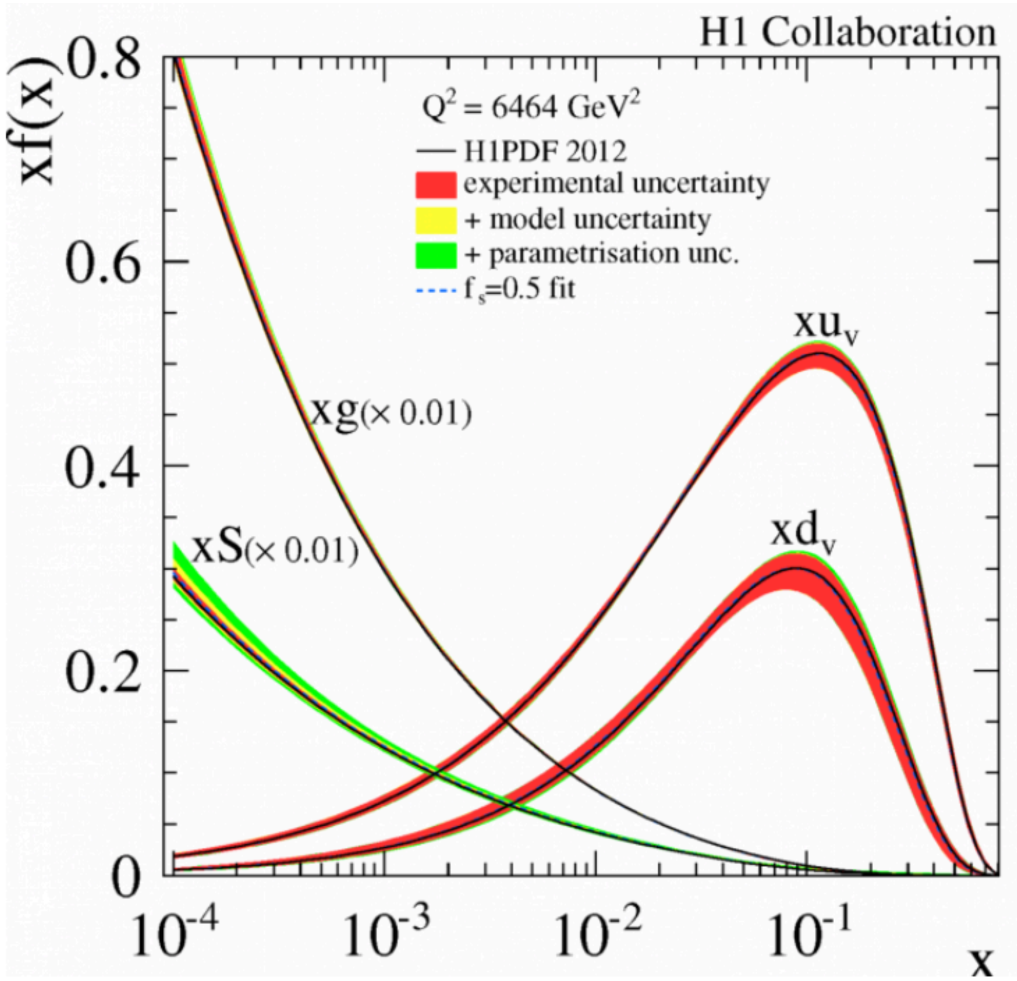
Massachusetts Institute of Technology

Gluon structure

Gluons offer a new window on nuclear structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluon structure also important
  - Unpolarised gluon PDF dominant at small longitudinal momentum fraction
- Other aspects of gluon structure relatively unexplored

Parton distributions in the proton



Longitudinal momentum fraction carried by parton

Gluon structure

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass
- D-term

1

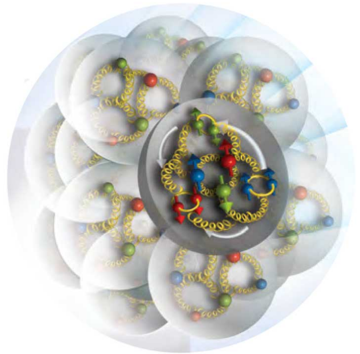
What is the gluon distribution in a proton

- PDFs, GPDs, TMDs
- Pressure, Shear
- 'Gluon radius'

2

How is the gluon structure of a proton modified in a nucleus

- Gluon 'EMC' effect
- Exotic glue



Gluon structure

First-principles QCD calculations

→ QCD benchmarks and predictions ahead of experiment

Cover image from EIC whitepaper arXiv::1212.1701

Energy-momentum tensor

Many gluon structure properties derived from **Energy-Momentum Tensor** (conserved Noether current associated with Lorentz translations)

Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G^{a}_{\{\mu\alpha} G^{a\alpha}{}_{\nu\}} | p, s \rangle = \bar{U}(p', s') \left(A_g(t)\, \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i\, P_{\{\mu} \sigma_{\nu\}\rho} \Delta^{\rho}}{2M_N} + D_g(t) \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor

Generalised gluon form factors

$\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2 \quad t = \Delta^2$

- Three generalised gluon form factors $A_g(t),\ B_g(t),\ D_g(t)$
- Sum rules with quark pieces in forward limit
  - Momentum fraction $A_a(0) = \langle x \rangle_a$ → $\sum_{a=q,g} A_a(0) = 1$
  - Spin $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t))$ → $\sum_{a=q,g} J_a(0) = \frac{1}{2}$
  - D-terms $D_a(0)$ unknown but equally fundamental!

D-term

D-term GFF encodes the pressure **and** shear **distributions in the nucleon** (Breit frame)

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r),$$

$$\widetilde{D}(r) = \int \frac{d^3\vec{p}}{2E(2\pi)^3}\, e^{-i\vec{p}\cdot\vec{r}}\, D(-\vec{p}^{\,2})$$

- Quark and gluon shear forces individually well-defined (i.e., scale-dependent partial contributions $s_{q,g}(r)$)
- Pressure defined from D only for the total system (pieces depend also on GFFs related to the trace terms of the EMT that cancel in the sum)

Generalised parton distributions

GFFs correspond to lowest moments of GPDs:

$$\int_0^1 \mathrm{d}x\, H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)\,, \qquad \int_0^1 \mathrm{d}x\, E_g(x,\xi,t) = B_g(t) - \xi^2 D_g(t)$$

$$\int_{-1}^1 \mathrm{d}x\; x\, H_q(x,\xi,t) = A_q(t) + \xi^2 D_q(t)\,, \qquad \int_{-1}^1 \mathrm{d}x\; x\, E_q(x,\xi,t) = B_q(t) - \xi^2 D_q(t)$$

- **Quark GPDs:** constraints from JLab, HERA, COMPASS, by DVCS, DVMP, future improvements from JLab 12GeV
- **Gluon GPDs:** almost unknown from experiment, future constraints are a central goal of EIC

Leading twist nucleon gluon GPDs:

Gluon field-strength tensor

$$\Delta_\mu = p'_\mu - p_\mu$$
$$P_\mu = (p_\mu + p'_\mu)/2$$
$$t = \Delta^2$$
$$n^2 = 0$$

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p',s'| G_a^{\{\mu\alpha}(-\frac{\lambda}{2}n) \left[\mathcal{U}^{(A)}_{[-\frac{\lambda}{2}n,\frac{\lambda}{2}n]}\right]_{ab} G_{b\alpha}^{\;\;\nu\}}(\frac{\lambda}{2}n)|p,s\rangle$$
$$= \frac{1}{2}\left(H_g(x,\xi,t)\bar{U}(p',s')P^{\{\mu}\gamma^{\nu\}}U(p,s) + E_g(x,\xi,t)\bar{U}(p',s')\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M}U(p,s)\right) + \ldots\,,$$

GPDs(Bjorken x, skewness, mom transfer)

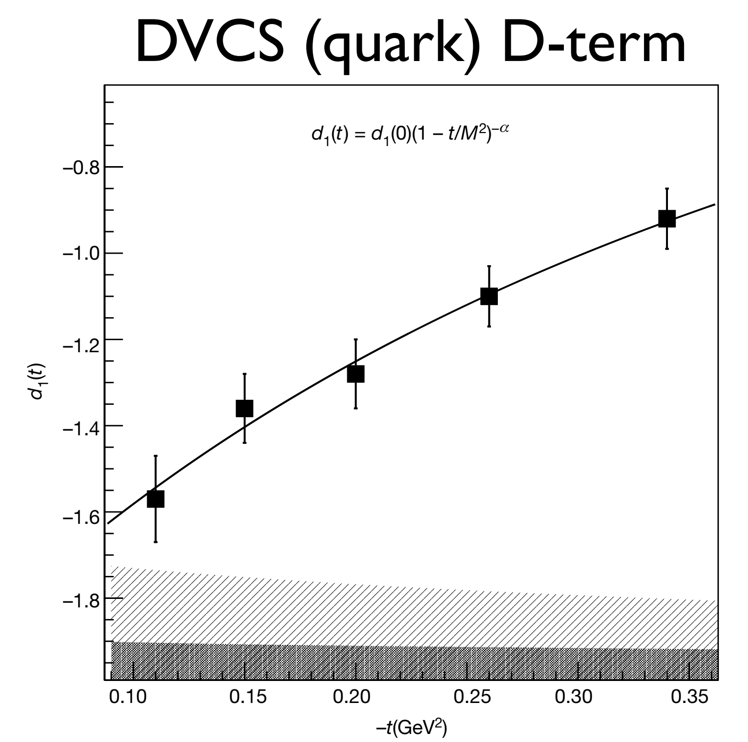
D-term from JLab DVCS

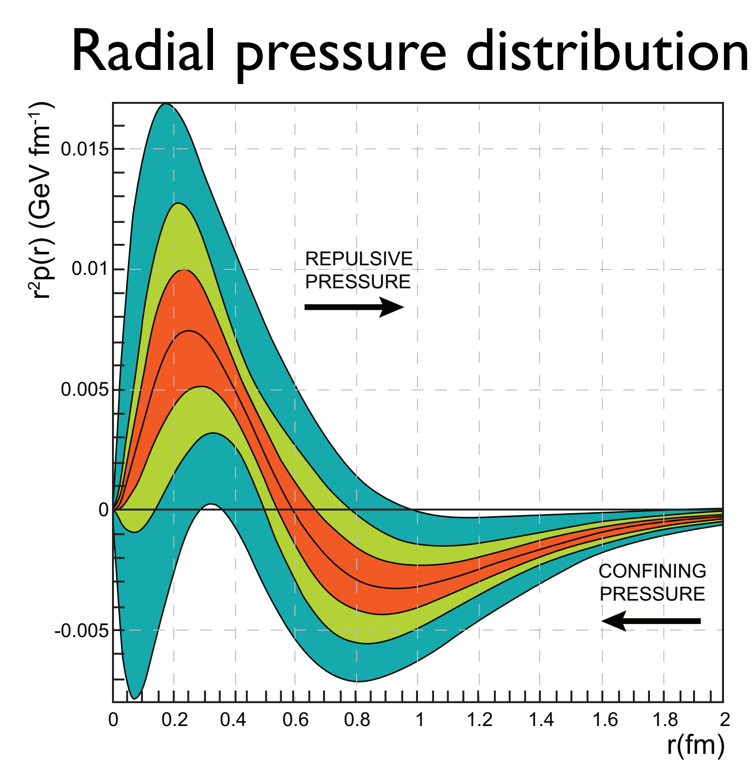
Recent experimental determination of DVCS D-term and extraction of proton pressure distribution

V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557, 396 (2018)

$$s(r) = -\frac{r}{2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{3}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r)$$

- Strong repulsive pressure near the centre of the proton
- Binding pressure at greater distances.
- Peak pressure near the centre ~10^{35} Pascal, greater than pressure estimated for neutron stars
- Key assumptions: **gluon D-term same as quark term, tripole form factor model,** $D_u(t,\mu) = D_d(t,\mu)$

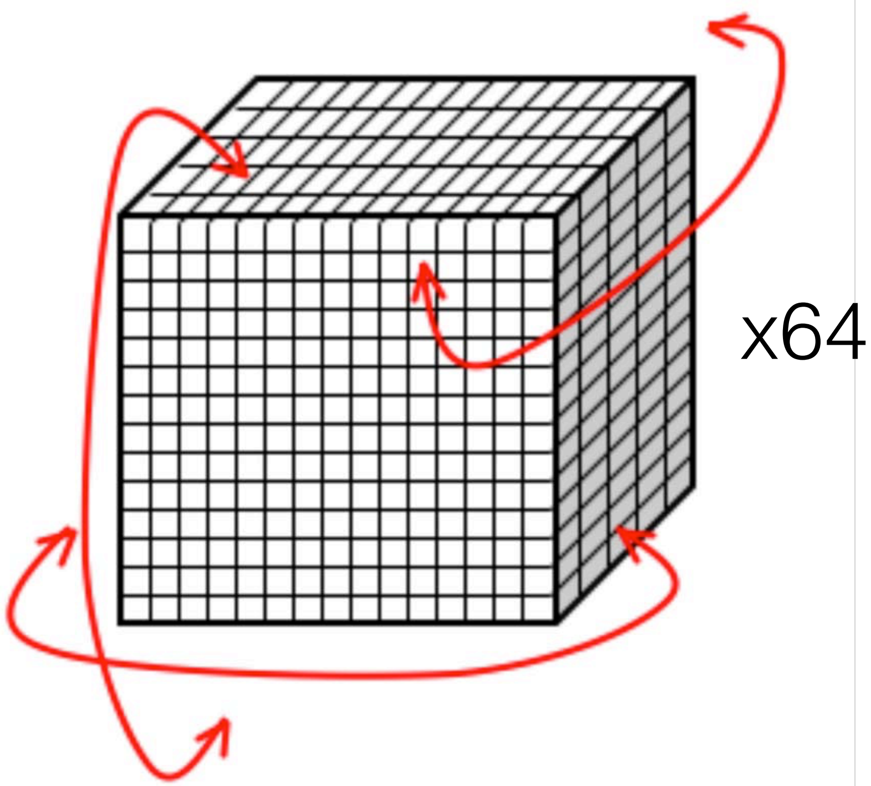




Use lattice QCD to test assumptions in pressure extraction

Lattice QCD

- Numerical first-principles approach to non-perturbative QCD
- Euclidean space-time $t \to i\tau$
  - Finite lattice spacing a
  - Volume $L^3 \times T \approx 32^3 \times 64$
  - Boundary conditions
- Some calculations use larger-than-physical quark masses (cheaper)



Approximate the QCD path integral by **Monte Carlo**

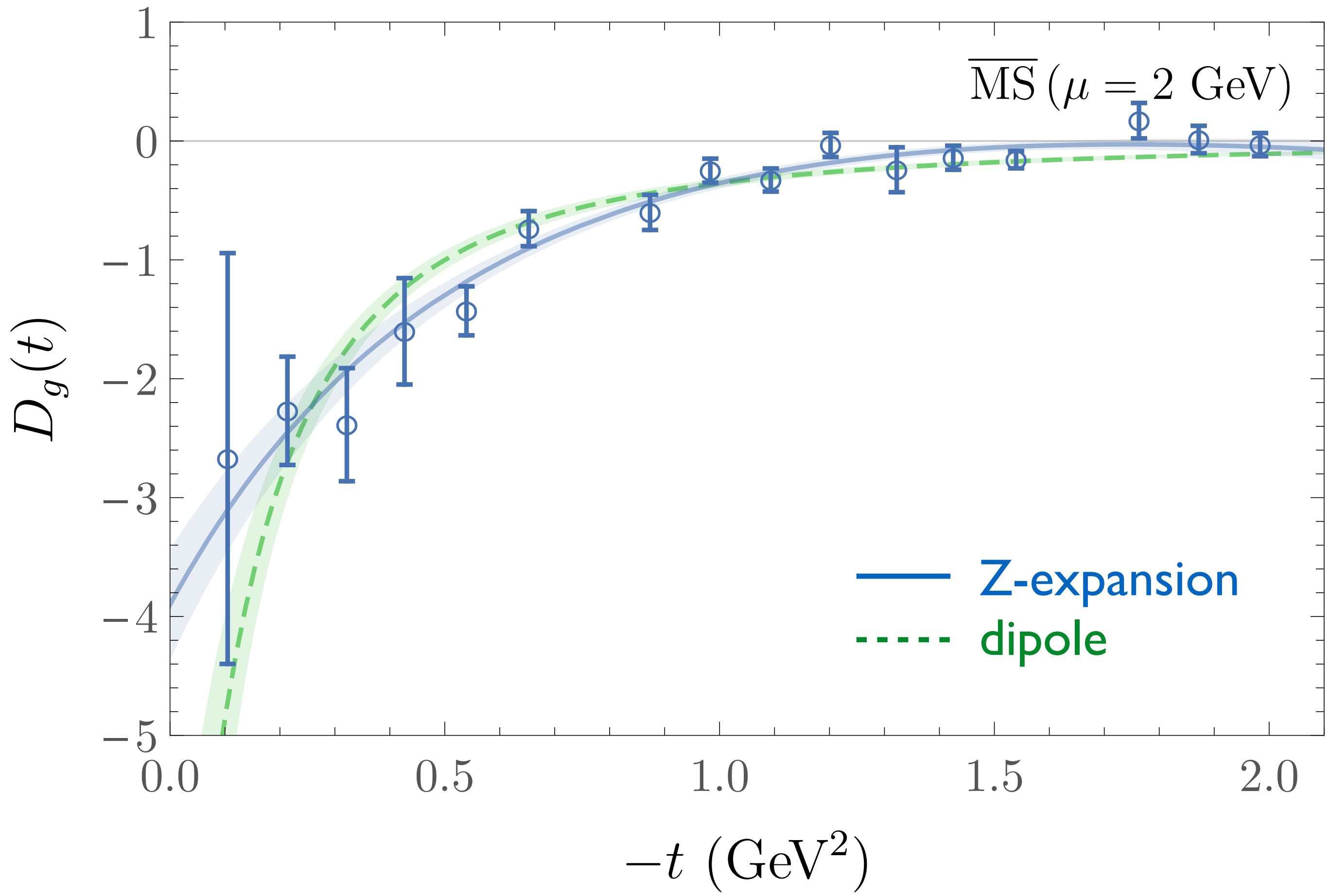
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

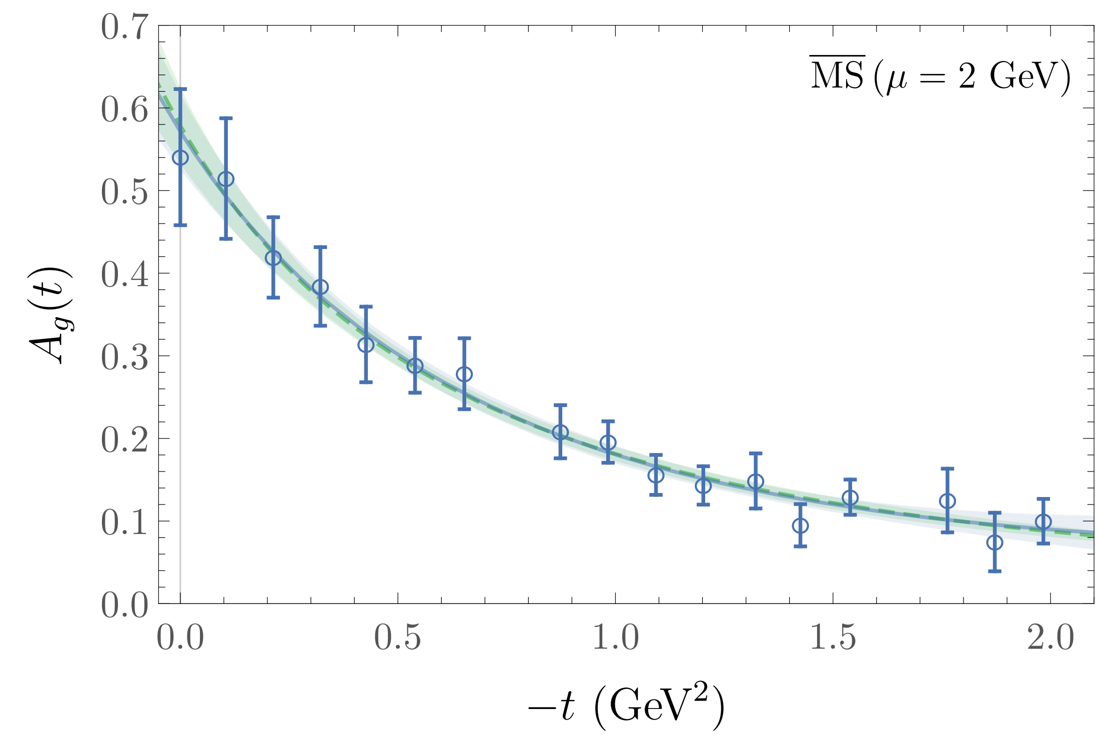
with field configurations U^i distributed according to $e^{-S[U]}$

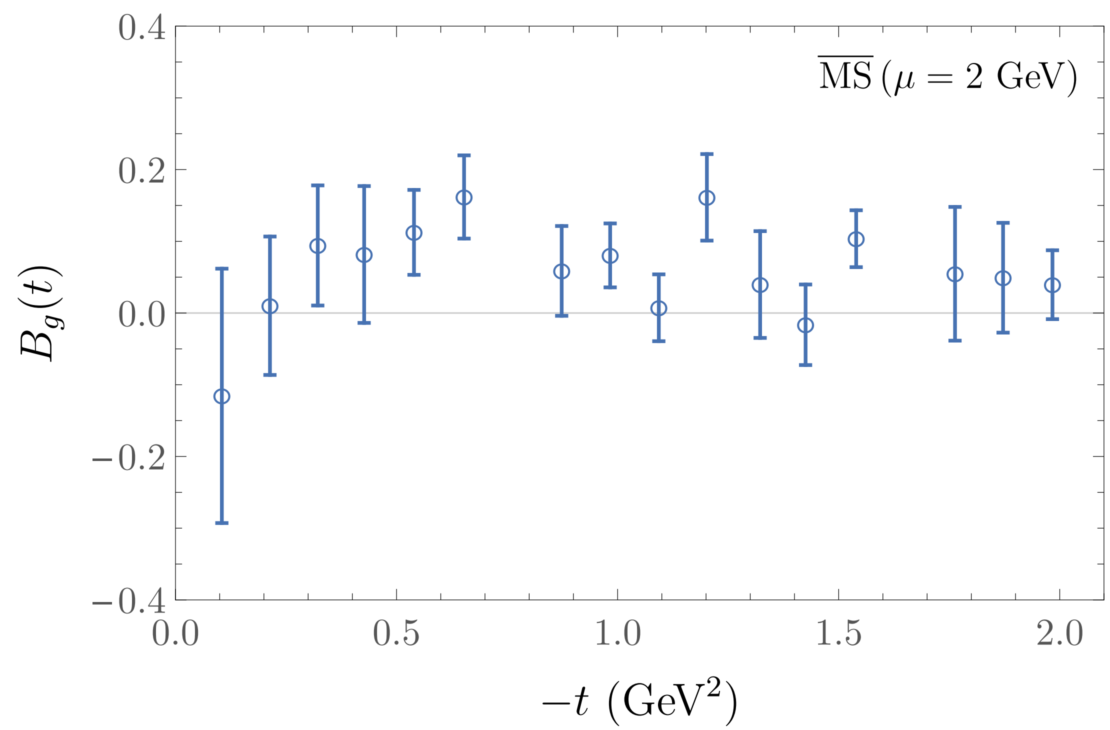
LQCD Nucleon GFFs

LQCD results for nucleon gluon GFFs
m_π ~450 MeV

Dipole-like fall-off with momentum transfer







Shanahan, Detmold, PRD99, 014511 & PRL122, 072003 (2019)

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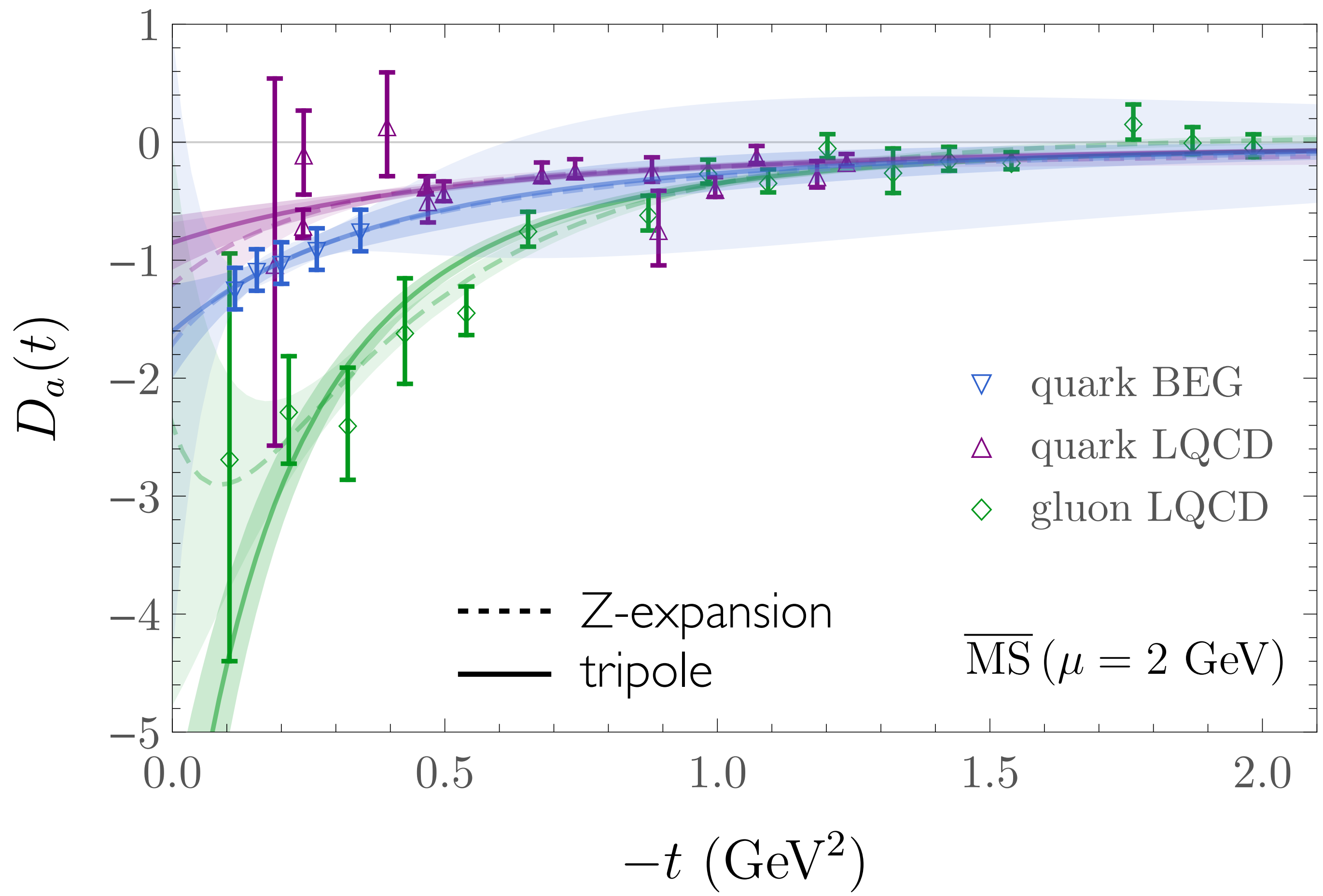

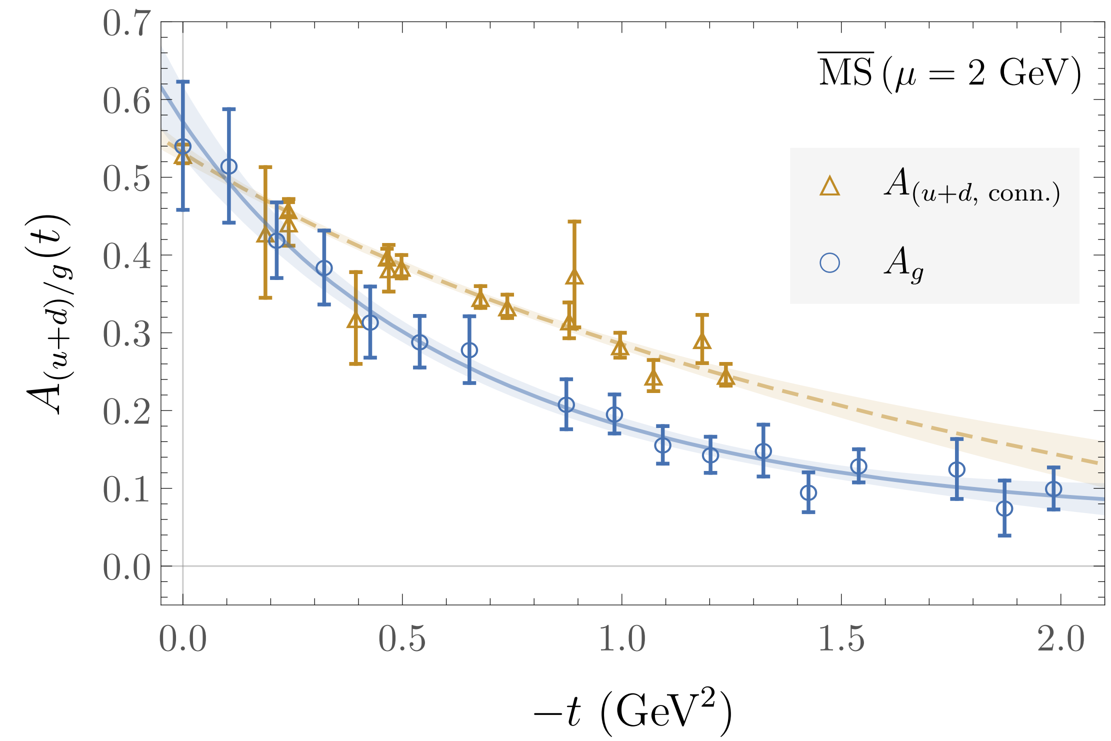

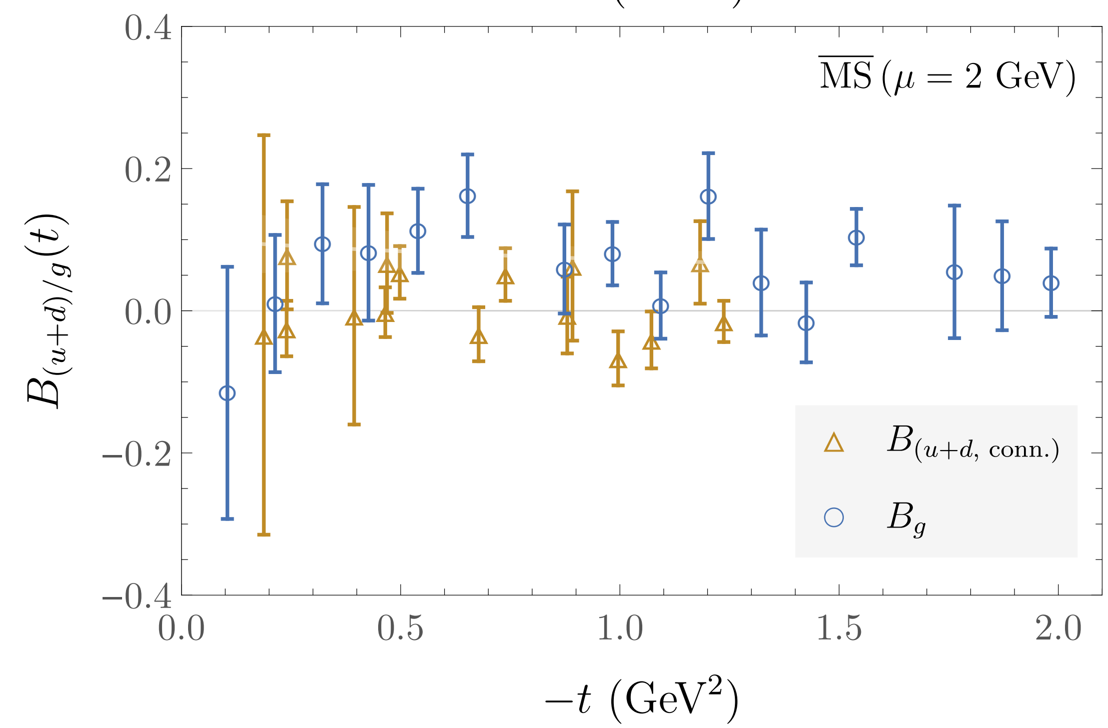

Gluon GFFs: Shanahan, Detmold, PRD99, 014511 & PRL122, 072003 (2019)
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

Nucleon D-term GFFs

LQCD results for nucleon gluon GFFs
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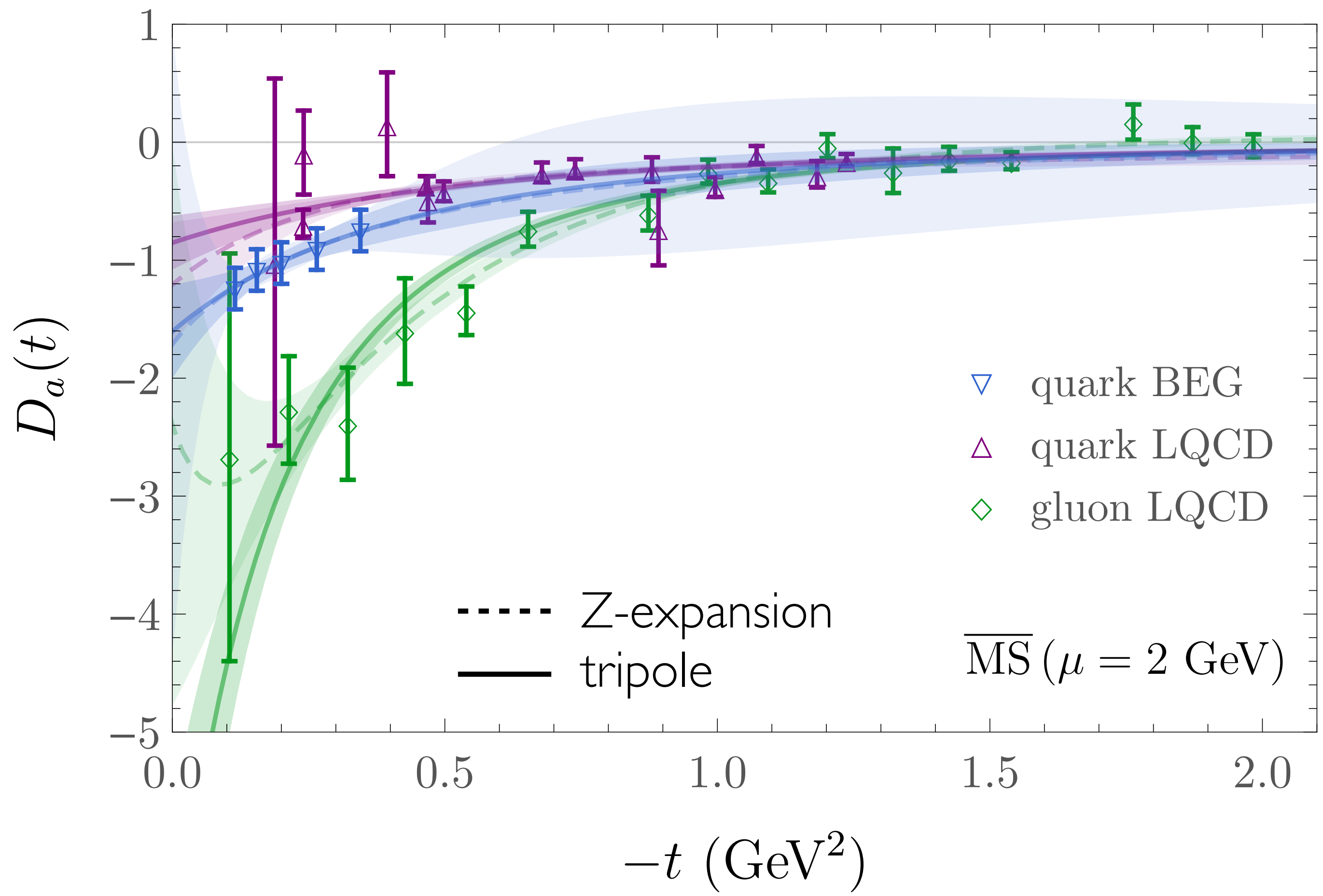

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Key assumptions in pressure extraction from DVCS

- **Gluon D-term same as quark term in magnitude and shape**

  Factor of ~2 difference in magnitude, somewhat different t-dependence

- **Tripole form factor model**

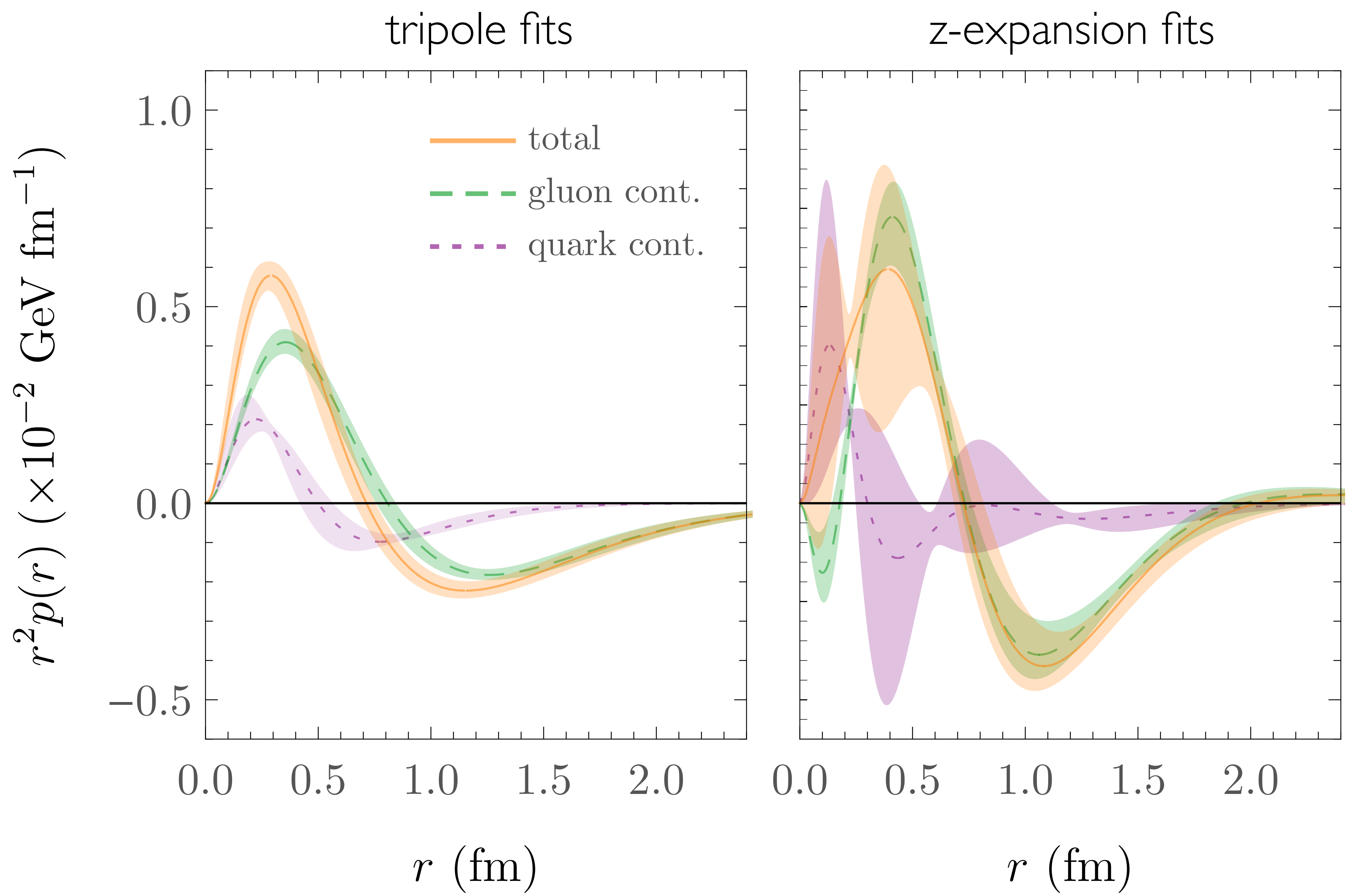
  LQCD results consistent with ansatz, but more general form is less well constrained

- **Isovector quark D-term vanishes**

  $D_{u-d}(t)$ ~0 from other LQCD studies

LQCD proton pressure

Nucleon pressure using LQCD results for quark and gluon GFFs, m_π ~450 MeV



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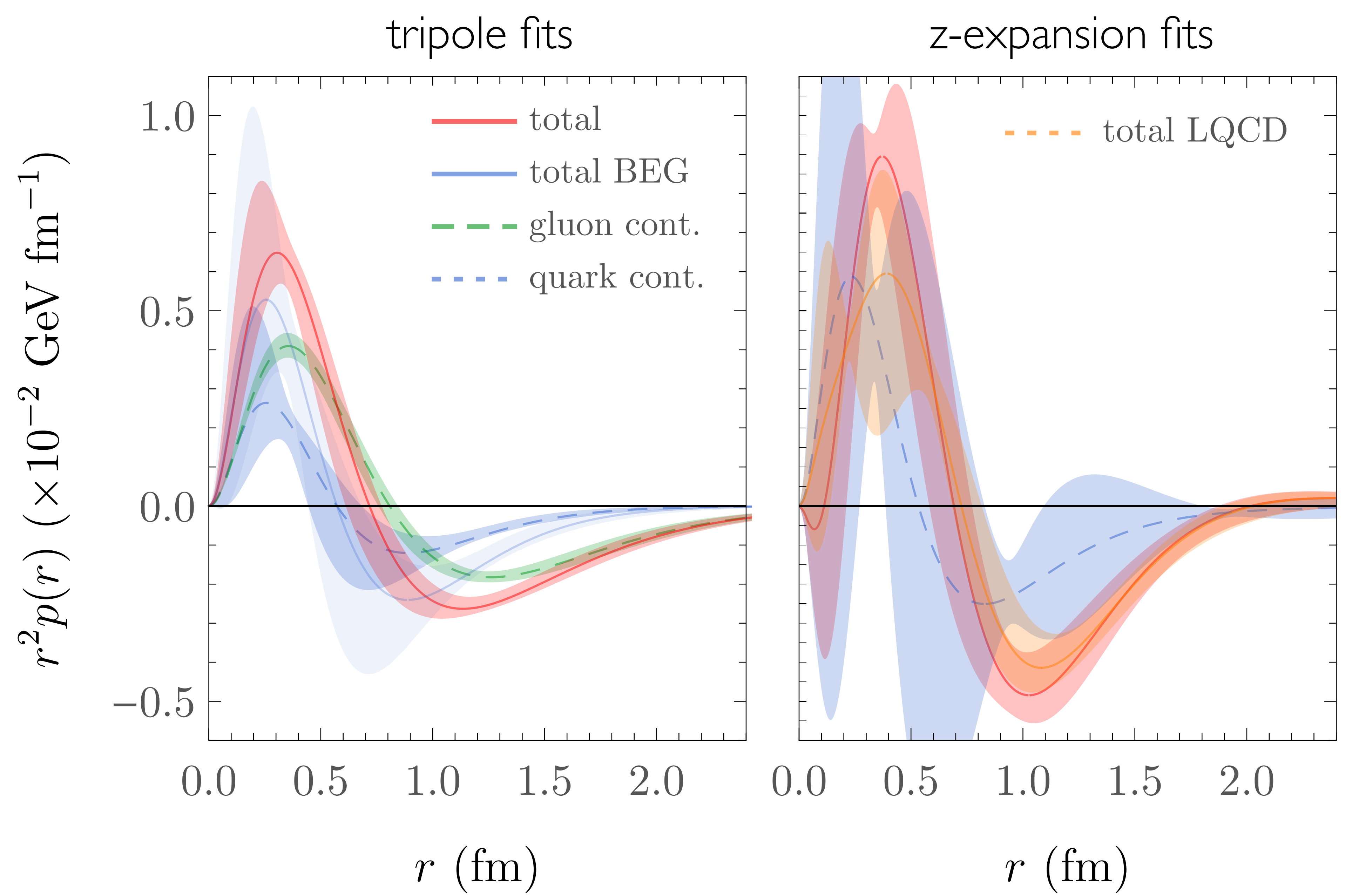
LQCD + EXP proton pressure

Nucleon pressure using LQCD results for gluon GFF, JLab results for quark GFF



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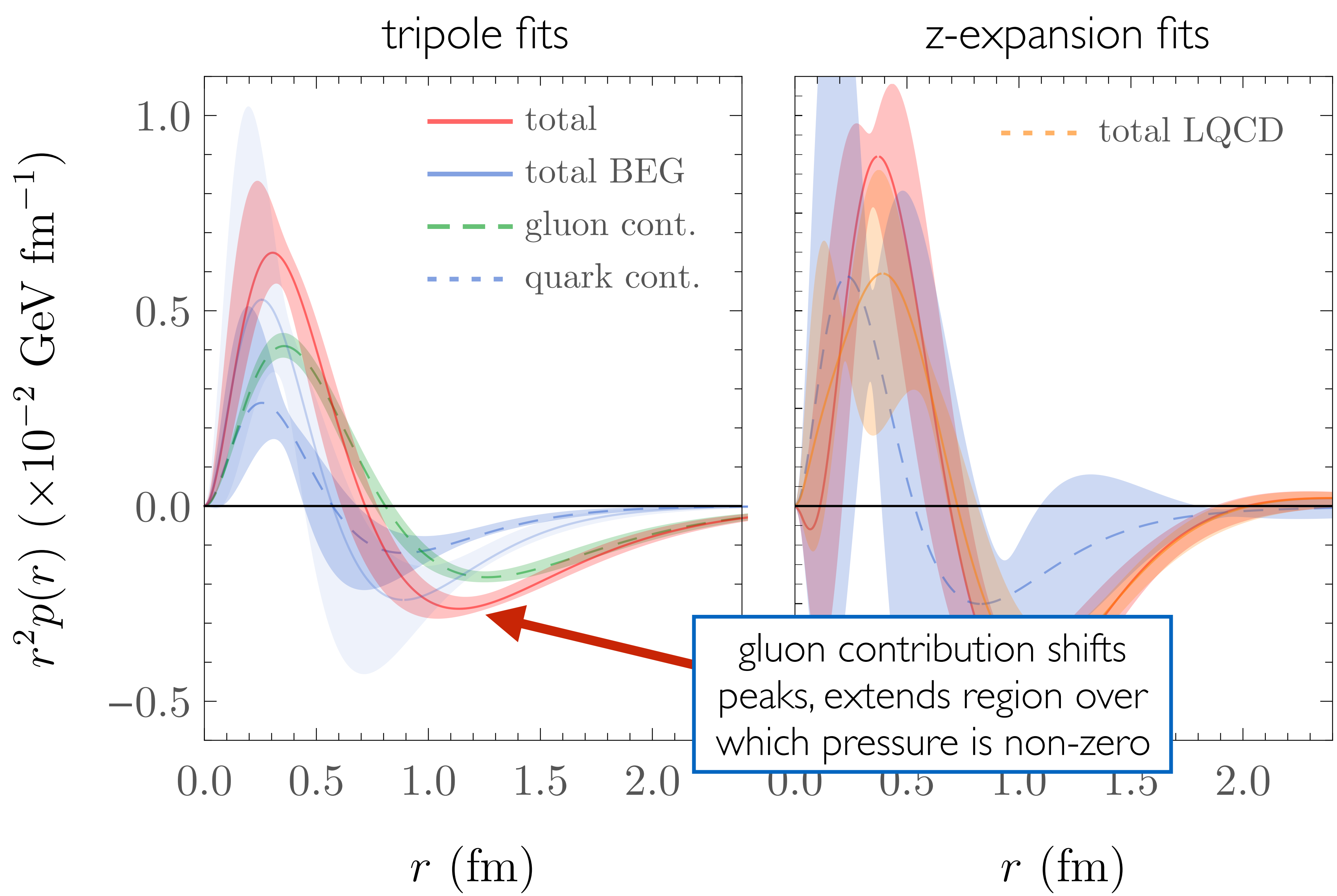

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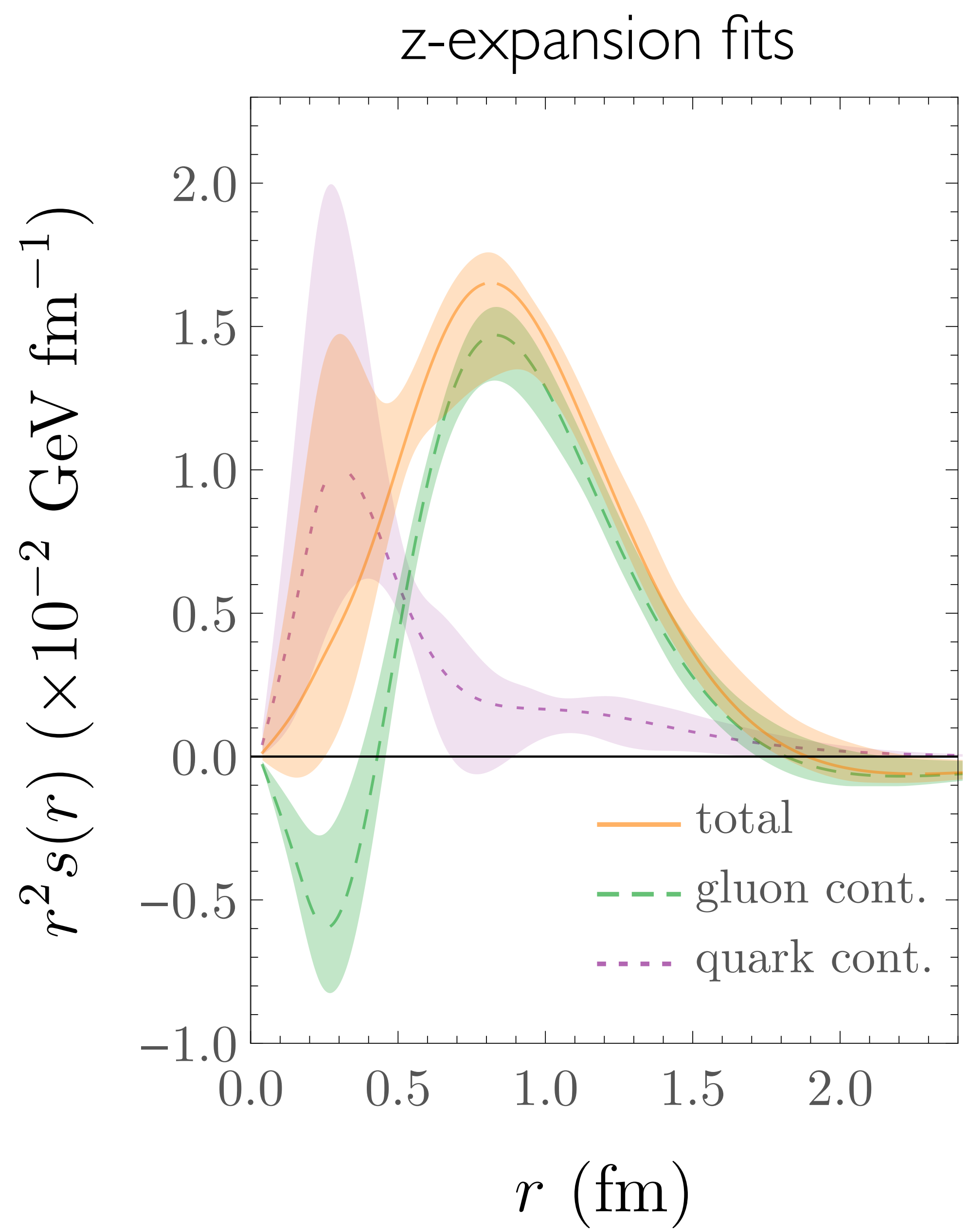
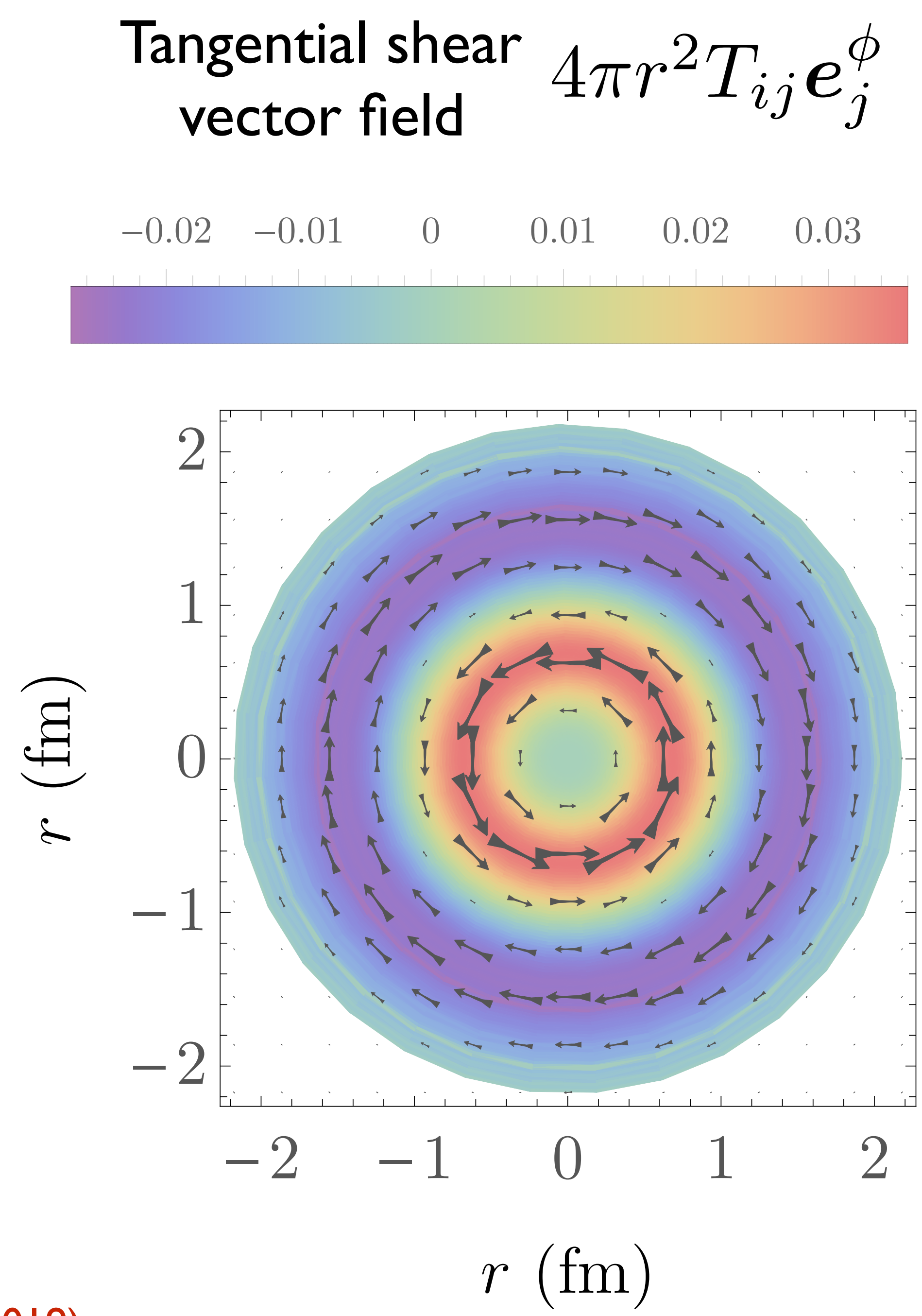
LQCD proton shear





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LQCD Pion GFFs

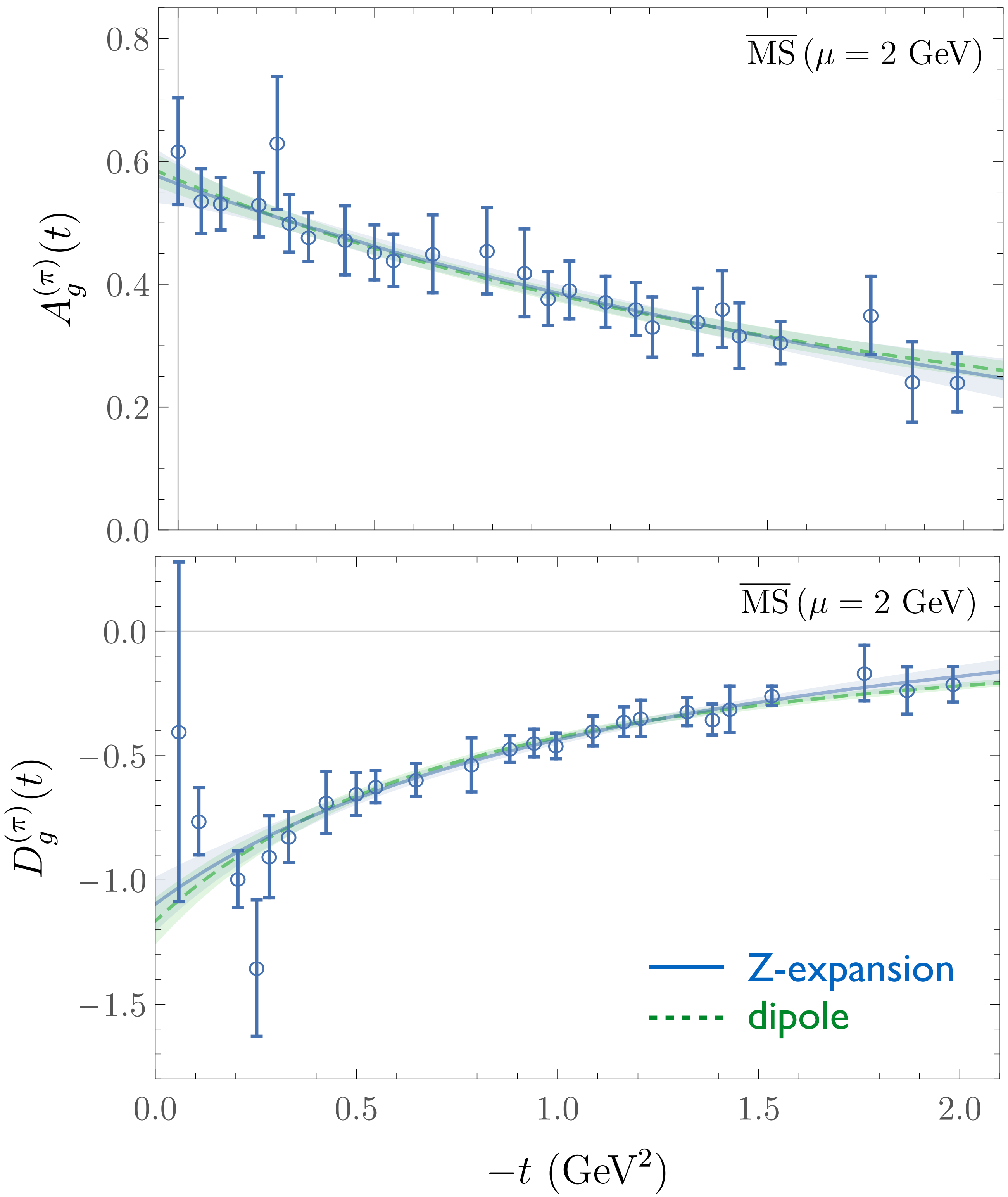
Pion gluon GFFs m_π ~450 MeV

Dipole-like fall-off with momentum transfer

- Momentum fraction $A_a(0) = \langle x \rangle_a$

$$\rightarrow \sum_{a=q,g} A_a(0) = 1$$

- D-terms $D_a(0)$ related to pressure and shear distributions



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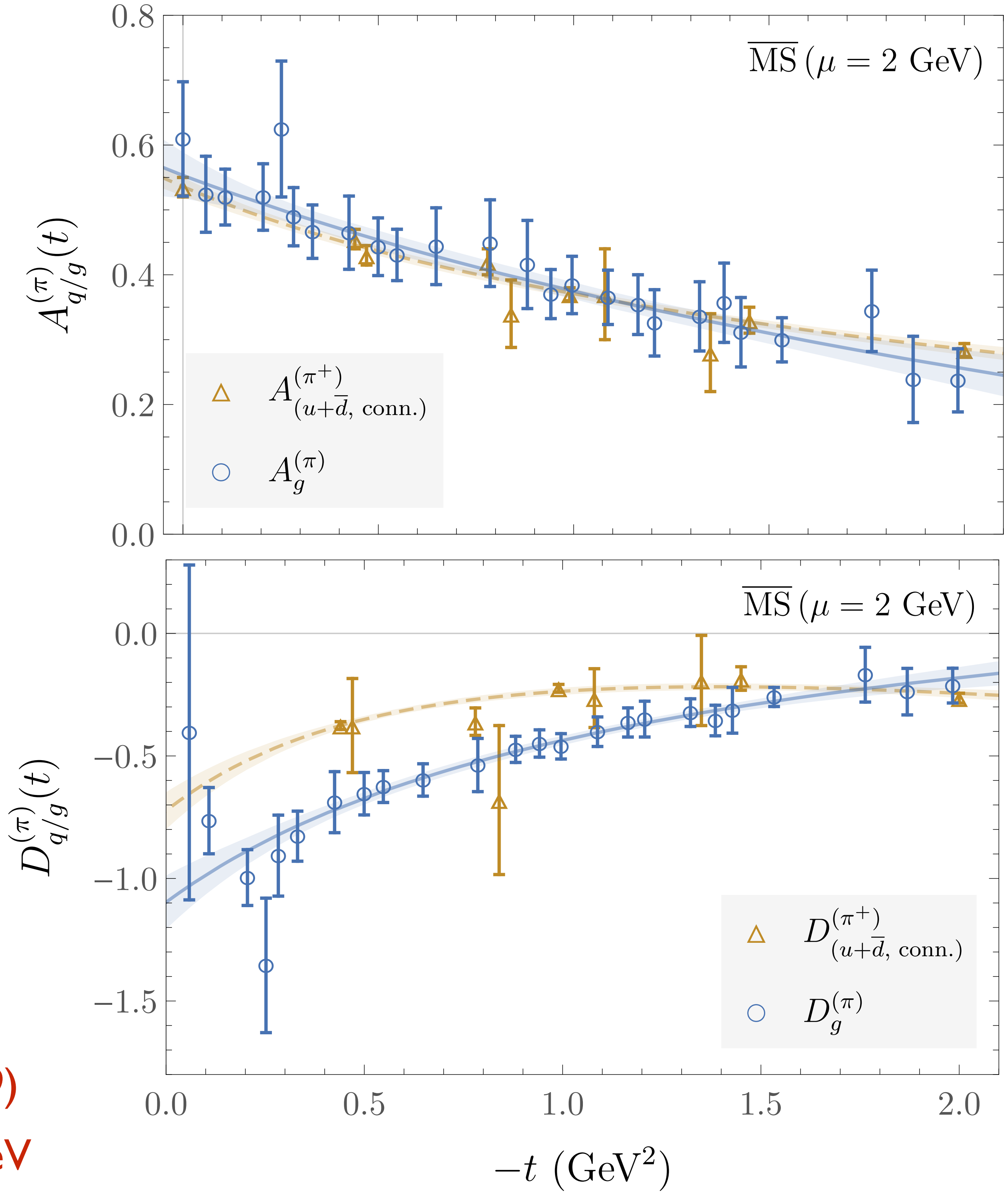


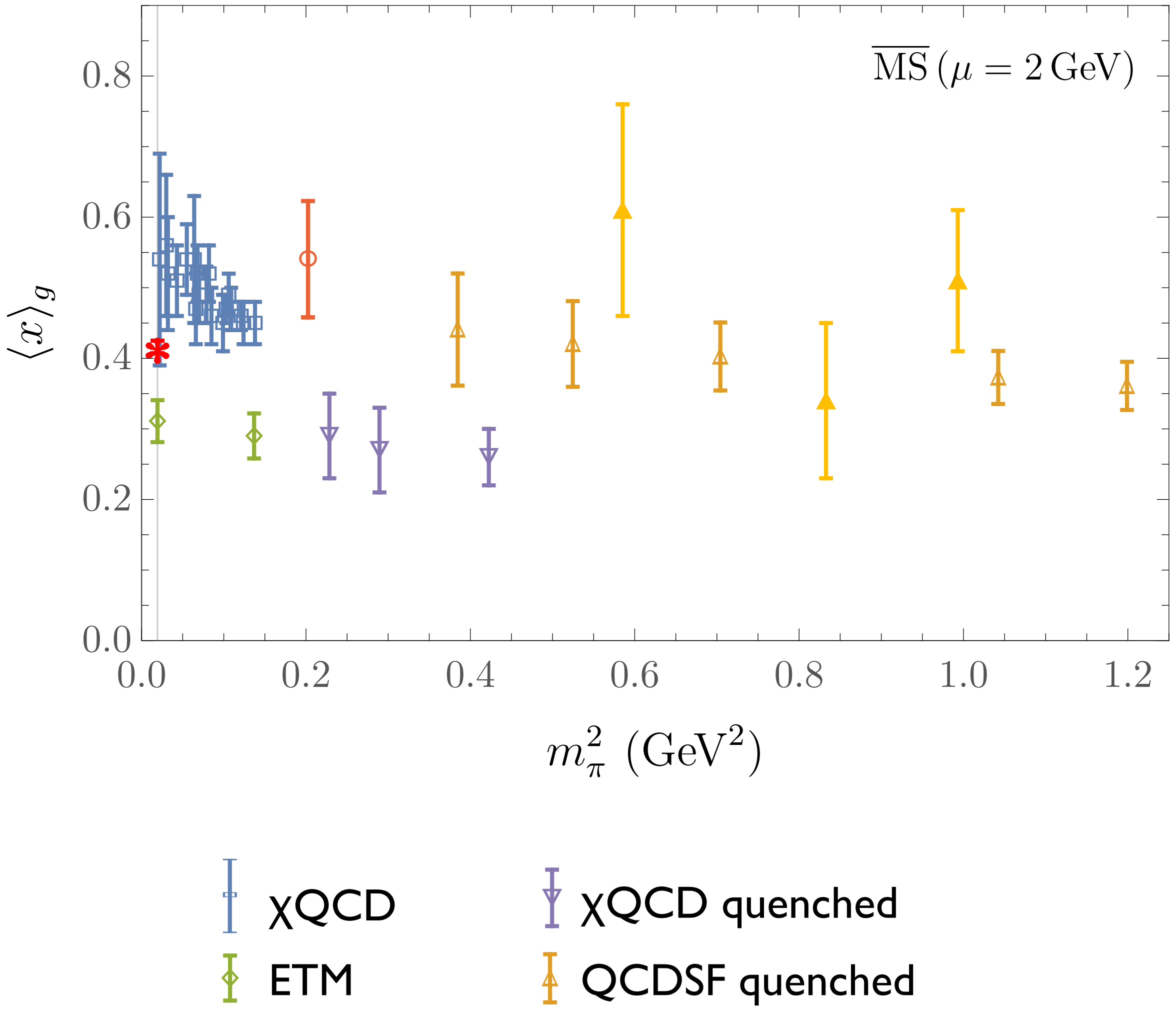
gluon: Shanahan, Detmold, PRD99, 014511 (2019)

quark: Brommel Ph.D. thesis (2007) m_π ~840 MeV

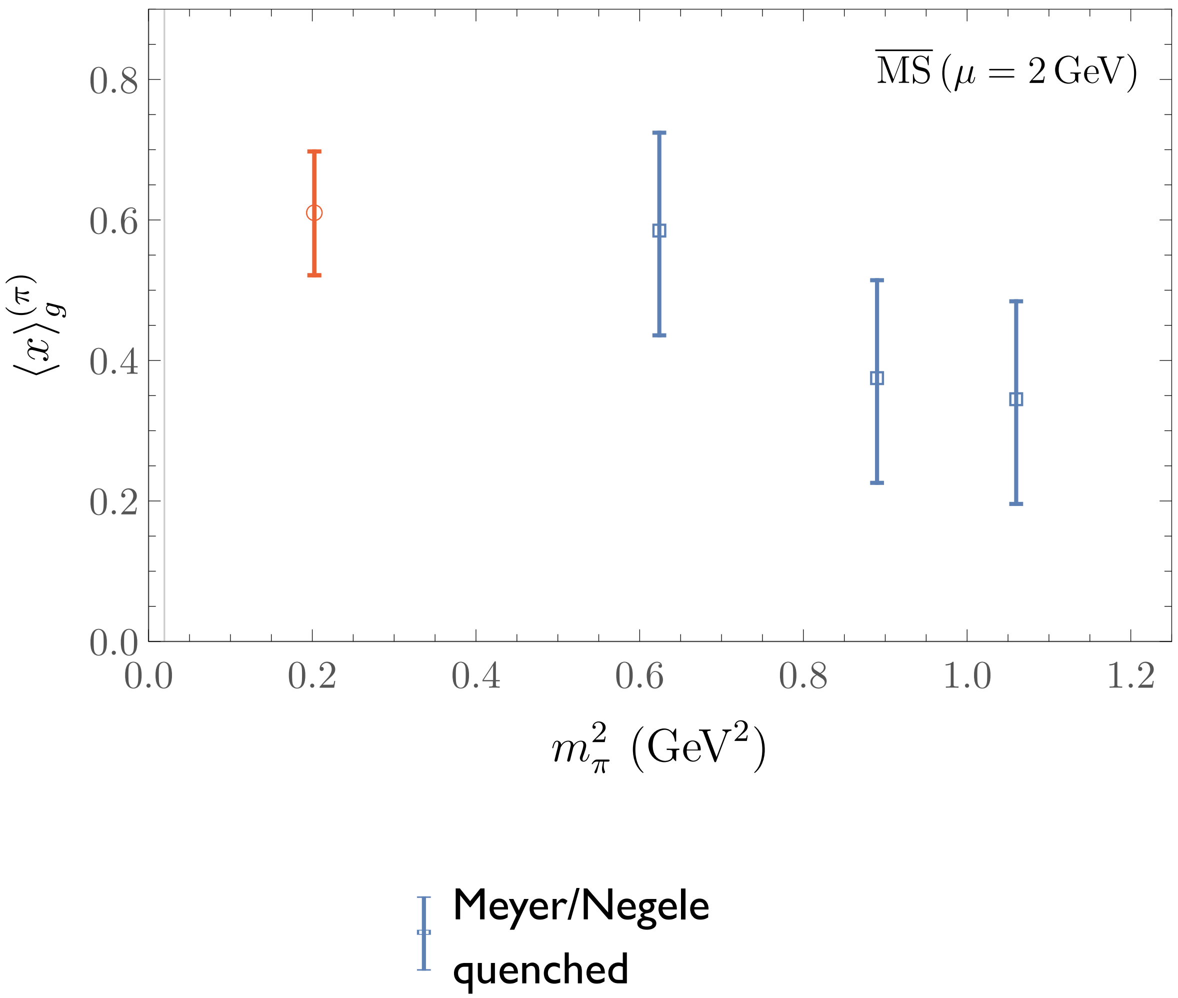
Gluon momentum fraction

Gluon momentum fraction $A_a(0) = \langle x \rangle_a$

Nucleon

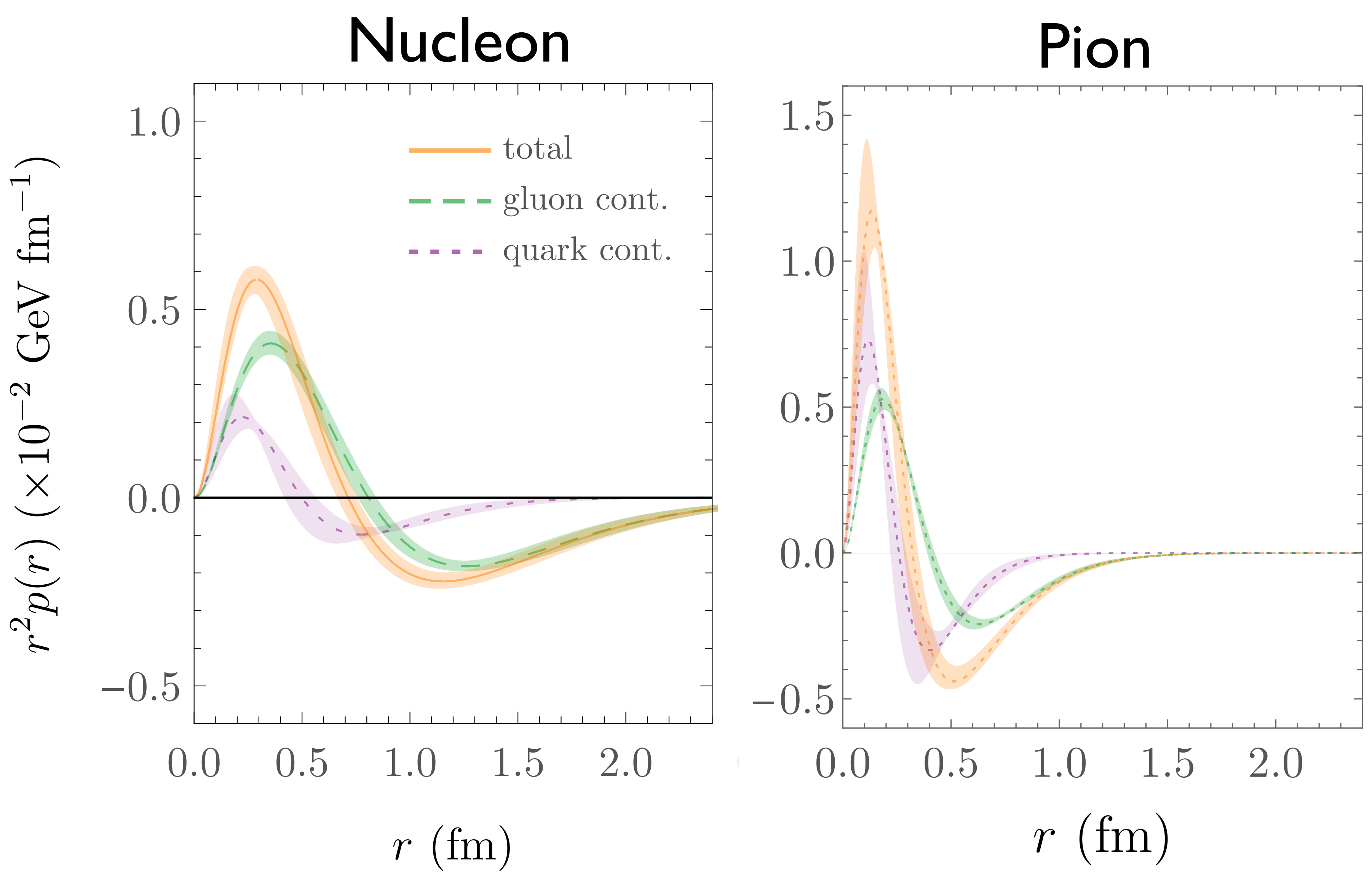


Pion



Very little pion-mass dependence within each set of calculations

LQCD pion pressure



Pion & Nucleon quark and gluon momentum fractions consistent within uncertainties, but very different pressure distributions!

m_π ~450 MeV, tripole fits

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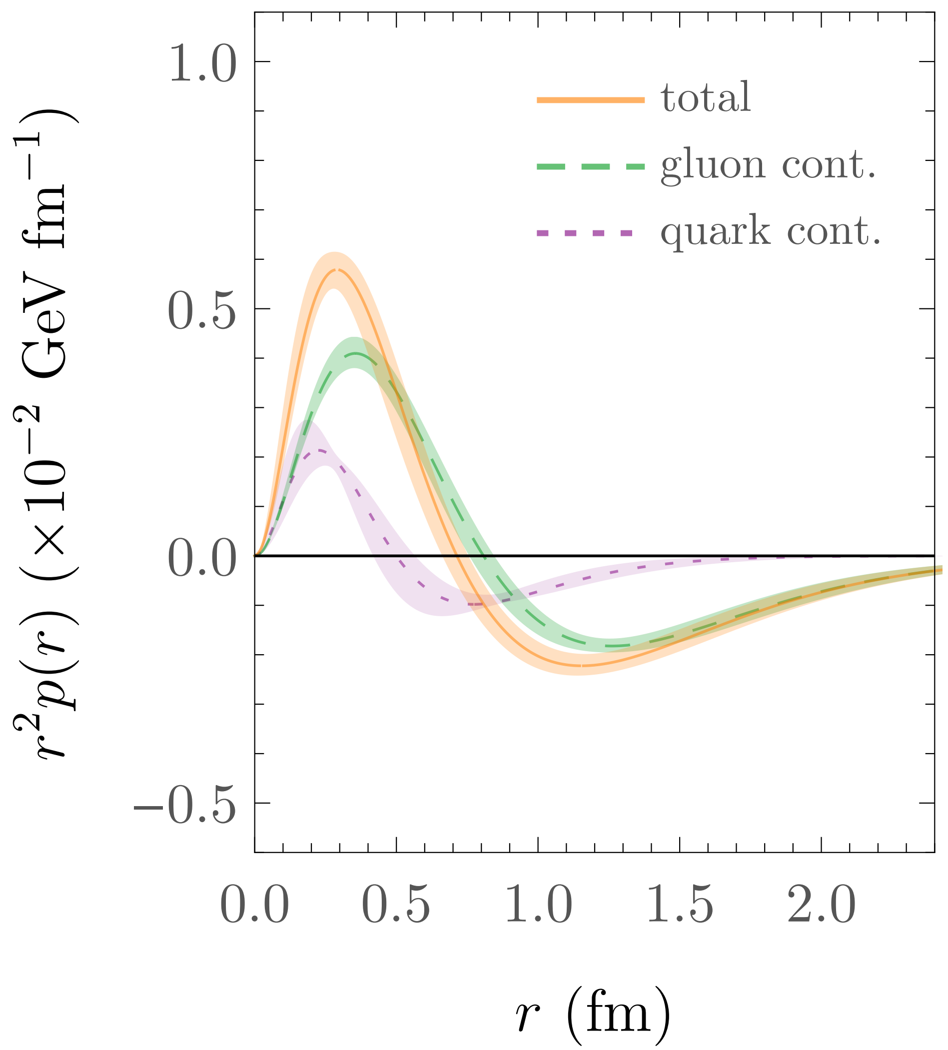
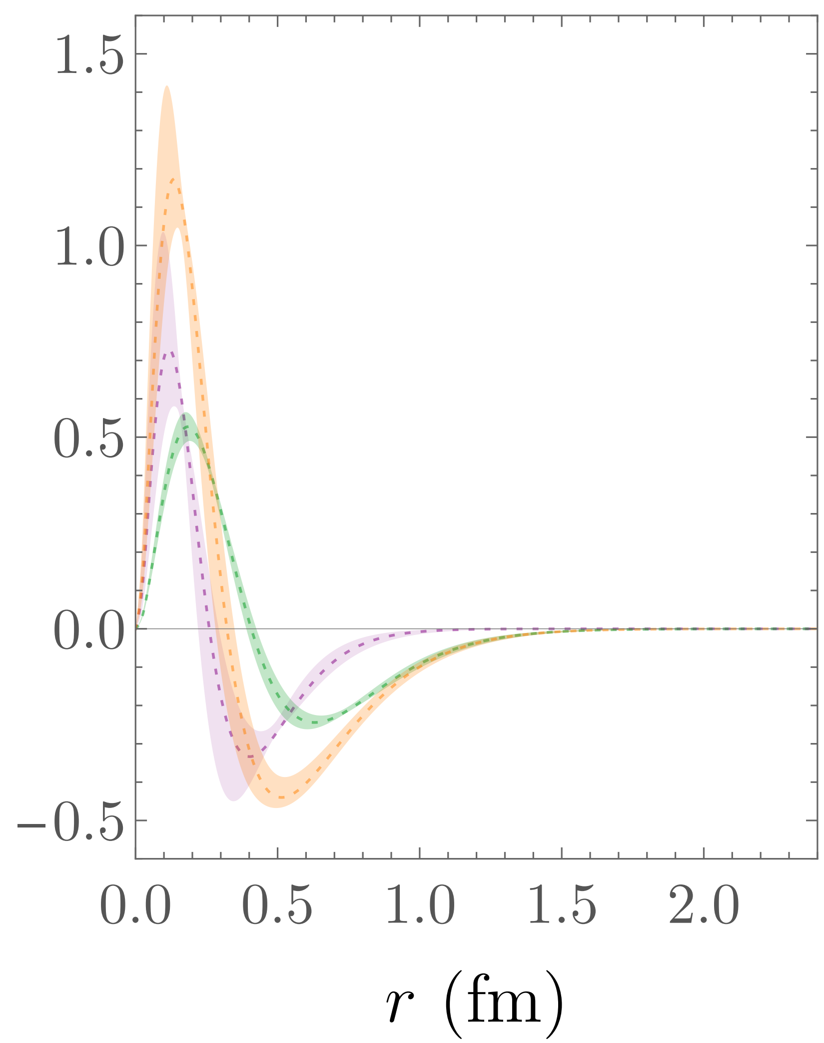
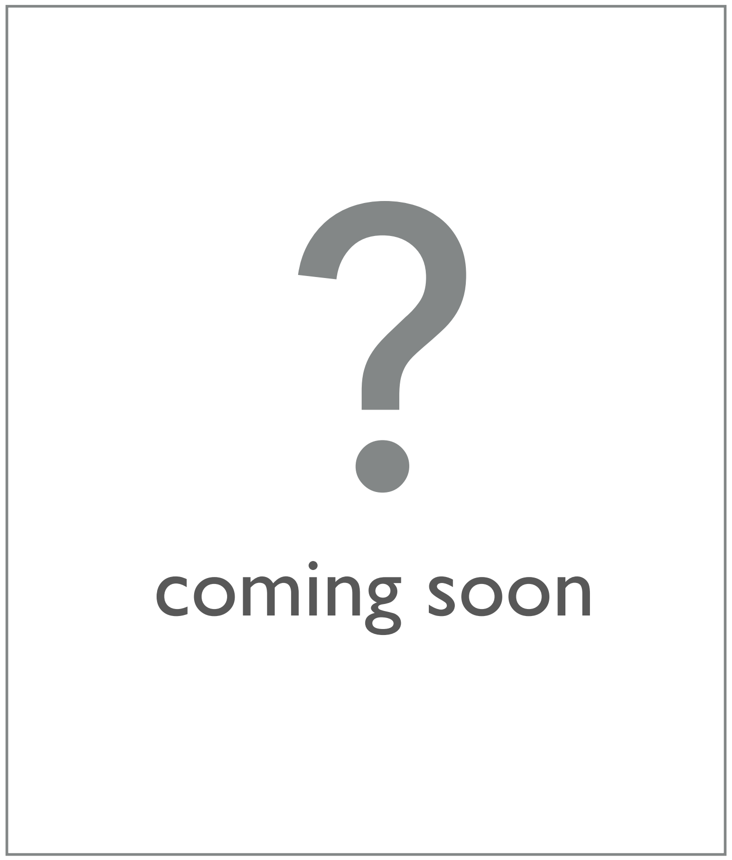
Nucleon



Pion



Light nuclei



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Gluon structure

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass
- D-term

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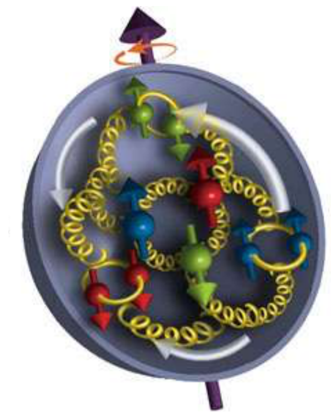
What is the gluon distribution in a proton

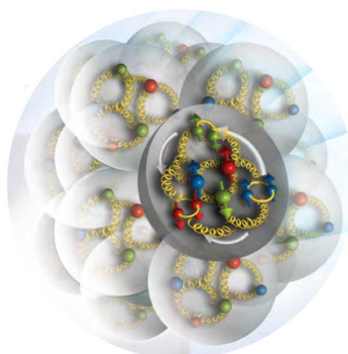
- PDFs, GPDs, TMDs
- Pressure, Shear
- 'Gluon radius'



2

How is the gluon structure of a proton modified in a nucleus

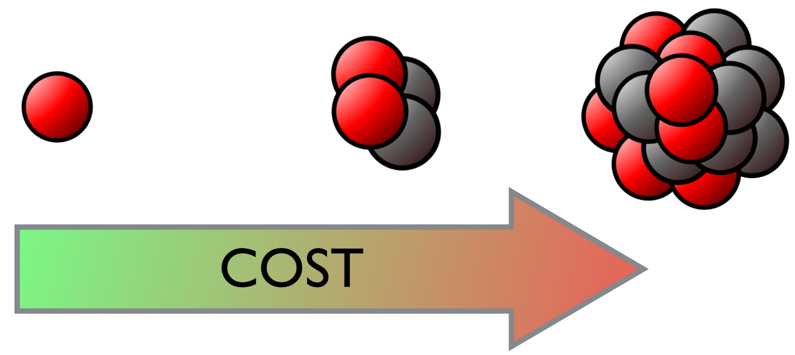
- Gluon 'EMC' effect
- Exotic glue



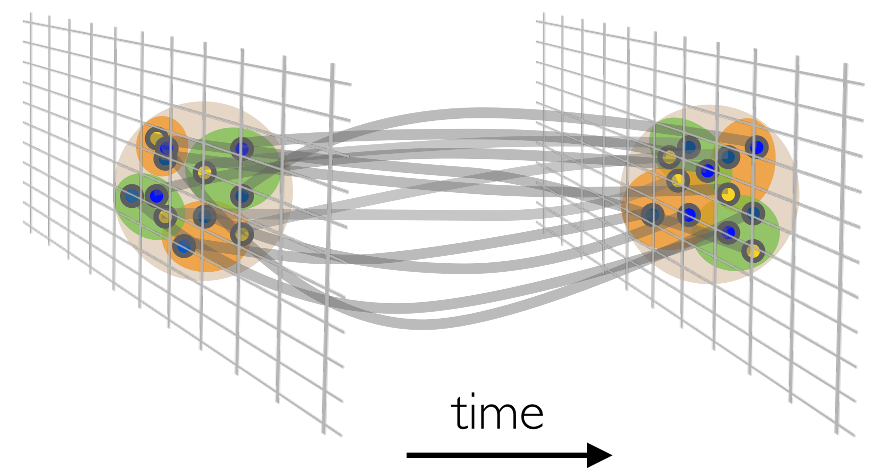
Nuclear physics from LQCD

Nuclei on the lattice: HARD

- **Noise:**
  Statistical uncertainty grows exponentially with number of nucleons



- **Complexity:**
  Number of contractions grows factorially



Calculations possible for A<5 (unphysically heavy quark masses)

Nuclear physics from LQCD

- Nuclei with A<5
- QCD with unphysical quark masses

  m_π~800 MeV, m_N~1,600 MeV

  m_π~450 MeV, m_N~1,200 MeV

- Nuclear structure: magnetic moments, polarisabilities

  [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]

- First nuclear reaction: np→dγ

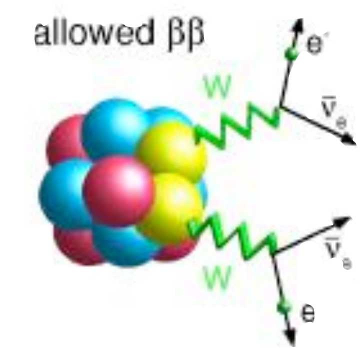
  [PRL **115**, 132001 (2015)]

- Proton-proton fusion and tritium β-decay

  [PRL **119**, 062002 (2017)]

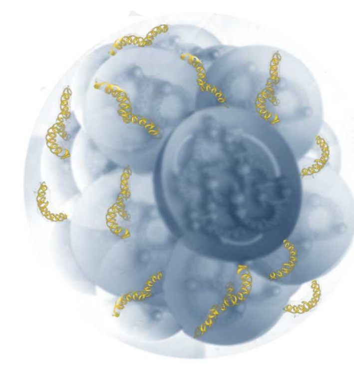
- Double β-decay

  [PRL **119**, 062003 (2017), PRD **96**, 054505 (2017)]



- Gluon structure of light nuclei

  [PRD **96** 094512 (2017)]



- Scalar, axial and tensor MEs

  [PRL **120,** 152002 (2018)]

Gluon structure of nuclei

2

How does the gluon structure of a nucleon change in a nucleus?

Ratio of structure function F_2 per nucleon for iron and deuterium

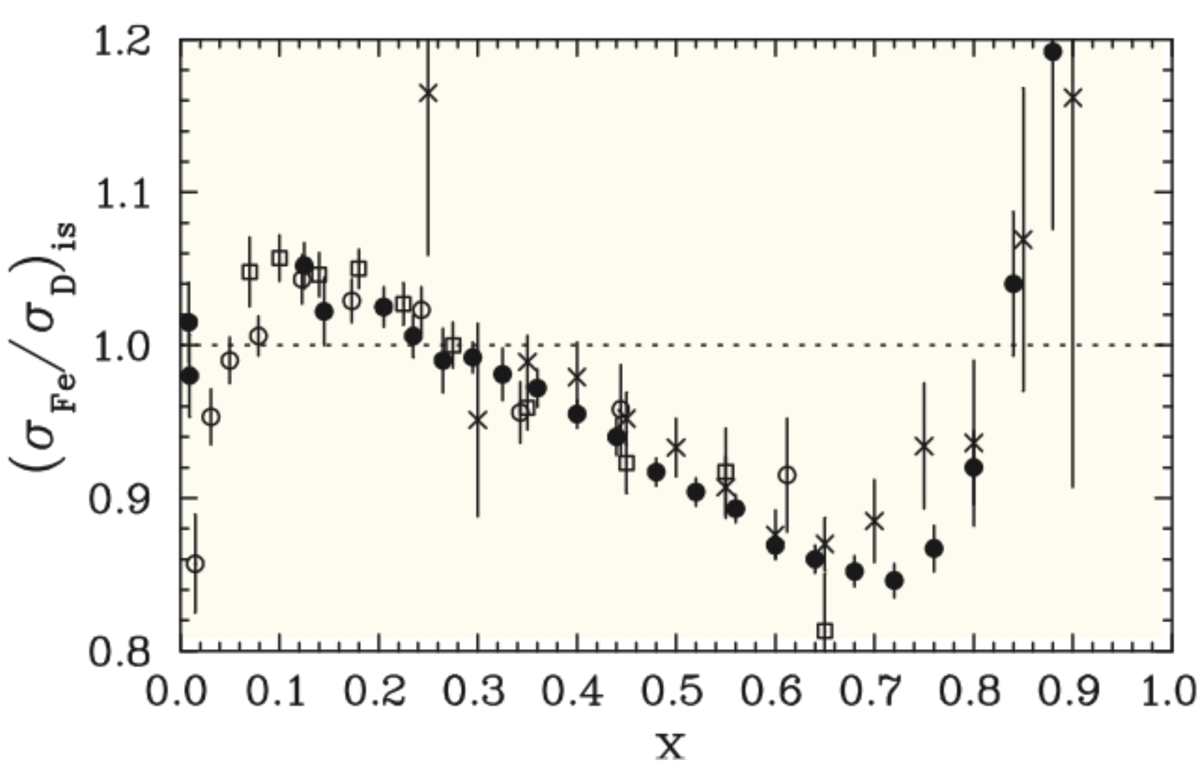
$$F_2(x, Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2 \left[q(x, Q^2) + \bar{q}(x, Q^2)\right]$$

Number density of partons of flavour q

European Muon Collaboration (1983): "EMC effect"

Modification of per-nucleon cross section of nucleons bound in nuclei

Gluon analogue?



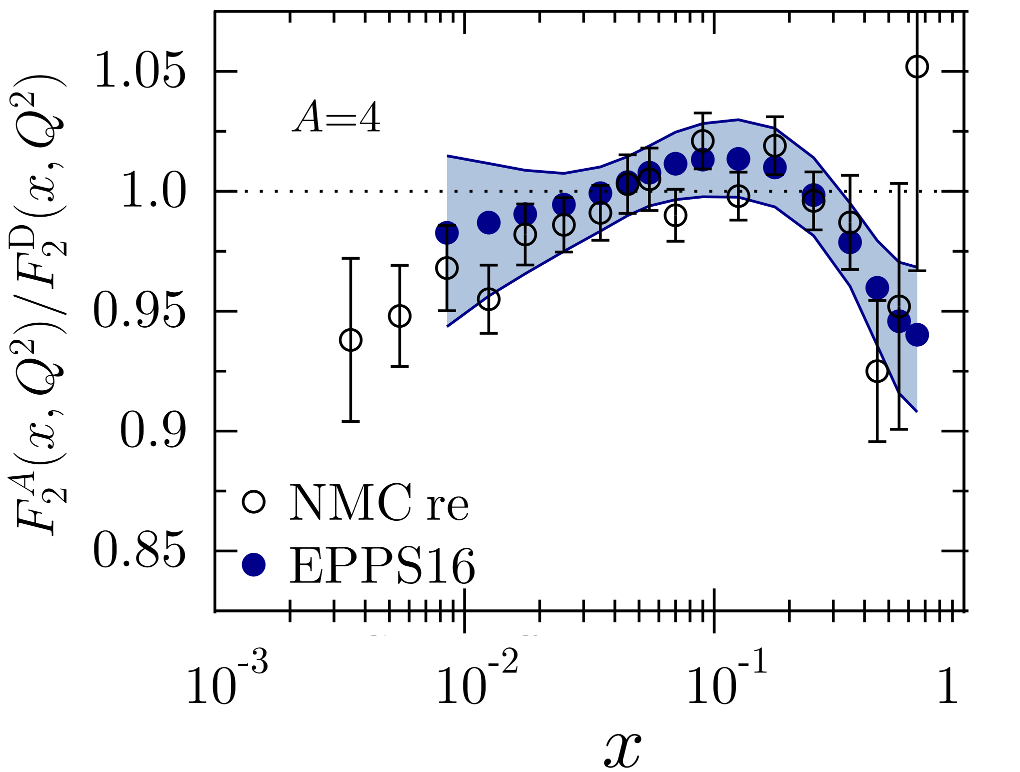
Longitudinal momentum fraction

EMC effects in Mellin moments

First investigation of EMC-type effects from LQCD: Nuclear effects in Mellin moments of PDFs

- Calculable from local operators
- **BUT** EMC effects in moments are very small

[Eskola et al., Eur. Phys. J. C 163 (2017)]



Classic EMC effect is defined in F_2:

$$F_2(x,Q^2) = \sum_{q=u,d,s\ldots} x\, e_q^2 \left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$

Number density of partons of flavour q

x-integrals of numerator and denominator $\int_0^1 dx\, x^n q(x,Q^2)$

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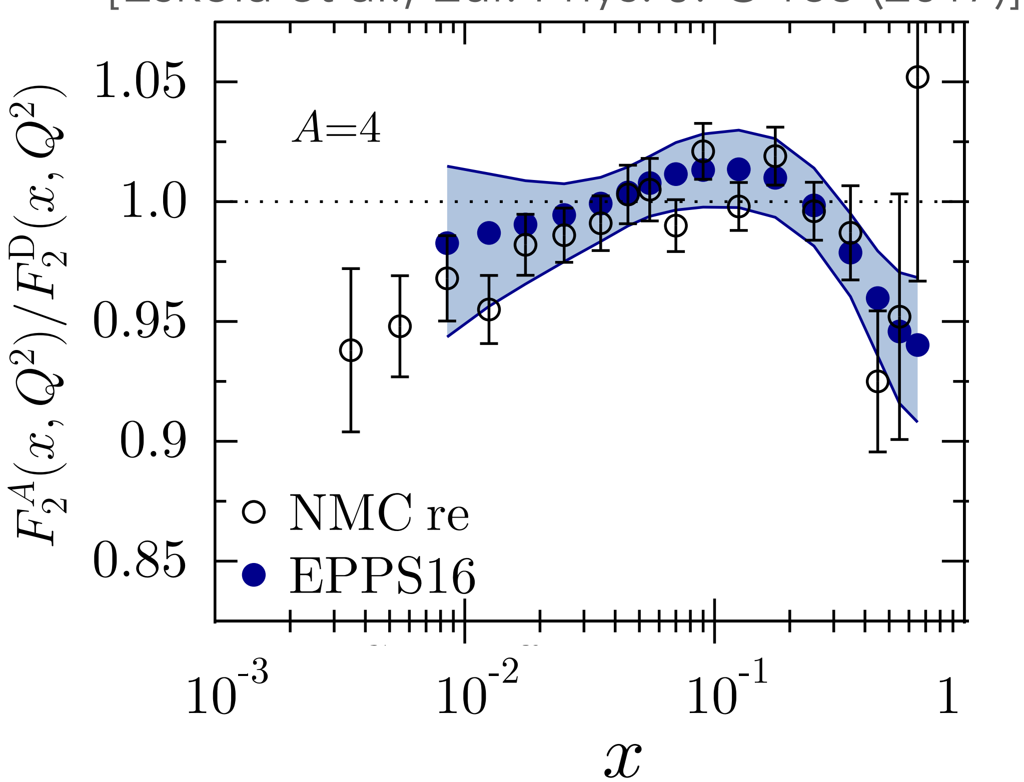
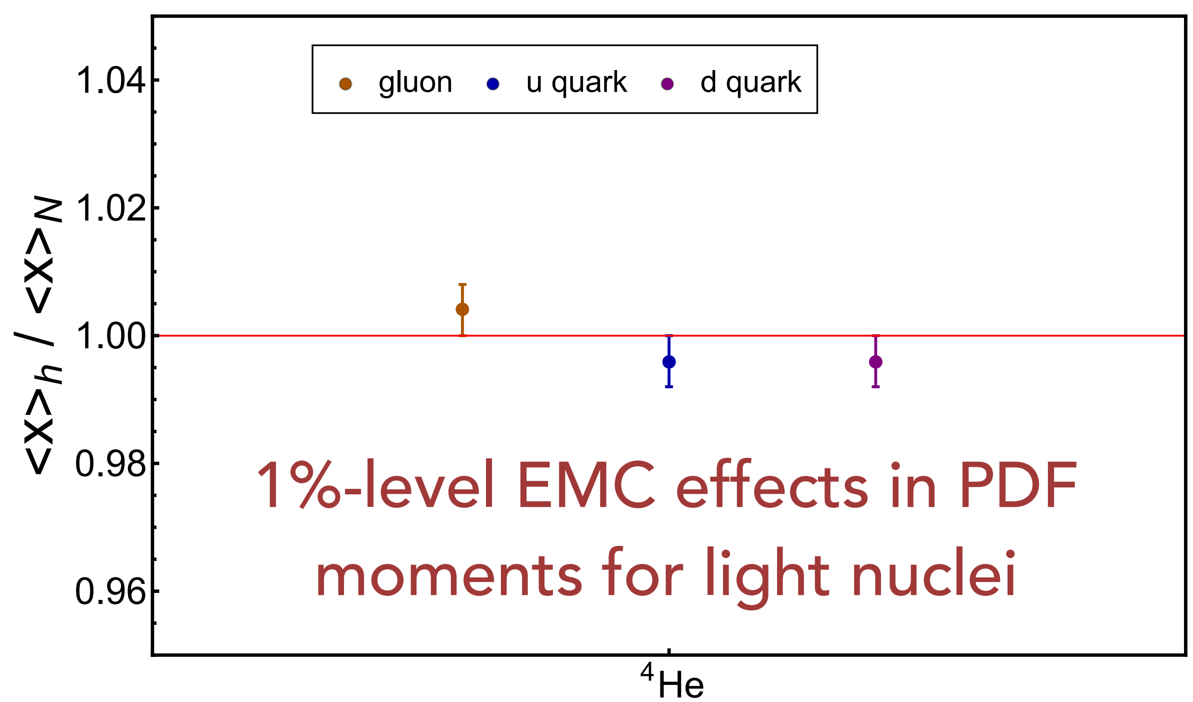




Momentum fraction of nuclei

First investigation of EMC-type effects from LQCD:
Nuclear effects in Mellin moments of PDFs

- Lowest Mellin moment of spin-independent PDF defines fraction of momentum of nucleus A carried by parton of type f

$$\langle x \rangle_A^f = \int_0^1 dx\, x f^A(x) \qquad \sum_{f=q,g} \langle x \rangle_h^f = 1$$

- Momentum sum rule implies **nucleus-independent ratio of quark and gluon EMC effects** in the first moment

$$\left(\frac{\langle x \rangle_A^f}{\langle x \rangle_p^f} - 1 \right) = E_A^f \qquad \frac{E_A^g}{E_A^q} = -\frac{\langle x \rangle_p^q}{\langle x \rangle_p^g} \approx -1.4$$

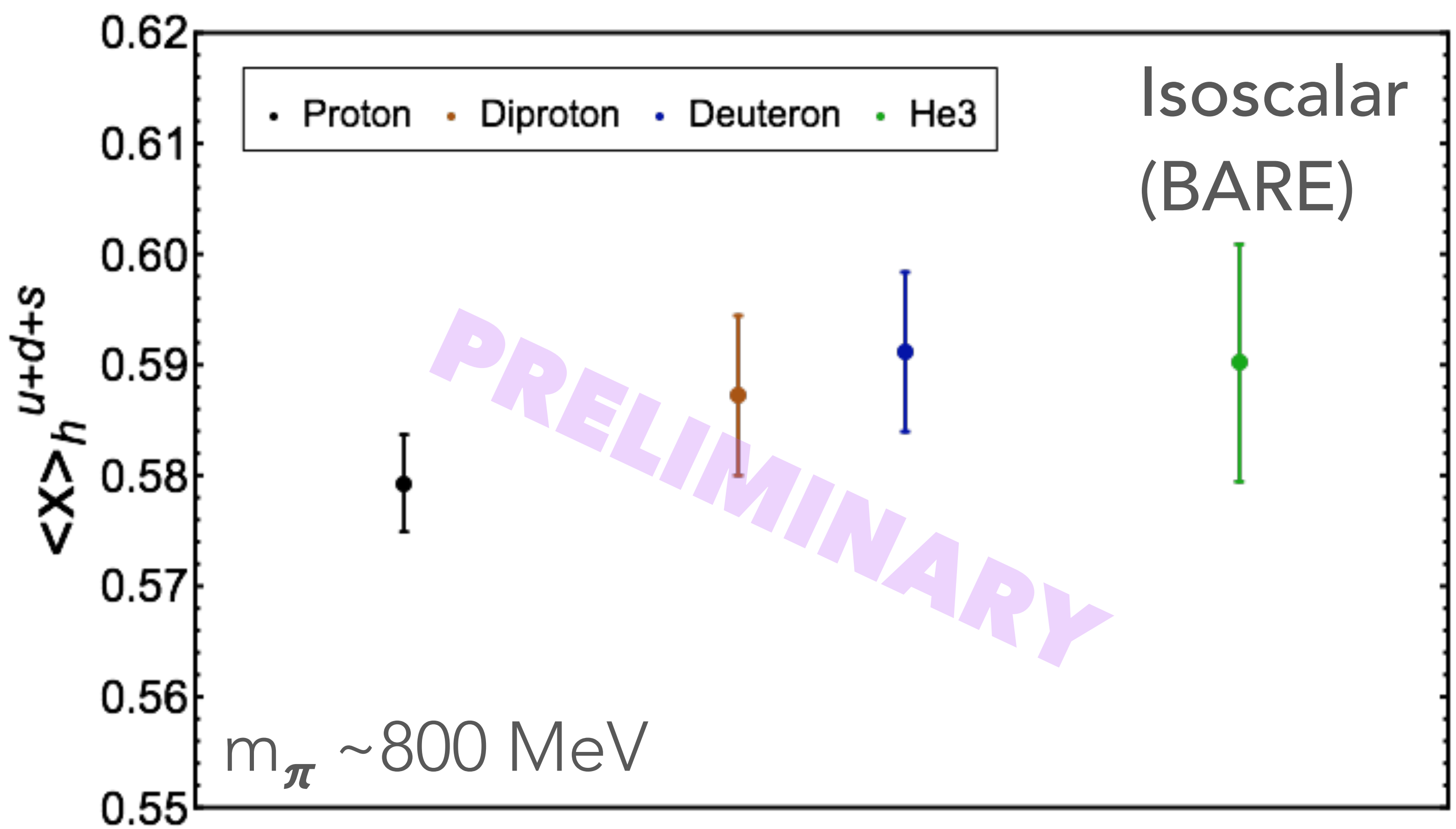
$\overline{\mathrm{MS}}\ (\mu = 2\mathrm{GeV})$

Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

- Few-percent determination of quark momentum fraction
  ~10% determination of strange quark contributions



NPLQCD

Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

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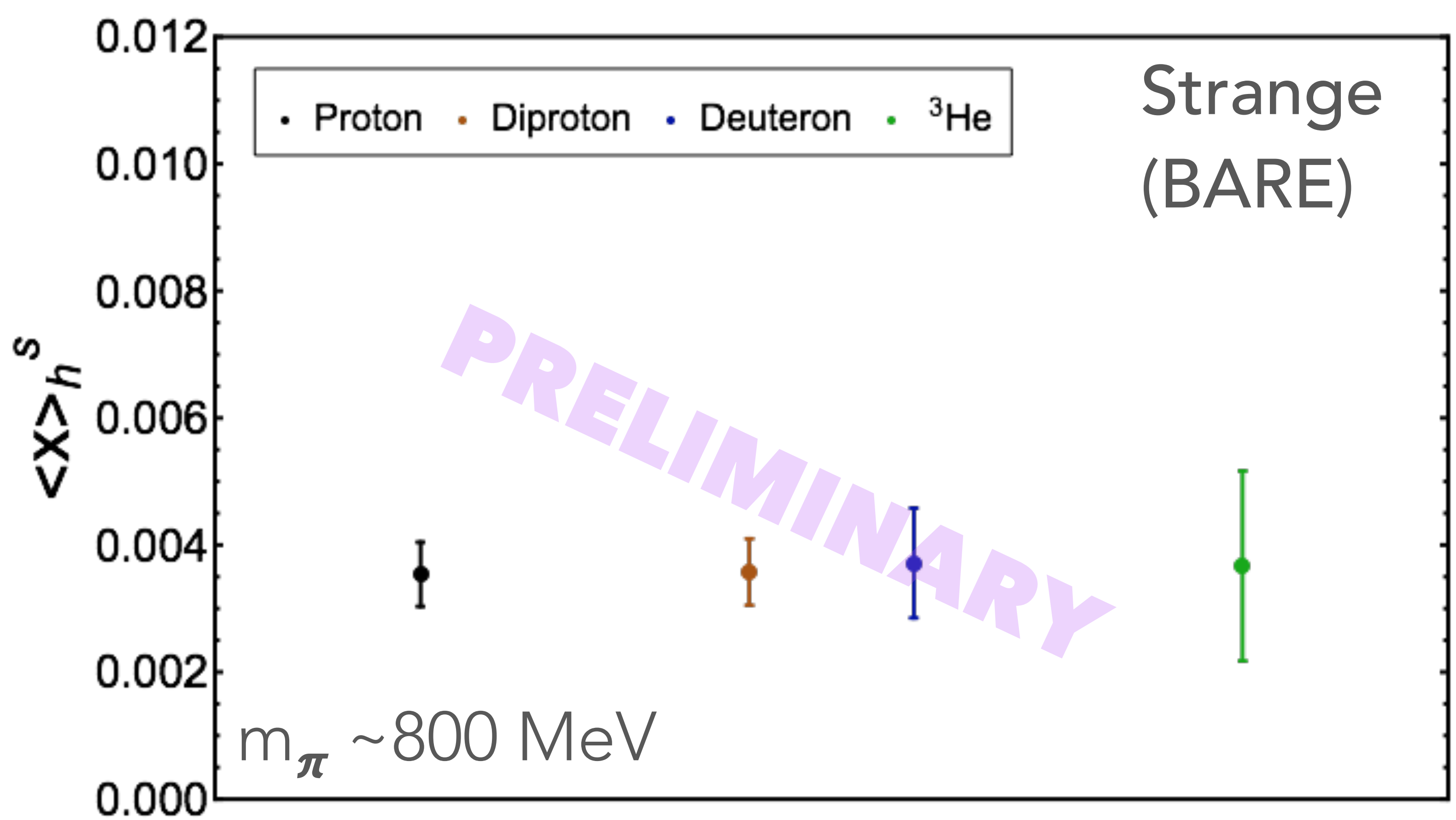


NPLQCD

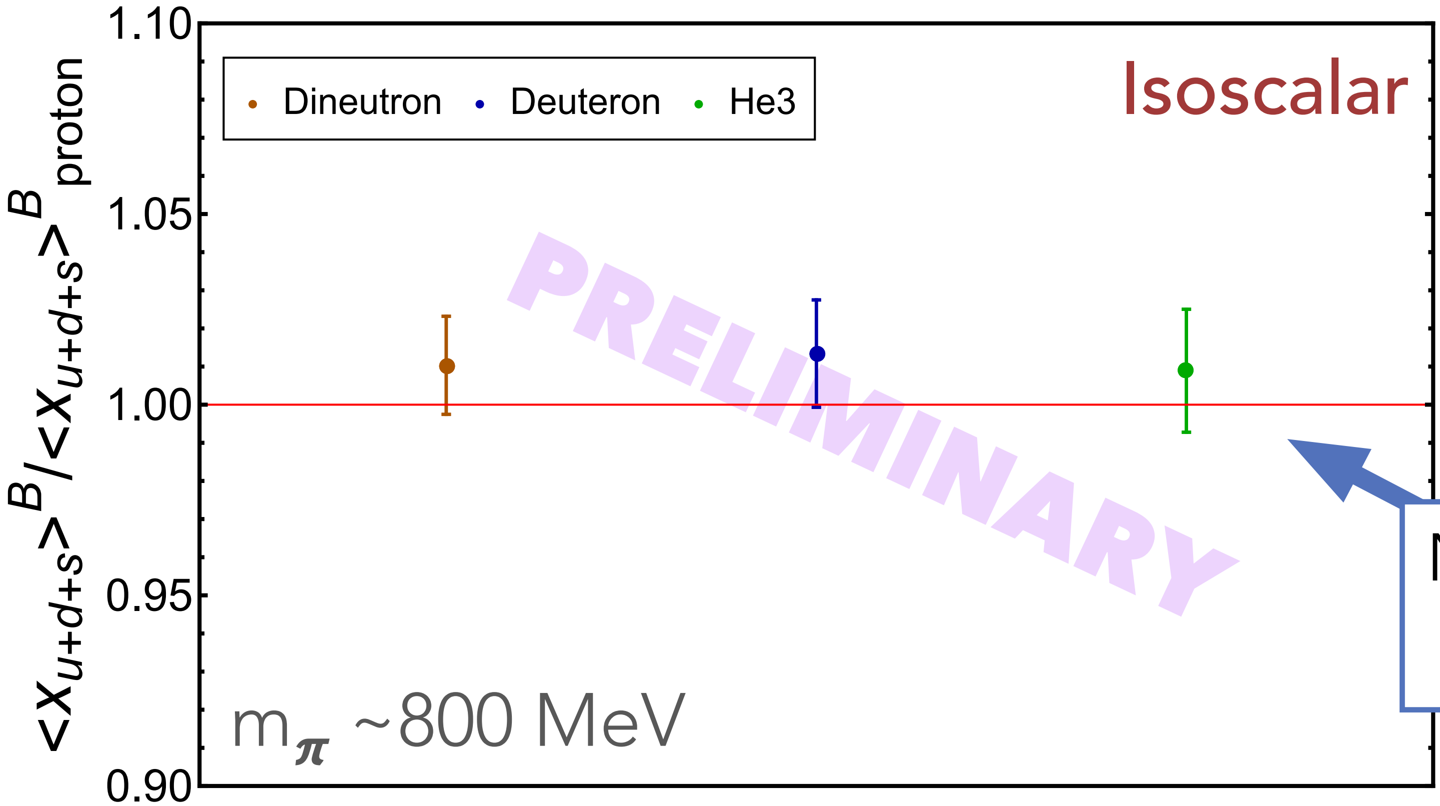
Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

- Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology

Ratio of quark momentum fraction in nucleus to nucleon



- Small mixing with gluon EMT operators (neglected)
- Sum rule constraint

Normalised to proton result

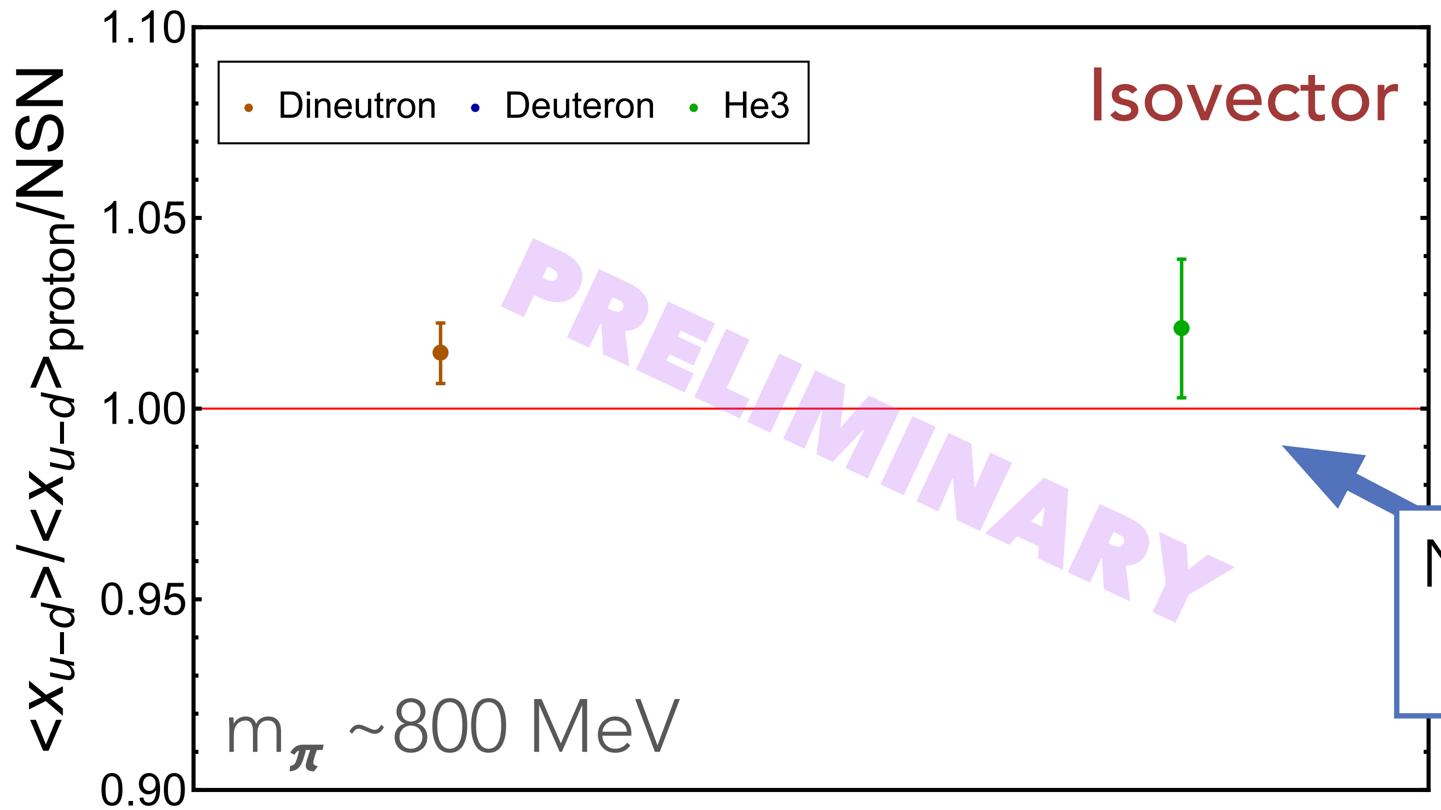
Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

- Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology

Ratio of quark momentum fraction in nucleus to nucleon



- No mixing
- No sum rule

Normalised to proton result

Gluon momentum fraction of nuclei

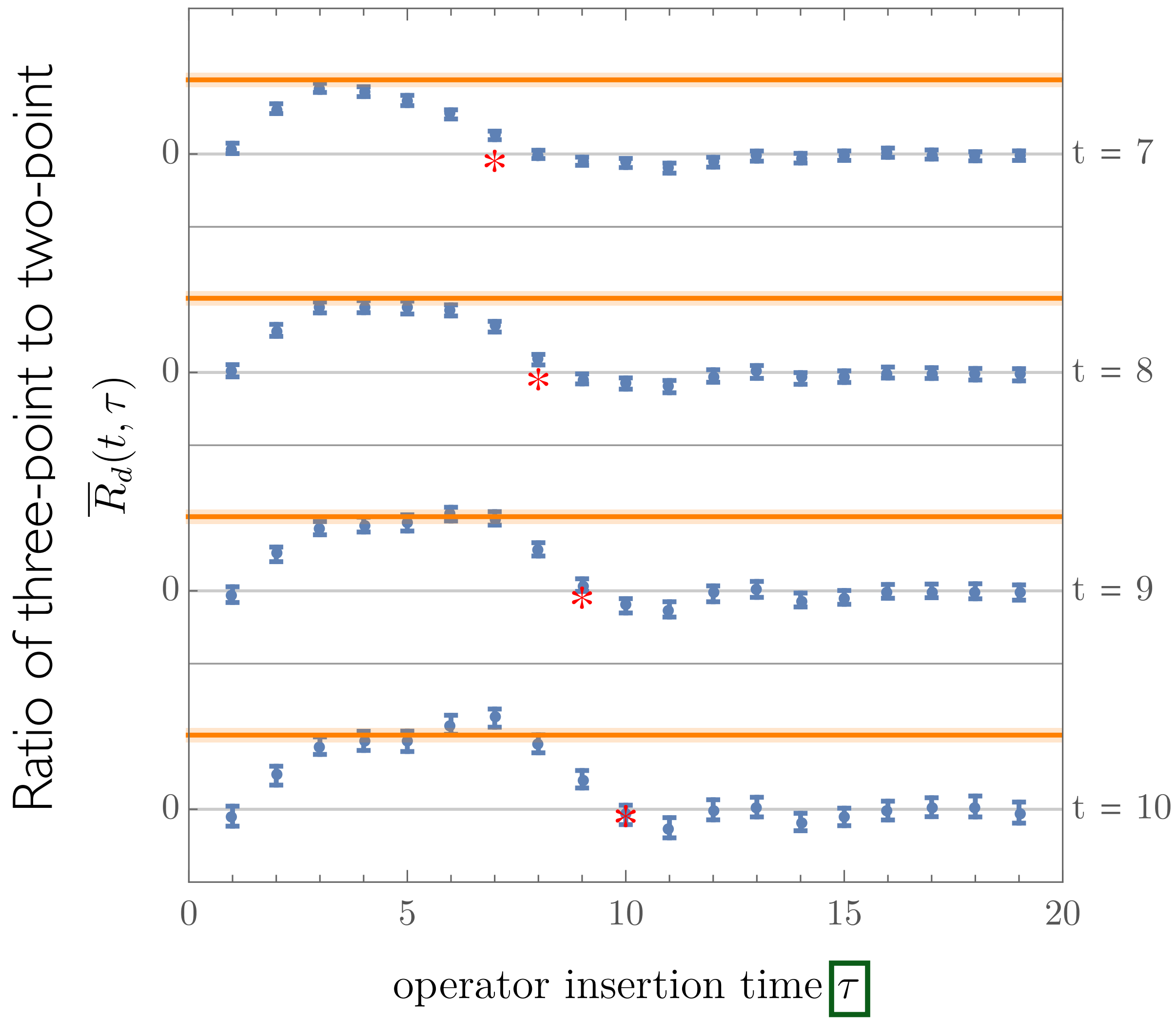
Matrix elements of the Energy-Momentum Tensor in light nuclei
→ **first QCD determination of momentum fraction of nuclei**

Doubly challenging:

- Nuclear matrix element
- Gluon observable (suffer from poor signal-to-noise)
- **BUT**: clean signals at ~5% precision

[NPLQCD PRD96 094512 (2017)]

Deuteron gluon momentum fraction



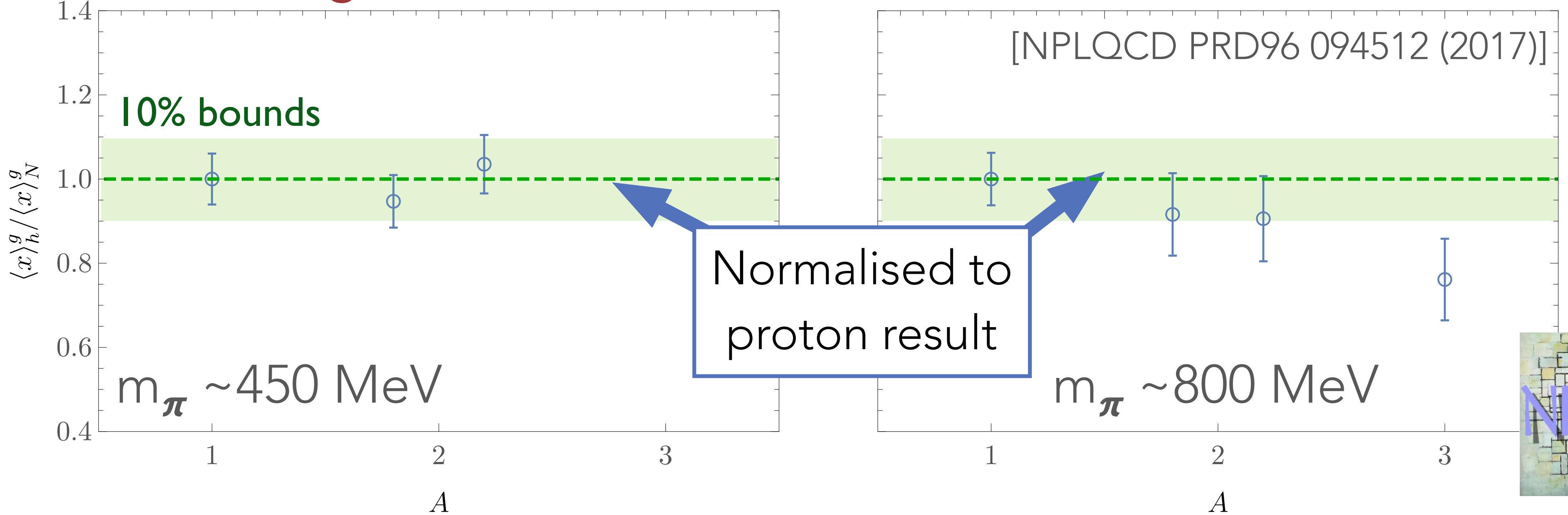
Gluon momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

→ **first QCD determination of momentum fraction of nuclei**

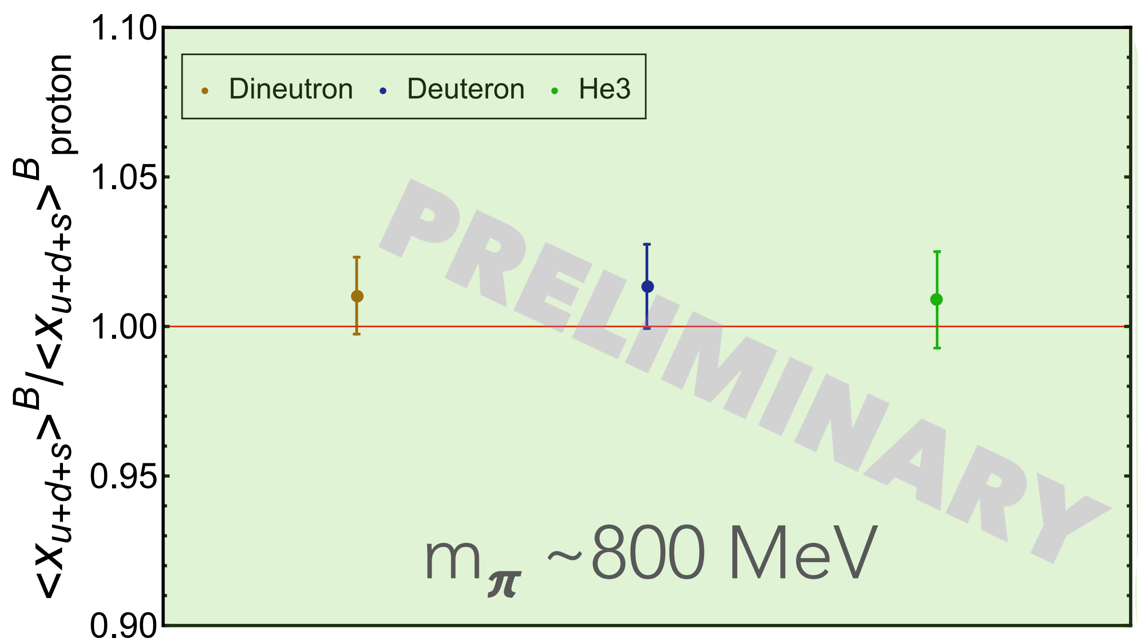
- **Constraints at ~10% level on EMC-effect in gluon momentum fraction**
- Small mixing with quark EMT operators (neglected)
- Sum rule constraint

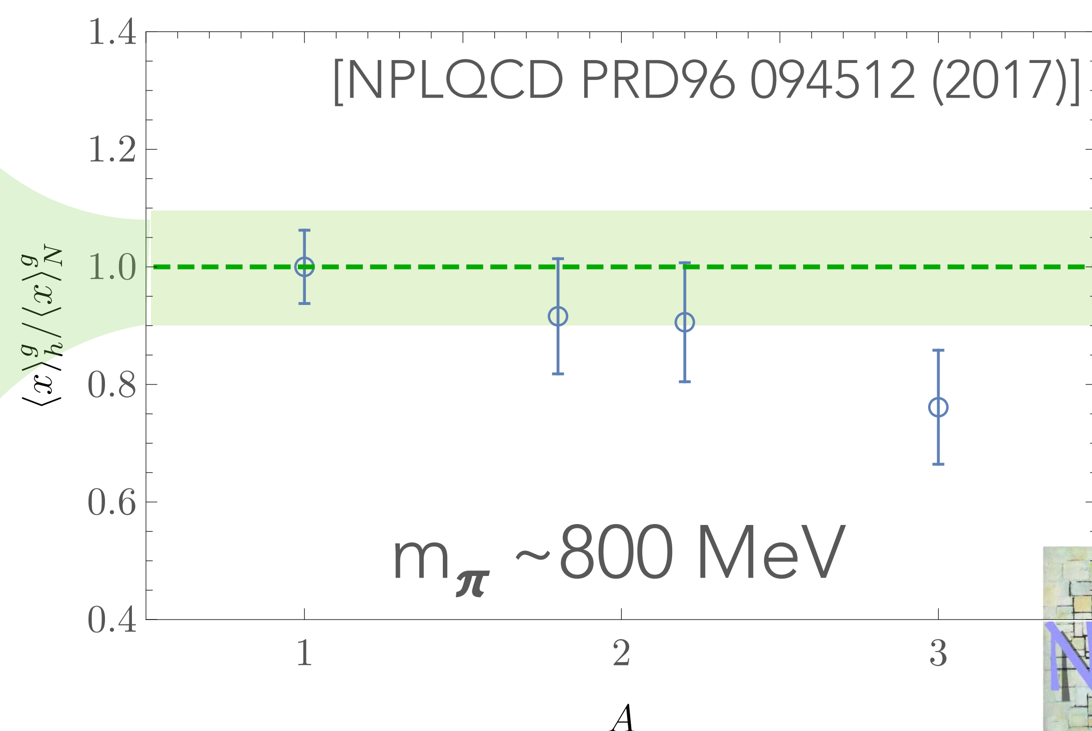
Ratio of gluon momentum fraction in nucleus to nucleon



Momentum fractions of nuclei

- First determination of all components of momentum decomposition of light nuclei
- Small mixing between quark and gluon EMT operators neglected
- Constraint on either quark or gluon EMC in this quantity implies constraint on the other from sum rules:





Gluon structure of nuclei

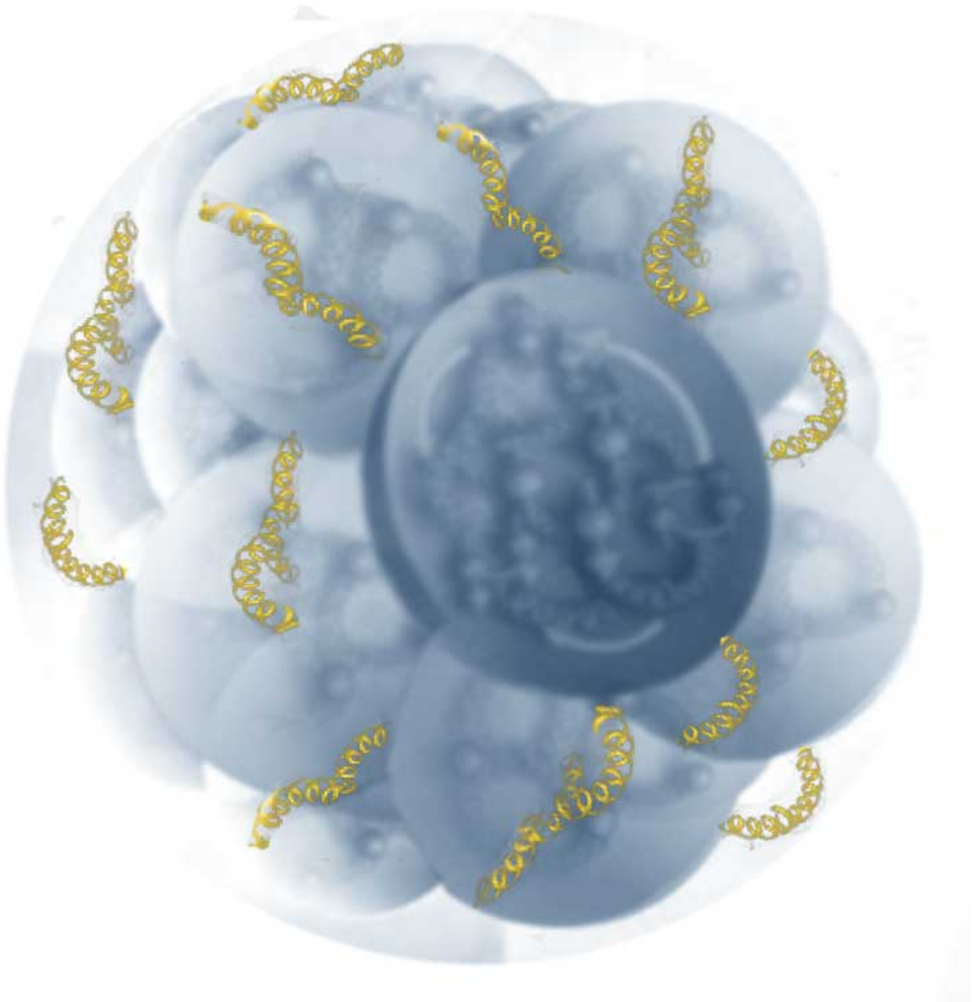
Exotic Glue

Contributions to nuclear structure from gluons not associated with individual nucleons in nucleus

Exotic glue operator:

nucleon $\langle p|\mathcal{O}|p\rangle = 0$

nucleus $\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$

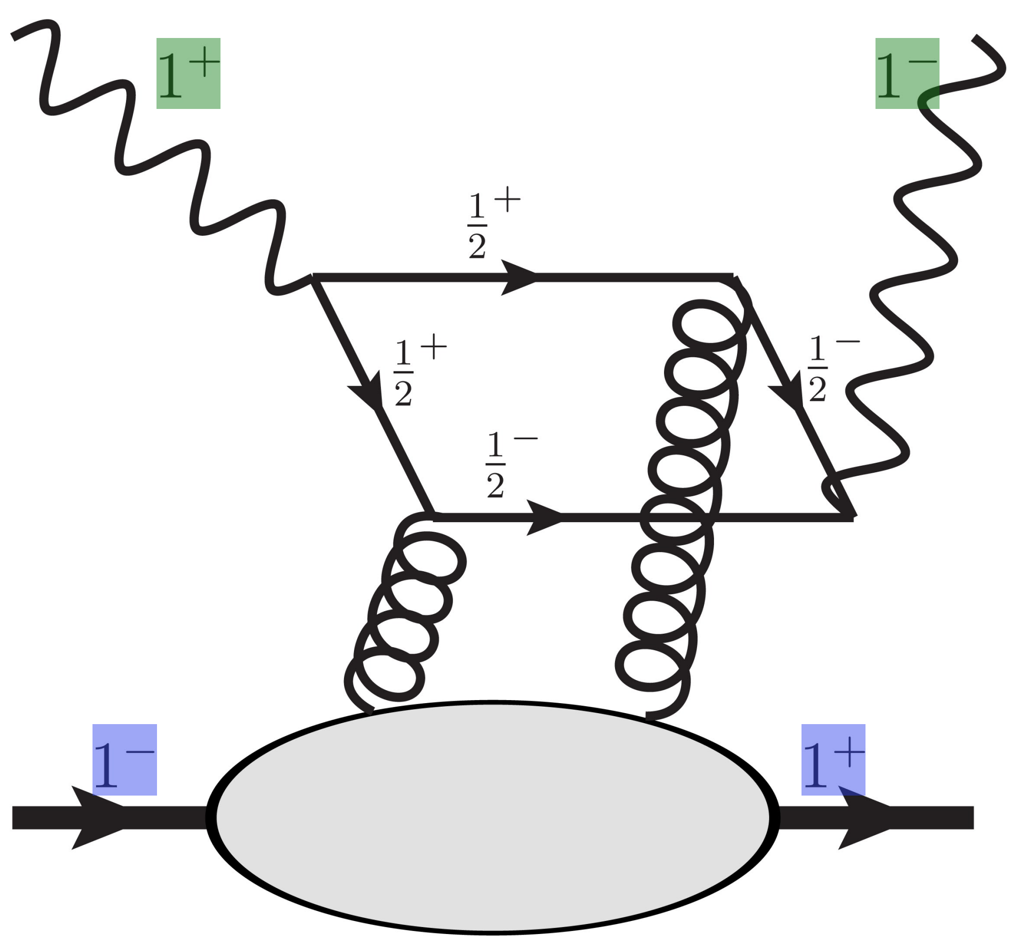


Jaffe and Manohar, "Nuclear Gluonometry"
Phys. Lett. B223 (1989) 218

Gluonic Transversity

Double helicity flip structure function Δ(x,Q²)

Changes both photon and target helicity by 2 units



- **Unambiguously gluonic:** no analogous quark PDF at twist-2
- Non-vanishing in forward limit for targets with spin≥1
- **Experimentally measurable** in unpolarised electron DIS on polarised target
  - Nitrogen target: JLab LoI 2015
  - Polarised nuclei at EIC
- **Moments calculable in LQCD**

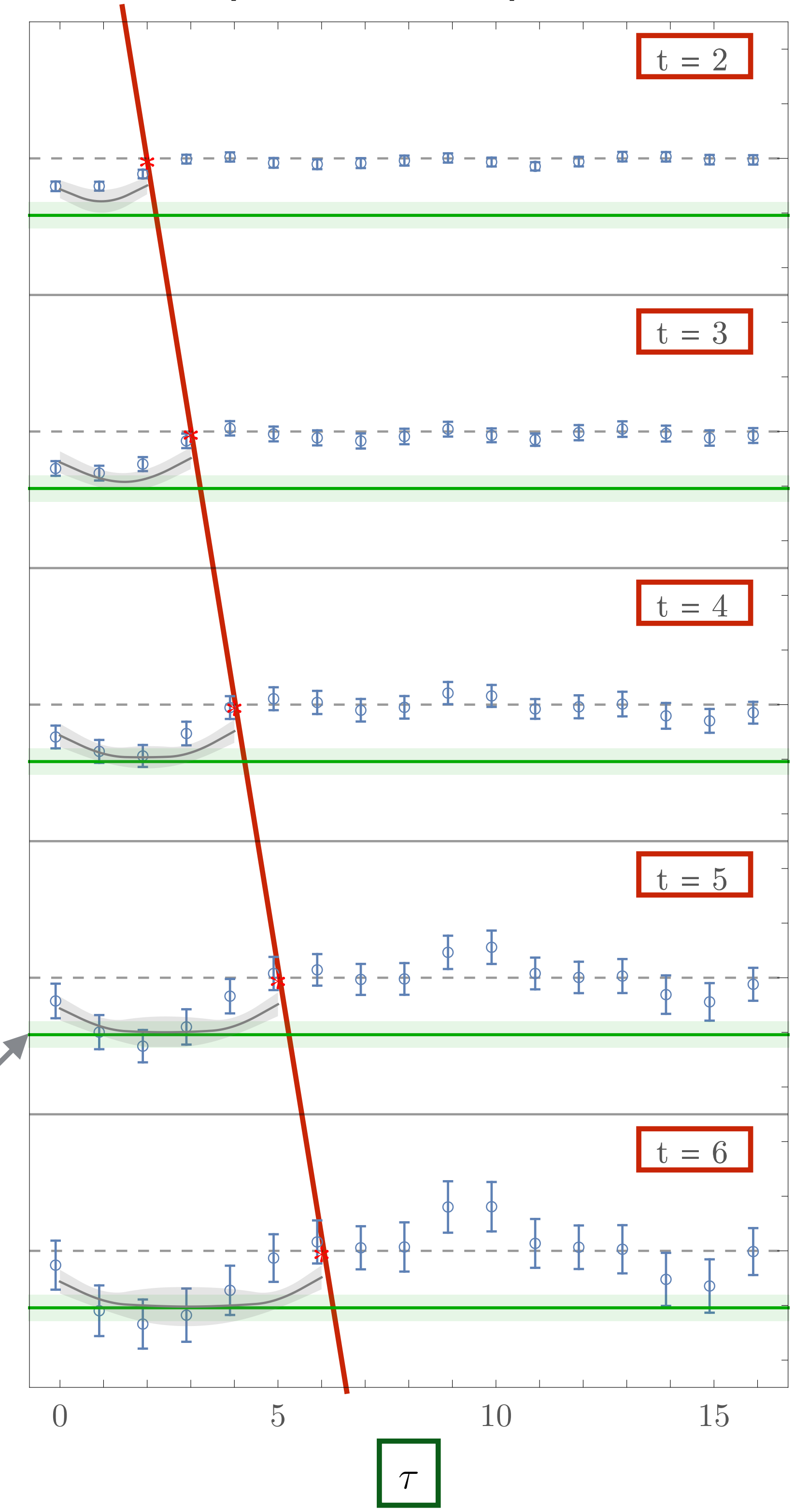
Non-nucleonic glue in deuteron

NPLQCD Collaboration PRD96 094512 (2017)

First moment of gluon transversity distribution in the deuteron,
m_π ~800 MeV

- **First evidence for non-nucleonic gluon contributions to nuclear structure**
- **Hypothesis of no signal ruled out to better than one part in 10^7**
- Magnitude relative to momentum fraction as expected from large-N_c

Ratio of 3pt and 2pt functions



Ratio ∝ matrix element for $0 \ll \tau \ll t$

Gluon structure from LQCD

- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
  - Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)
  - **First determination of gluon contributions to shear and pressure distributions in the proton**
    - Supports analysis assumptions in recent experimental determination
    - Suggests target kinematics for future model-independent extractions at JLab12 and EIC
  - Compare quark and gluon distributions in hadrons and nuclei
- **Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature**