First order QED corrections to the Bethe-Heitler process in the $\gamma p \to l^+ l^- p$ reaction

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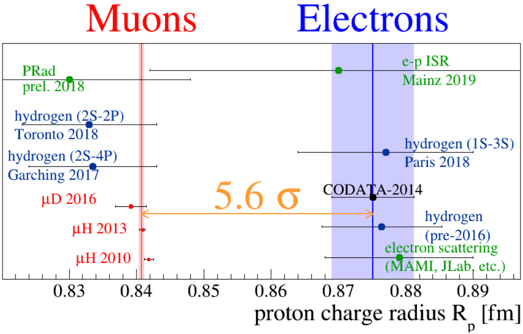
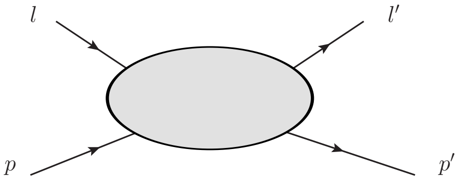
Proton radius puzzle

- talks by Nilanga Liyanage and Randolph Pohl
- discrepancy between measurements from muonic spectroscopy and electron data
- at least 4 standard deviations difference
- physics beyond the Standard model?



Test of lepton universality

- Assume universal proton form factor for all leptons
  $\rightarrow$ same proton radius for muons and electrons
- Broken universality could be an explanation for proton radius puzzle
- Test this with upcoming experiments:
  - MUSE @ PSI (electron vs muon scattering)
  - COMPASS @ CERN (high energy muon scattering off the proton)
  - MAMI $\gamma p \rightarrow e^- e^+ p$ vs. $\gamma p \rightarrow \mu^- \mu^+ p$



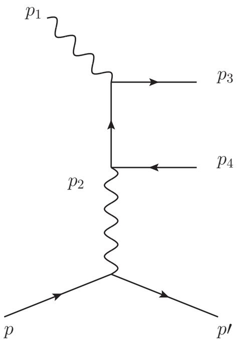
Outline

1. Bethe-Heitler process at leading order
2. Corrections in soft-photon approximation
3. First order QED corrections to Lepton tensor
4. Hadronic corrections
5. Results

Bethe-Heitler process

$2 \to 3$ process has 5 indepenedent kinematic invariants

$$(p_3 + p_4)^2 = s_{ll}$$
$$(p_3 - p_1)^2 = t_{ll}$$
$$(p_3 - p_2)^2 = u_{ll}$$
$$(p_1 + p)^2 = s$$
$$(p - p_3)^2 = u$$
$$p_2^2 = (p - p')^2 = t$$
$$s_{ll} + t_{ll} + u_{ll} = 2m^2 + t$$



Lab quantities: $E_\gamma, E_{p'}, \theta_{p'}, \phi_{ll}^{\text{lab}}, \theta_{ll}^{\text{lab}}$

Differential cross section

Fully differential cross section is given by:

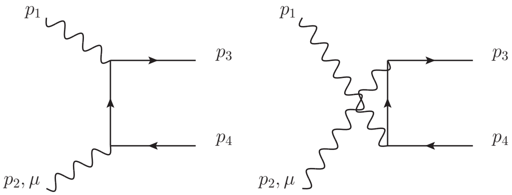
$$\left(\frac{d\sigma}{dt\, ds_{ll}\, d\Omega_{ll}^{CM_{l^+l^-}}} \right)_0 = \frac{\alpha^3 \sqrt{1 - \frac{4m^2}{s_{ll}}}}{16\pi (2ME_\gamma)^2\ t^2} L_0^{\mu\nu} H_{\mu\nu}$$

The unpolarized hadron tensor can be expressed in terms of two form factors:

$$H^{\mu\nu} = (-g^{\mu\nu} + \frac{p_2^\mu p_2^\nu}{p_2^2})[4M^2 \tau G_M^2(t)] + \tilde{p}^\mu \tilde{p}^\nu [G_E^2(t) + \tau G_M^2],$$

where $\tilde{p} \equiv (p + p')/2$ and $\tau \equiv -t/(4M^2)$.

Lepton tensor



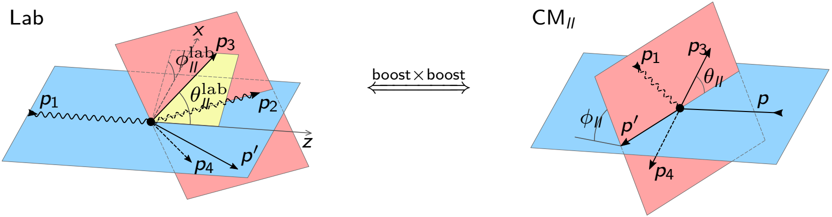
The leading order unpolarized lepton tensor is given by:

$$L_0^{\mu\nu} = -\frac{1}{2} \operatorname{Tr} \left[(\not{p}_3 + m) \left(\gamma^\alpha \frac{(\not{p}_3 - \not{p}_1 + m)}{(p_3 - p_1)^2 - m^2} \gamma^\mu + \gamma^\mu \frac{(\not{p}_1 - \not{p}_4 + m)}{(p_1 - p_4)^2 - m^2} \gamma^\alpha \right) \right.$$
$$\left. \times (\not{p}_4 - m) \left(\gamma^\nu \frac{(\not{p}_3 - \not{p}_1 + m)}{(p_3 - p_1)^2 - m^2} \gamma_\alpha + \gamma_\alpha \frac{(\not{p}_1 - \not{p}_4 + m)}{(p_1 - p_4)^2 - m^2} \gamma^\nu \right) \right]$$

Integrated cross section

Only the recoiled proton is observed
$\rightarrow$ integration over lepton angles in center-of-mass frame of dilepton-pair

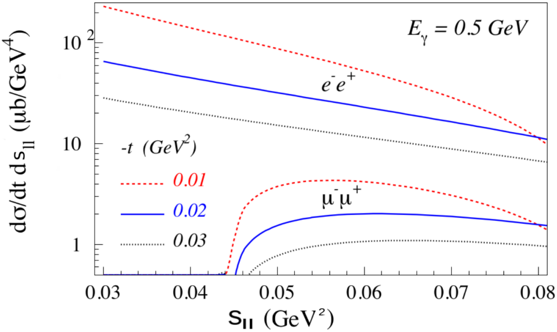
$$\left(\frac{d\sigma}{dt\, ds_{ll}}\right)_0 = \frac{\alpha^3\beta}{16\pi(2ME_\gamma)^2\ t^2} \cdot \int d\Omega_{ll}^{CM_{l^+l^-}}\ (L_0)_{\mu\nu}\, H^{\mu\nu}.$$



Bethe-Heitler process

Comparison of cross sections for $l = e$ and $l = \mu$ can be used to test lepton universality:

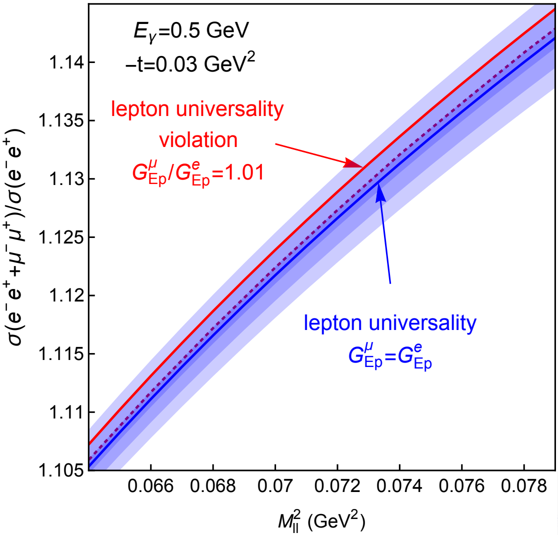
$$R(s_{ll}, s_{ll}^0) \equiv \frac{[\sigma(\mu^+\mu^-)](s_{ll}) + [\sigma(e^+e^-)](s_{ll})}{[\sigma(e^+e^-)](s_{ll}^0)}$$



Pauk and Vanderhaeghen, PRL 115 (2015)

Ratio at tree level

aim: measure ratio with absolute precision of $\sigma = 7 \times 10^{-4}$



Soft-photon approximation

Assume scaling of loop momentum

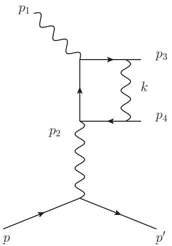
$$k \sim \lambda$$
$$d^4k \sim \lambda^4,$$

where λ is small compared to all external scales
$\rightarrow$ sensitive to infrared divergences

propagator denominator	scaling at least as
$(k+p_3)^2 - m^2$	λ
$(k-p_4)^2 - m^2$	λ
k^2	λ^2
$(p_3 - p_1 + k)^2 - m^2$	1

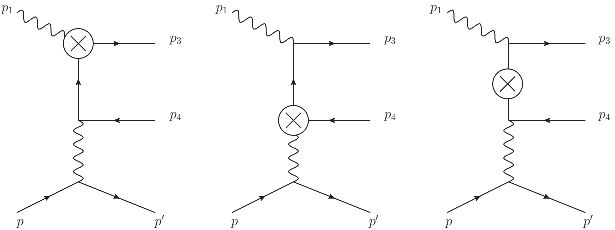
$\rightarrow$ only box graph contributes

Virtual soft-photon corrections

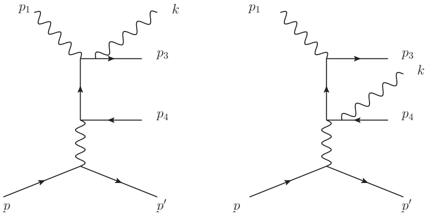


$$\mathcal{M}^{\text{box}} = (ie^2)\, 4 \cdot (p_3 p_4) \cdot \mathcal{M}_0\, \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(p_3+k)^2 - m^2} \times \frac{1}{(k-p_4)^2 - m^2} \frac{1}{k^2} + \mathcal{O}(\lambda) = -\frac{e^2}{8\pi^2} \left(s_{ll} - 2m^2 \right) \cdot \mathcal{M}_0 \cdot C_0 \left(m^2, s_{ll}, m^2, 0, m^2, m^2 \right)$$

In on-shell scheme, we need infrared divergent parts of counter terms:



Real soft-photon corrections



$$\left|\mathcal{M}(\gamma p \to \gamma_s\, l^+ l^- p)\right|^2 = \left|\mathcal{M}(\gamma p \to l^+ l^- p)\right|^2 (-e^2) \left[\frac{p_3^\mu}{p_3 \cdot k} - \frac{p_4^\mu}{p_4 \cdot k}\right] \left[\frac{p_{3\mu}}{p_3 \cdot k} - \frac{p_{4\mu}}{p_4 \cdot k}\right] + \mathcal{O}(\lambda)$$

Integration in frame $\mathcal{S}$, in which $\vec{p}_3 + \vec{p}_4 + \vec{k} = 0$, over soft-photon energy up to ΔE_s:

$$\left(\frac{d\sigma}{dtds_{ll}}\right)_{s;R} = -\left(\frac{d\sigma}{dtds_{ll}}\right)_0 \frac{e^2}{(2\pi)^3} \int_{|\vec{k}|<\Delta E_s} \frac{d^3\vec{k}}{2k^0} \left[\frac{m^2}{(p_3 k)^2} + \frac{m^2}{(p_4 k)^2} - \frac{2(p_3 p_4)}{(p_3 k)(p_4 k)}\right]$$

Detector resolution

- measure energy $E_{p'}$ and scattering angle $\theta_{p'}$ of recoiling proton
- this gives s_{ll} and $\tau = -t/(4M^2)$:

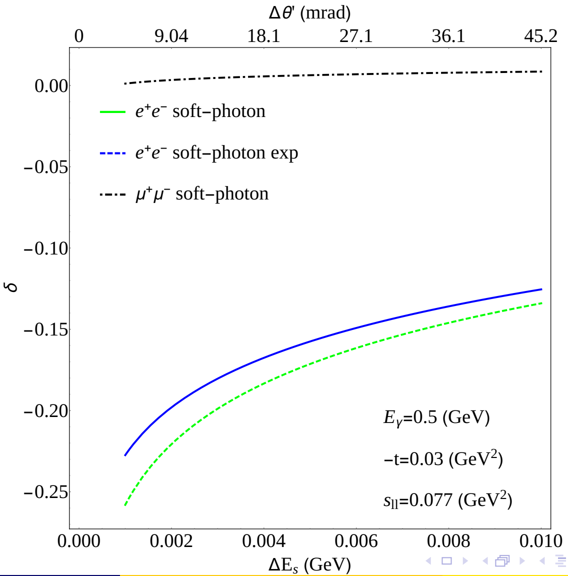
$$E' = M(1 + 2\tau)$$

$$\cos\theta_{p'} = \frac{s_{ll} + 2(s + M^2)\tau}{2(s - M^2)\sqrt{\tau(1+\tau)}}$$

Due to bremsstrahlung s_{ll} gets shifted, resulting in relation between maximum energy of undetected soft photon and experimental recoiling proton angular resolution

$$\Delta E_s = \frac{2ME_\gamma\sqrt{\tau(1+\tau)}}{\sqrt{s_{ll}}} \sin\theta_{p'}\, \Delta\theta_{p'}$$

Detector resolution



Corrections in soft-photon approximation

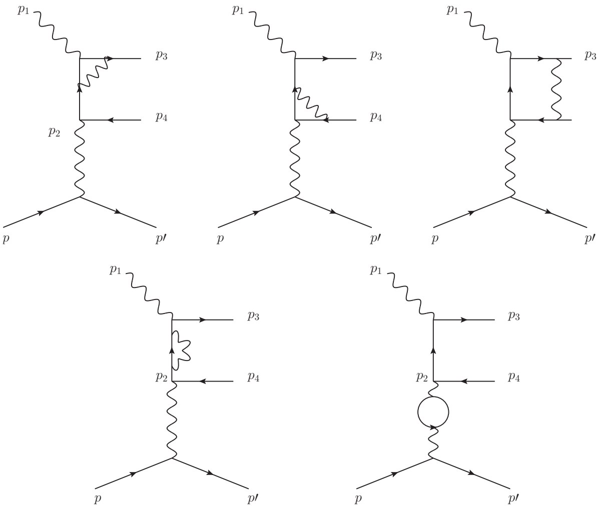
$$\left(\frac{d\sigma}{dt\ ds_{ll}}\right)_s \equiv \left(\frac{d\sigma}{dt\ ds_{ll}}\right)_0 (1+\delta)$$
$$= \left(\frac{d\sigma}{dt\ ds_{ll}}\right)_0 \left\{1 - \left(\frac{\alpha}{\pi}\right)\left\{\left[\ln\left(\frac{4\Delta E_s^2}{m^2}\right) + \ln\left(\frac{1-\beta}{1+\beta}\right)\right]\left[1 + \left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right)\right]\right.\right.$$
$$\left.\left. + \left(\frac{1-\beta}{\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{1+\beta^2}{2\beta}\right)\left[4\ \mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) + \ln^2\left(\frac{1+\beta}{1-\beta}\right) - \pi^2\right]\right\}\right\}$$

Following [Yennie, Frautschi, Suura, 61] we can exponentiate the "double logs":

$$\left(\frac{d\sigma}{dt\ ds_{ll}}\right)_{s;tot} \equiv \left(\frac{d\sigma}{dt\ ds_{ll}}\right)_0 (1+\delta_{\exp})$$
$$= \left(\frac{d\sigma}{dt\ ds_{ll}}\right)_0 F\ \exp\left\{-\frac{\alpha}{\pi}\left[\ln\left(\frac{4\Delta E_s^2}{m^2}\right) + \ln\left(\frac{1-\beta}{1+\beta}\right)\right]\left[1 + \left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right)\right]\right\}$$
$$\times \left\{1 - \frac{\alpha}{\pi}\left[\left(\frac{1-\beta}{\beta}\right)\ln\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{1+\beta^2}{2\beta}\right)\left[4\ \mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) + \ln^2\left(\frac{1+\beta}{1-\beta}\right) - \pi^2\right]\right]\right\}$$

with $\beta = \sqrt{1 - \frac{4m^2}{s_{ll}}}$, $F = 1 + \mathcal{O}(\alpha^2)$ soft-photon phase-space factor

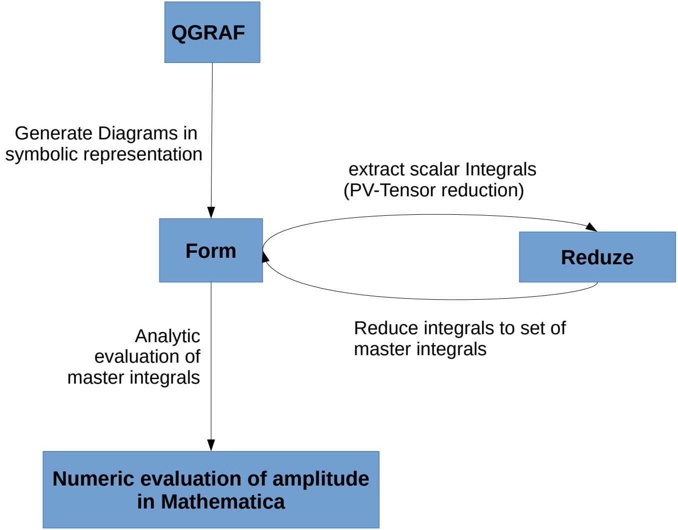
One-loop diagrams



Strategy of calculation

- after applying Feynman rules, map loop integration to scalar integrals
- use Integration-By-Part (IBP) identities to relate different integrals to set of master integrals
- use of computer algebra to do this:
  - `QGRAF` [Nogueira '93]
  - `FORM 4.1` [Kuipers, Ueda, Vermaseren, Vollinga '12]
  - `Reduze 2` [Studerus, von Manteuffel '12]
- dimensional regularization [t'Hooft and Veltman, '72] for UV and IR divergences

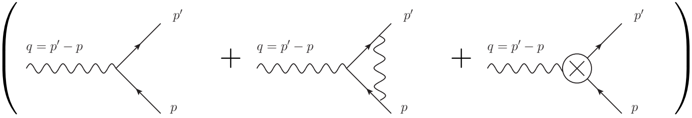
Automated evaluation of Feynman diagrams



On-shell renormalization

- amplitude is divergent for high loop momenta
- absorb infinities in counter terms
- renormalization procedure to fix finite piece of counter terms

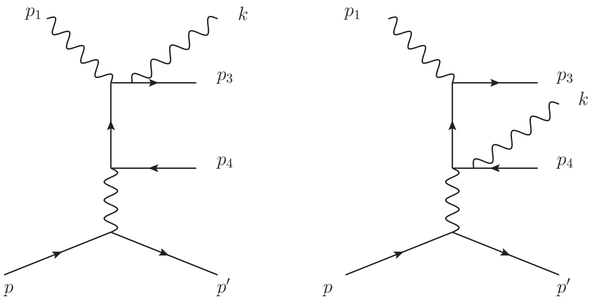
e.g. vertex renormalization:



$$\Big(\ldots \Big)_{q=0} = ie\bar{u}\gamma_\mu u$$

IR divergences

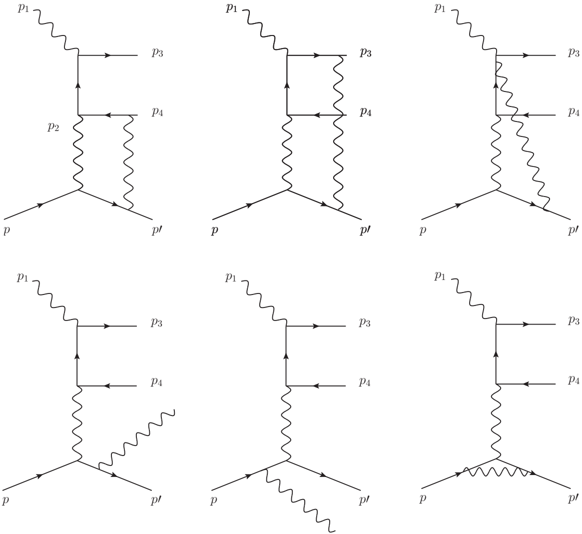
- full one-loop calculation reproduces IR divergences as predicted from soft-photon limit
- cancellation with IR divergences from real radiation
- still use soft-photon approximation for these (no hard radiation)



Checks of the calculation

- Virtual corrections have the correct infrared structure
- Gauge invariance of lepton tensor
- Reproduction of soft-photon double logarithms
- Exact agreement with known result [Huld, 68] for $m^2 << s_{ll}, t, t_{ll}$
- Comparison with independent, numerical calculation in `FormCalc`

Hadronic corrections



Cancellation of Box type diagrams

Consider parity transformation of di-lepton pair

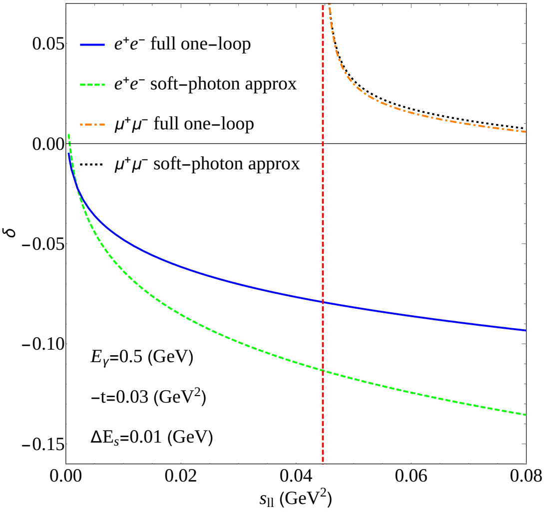
$$\theta_{ll} \to \pi - \theta_{ll}, \qquad \phi_{ll} \to \pi + \phi_{ll}$$

This corresponds to interchange of lepton and anti-lepton, resulting in a symmetry property of the cross section:

$$\mathrm{d}\sigma_{\mathrm{pBox}}^{\gamma p \to p l_+ l_-}(\pi - \theta_{ll},\ \phi_{ll} + \pi) = -\mathrm{d}\sigma_{\mathrm{pBox}}^{\gamma p \to p l_+ l_-}(\theta_{ll},\ \phi)$$

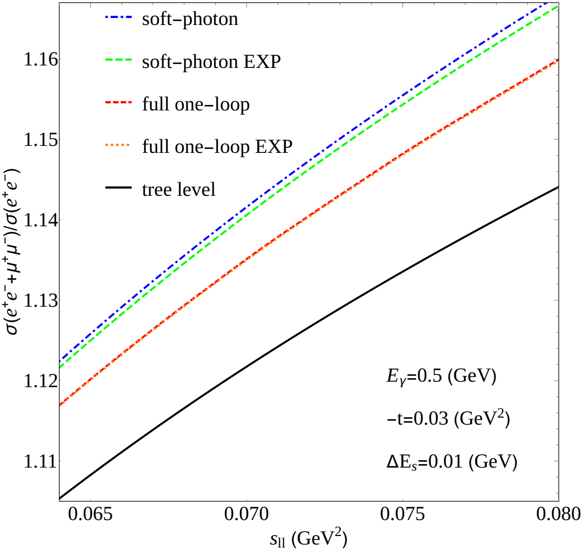
Integration over lepton angles yields therefore exactly zero

Correction to cross section



[M.H., Tomalak, Vanderhaeghen, 18, M.H., Tomalak, Wu, Vanderhaeghen, 19]

Effect on ratio of cross sections



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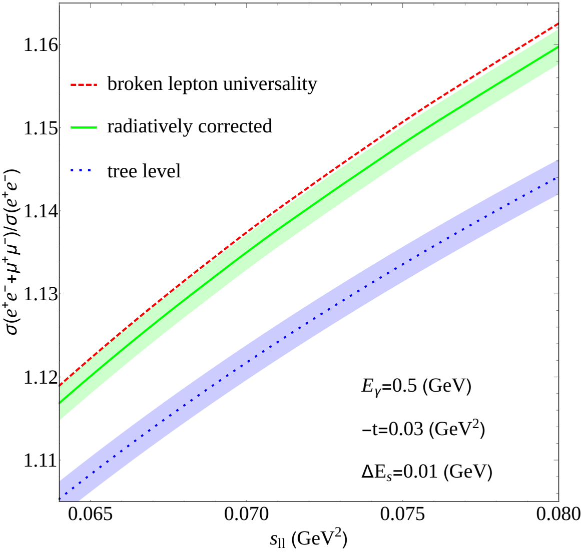

Conclusion and Outlook

- we calculated full one-loop QED corrections on lepton side of Bethe-Heitler process
- hadronic corrections are negligible at required level of precision
- upcoming experiment at MAMI (Mainz) aims to test lepton universality
- complementary $\gamma p \to l^+ l^- p$ experiment could shed light on the proton radius puzzle
- next step: calculation for initial photon off-shell (electron-scattering)