13th European Research Conference on Electromagnetic Interactions with Nucleons and Nuclei

27 October – 02 November 2019

Paphos, Cyprus

Summary of Workshop 1

Distribution Functions: Lattice QCD meets phenomenology

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Jefferson Lab

TMD Collaboration

U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

One-page summary: QCD and Hadron Structure

- 14 talks on Theory: Lattice meets Phenomenology

7 Lattice:

Richards – Extraction of pseudo-PDFs
Scapellato – Quasi-GPDs using twisted mass fermions
Koutsou – Overview of lattice results on nucleon moments
Urbach – Overview of meson results
Engelhardt – Overview of lattice computations of TMDs
Jansen – Quasi-PDFs
Zafeiropoulos – Extracting pseudo-PDFs

7 Phenomenology:

Nocera – Connecting PDFs from phenomenology and lattice QCD
Qiu – Overview of direct evaluation of parton distribution functions
Harland-Lang – PDFs from phenomenology
Sato – Polarized PDFs from phenomenology
Thomas – New insights into the EMC effects
Ji – Large momentum effective theory (remote)
Zhang – Renormalization of non-local operators

- 5 Talks on Experiment: Hadron structure

Cividini – Measurement of helicity dependence of π^0 photoproduction on deuteron
Martel – Accessing nucleon polarizabilities with Compton scattering
Mornacchi – Proton scalar polarizabilities at MAMI
Ahmed – Study of time-like nucleon form factors at BESIII
Wang – Studies of time-like hyperon form factors at BESIII

A2 collaboration @ MAMI

- **3 talks:**

  Also see Theory talk by B. Pasquini

  - Measurement of helicity dependence of π^0 photoproduction on deuteron – Cividini
  - Accessing nucleon polarizabilities with Compton scattering – Martel
  - Proton scalar polarizabilities at MAMI – Mornacchi

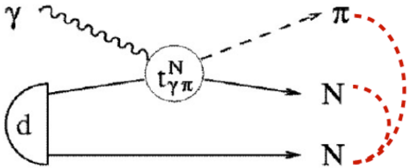
- **Double polarization observable E:**

  Polarized photon beam on polarized hadron targets

$$\frac{d\sigma}{d\Omega} = \sigma_0 \{1 \pm P_z^T P_\odot^\gamma E\}$$

$$E = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}} = \frac{N^{\uparrow\uparrow} - N^{\uparrow\downarrow}}{N^{\uparrow\uparrow} + N^{\uparrow\downarrow}} \cdot \frac{1}{P_t} \cdot \frac{1}{P_\gamma}$$

- **Deuteron as a neutron source:**

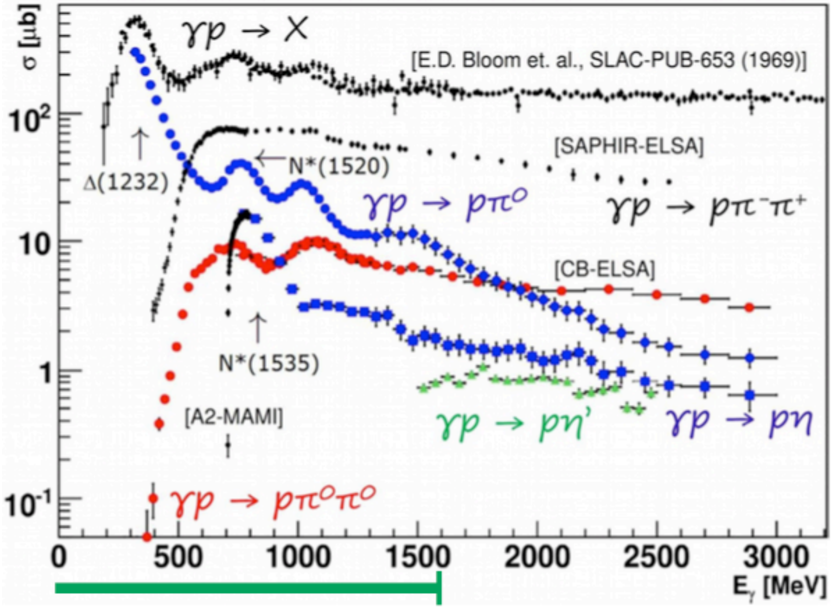


Final-state interaction

- **Single π0 on deuteron:**

$$[\gamma + d \rightarrow \pi^0 + X]$$

Construct E on proton or neutron



A2 energy range

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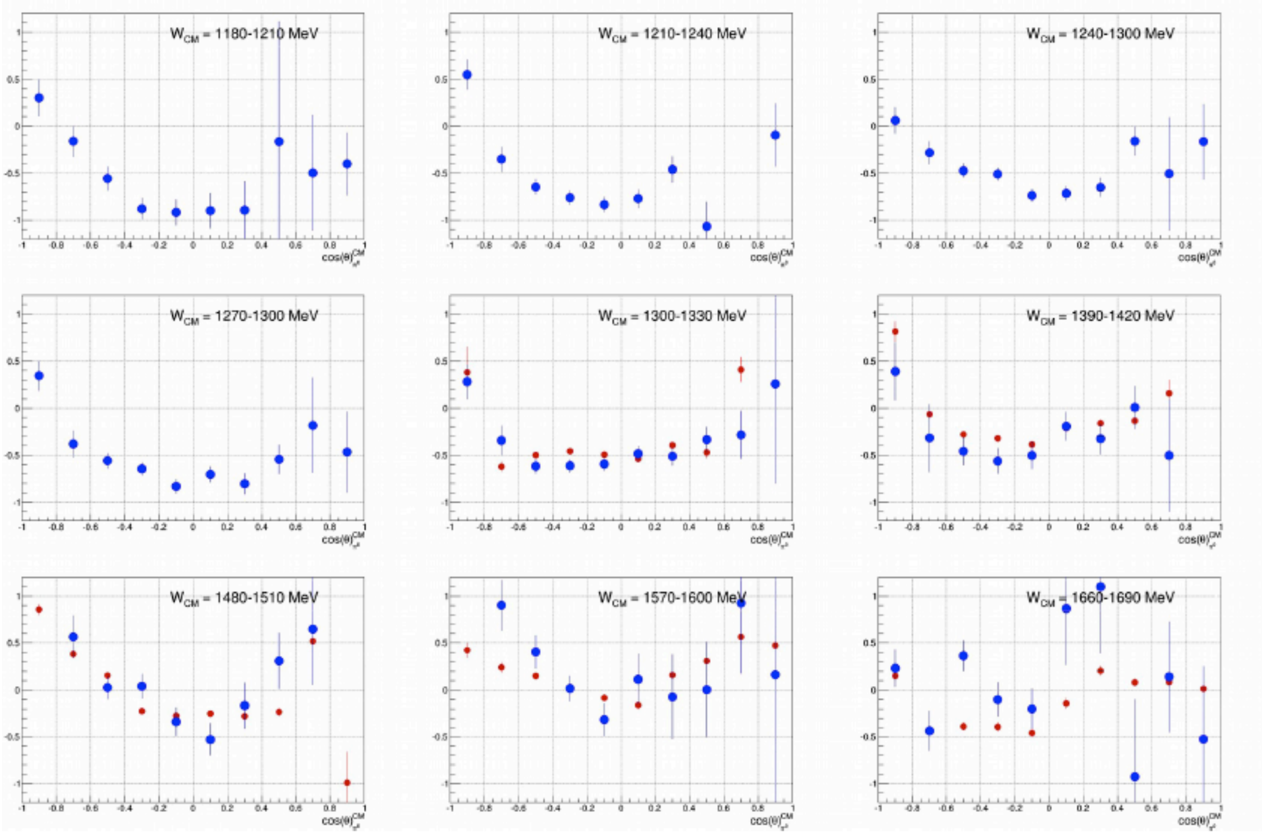
❑ **On-neutron:**

Also see Theory talk by B. Pasquini



- ● This work
- ● Dieterle et al., Phys Lett B 770, 523, 2017

Summary:

Measurement of:

- **inclusive polarized single π0 photoproduction on the deuteron**
- **exclusive E asymmetry for π0 from quasi-free proton and quasi-free neutron**

A2 collaboration @ MAMI

❑ Proton scalar polarizability:

Also see Theory talk by B. Pasquini

- Electric dipole moment:

$$\vec{p} = \boxed{\alpha_{E1}} \times \vec{E}$$

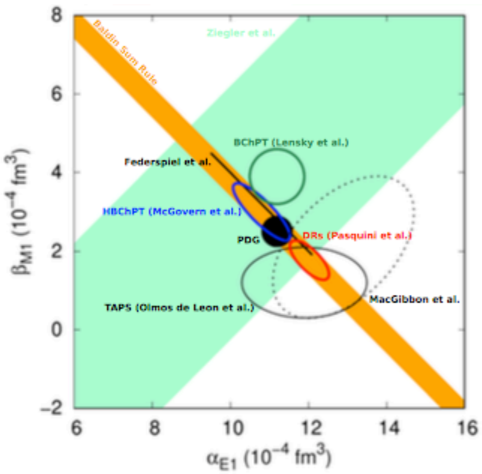
Electric polarizability

- "Stretchability" of the proton
- Magnetic dipole moment:

$$\vec{m} = \boxed{\beta_{M1}} \times \vec{H}$$

Magnetic polarizability

- "Alignability" of the proton



B. Pasquini, P. Pedroni and S. Sconfietti, J. Phys. G 46, no. 10, 104001 (2019).

⇒ New high-precision dataset is needed!

PDG (2012) values:

$\alpha_{E1} = (12.0 \pm 0.6)\ 10^{-4}$ fm^3
$\beta_{M1} = (1.9 \pm 0.5)\ 10^{-4}$ fm^3

Current PDG values:

$\alpha_{E1} = (11.2 \pm 0.4)\ 10^{-4}$ fm^3
$\beta_{M1} = (2.5 \pm 0.4)\ 10^{-4}$ fm^3

Significant change between reviews without new experimental data
⇒ Dataset not fully consistent!

❑ Beam asymmetry Σ_3 for extracting β_{M1}:

$$\Sigma_3 = \frac{d\sigma_{\parallel} - d\sigma_{\perp}}{d\sigma_{\parallel} + d\sigma_{\perp}}$$

Analysis is almost finalized and a publication is expected soon

Preliminary systematic errors included: 3% on the unpolarized cross-section and 5% on the beam asymmetry.

Baldin SR	Yes		No	
γ_π	Fix	Fit	Fix	Fit
α_{E1}	±0.47	±0.60	±0.75	±0.84
β_{M1}	±0.29	±0.46	±0.31	±0.48
$\alpha_{E1} + \beta_{M1}$	±0.32	±0.32	±0.59	±0.59
γ_π	8.00	±1.29	8.00	±1.26
χ^2/DOF	1.18	1.15	1.14	1.10

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- **Nucleon spin polarizability:**

Also see Theory talk by B. Pasquini

Polarized photon on polarized nuclon

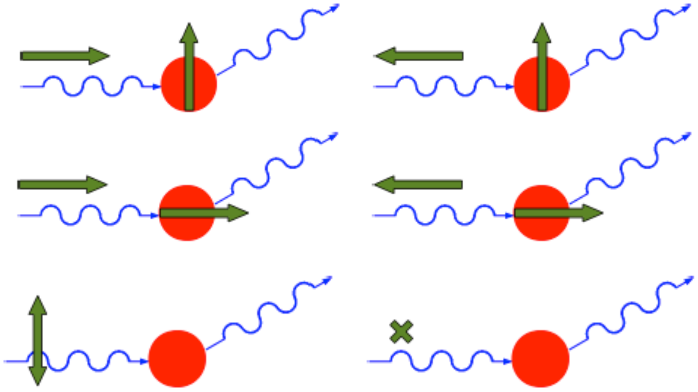
$$H^{(3)}_{\text{eff}} = -4\pi\left[\frac{1}{2}\gamma_{E1E1}\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \frac{1}{2}\gamma_{M1M1}\vec{\sigma}\cdot(\vec{H}\times\dot{\vec{H}}) - \gamma_{M1E2}E_{ij}\sigma_i H_j + \gamma_{E1M2}H_{ij}\sigma_i E_j\right]$$

- **Asymmetries:**

$$\Sigma_{2x} = \frac{N^R_{+x} - N^L_{+x}}{N^R_{+x} + N^L_{+x}}$$

$$\Sigma_{2z} = \frac{N^R_{+z} - N^L_{+z}}{N^R_{+z} + N^L_{+z}}$$

$$\Sigma_3 = \frac{N_\parallel - N_\perp}{N_\parallel + N_\perp}$$



- **Preliminary results:**

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Nucleon spin polarizability:

Also see Theory talk by B. Pasquini

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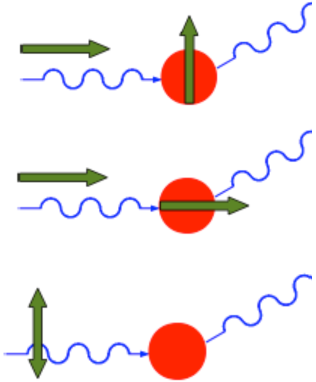
Asymmetries:

$$\Sigma_{2x} = \frac{N^R_{+x} - N^L_{+x}}{N^R_{+x} + N^L_{+x}}$$

$$\Sigma_{2z} = \frac{N^R_{+z} - N^L_{+z}}{N^R_{+z} + N^L_{+z}}$$

$$\Sigma_3 = \frac{N_{\parallel} - N_{\perp}}{N_{\parallel} + N_{\perp}}$$



	Σ_3^{MAMI}		Σ_3^{LEGS}	
	HDPV	BχPT	HDPV	BχPT
γ_{E1E1}	-3.99 ± 0.66	-3.53 ± 0.58	-3.18 ± 0.52	-2.65 ± 0.43
γ_{M1M1}	3.33 ± 0.45	2.71 ± 0.46	2.98 ± 0.43	2.43 ± 0.42
γ_{E1M2}	0.70 ± 0.82	0.19 ± 0.90	-0.44 ± 0.67	-1.32 ± 0.72
γ_{M1E2}	0.89 ± 0.49	1.56 ± 0.51	1.58 ± 0.43	2.47 ± 0.42
γ_0	-0.93 ± 0.11	-0.93 ± 0.11	-0.93 ± 0.11	-0.94 ± 0.11
γ_π	7.51 ± 1.62	7.61 ± 1.68	8.17 ± 1.60	8.86 ± 1.57
$\chi 2/\text{DOF}$	1.11	1.79	1.14	1.36

Preliminary results:

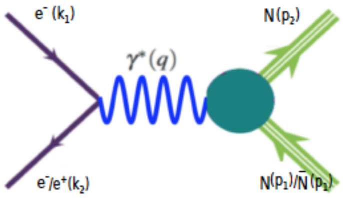
- Spin polarizabilities have been individually extracted for the first time
- Analyses finished: one published, one submitted, one being written
- More data on tape from which Σ_3 can be extracted $\rightarrow$ LEGS vs MAMI

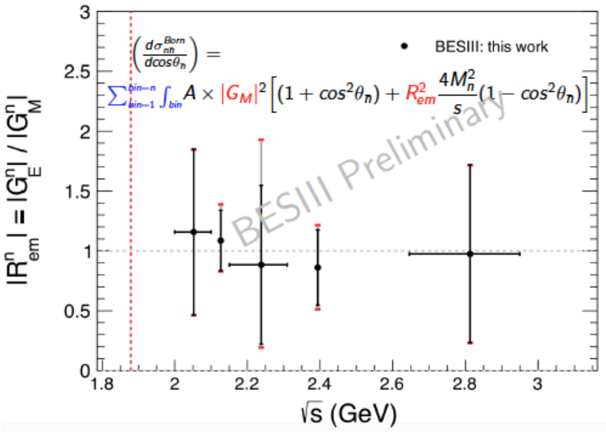
BESIII collaboration @ BEPCII

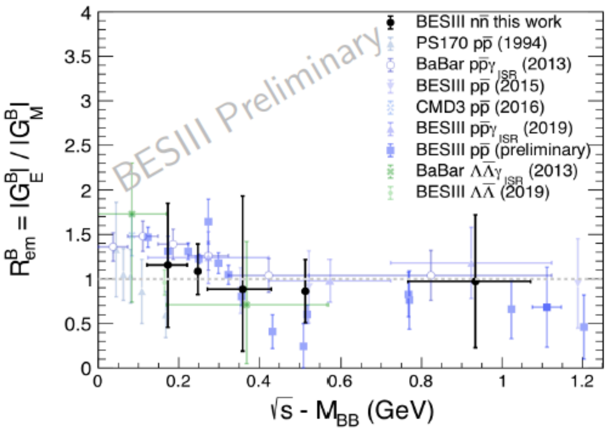
- **2 talks:**

  Study of time-like nucleon form factors at BESIII – Ahmed
  Study of time-like hyperon form factors at BESIII – Wang

- **Time-like form factors in e+e-:**







- **Hyperon form factors:**
  - With the large data set, precise results on Hyperon FFS and the first full measurement of

    $$e^+e^- \to \Lambda\bar{\Lambda}$$

    have been done.
  - Determination of Hyperon form factors could be measured at BESIII

Results for the Neutron Time-like Form Factors at BESIII:

- The neutron form factor ratio R_{em} has been determined for the first time in the TL region.
- The uncertainty of the extracted results for the form factor ratio is dominated by the statistical one.
- The statistical precision of the R_{em} is 35.7% and 52.2% at $\sqrt{s}$= 2.125 and $\sqrt{s}$= 2.394 GeV.

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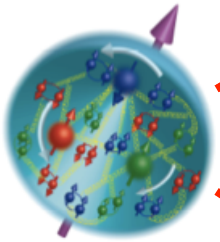
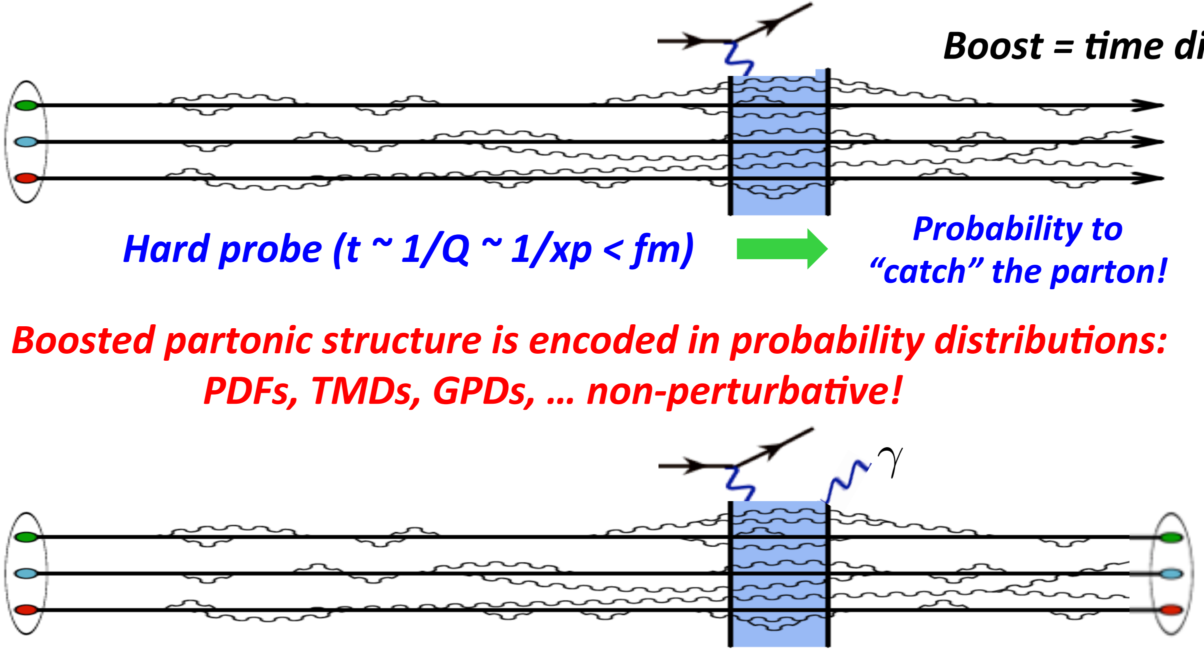
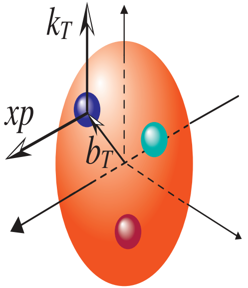
Distribution Functions: Lattice meets Phenomenology

Ji, Qiu







- ❑ **Phenomenology:** QCD global analyses of existing data in terms of QCD factorization

Meet at the distribution functions!

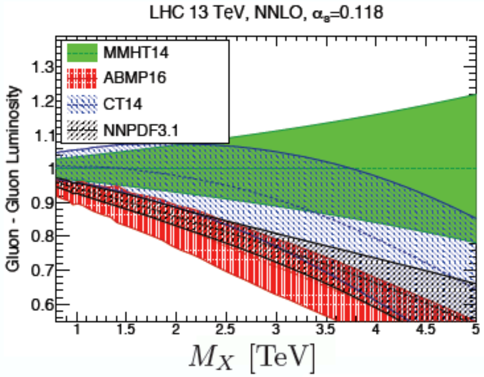
- ❑ **Lattice QCD:** Only consistent method to calculate non-perturbative QCD quantities

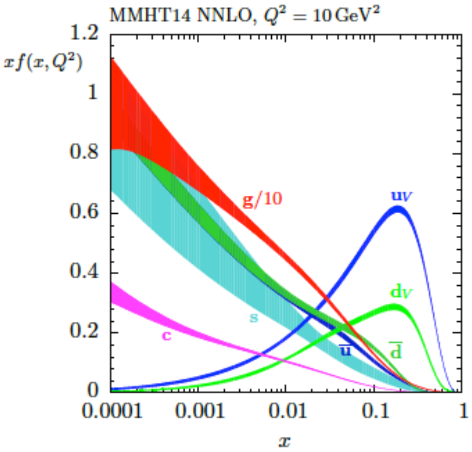
Phenomenology: Unpolarized PDFs

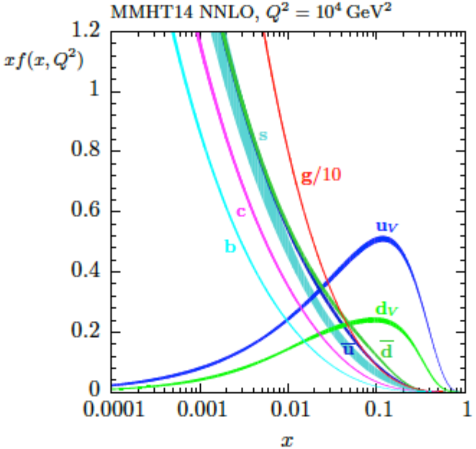
Why PDFs?

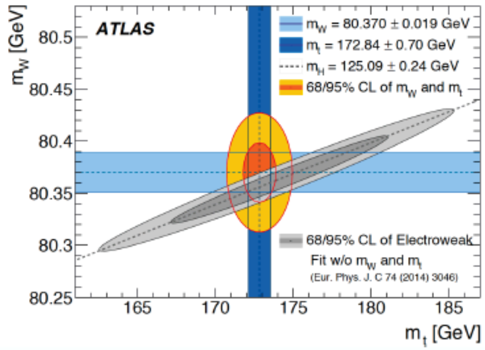
- Ultimate reach of LHC limited by knowledge of PDFs.

- **High mass searches** - PDFs in high region (currently constraints poor)

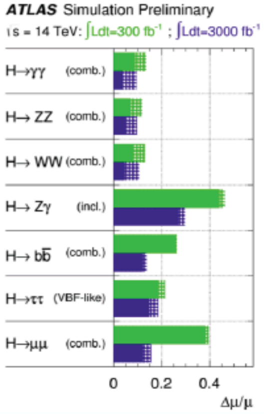








- **Higgs couplings** $\longrightarrow$ need to model SM production precisely.



Combined categories	Value [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.
m_T-p_T^ℓ, $W^\pm$, e-μ	80369.5	6.8	6.6	6.4	2.9	4.5	8.3	5.5	9.2	18.5

- **Precision SM** measurements - PDFs dominant uncertainty for e.g. W mass.

Phenomenology: Unpolarized PDFs

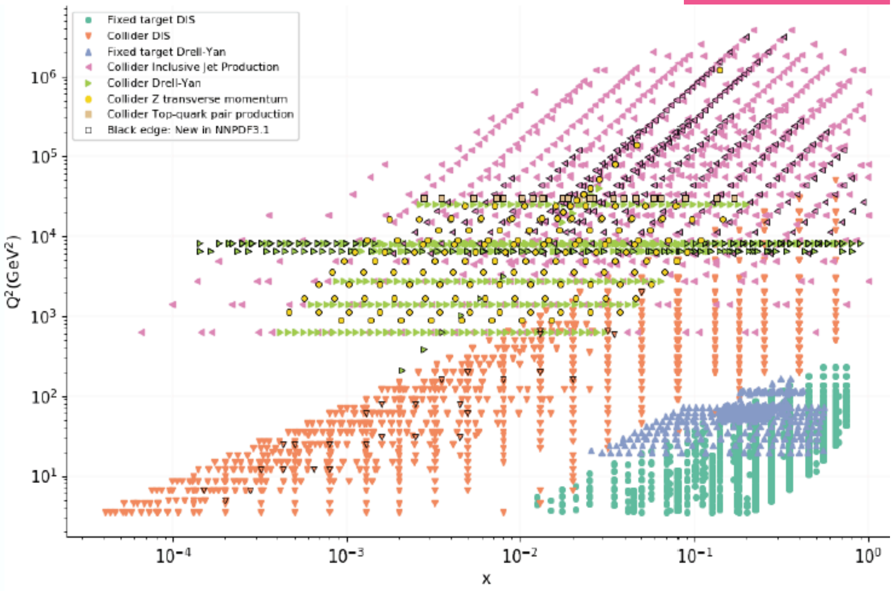
Harland-Lang

❑ Data sets for Global fits:

	Process	Subprocess	Partons	x range
Fixed Target	$\ell^\pm \{p,n\} \to \ell^\pm + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^\pm n/p \to \ell^\pm + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu}) N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
Collider DIS	$e^\pm p \to e^\pm + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} + X$	$W^+ \{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^\pm p \to e^\pm c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to e^\pm b\bar{b} + X$	$\gamma^* b \to b, \gamma^* g \to b\bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to$ jet $+ X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \to$ jet $+ X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^\pm \to \ell^\pm \nu) + X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$p\bar{p} \to (Z \to \ell^+\ell^-) + X$	$uu, dd \to Z$	u, d	$x \gtrsim 0.05$
	$p\bar{p} \to t\bar{t} + X$	$qq \to t\bar{t}$	q	$x \gtrsim 0.1$
LHC	$pp \to$ jet $+ X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
	$pp \to (W^\pm \to \ell^\pm \nu) + X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X$	$q\bar{q} \to Z$	$q, \bar{q}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X, p_\perp$	$gq(\bar{q}) \to Zq(\bar{q})$	$g, q, \bar{q}$	$x \gtrsim 0.01$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, Low mass	$q\bar{q} \to \gamma^*$	$q, \bar{q}, g$	$x \gtrsim 10^{-4}$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, High mass	$q\bar{q} \to \gamma^*$	$\bar{q}$	$x \gtrsim 0.1$
	$pp \to W^+\bar{c}, W^- c$	$sg \to W^+ c, \bar{s}g \to W^- \bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \gtrsim 0.01$
	$pp \to D, B + X$	$gg \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to J/\psi, \Upsilon + pp$	$\gamma^*(gg) \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to \gamma + X$	$gq(\bar{q}) \to \gamma q(\bar{q})$	g	$x \gtrsim 0.005$

❑ Kinematic coverage:

NNPDF3.1



❑ Fit quality:

$\chi^2/\mathrm{dof} \sim 1$ $\Rightarrow$ Non-trivial check of QCD

	LO	NLO	NNLO
All data sets	3706 / 2763	3267 / 2996	2717 / 2663

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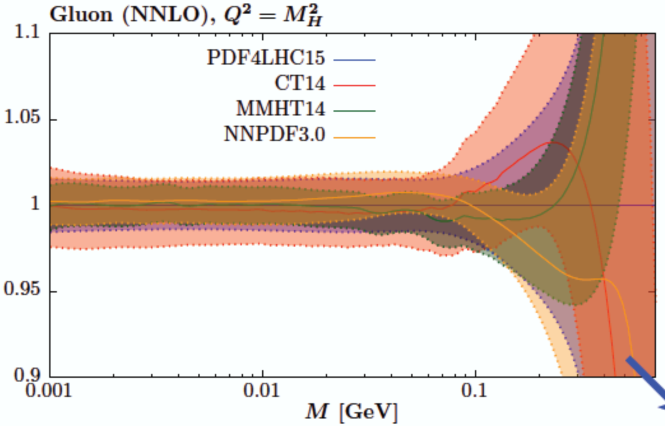
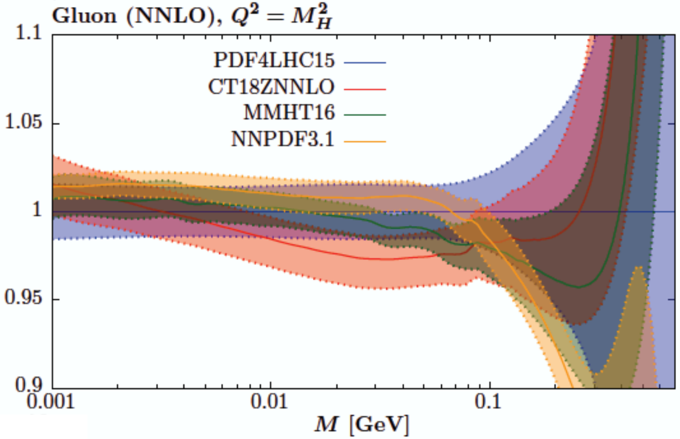
Phenomenology: Unpolarized PDFs

Harland-Lang

❑ Status in 2019:



- Spread between groups has increased! Not always straightforward picture of ever decreasing PDF errors.
- To understand this: detailed benchmarking + combination exercise in early stages.



- Note preliminary: updated 'MMHT19' release coming soon.
- Similar situation for other partons (backup).

❑ Goal:

✧ Current fits very much aiming for (and in some cases achieving) high precision (~ 1% level) PDF determination in some regions.

✧ LHC data now playing a key role in all fits

❑ Challenges:

Cracks start to appear in data/theory comparison as collider data becomes increasingly precise, even for "textbook" – benchmark processes!

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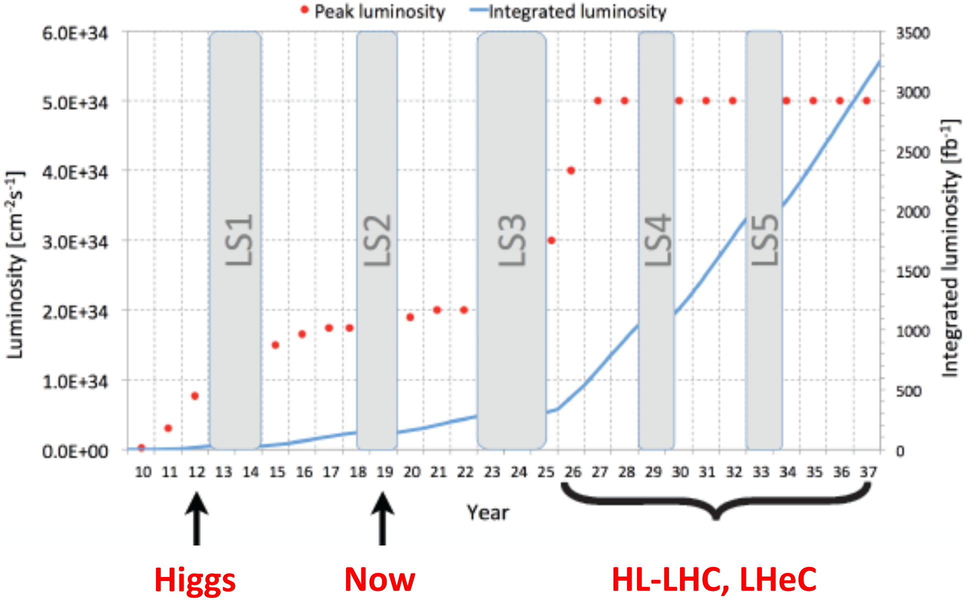
Phenomenology: Unpolarized PDFs

Harland-Lang

❑ Looking to the Future:



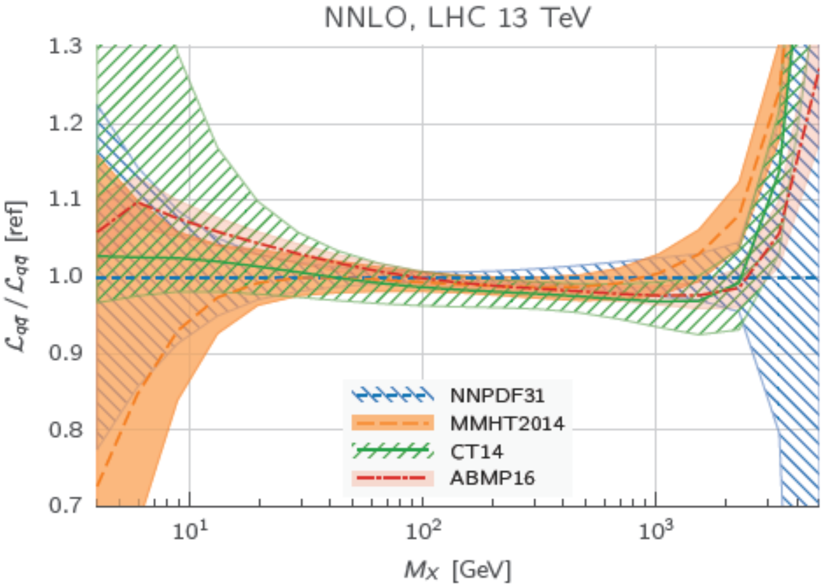
✧ **Question:** **what exactly can we expect that impact to be?**

Fit the pseudodata with statistical + systematic errors

Sub percent level uncertainty

LHeC placing very clean constraints across range

❑ Ultimate PDFs:



❑ Summary:

LHC phenomenology and PDF determination has entered high precision era. Percent level (and below) uncertainties possible

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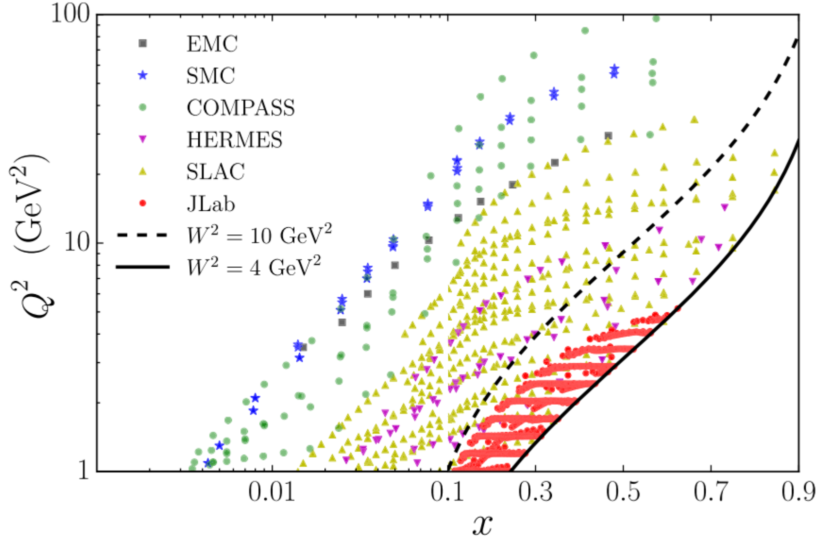
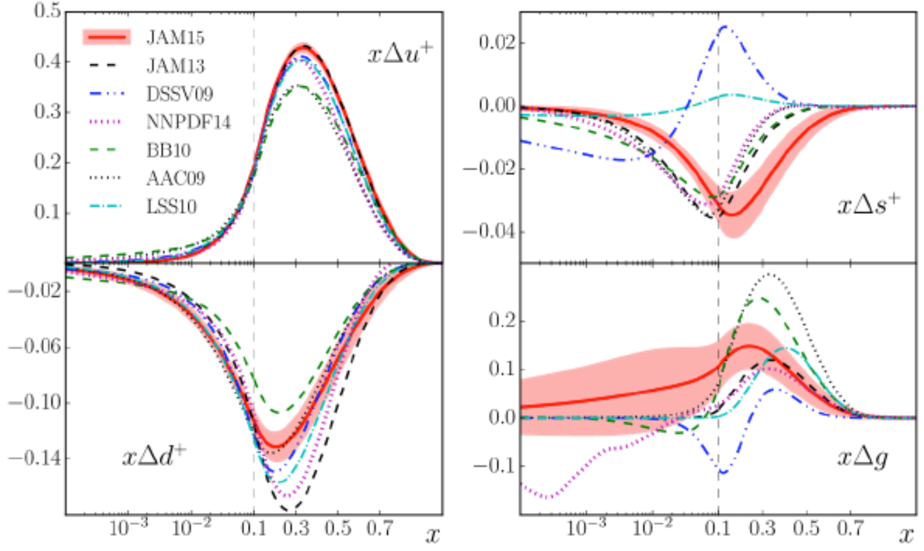
Phenomenology: Polarized PDFs

Sato

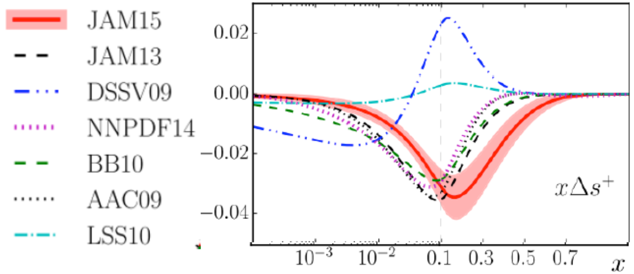
❑ Kinematic coverage:



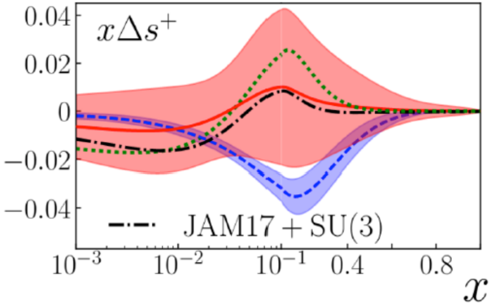
❑ Polarized PDFs:



❑ Strange polarization "puzzle":



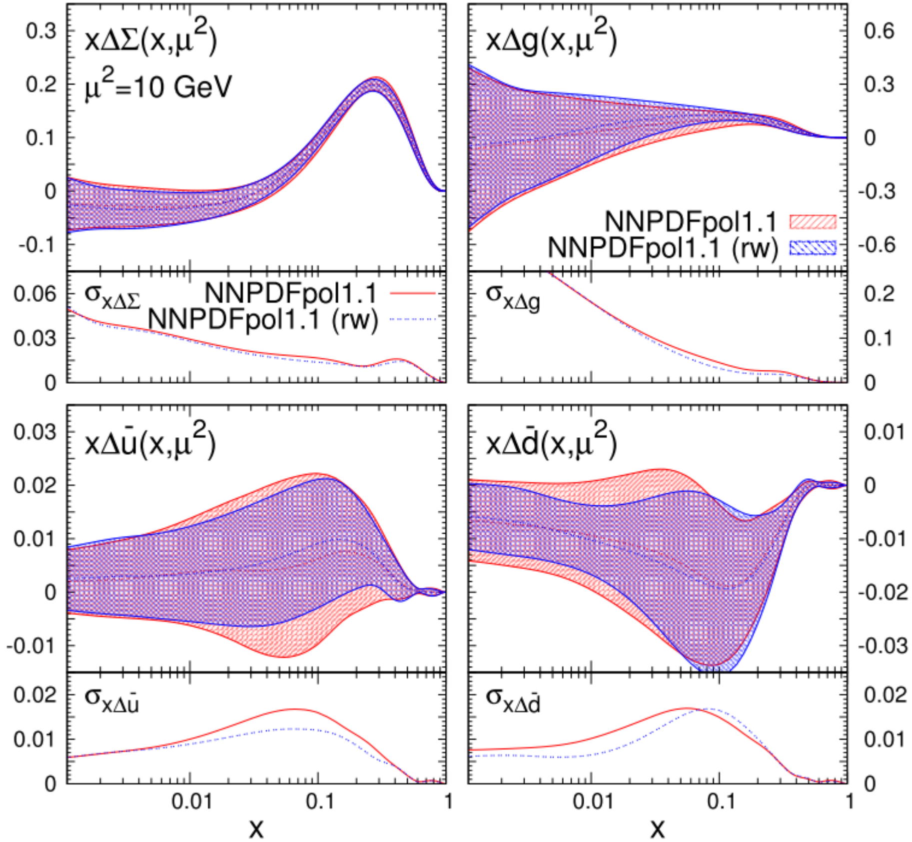
First simultaneous extraction of polarized PDFs and FFs



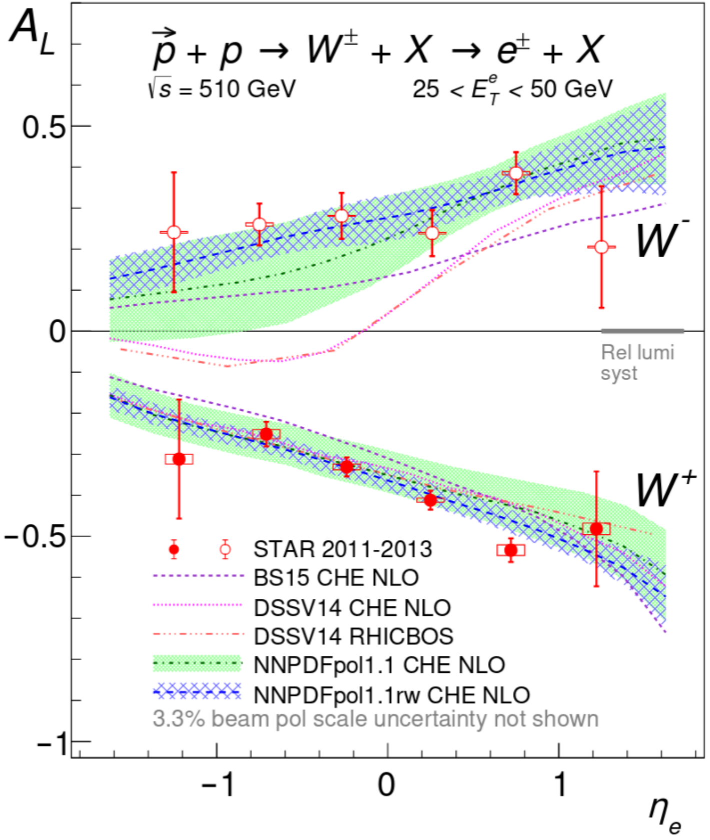
J.J. Ethier et al. PRL 119 (2019) 132001

Phenomenology: Polarized PDFs

- Impact of recent RHIC data:



- Light sea polarization:



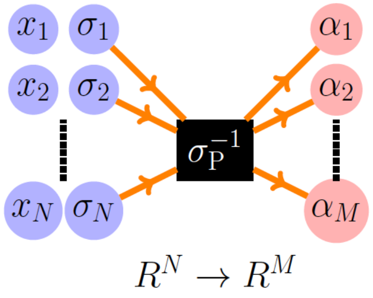
Phenomenology: Polarized PDFs

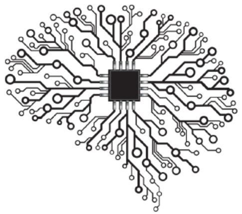
- We know how to go from $\boldsymbol{a}$ to cross sections

$$\frac{d\sigma}{dx dQ^2} = \sum_q \int_x^1 \frac{d\xi}{\xi} H(\xi) f_q\left(\frac{x}{\xi}, \mu; \boldsymbol{a}\right)$$

- We **DON'T** have the inverse function to go from cross sections to $\boldsymbol{a}$

- The inverse mapper:





Can we use Machine Learning?

Phenomenology: Polarized PDFs

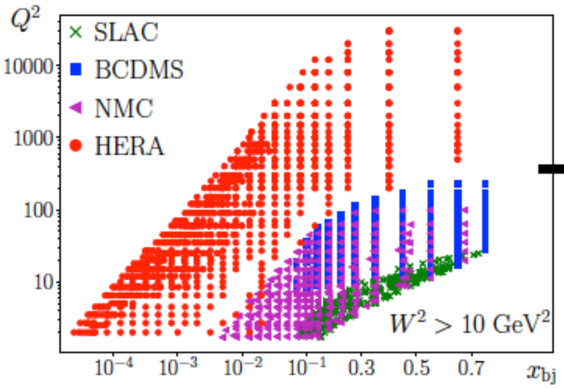
Sato

- We know how to go from a to cross sections

$$\frac{d\sigma}{dxdQ^2} = \sum_q \int_x^1 \frac{d\xi}{\xi} H(\xi) f_q\left(\frac{x}{\xi}, \mu; a\right)$$

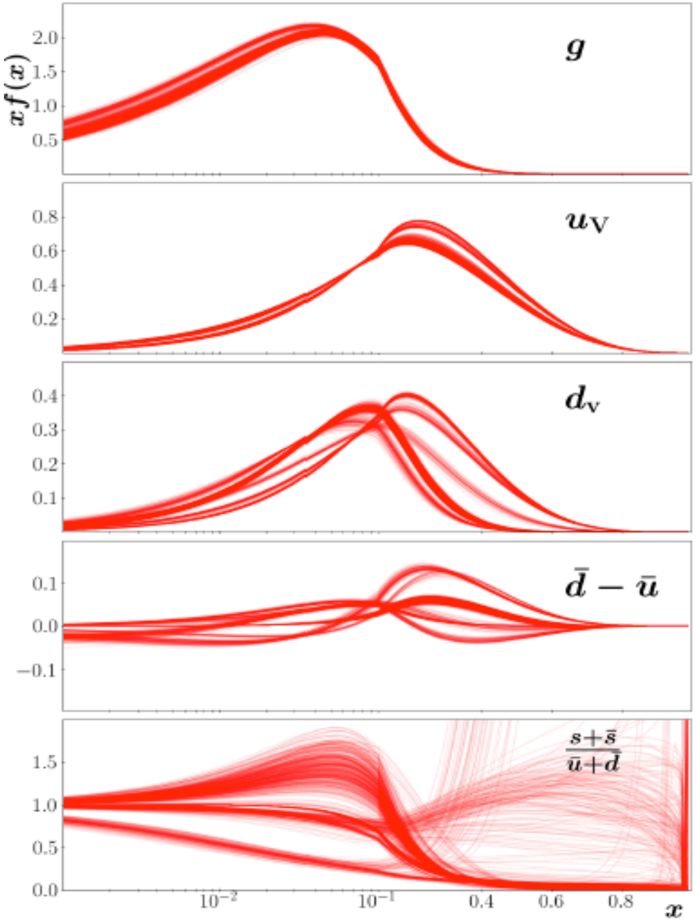
- We **DON'T** have the inverse function to go from cross sections to a

❑ **Next generation analyses tools:**



→

→



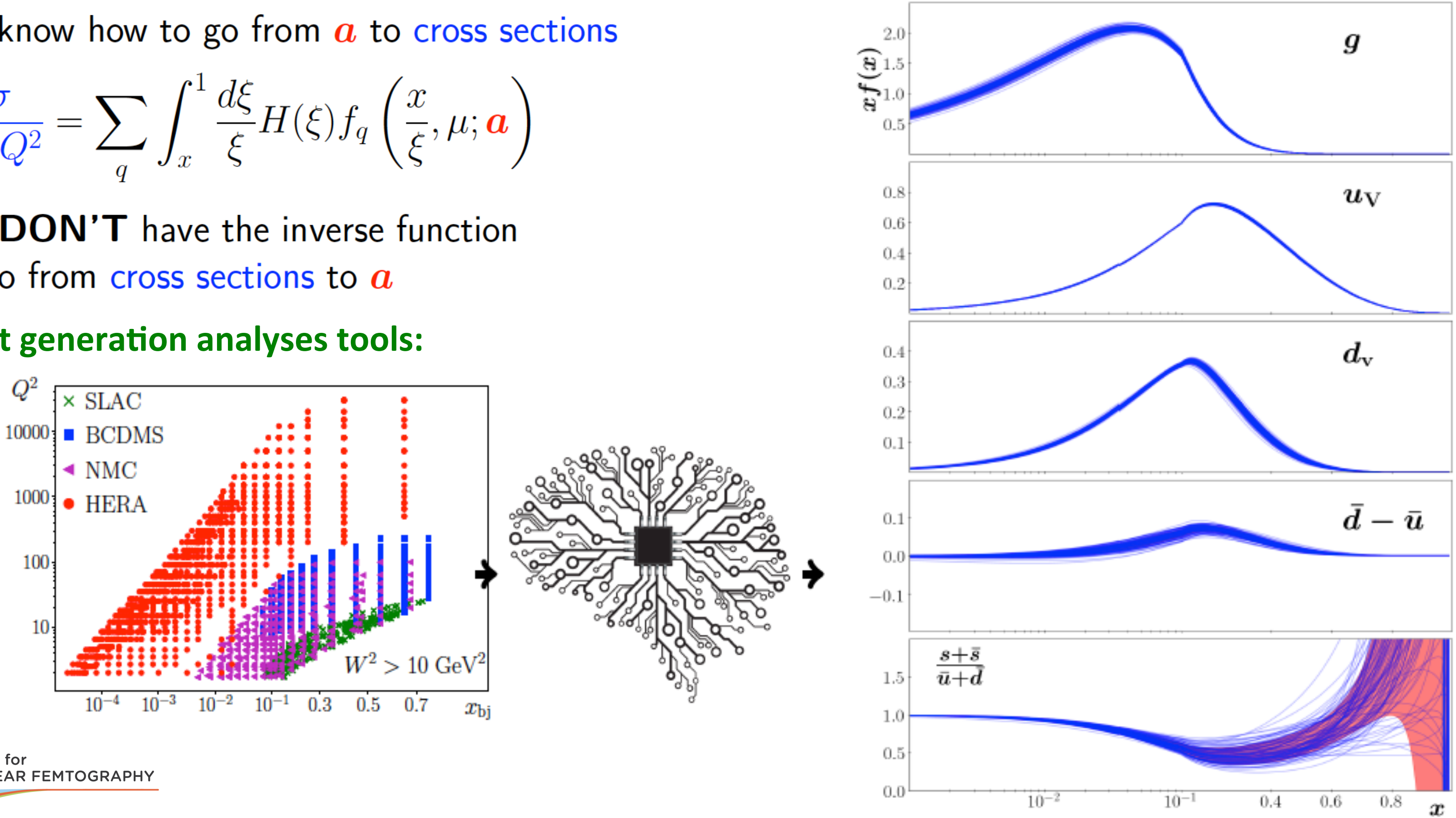
Phenomenology: Polarized PDFs

- We know how to go from a to cross sections

$$\frac{d\sigma}{dxdQ^2} = \sum_q \int_x^1 \frac{d\xi}{\xi} H(\xi) f_q\left(\frac{x}{\xi}, \mu; a\right)$$

- We **DON'T** have the inverse function to go from cross sections to a

❑ **Next generation analyses tools:**



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Phenomenology: Nuclear structure of origin of the EMC effect

Thomas

- ❑ **Approach differs from the SRC:**

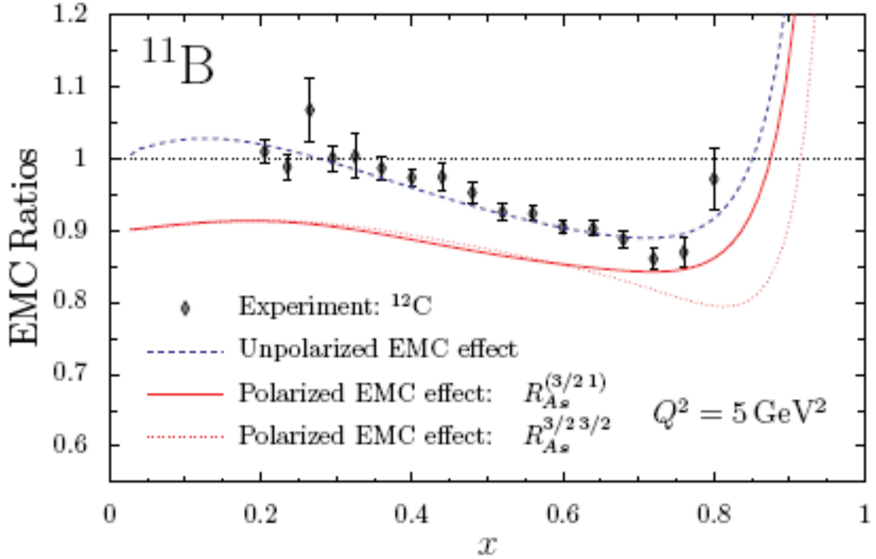


FIG. 7: The EMC and polarized EMC effect in ^{11}B. The empirical data is from Ref. [31].

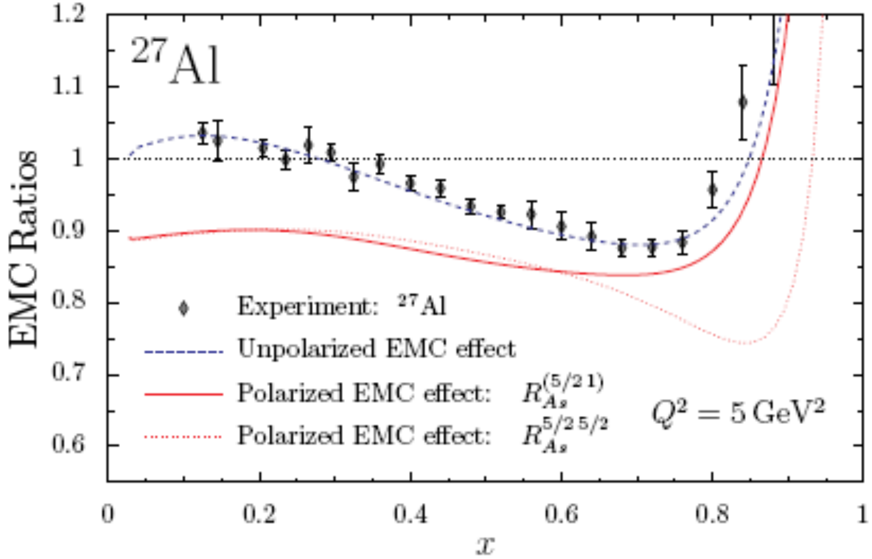


FIG. 9: The EMC and polarized EMC effect in ^{27}Al. The empirical data is from Ref. [31].

- ✧ **Quark-Meson Coupling Model: the change in nucleon structure due to STRONG Lorentz scalar mean field**
- ✧ **Prediction: There is also a spin dependent EMC effect - as large as unpolarized one**

Cloët, Bentz &Thomas, Phys. Lett. B642 (2006) 210 (nucl-th/0605061)

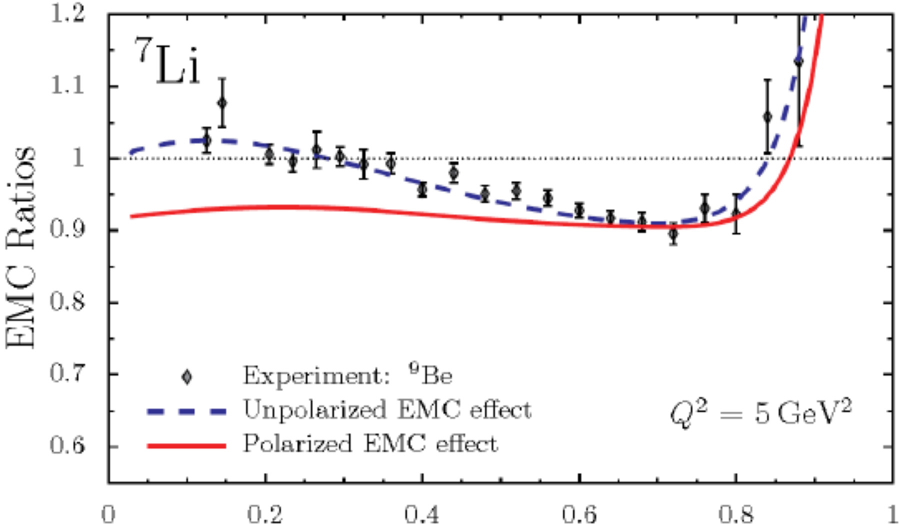
Jefferson Lab

Phenomenology: Nuclear structure of origin of the EMC effect

Cloët, Bentz &Thomas, Phys. Lett. B642 (2006) 210 (nucl-th/0605061)

- **Approved JLab experiment:**
  - Effect in ^{7}Li is slightly suppressed because it is a light nucleus and proton does not carry all the spin (simple WF: $P_p = 13/15$ & $P_n = 2/15$)
  - Experiment now approved at JLab [E12-14-001] to measure spin structure functions of ^{7}Li (GFMC: $P_p = 0.86$ & $P_n = 0.04$)
  - *Everyone with their favourite explanation for the EMC effect should make a prediction for the polarized EMC effect in ^{7}Li*
  - Nucleons in SRC are depolarized – simple Clebsch-Gordan coefficients - and cannot contribute to spin-EMC effect
  - SRC idea gives essentially NO spin-EMC effect

Propose the spin-EMC effect as a vital test



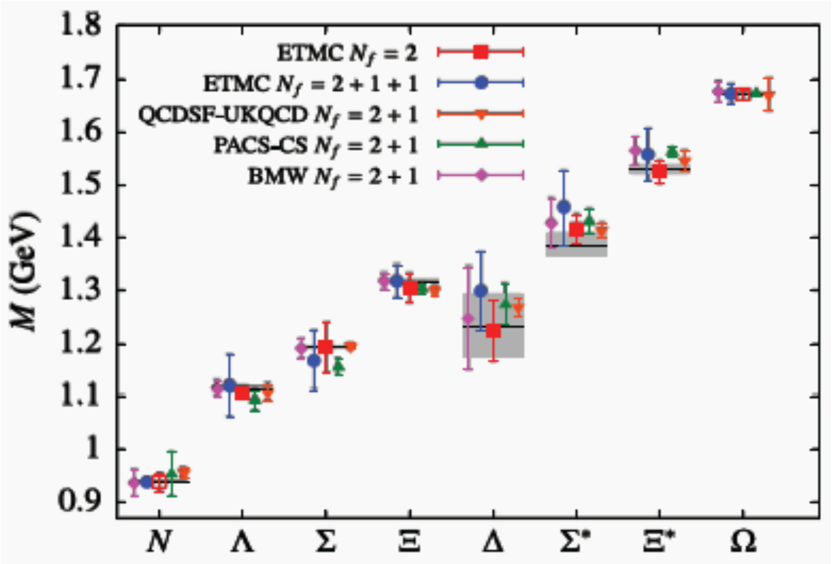
A.W. Thomas, Int J Mod Phys 27 (2018) 1840001
(Ernest Henley Memorial)

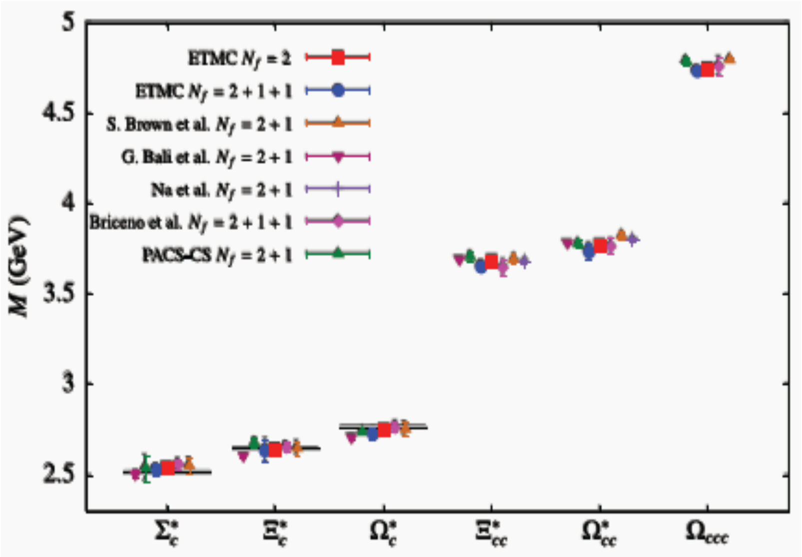
Lattice QCD – ab initio simulation of QCD (or numerical solution of QCD)

Koutsou

- **Baryon spectrum:**

arXiv:1704.02647





Reproduction of light baryon masses:

- Agreement between lattice discretizations
- Reproduction of experimental results

Prediction of yet to be observed baryons

- Agreement between lattice schemes

- **Lattice "time" is Euclidean:** $\tau = i\,t$

Lattice cannot calculate PDFs, TMDs, GPDs, …, directly, whose operators are time-dependent!

Jefferson Lab

Lattice QCD – Moments of Nucleon PDFs:

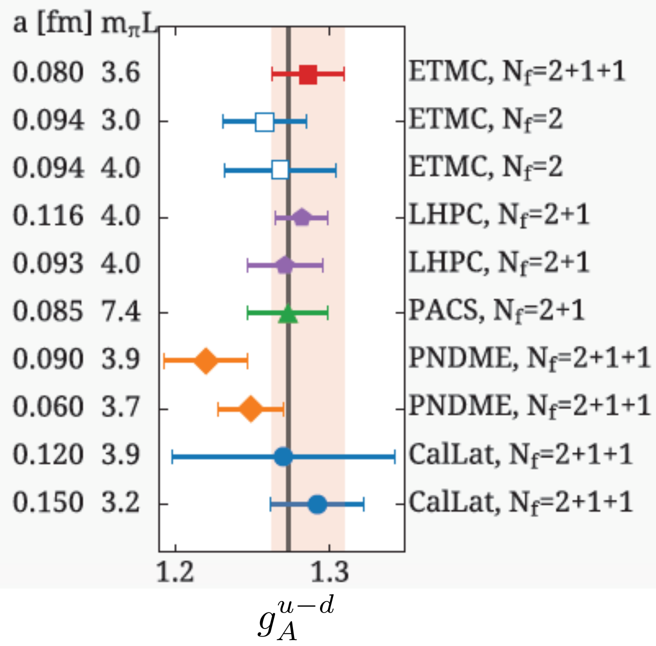
Koutsou

- **Moments of PDFs – matrix elements of local operators:**

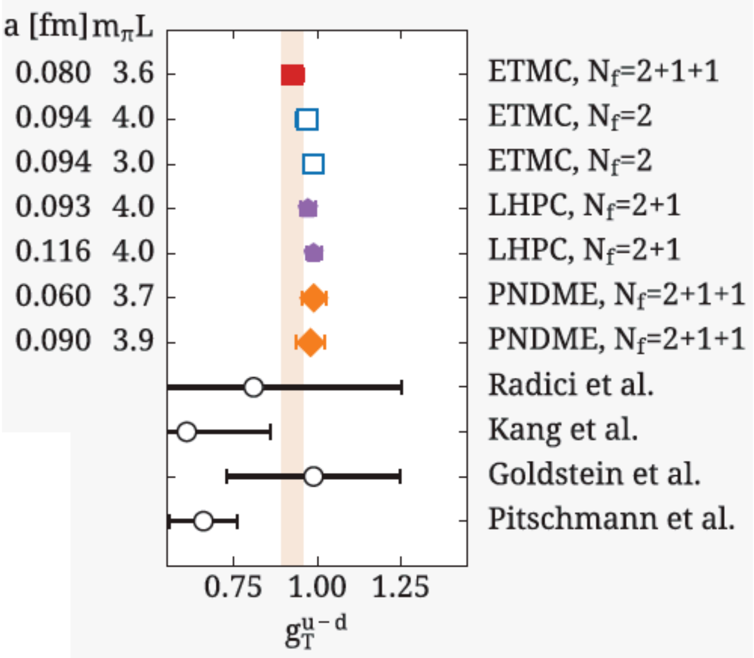
$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2)$$

$$q^{\pm} \equiv q \pm \bar{q} \quad \text{and} \quad \Delta q^{\pm} \equiv \Delta q \pm \Delta \bar{q}$$

- **Axial charge:** $\mathcal{O}^A = \bar{u}\gamma_5\gamma_k u - \bar{d}\gamma_5\gamma_k d$



- **Tensor charge:** $\mathcal{O}^T = \bar{u}\sigma_{\mu\nu}u - \bar{d}\sigma_{\mu\nu}d$



ETMC, arXiv:1909.00485

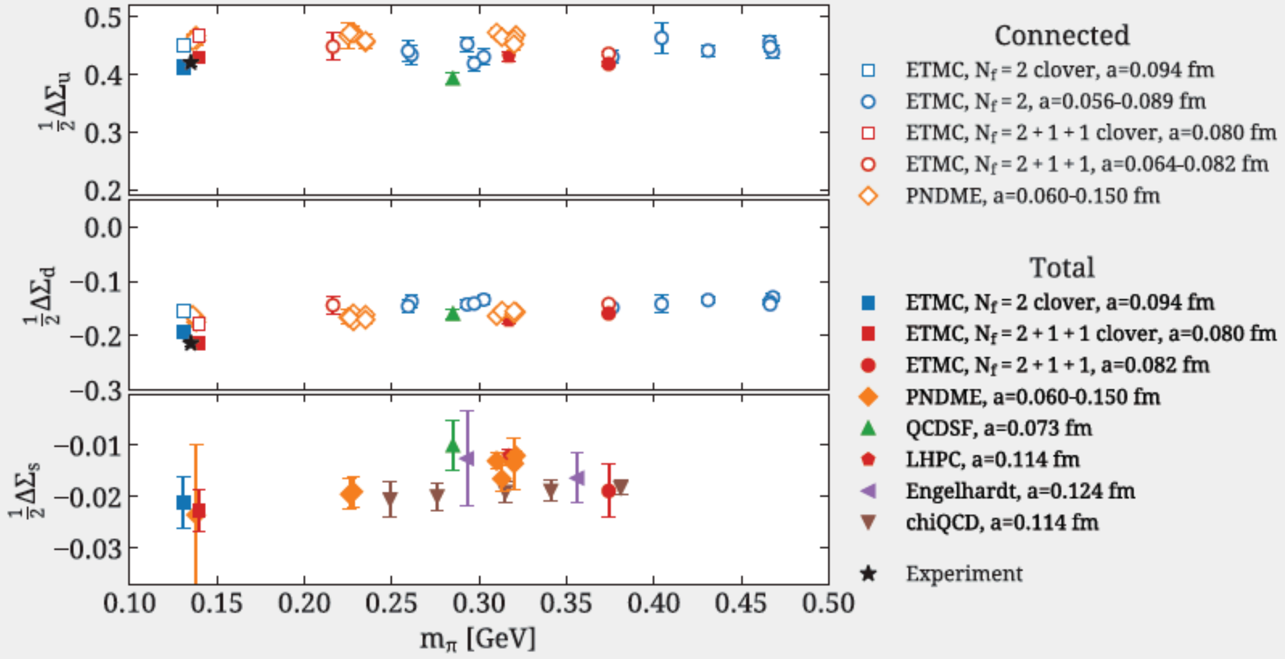
Jefferson Lab

Lattice QCD – Moments of Nucleon PDFs:

Koutsou

- Nucleon spin – quark contribution: $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{q=u,d,s,\ldots} g_A^q$

ETMC, arXiv:1909.00485



- Nucleon generalized form factors – parton angular momentum:

$$J^{u-d} = \frac{1}{2}[A_{20}^{u-d}(0) + B_{20}^{u-d}(0)] = 0.167(24)(04)$$

ETMC, arXiv:1908.10706

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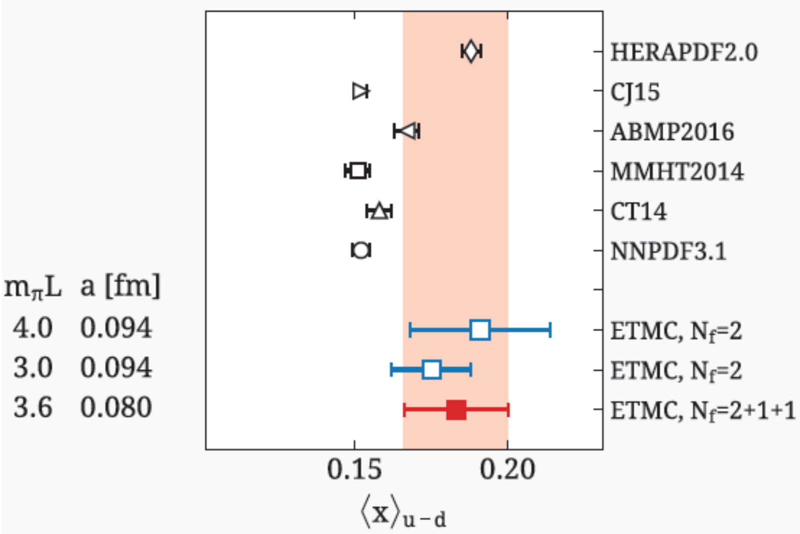
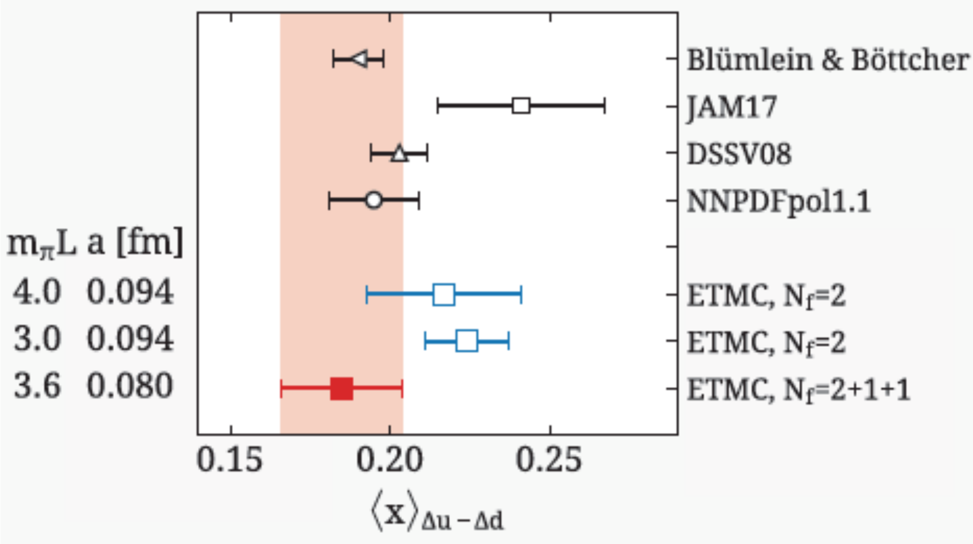
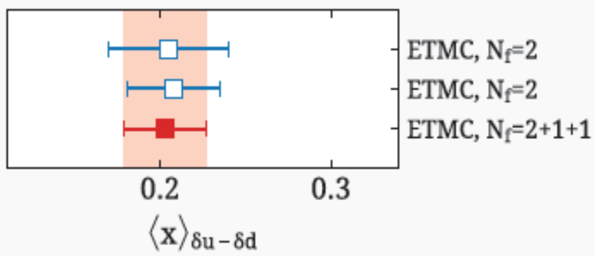
Lattice QCD – Moments of Nucleon PDFs:

❑ The first moment:



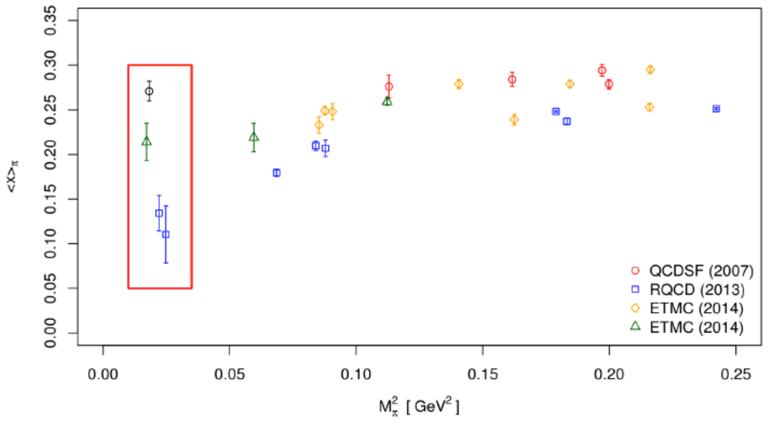


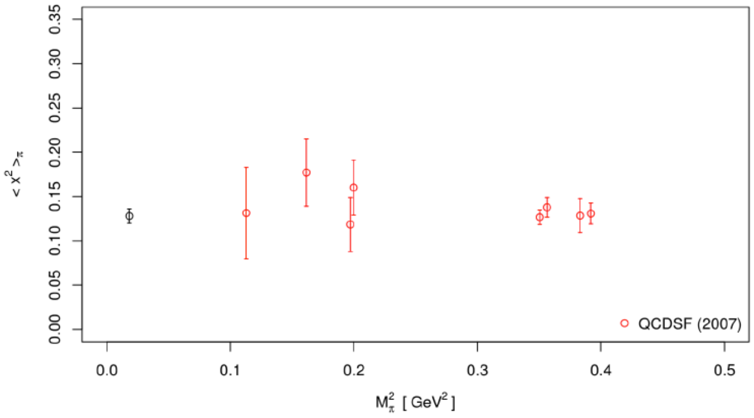


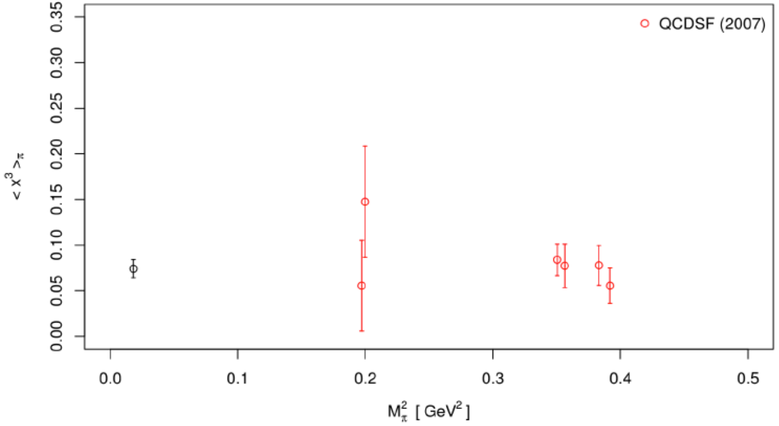
ETMC, arXiv:1908.10706

Lattice QCD – Moments of meson PDFs:

❑ Pion $\langle x^n \rangle$ from $N_f=2$ LQCD:





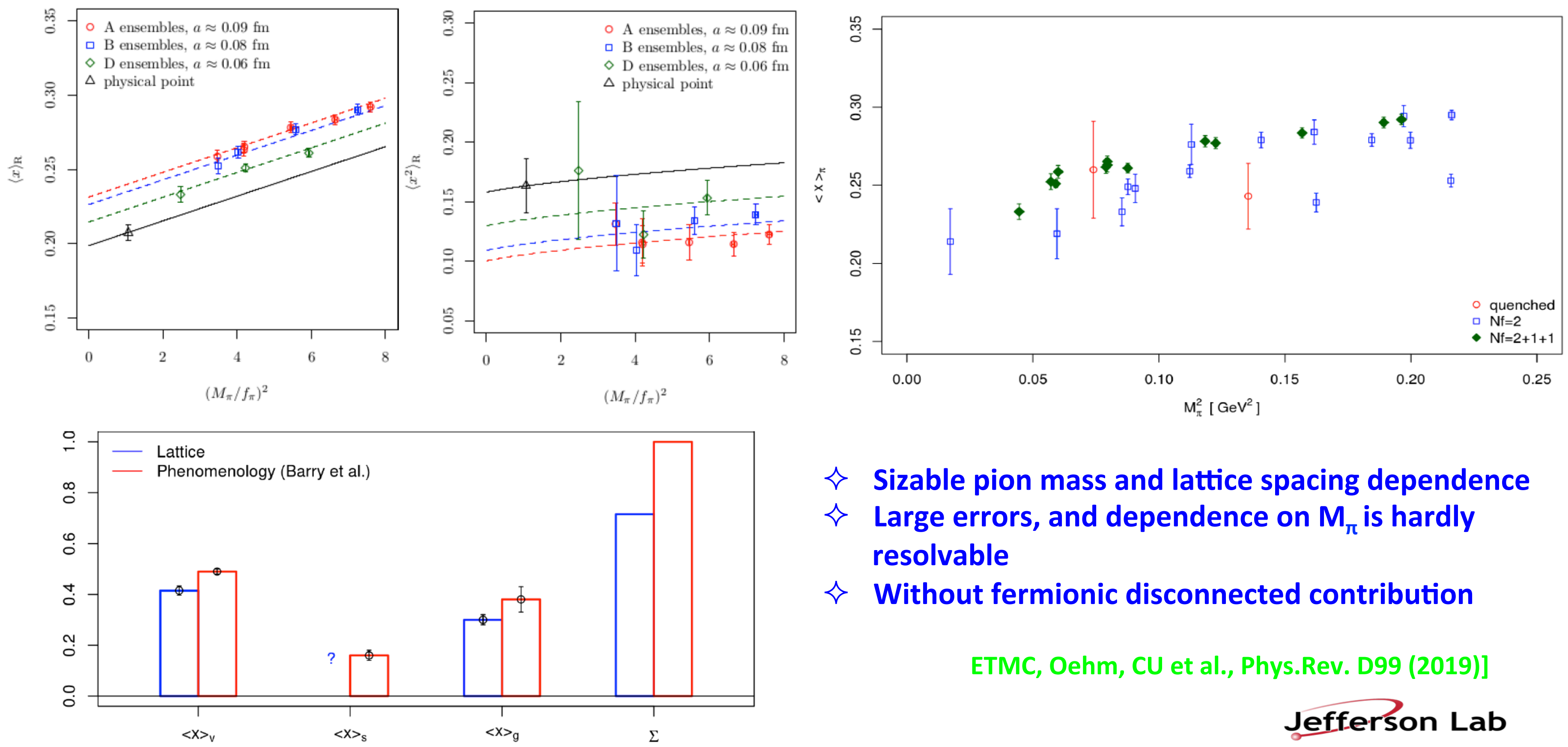


Earlier results

ETMC, Abdel-Rehim et al., Phys.Rev. D92 (2015)]

Lattice QCD – Moments of meson PDFs:

❑ Pion $\langle x^n \rangle$ from N_f=2+1+1 LQCD:

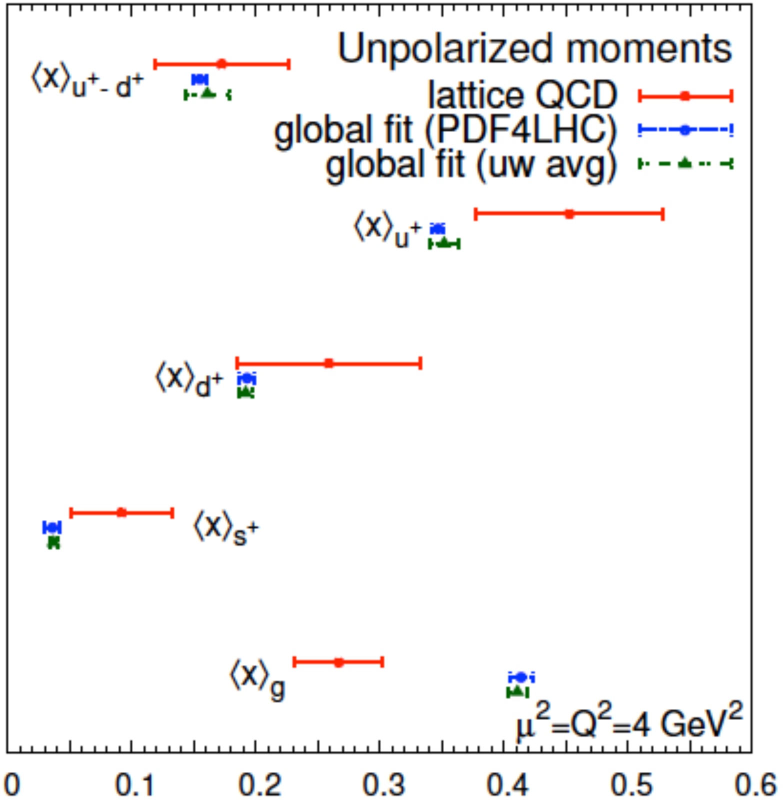


- ✧ Sizable pion mass and lattice spacing dependence
- ✧ Large errors, and dependence on M_π is hardly resolvable
- ✧ Without fermionic disconnected contribution

ETMC, Oehm, CU et al., Phys.Rev. D99 (2019)]

Lattice meets Phenomenology: Moments

❑ Unpolarized:



Moment	Lattice QCD	Global Fit	PDF4LHC
$\langle x \rangle_{u^+-d^+}$	0.119–0.226	0.161(18)	0.155(5)
$\langle x \rangle_{u^+}$	0.453(75)†	0.352(12)	0.347(5)
$\langle x \rangle_{d^+}$	0.259(74)†	0.192(6)	0.193(6)
$\langle x \rangle_{s^+}$	0.092(41)†	0.037(3)	0.036(6)
$\langle x \rangle_{g}$	0.267(35)†	0.411(8)	0.414(9)

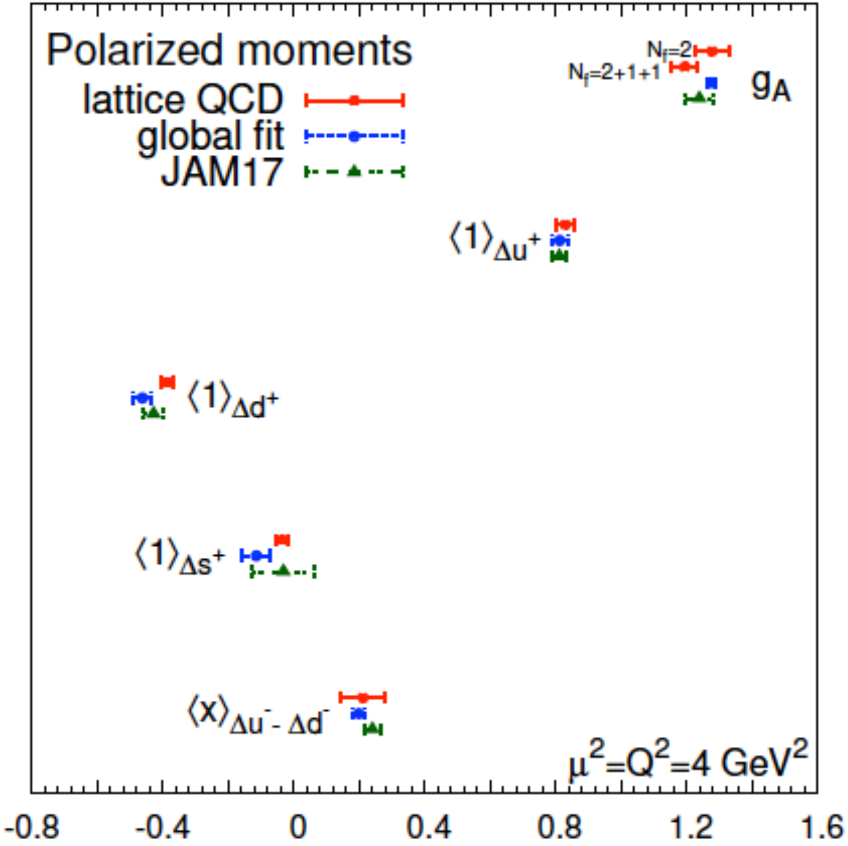
† Single lattice result [PRL 119 (2017) 142002].

$q^{\pm} = q \pm \bar{q}$, $q = u, d, s$; $Q = 2$ GeV.

For details, see [Prog.Part.Nucl.Phys. 100 (2018) 107].

Lattice meets Phenomenology: Moments

❑ Polarized:



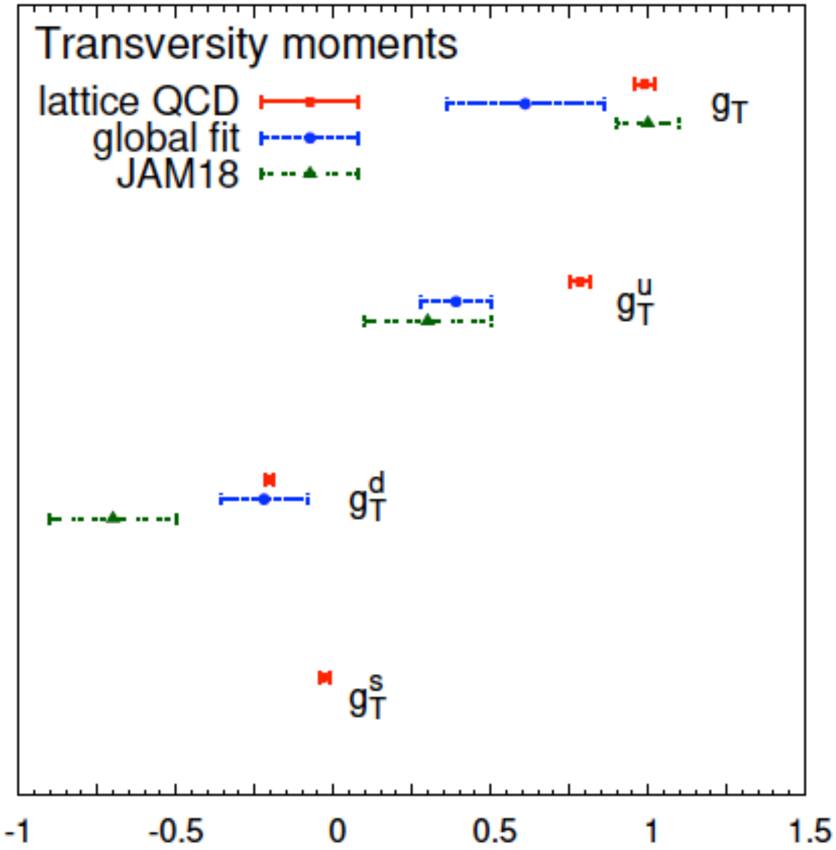
Moment	Lattice QCD	Global Fit	JAM17
g_A	1.195(39)* 1.279(50)**	1.275(12)	1.240(41)
$\langle 1 \rangle_{\Delta u^+}$	0.830(26)†	0.813(25)	0.812(22)
$\langle 1 \rangle_{\Delta d^+}$	-0.386(17)†	-0.462(29)	-0.428(31)
$\langle 1 \rangle_{\Delta s^+}$	-0.052−-0.014	-0.114(43)	-0.038(96)
$\langle x \rangle_{\Delta u^- - \Delta d^-}$	0.146–0.279	0.199(16)	0.241(26)

* $N_f = 2.$
** $N_f = 2 + 1 + 1.$
† Single lattice result [PRL 119 (2017) 142002].

$\Delta q^{\pm} = \Delta q \pm \Delta \bar{q}$, $q = u, d, s$; $Q = 2$ GeV.
For details, see [Prog.Part.Nucl.Phys. 100 (2018) 107]

Lattice meets Phenomenology: Moments

❑ Transversity:



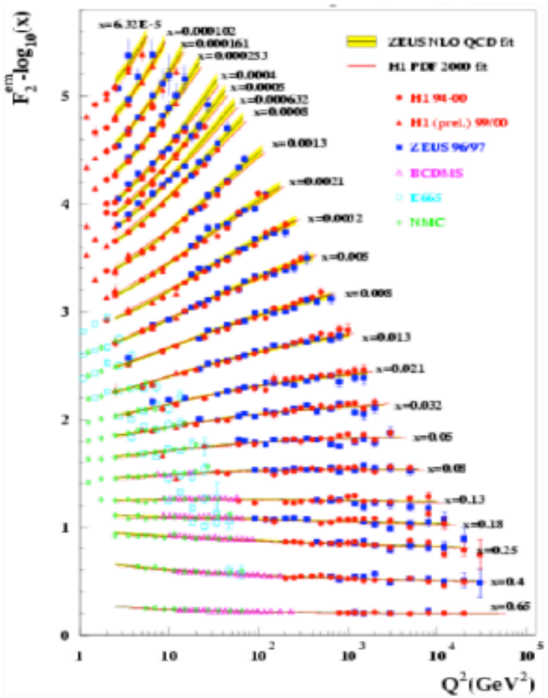
Moment	Lattice QCD	Global Fit	JAM18
g_T	0.989(32)(10)	0.61(25)	1.0(1)
g_T^u	0.784(28)(10)	0.39(11)	0.3(2)
g_T^d	-0.204(11)(1)	-0.22(14)	-0.7(2)
g_T^s	-0.027(16)	—	—

$q^+ = q + \bar{q}$, $q = u, d, s$; $Q = 2$ GeV.
Lattice results from the 2019 FLAG review.
Global fit [PRD 93 (2016) 014009]
JAM18 [PRL 120 (2018) 152502]

Lattice meets Phenomenology: Data accuracy

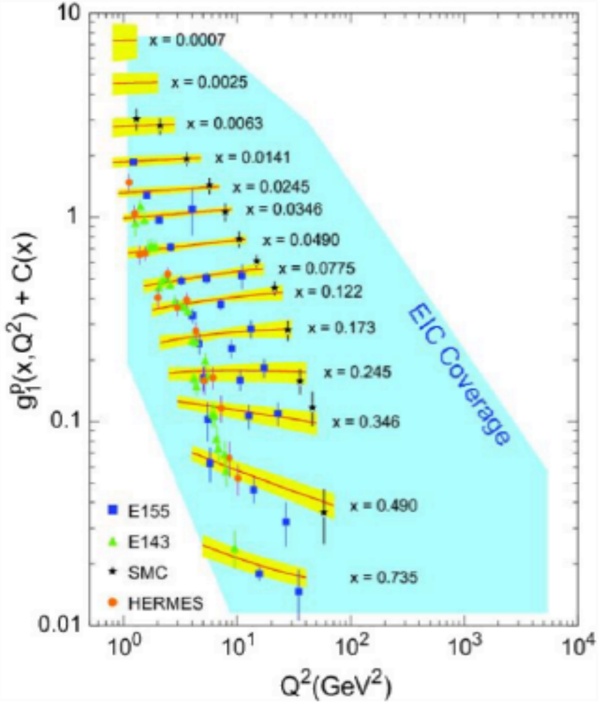
Nocera

World data for F_2^p



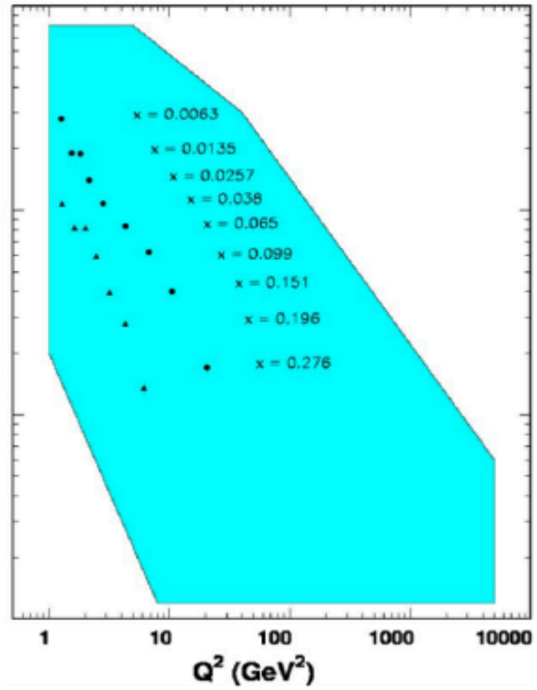
Fits of f
from **thousands** of data
CT, MMHT, NNPDF, . . .

World data for g_1^p



Fits of Δf
from hundreds of data
DSSV, JAM, NNPDF, . . .

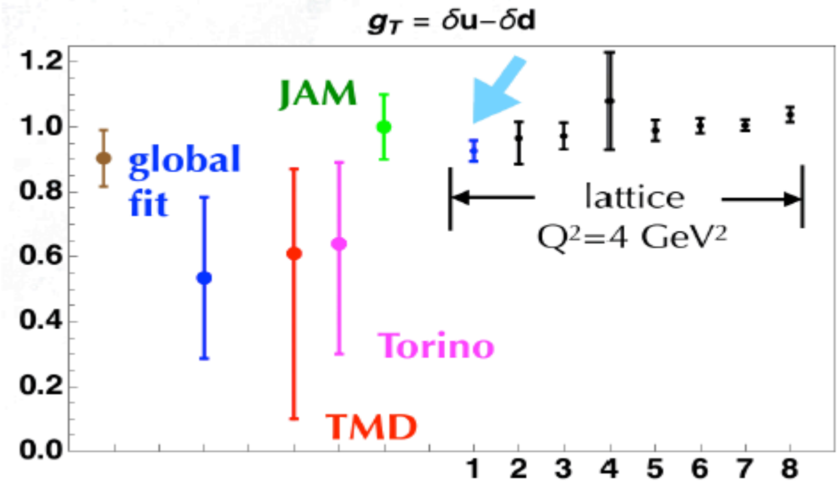
World data for h_1

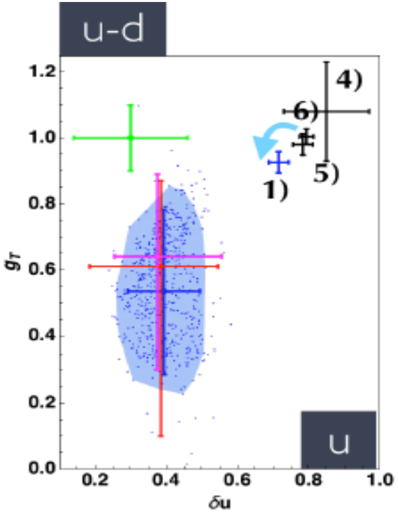


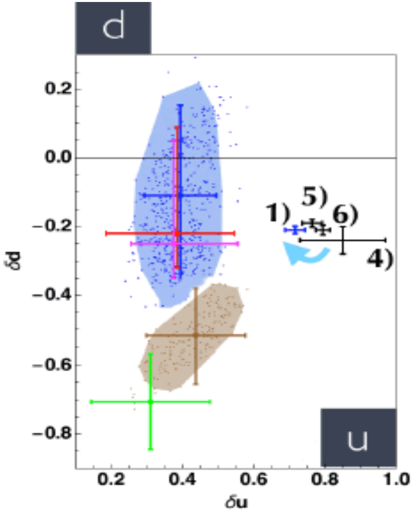
Fits of δf
from tens of data
Kang; Anselmino; Bacchetta

Lattice meets Phenomenology: Impact of lattice QCD moments on δf

Nocera, Qiu



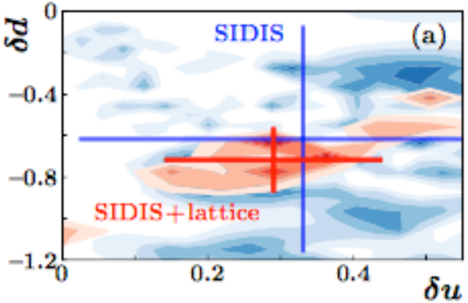


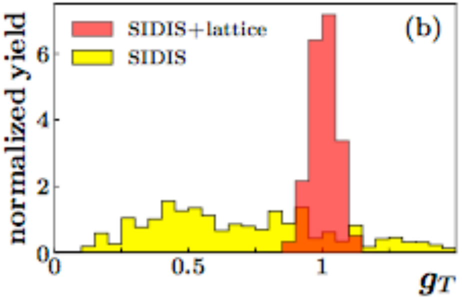


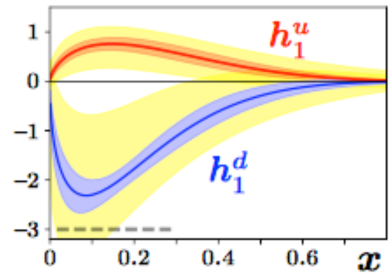
[1] ETMC 19; [2] Mainz 19; [3] LHPC 19; [4] JLQCD 18; [5] PNDME 18; [6] ETMC 17; [7] RQCD 14; [8] LHPC 12

global fit [Radici, in progress]; JAM [PRL 120 (2018) 152502]; TMD [PRD 93 (2016) 014009]; Torino [PRD 92 (2015) 114023]

JAM:







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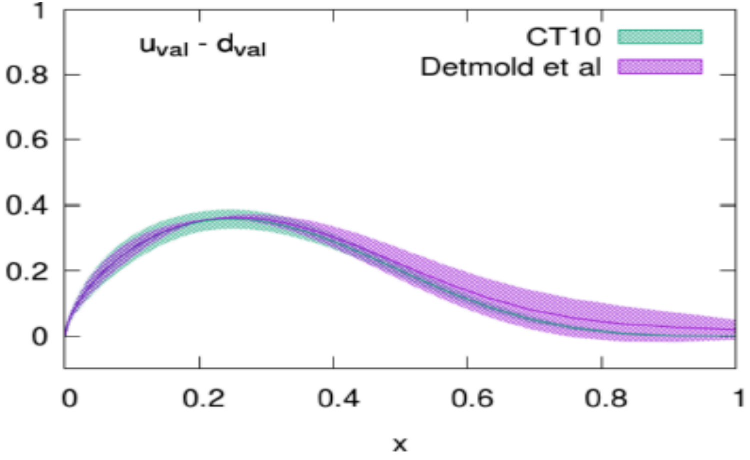
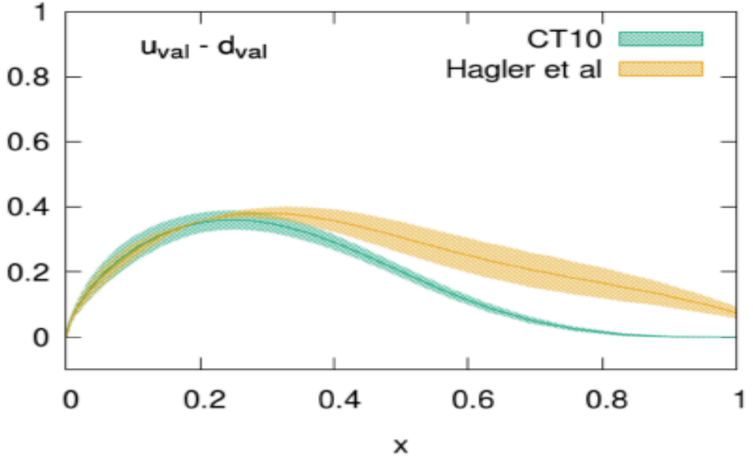
Lattice meets Phenomenology: PDFs from Lattice Moments

Nocera, Qiu





Detmold *et al.* [EPJ direct 3 (2001) 13]

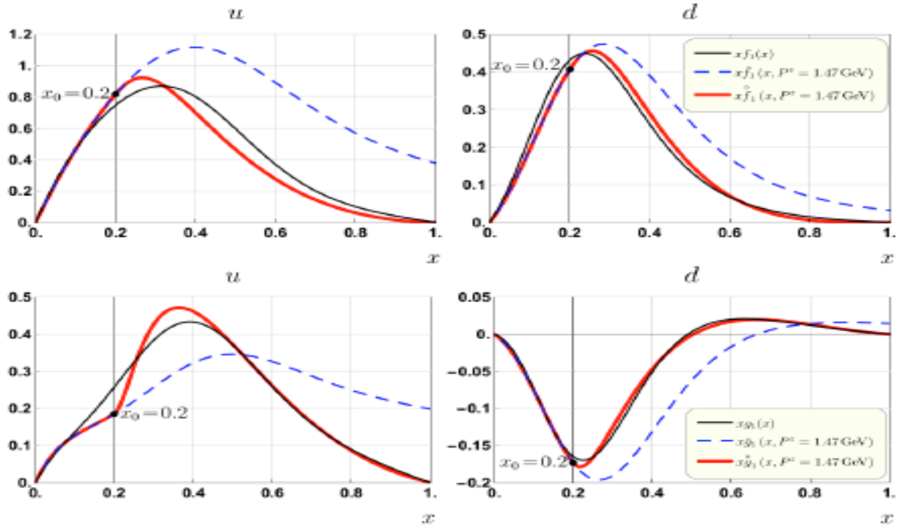
$u - d$ from the lowest few lattice moments, ensure the correct behavior in the chiral and heavy quark limits

Haegler *et al.* [PRD 77 (2008) 094502]

non-perturbative renormalization factor for the axial vector current, only connected diagrams are included

Bacchetta *et al.* [PRD 95 (2017) 014036]

supplement lattice moments with quasi-PDFs (using results of a diquark spectator model) matched at a fixed point x_0



Lattice QCD: Distribution functions

Qiu

- **X-dependent PDFs:**

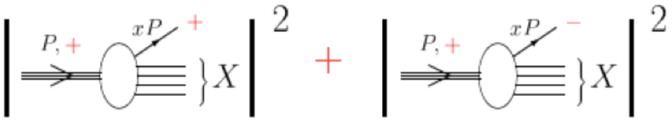
$$f_q(x,\mu^2) \equiv \int \frac{dP^+\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \langle P|\overline{\psi}(\xi^-)\frac{\gamma^+}{2P^+}\exp\left\{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right\}\psi(0)|P\rangle$$

Boost invariant – ***Dominated by the region*:**

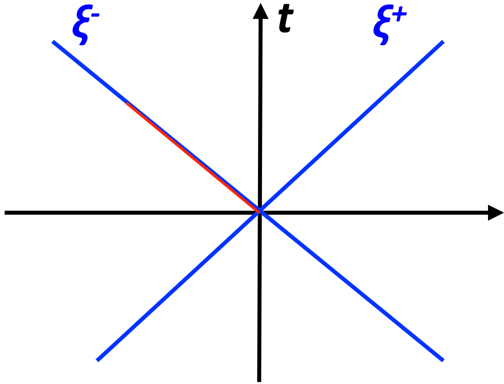
$$\xi^- \lesssim 1/(xP^+) \sim 1/Q$$

Interpreted as:

$q(x) =$



Quantum correlation of quark fields along ξ^- direction! (Conjugated to the large P^+)



Probability density to find a quark with a momentum fraction x

Lattice cannot calculate PDFs directly!

Lattice QCD: x-dependent PDFs

Qiu

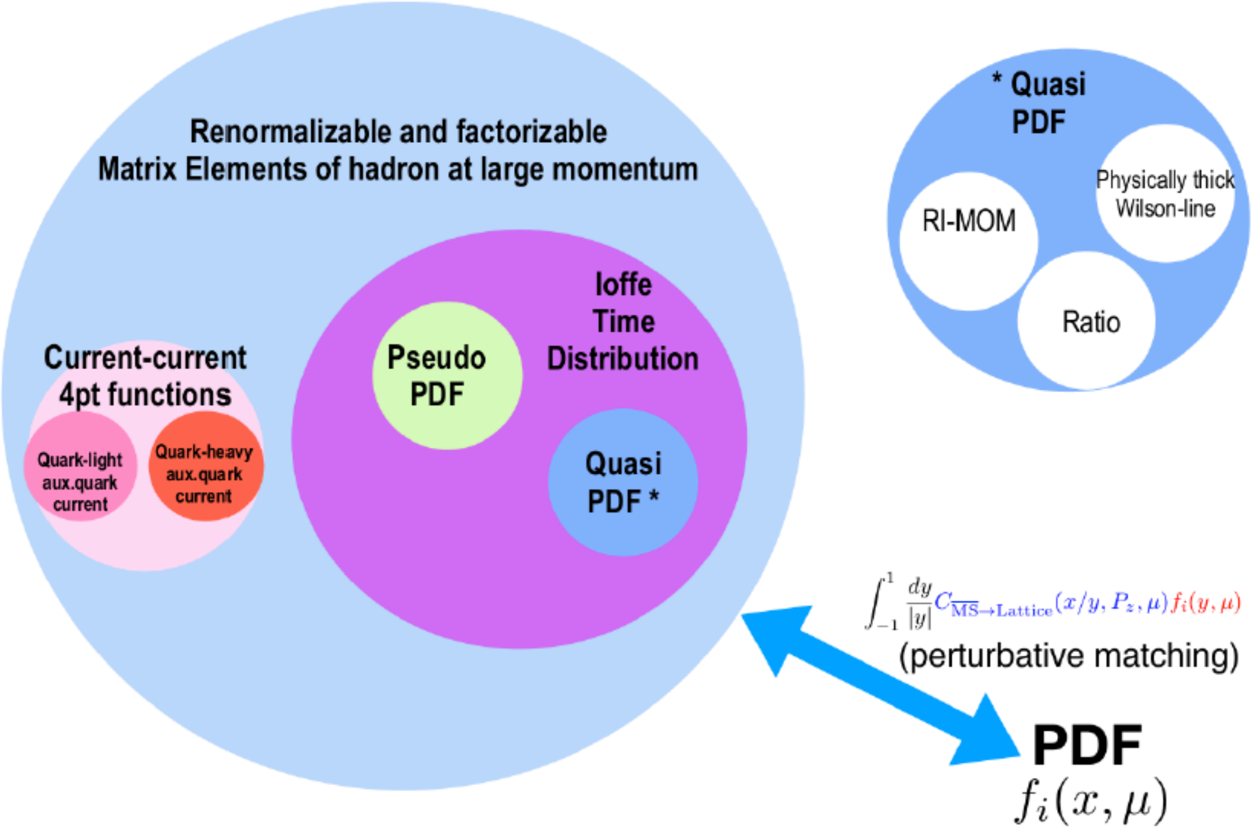
- X-dependent PDFs:
  - Hadronic tensor
    [K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
  - Auxiliary scalar quark
    [U. Aglietti et al., arXiv:hep-ph/9806277, Phys. Lett. B441, 371 (1998)]
  - Fictitious heavy quark
    [W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
  - Auxiliary scalar quark
    [V. Braun & D. Mueller, arXiv:0709.1348, Eur. Phys. J. C55, 349 (2008)]
  - Higher moments
    [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
  - Quasi-Parton Distributions (LaMET)
    [X. Ji, arXiv:1305.1539, PRL 110 (2013) 262002; Sci. China PPMA. 57, 1407 (2014)]
  - Good Lattice Cross Sections
    [Y-Q Ma & J. Qiu, arXiv:1404.6860, arXiv:1709.03018, PRL 120, 022003 (2018)]
  - Compton amplitude and OPE
    [A. Chambers et al. (QCDSF), arXiv:1703.01153, PRL 118, 242001 (2017)]
  - Pseudo-Parton Distributions
    [A. Radyushkin, arXiv:1705.01488, Phys. Rev. D 96, 034025 (2017)]

arXiv:1811.07248

All approaches are under investigation in lattice QCD!

Lattice QCD: x-dependent PDFs

[Courtesy of N. Karthik]



Y.Q.Ma and J.W.Qiu, PRL120 (2018),022003

X. Ji, PRL110 (2013), 262002

A. Radyushkin, PRD98 (2017),034025

W. Detmold and C.J. D. Lin, PRD73 (2006) 014501
V. M. Braun and D. Muller, EPJ C55, 349 (2008)

R.S. Sufian et al, PRD (2019), 074507

Lattice QCD: Quasi-PDFs

Ji, Qiu

- Quasi-PDFs:

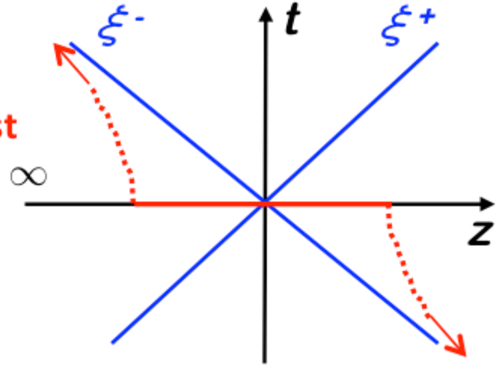
Ji, arXiv:1305.1539

$$\tilde{q}(\tilde{x},\mu_R^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-i\tilde{x}P_z\xi_z} \langle P|\overline{\psi}(\frac{\xi_z}{2})\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(\frac{-\xi_z}{2})|P\rangle$$

Idea – Large Momentum Effective Theory (LaMET):

Quasi-PDFs are not boost invariant.

$\tilde{q}(\tilde{x},\mu_R^2,P_z) \longrightarrow q(x,\mu^2)$ **when** $P_z \to \infty$



Note: **In Lattice QCD calculation, difficult to take** $P_z \to \infty$ **limit**

Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Caution: **The matching formalism was proved to all orders by Ma and Qiu**
But, only for z << $1/\Lambda_{QCD}$ ~ fm!

Ma, Qiu arXiv:1404.6860

Power UV divergence - μ **does not obey DGLAP**

Power corrections could be large:

$$\frac{\Lambda_{QCD}^2 R}{x^2(1-x)P_z^2}$$

V. Braun et al, arXiv: 1810.00048

Jefferson Lab

Lattice QCD: Quasi-PDFs - Renormalization

Qiu, Zhang

- Both quasi-quark and quasi-gluon operators are multiplicative renormalizable!

$$\mathcal{O}^{\nu}_{bq}(\xi) = \overline{\psi}_q(\xi)\,\gamma^\nu \Phi^{(f)}(\xi,0)\,\psi_q(0) \quad \Rightarrow \quad \mathcal{O}^{\nu}_{q}(\xi) = e^{-C_q|\xi_z|} Z^{-1}_{wq} Z^{-1}_{vq} \mathcal{O}^{\nu}_{bq}(\xi)$$

$$\mathcal{O}^{\mu\nu\rho\sigma}_{bg}(\xi) = F^{\mu\nu}(\xi)\Phi^{(a)}(\{\xi,0\})F^{\rho\sigma}(0) \quad \Rightarrow \quad \mathcal{O}^{\mu\nu\rho\sigma}_{g}(\xi) = e^{-C_g|\xi_z|} Z^{-1}_{wg} Z^{-s/2}_{vg1} Z^{-(2-s)/2}_{vg2} \mathcal{O}^{\mu\nu\rho\sigma}_{bg}(\xi)$$

  - For a given combination of Lorentz indices – no sum!
  - Not all terms contribute to the leading power PDFs

  S = number of z-components from all Lorentz indices

Quark: Dorn, Fortsch. Phys. 86', Ji, JHZ, Zhao, PRL 18', Green, Jansen and Steffens, PRL 18']
Gluon: Wang, Zhao, JHEP 18', JHZ, Ji, Schaefer, Wang, Zhao, PRL 19

- Auxiliary field approach:
  - Spacelike Wilson line replaced by two-point function of auxiliary heavy quark field:

$$O(x,y) = \overline{\psi}(x)\Gamma L(x,y)\psi(y) \quad \Rightarrow \quad O(x,y) = \overline{\psi}(x)\Gamma Q(x)\overline{Q}(y)\psi(y)$$

  - Integrating out the auxiliary field (taking into account potential mass term generated by radiative corrections)

Lattice QCD: Quasi-PDFs - Renormalization

Qiu, Zhang

- Both quasi-quark and quasi-gluon operators are multiplicative renormalizable!

$$\mathcal{O}^{\nu}_{bq}(\xi) = \overline{\psi}_q(\xi)\,\gamma^{\nu}\Phi^{(f)}(\xi,0)\,\psi_q(0) \quad \Rightarrow \quad \mathcal{O}^{\nu}_{q}(\xi) = e^{-C_q|\xi_z|} Z^{-1}_{wq} Z^{-1}_{vq}\mathcal{O}^{\nu}_{bq}(\xi)$$

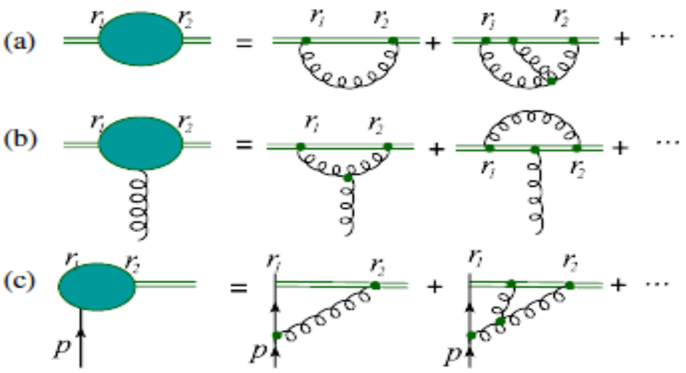
$$\mathcal{O}^{\mu\nu\rho\sigma}_{bg}(\xi) = F^{\mu\nu}(\xi)\Phi^{(a)}(\{\xi,0\})F^{\rho\sigma}(0) \quad \Rightarrow \quad \mathcal{O}^{\mu\nu\rho\sigma}_{g}(\xi) = e^{-C_g|\xi_z|} Z^{-1}_{wg} Z^{-s/2}_{vg1} Z^{-(2-s)/2}_{vg2}\mathcal{O}^{\mu\nu\rho\sigma}_{bg}(\xi)$$

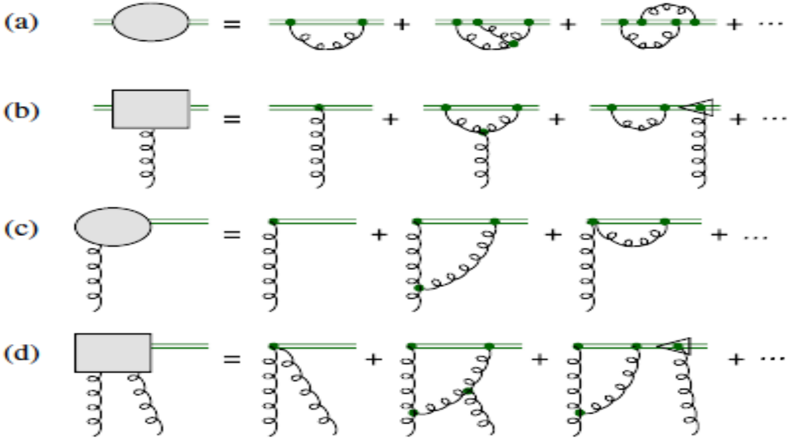
- ✧ For a given combination of Lorentz indices – no sum!
- ✧ Not all terms contribute to the leading power PDFs

S = number of z-components from all Lorentz indices

- All order Feynman diagram approach:

Quark: Ishikawa, Ma, Qiu, Yoshida, arXiv: 1701.03108
Gluon: Li, Ma and Qiu, arXiv: 1809.01836





Lattice QCD: Pseudo-PDFs

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time*. $\nu = p \cdot z$

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\quad$ $z = (0, z_-, 0_T)$ $\quad$ Ioffe-Time Distribution

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Lattice "building blocks" that of quasi-PDF approach.

$\Downarrow$ *Lorentz covariant*

$$\mathcal{M}(\nu, z^2) = \int_{-1}^{1} dx\, e^{i\nu x}\mathcal{P}(x, z^2)$$ ← *pseudo-PDF*

$$\Downarrow$$

$$f(x) = \mathcal{P}(x,0) \underset{z_3^2 \to 0}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, -z_3^2)$$

Lattice QCD: Quasi-PDFs vs Pseudo-PDFs

Qiu, Richards, Zafeiropoulos

$$\frac{\langle H(t_{\text{sink}})\, \overline{\psi}(0)\, W(0,z)\, \gamma_t \psi(z)\, H^\dagger(0)\rangle}{\langle H(t_{\text{sink}}) H^\dagger(0)\rangle} \quad \xrightarrow{t_{\text{sink}} \to \infty} \quad h(z, P_z) = \langle P_z | \overline{\psi}(0)\, W(0,z)\, \gamma_t \psi(z) | P_z \rangle$$

Renormalization

$$h_R(z, P_z, \mu_R) \quad \xrightarrow{\text{Fourier } z,\ \text{Fixed } P_z} \quad \widetilde{q}(\tilde{x}, P_z, \mu_R)$$

$$\mathcal{M}(zP_z, z^2) = \frac{h(z, P_z)}{h(z, 0)} \quad \xrightarrow{\text{Fourier } zP_z,\ \text{Fixed } z^2} \quad \mathcal{P}(x, z^2)$$

LaMET: $P_z \to \infty$

$$f(x, \mu^2)$$

$z^2 \to 0$: Light-cone OPE

Each step has systematic uncertainties and challenges!

See also Constantinou, Nikhil @ DNP2019

Lattice QCD: Good "Lattice Cross Section"

- "Good lattice cross section" = Any hadronic matrix elements:
  1) can be calculated in lattice QCD with precision, has a well-defined continuum limit (UV+IR safe perturbatively), and
  2) can be factorized into universal matrix elements of quarks and gluons *with controllable approximation*

$$P \to \sqrt{s}$$
$$\xi \to 1/Q$$

define collision kinematics

$$\sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P\rangle$$

Ma and Qiu, Phys. Rev. Lett. 120 022003

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

Calculated in LQCD

Parton Distribution function

Calculated in perturbation theory ("process dependent")

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

← Encompasses qPDF/pPDF

Gauge-Invariant Currents

$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0)$$
$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0)$$

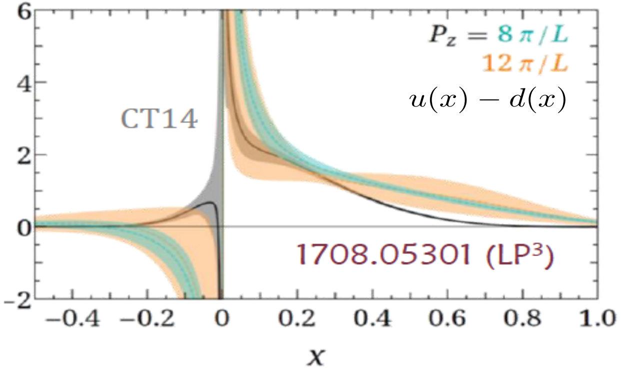
← Flavor-changing

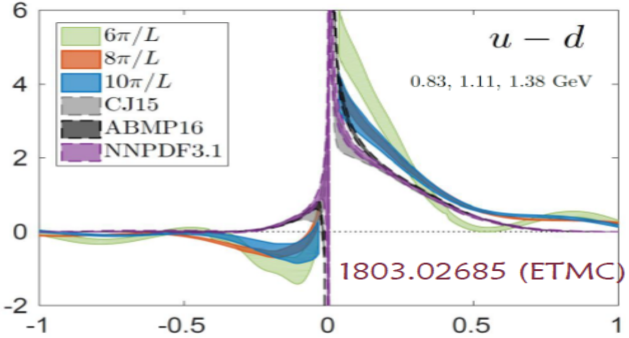
+ analogous gluon operators

Lattice QCD calculated PDFs – Quasi-PDFs approach

Nocera, Qiu, Richards, Zafeiropoulos

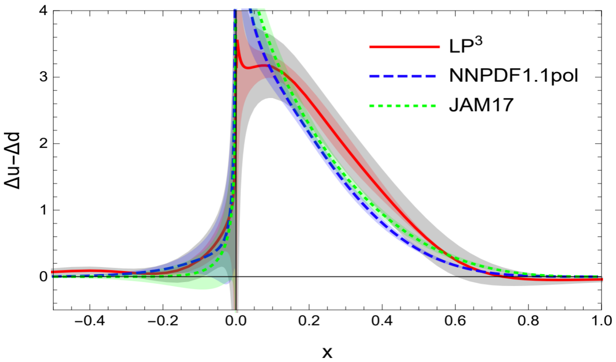
- Unpolarized: Both LP3 and ETMC obtained their results at physical pion mass!

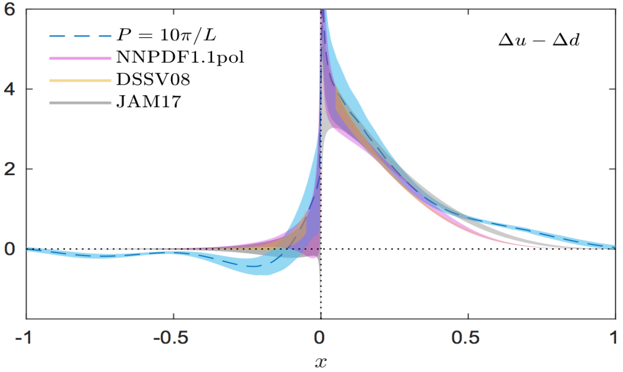




One-loop matching
Target mass corrections

- Helicity distributions:





[H.-W. Lin et al. (LP3), PRL 121 (2018) 242003]

[C. Alexandrou et al. (ETMC), PRL 121 (2018) 112001]

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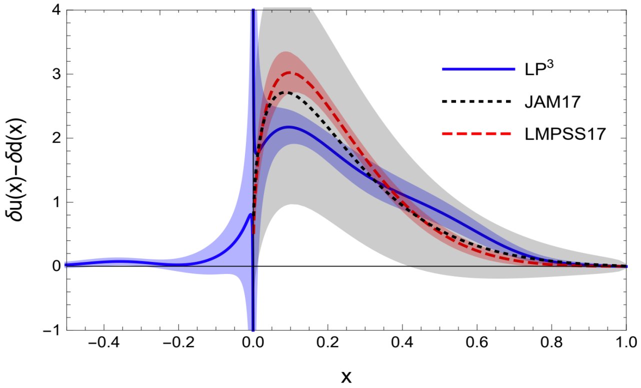
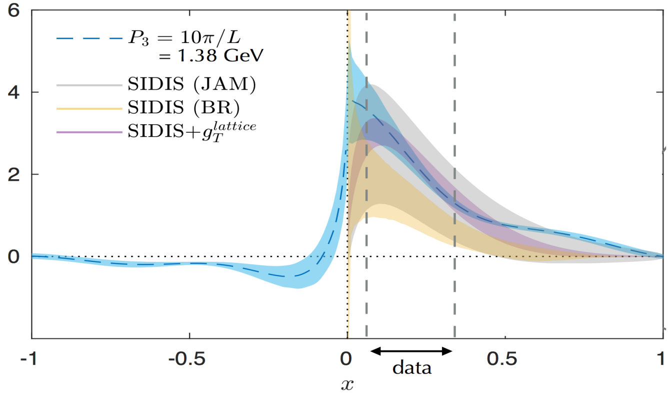
Lattice QCD calculated PDFs – Quasi-PDFs approach

Nocera, Qiu, Richards, Zafeiropoulos

❑ Transversity distribution:



[Y.-S. Liu et al. (LP3), arXiv:1807.00232]



[C. Alexandrou et al. (ETMC), arXiv:1807.00232]

Lattice QCD calculation of GPDs:

Qiu, Scapeliato

- **Quark-GPDs:**

$$\tilde{q}_{\Gamma}^{\rm GPD}(x,\xi,t,P_z,\mu_R) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle h(P_z+\Delta/2)|\overline{\psi}(z)\Gamma\Phi_z(z,0)\phi(0)|h(P_z-\Delta/2)\rangle_{\mu_R}$$

More variables:

- Length of the Wilson line (z converts to x in momentum space)
- Hadron momentum: P_z
- Momentum transfer: $t = \Delta^2 = -Q^2$
- Skewness: $\xi = -\frac{Q_z}{P_z}$ Quasi-skewness = light-cone skewness $+\mathcal{O}(1/P_Z^2)$

Much higher computational cost compared to PDFs

- **Matching:**

- Perturbative matching depends on skewness, but not on momentum transfer
- For ξ=0, the matching is the same as that of PDFs
- Matching for general ξ

[X. Ji et al., PRD 92 (2015) 014039, arXiv:1506.00248]

[Y.-S. Liu et al., PRD 100, 034006 (2019), arXiv:1902.00307]

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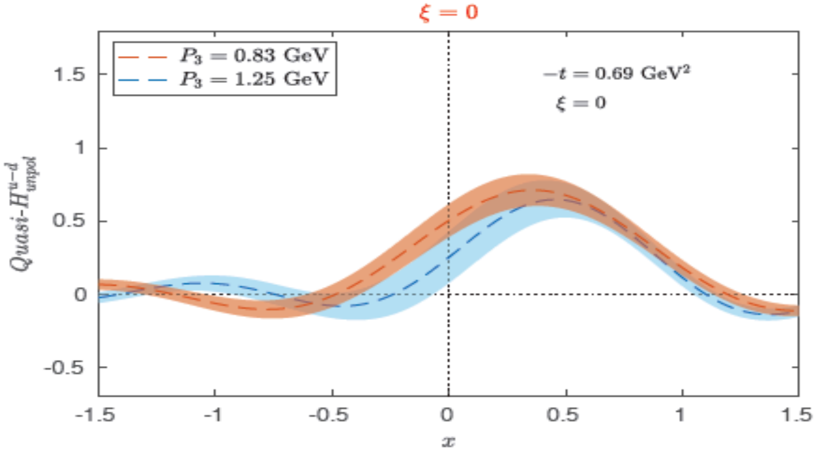
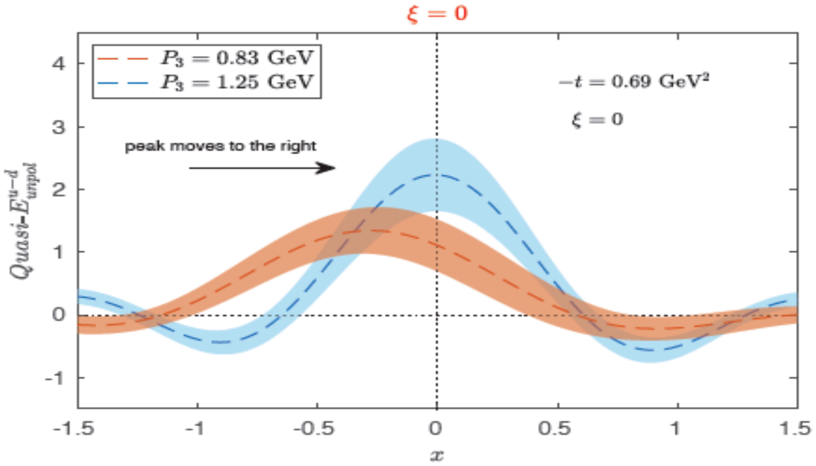
Unpolarized quasi-GPDs

Upon Fourier transform

$$\tilde{q}^{GPD} = \frac{2P_3}{4\pi} \sum_{z=-z_{max}}^{z=z_{max}} e^{-ixP_3 z} ME^{GPD}(z,t,\xi)$$





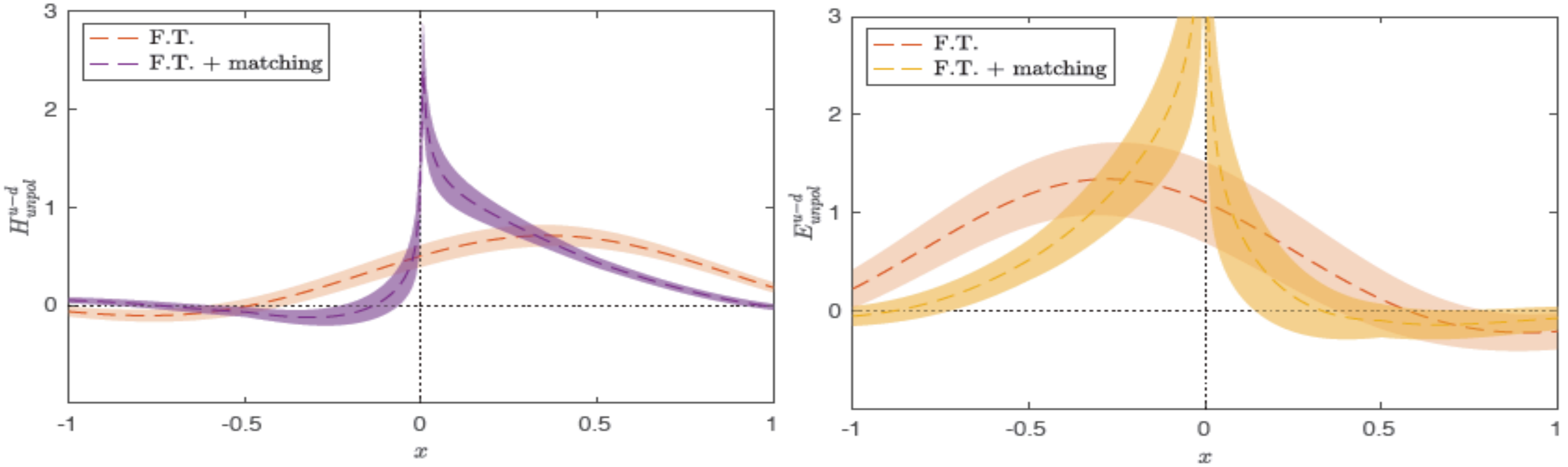
- Quasi-H and -E affected differently on the momentum boost
  - quasi-H is compatible within errors
  - quasi-E becomes symmetric in x (larger momenta will shed light on the behavior of the quasi-E)

Still non-physical results, matching is needed

Matching effect on the GPDs

- We apply the RI→ $\overline{\text{MS}}$ matching [Y-S Liu et al., Phys.Rev. D100 (2019) no.3, 034006]

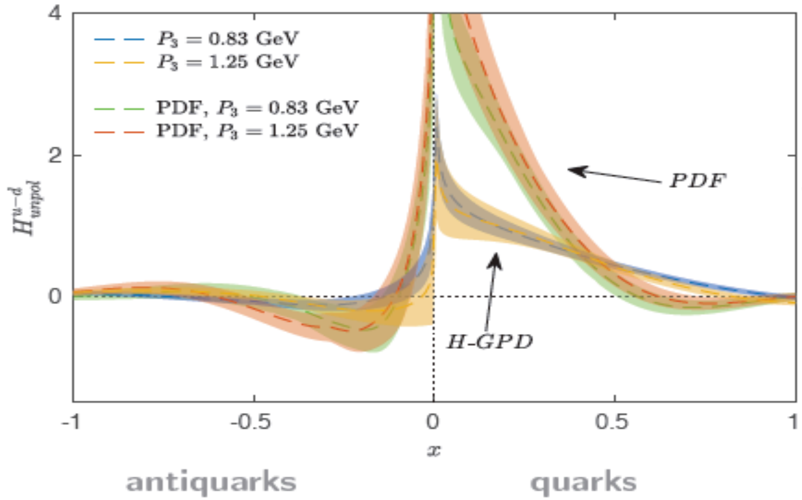
$P_3 = 0.83$ GeV, $-t = 0.69$ GeV^2, $\xi = 0$

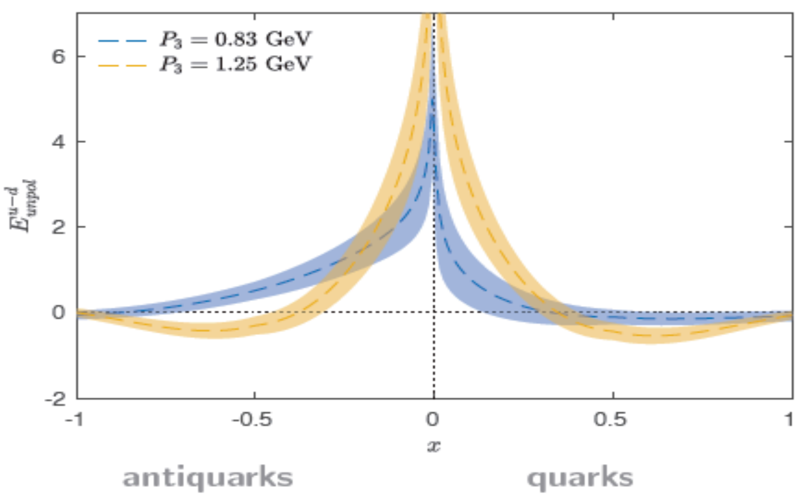


- Matching affects both H and E largely

Unpolarized GPDs (at $\xi = 0$)

Momentum dependence on $H(x,\xi,t)$ and $E(x,\xi,t)$, at $-t = 0.69$ GeV^2





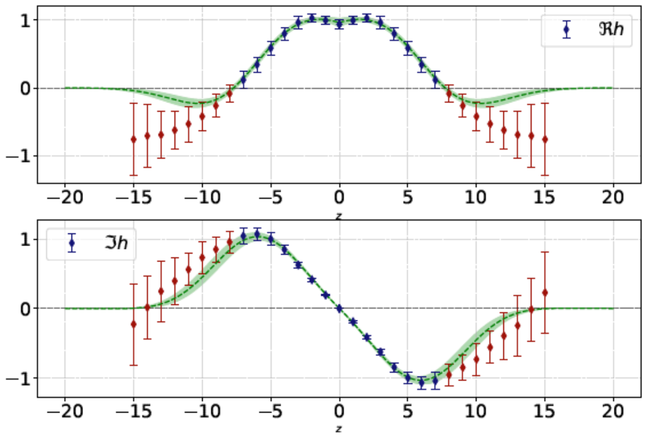
- Compatible results in H and PDF at $P_3 = 0.83$ GeV and $P_3 = 1.25$ GeV
- H-GPD suppressed with respect to the PDF, as expected
- Remarkable P_3-dependence in E-GPD
- E-GPD becomes symmetric in x at the larger P_3

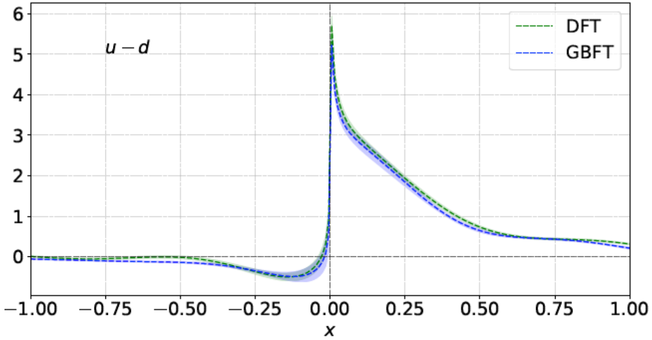
★ Lattice results will be compared to global fits to DVCS data

Lattice QCD: Quasi-PDFs approach - Challenges

Jansen

- continuum limit
- 2-loop formulae
  - matching formulae
  - conversion factors
- understanding and removing the oscillations
- reach a quantitative understanding of quasi PDFs
  - control systematic effects
  - come to prediction level
- we are on a very way to come there



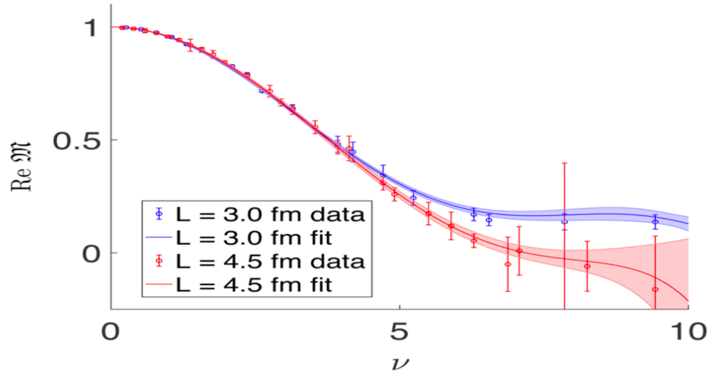


Lattice QCD calculated PDFs – Pseudo-PDFs approach

Nocera, Qiu, Richards, Zafeiropoulos

- Volume effect:

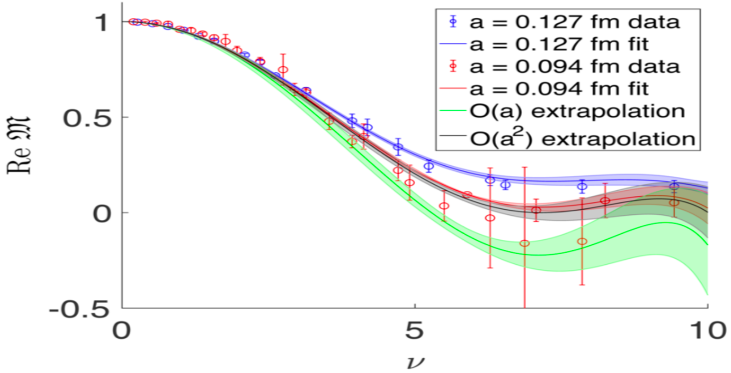
[B. Joo et al. (JLab-W&M), arXiv:1908.09771]



a(fm)	M_π(MeV)	$L^3 \times T$
0.127(2)	415(23)	$24^3 \times 64$
0.127(2)	415(23)	$32^3 \times 96$
0.094(1)	390(71)	$32^3 \times 64$

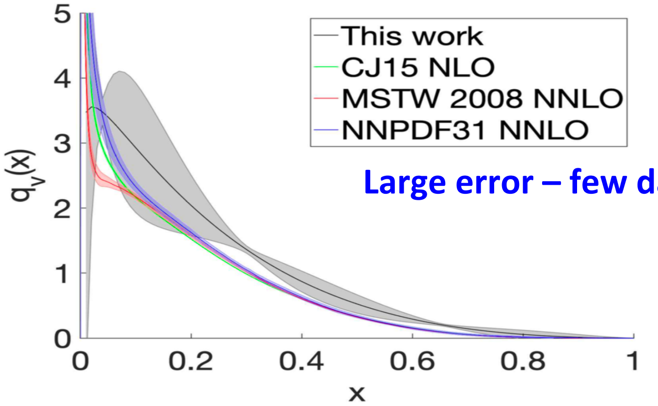
N_f=2+1 clover fermions (3 ensembles):

- Discretization effect:



Too strong "L" & "a" dependence – limited the range of ν!

- Extract/fit PDF from lattice data with a functional form similar to CJ and MSTW

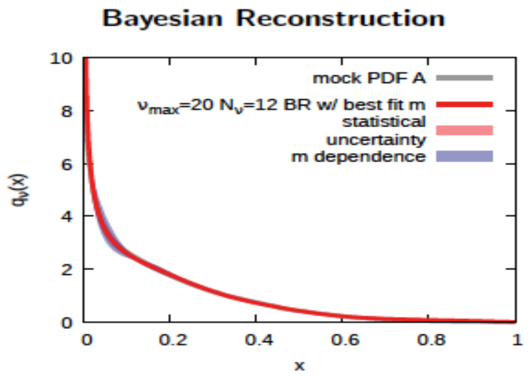


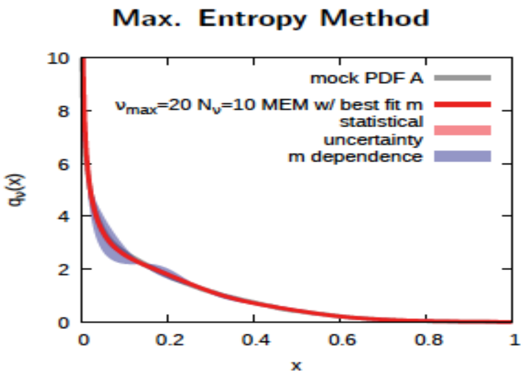
Challenges due to lattice limitation
Results are encouraging!

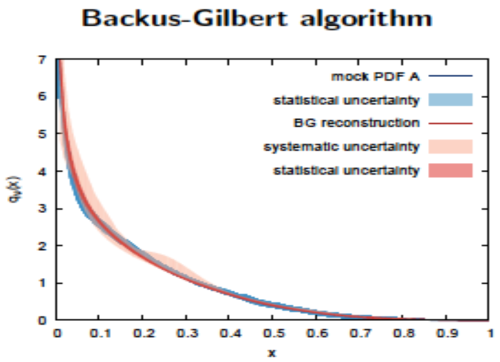
Lattice QCD calculated PDFs – Pseudo-PDFs approach

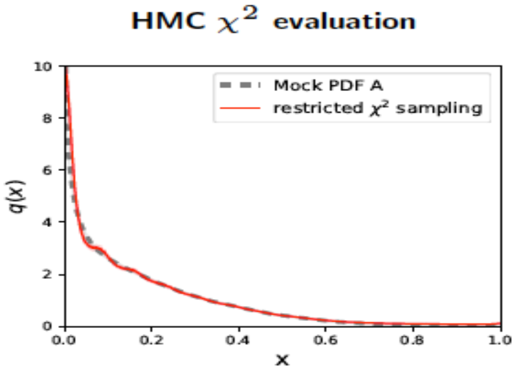
Richards, Zafeiropoulos

- ❑ **Various inverse approaches:**

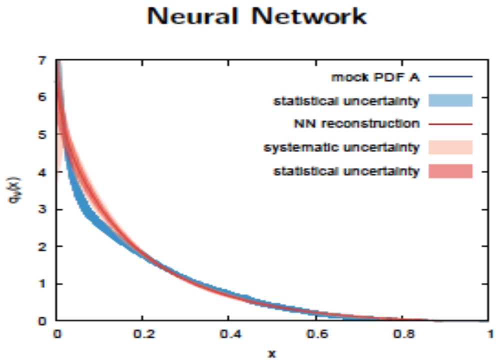








Capitalize of the good scanning in Ioffe time and use advanced reconstruction methods to extract the maximum amount of information also for the small-x region.

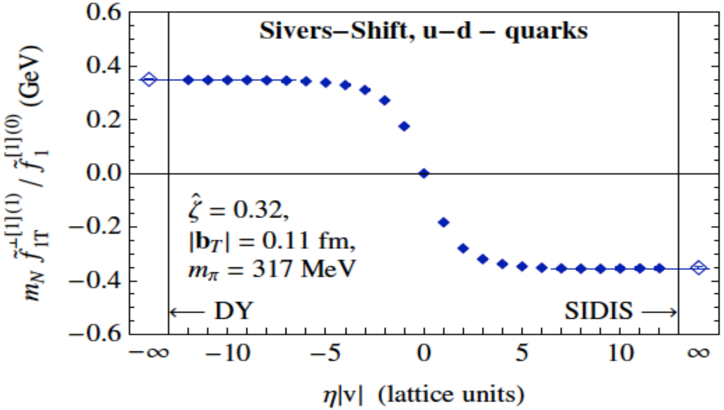


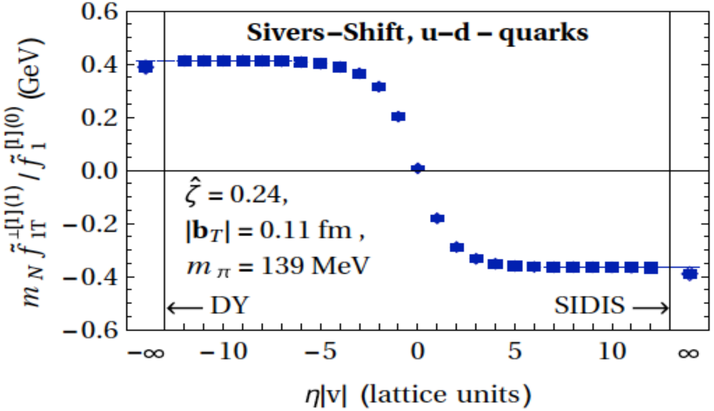
Karpie, Orginos, Rothkopf, S.Z. JHEP 1904 (2019) 057

Lattice QCD calculation of TMDs:

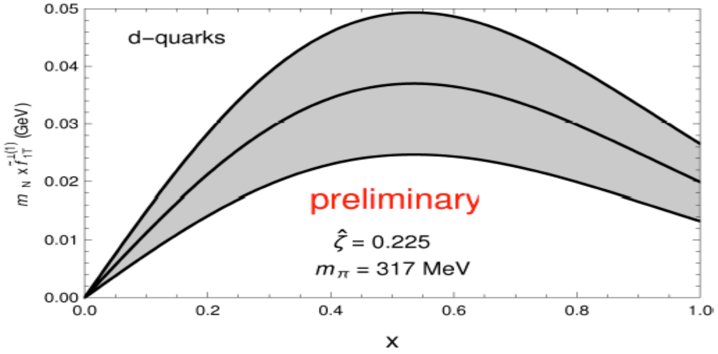
Engelhardt

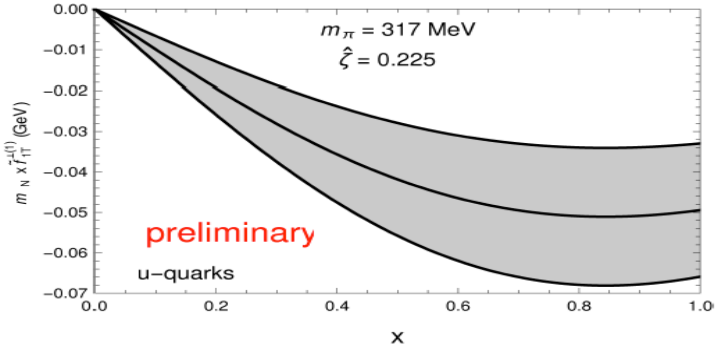
❑ Sivers' sign change:





❑ Advertisement – x-dependent TMDs:



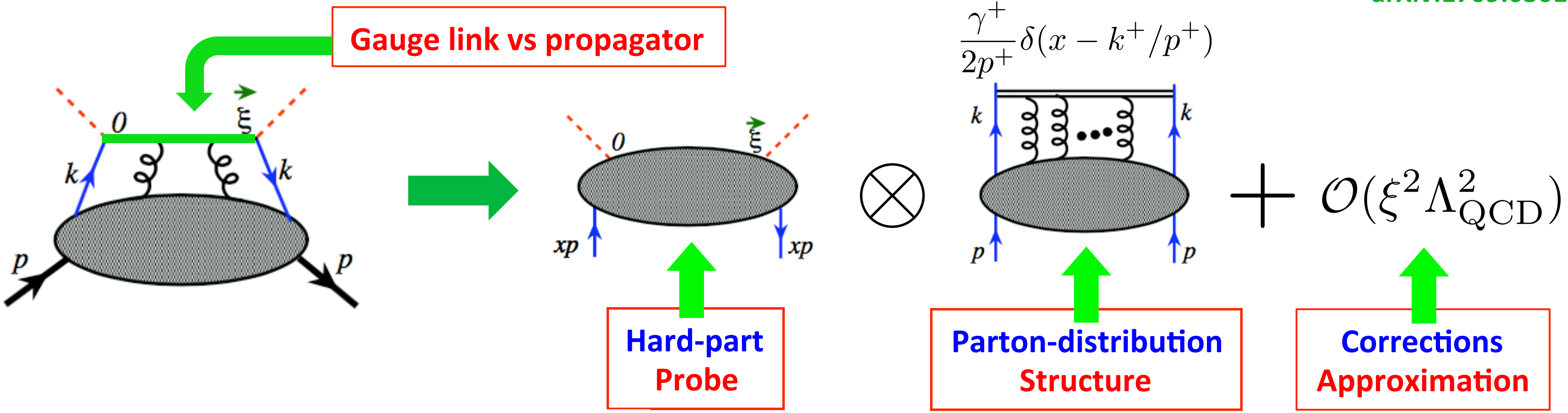


Lattice QCD calculated PDFs – current-current approach

Qiu, Richards

❑ Factorization:

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018



$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda^2_{\mathrm{QCD}})$$

with $f_a(x,\mu^2) = -f_a(-x,\mu^2)$

❑ Tremendous potentials:

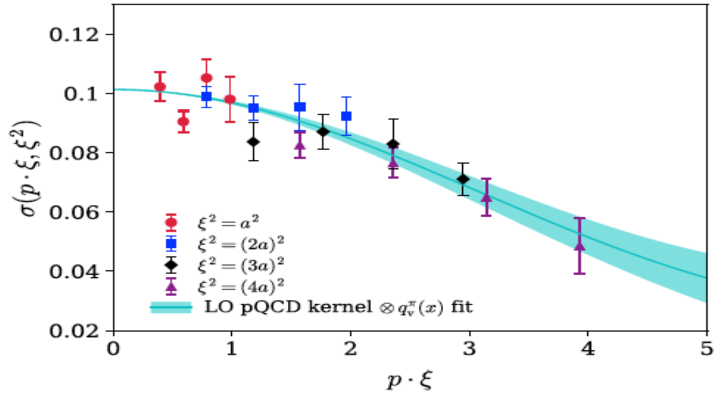
- ✧ *Neutron PDFs, … (no free neutron target!)*
- ✧ *Meson PDFs, such as pion, …*
- ✧ *More direct access to gluons – gluonic current, quark flavor, …*

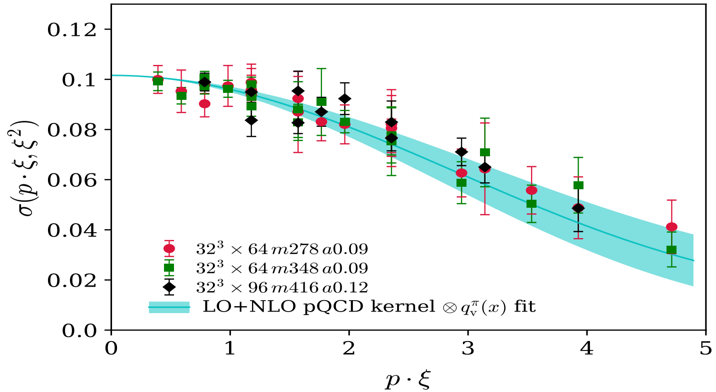
Lattice QCD calculated pion distribution – current-current approach

Qiu, Richards

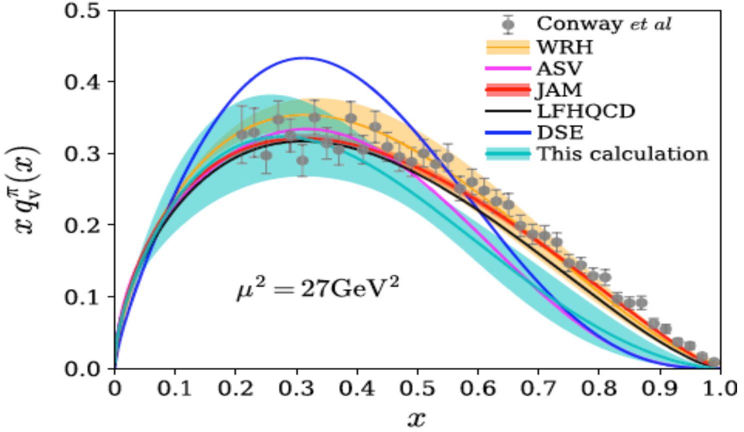
- "Lattice cross section" of V-A current correlator:

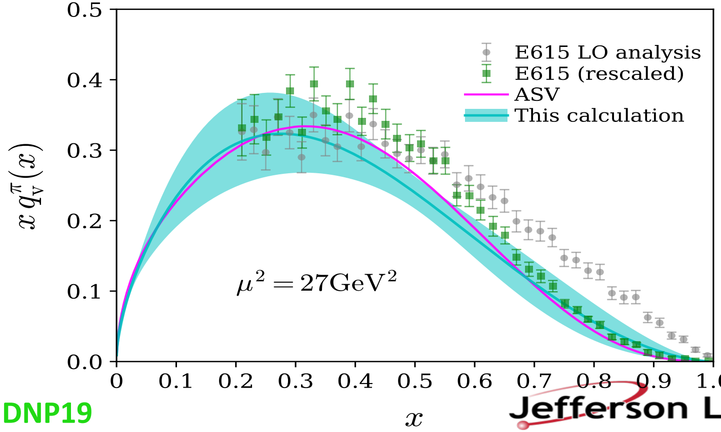
Sufian et al. JLab
PRD99 (2019) 074507





- Extracted pion valence quark distribution:





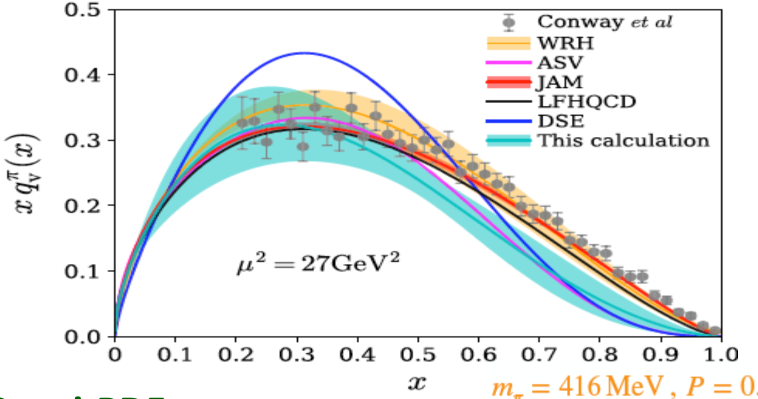
Sufian et al. @ DNP19

Jefferson Lab

Lattice QCD calculated pion distribution

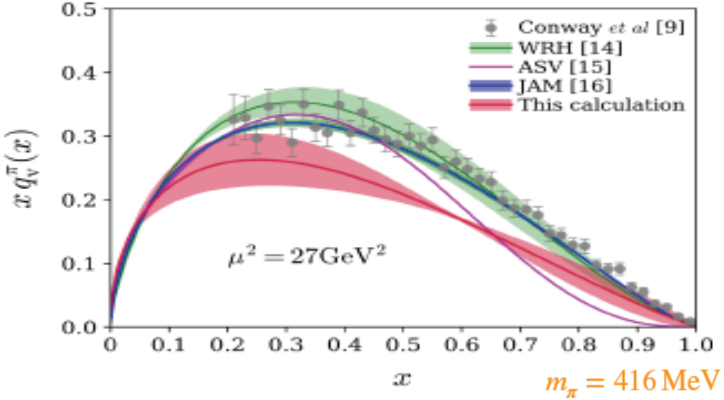
Qiu, Richards, Zafeiropoulos

- "Lattice cross section":
  [R. Sufian et al. (JLab - W&M), Phys. Rev. D 99 (2019) 074507]



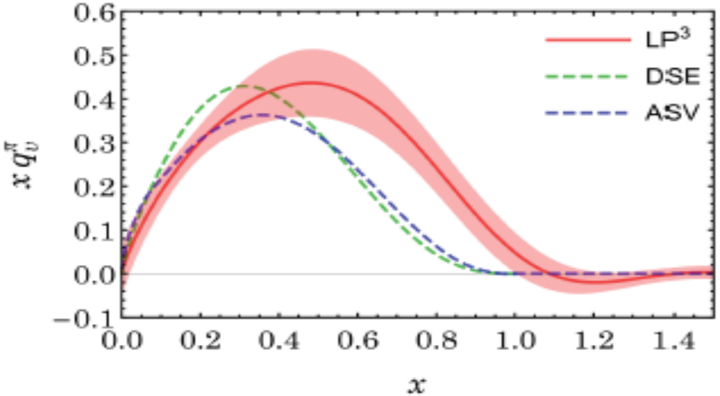
$m_\pi = 416\,\text{MeV}$, $P = 0.6 - 1.5$

- Pseudo-PDF:
  [B. Joo et al. (JLab-W&M), arXiv:1909.08517]



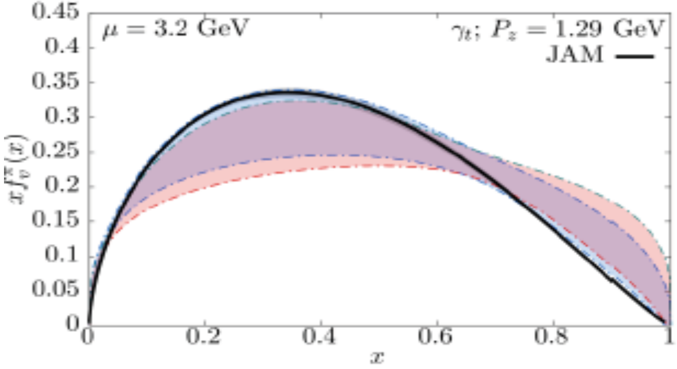
$m_\pi = 416\,\text{MeV}$, $\nu \leq 4.7$

- Quasi-PDF:
  [J.-H. Zhang et al. (LP3),Phys. Rev. D 100, 034505 (2019)]



GeV

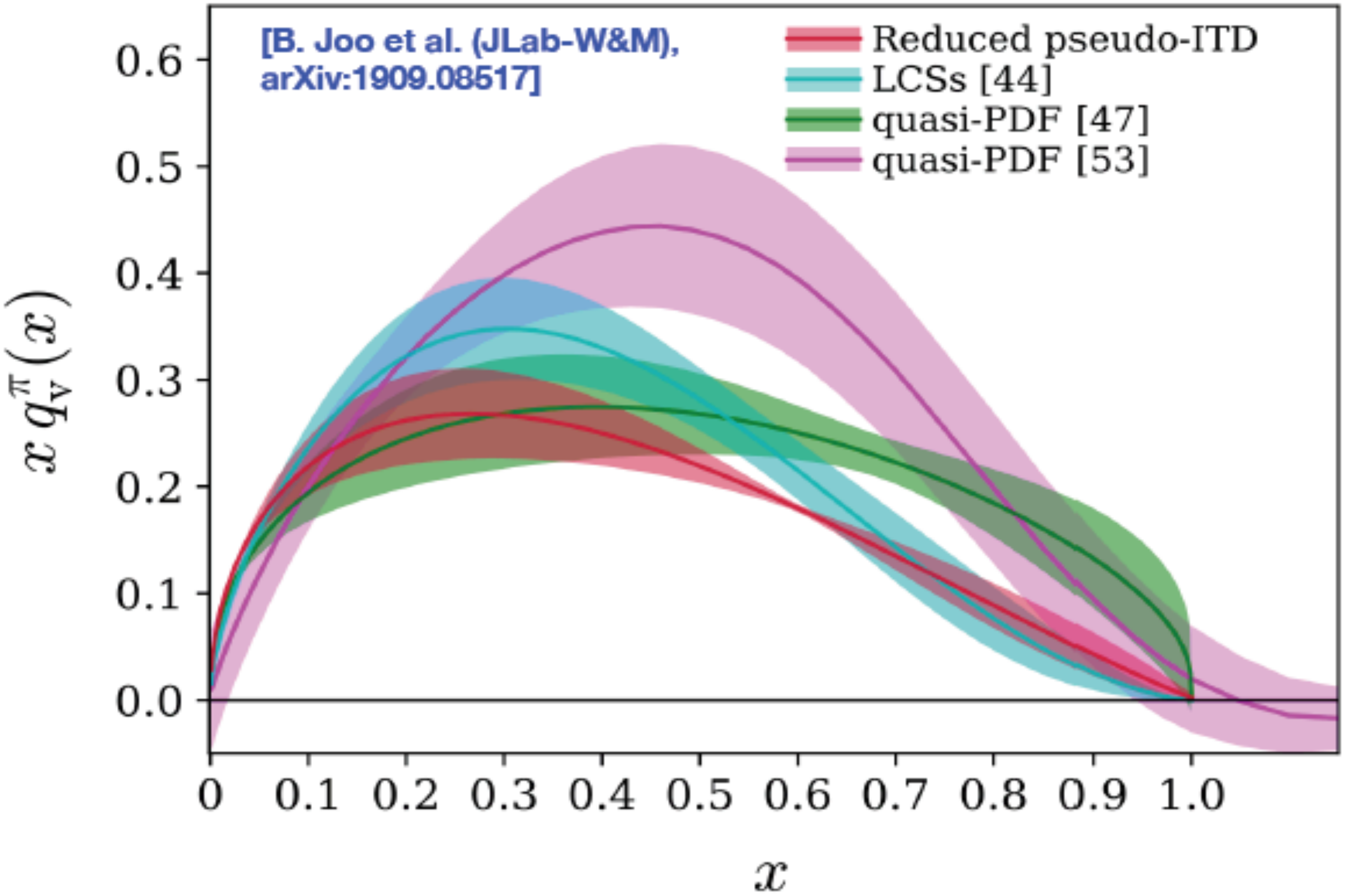
[T. Izubuchi et al. (BNL-SBU-UConn), Phys. Rev. D 100, 034516 (2019)]



$m_\pi = 300\,\text{MeV}$, $P = 1.29\,\text{GeV}$

Lattice QCD calculated pion distribution

Qiu, Richards



Summary

- Although lattice QCD cannot calculate parton distributions directly, many new ideas and approaches make it possible to extract PDFs, GPDs, TMDs, … from lattice QCD calculations

- Like extracting PDFs and partonic structure from hadronic cross sections, PDFs and non-perturbative partonic structure can be extracted from:

   1) Lattice QCD calculable hadronic matrix elements (quasi-, pseudo-, current-current correlators, … , which
   2) can be factorized/matched into PDFs or any universal partonic distributions

- Tremendous progresses have been made for extracting PDFs from lattice QCD calculations, with various and complementary approaches

- Lattice QCD can be used to study hadron structure, including PDFs, GPDs and TMDs, and will meet and complement to our phenomenology approaches. But, more works are still needed for understanding the systematic uncertainties, lattice artifacts, …

Thank you!