PDFs from Phenomenology

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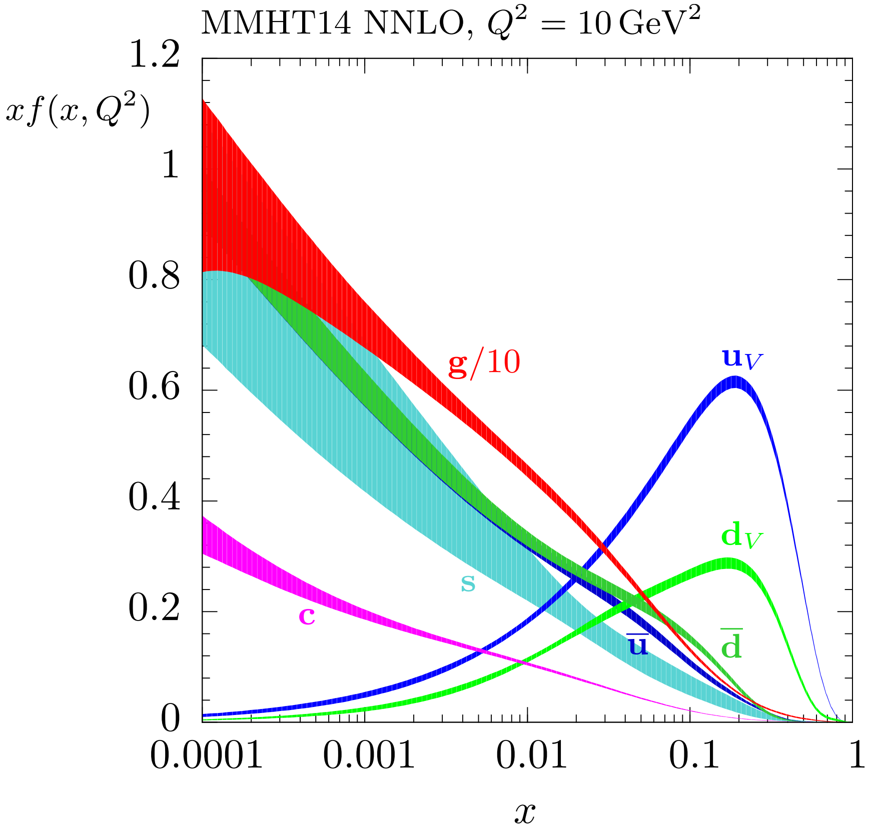
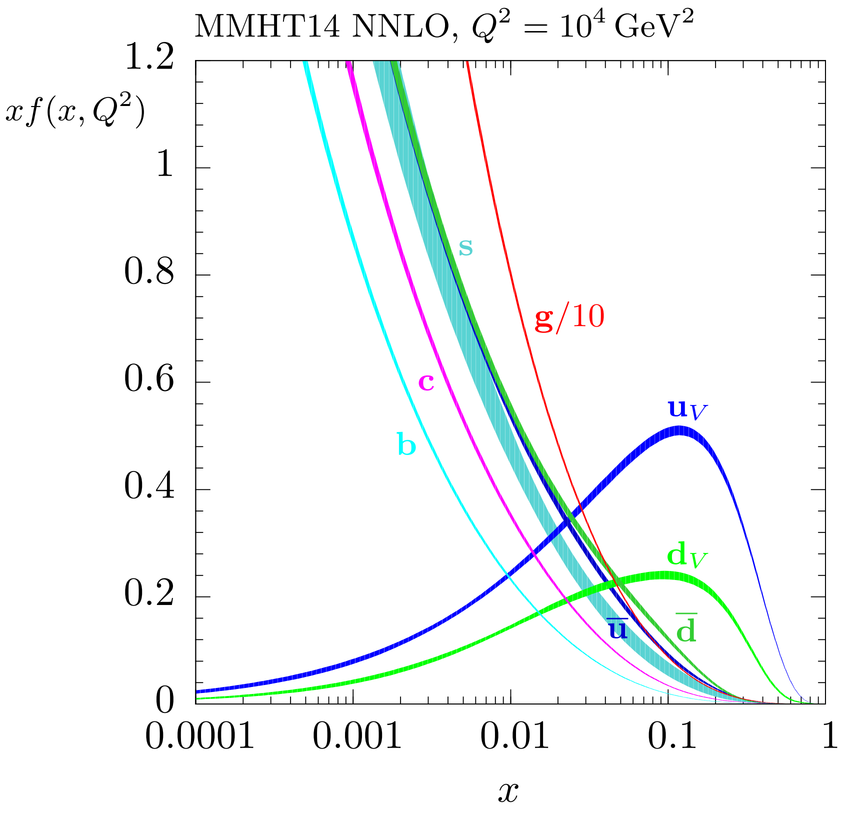
Outline

- ★ Why are PDFs important for collider phenomenology and how do we extract them?
- ★ Status of current global fits and role of LHC.
- ★ Outlook for future: HL-LHC and LHeC.
- ★ Challenges of current high precision LHC era.
- ★ Other topics of interest: theory uncertainties and the photon PDF.

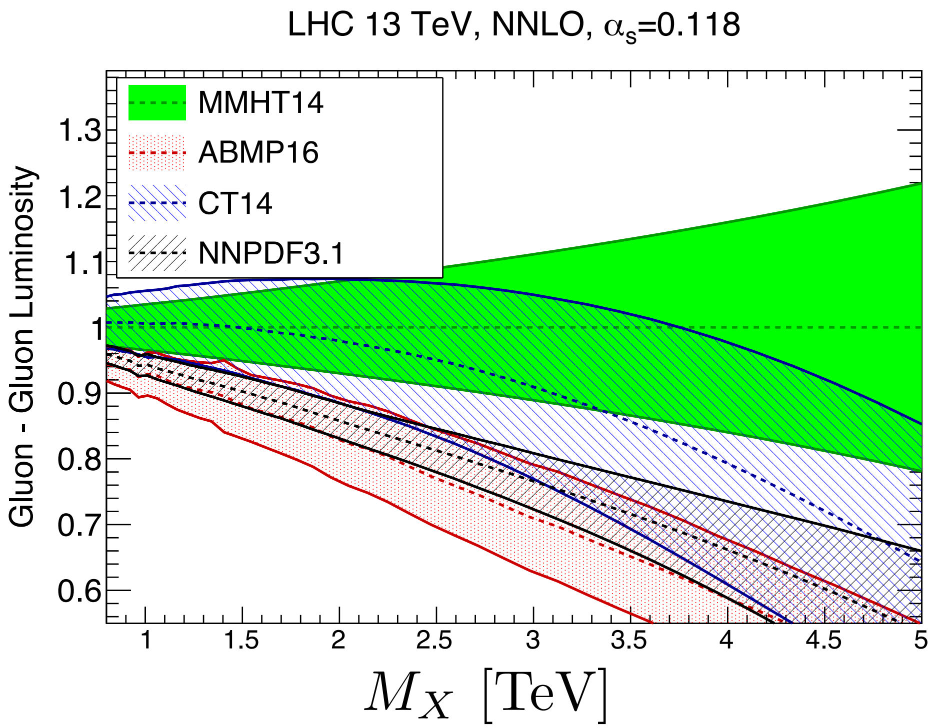


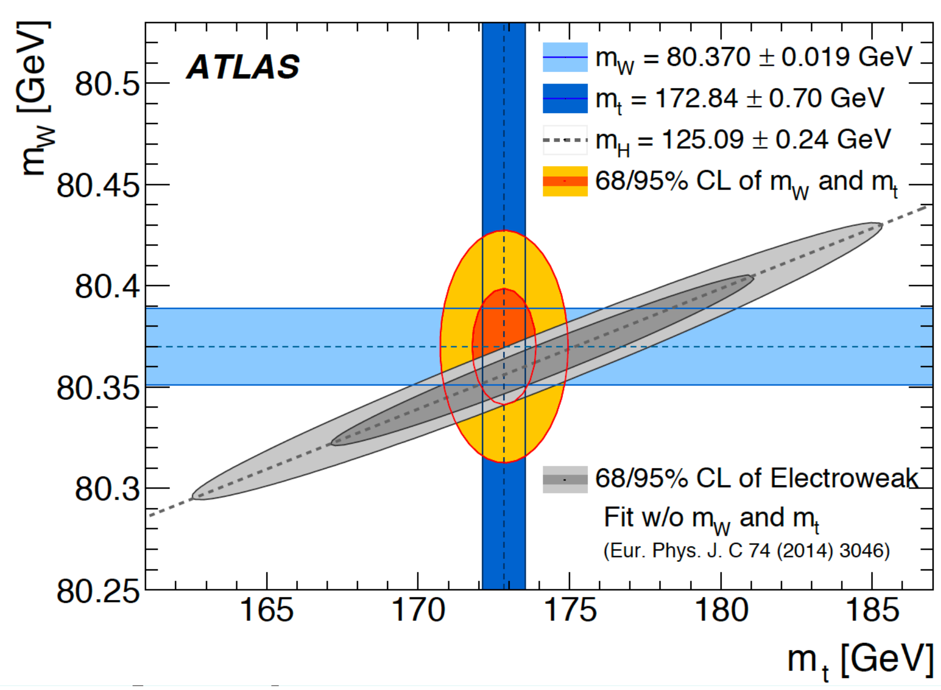


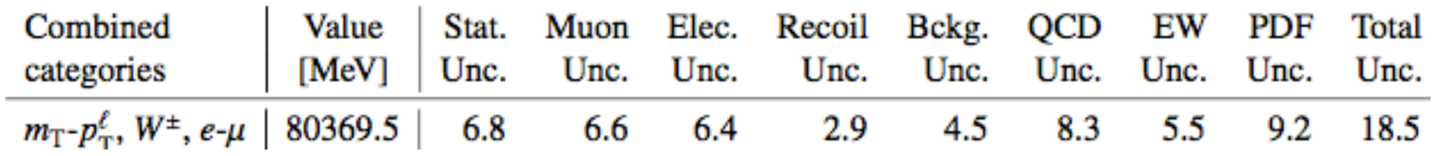
Motivation

- Ultimate reach of LHC limited by knowledge of PDFs.

- **High mass searches** - PDFs in high region (currently constraints poor)

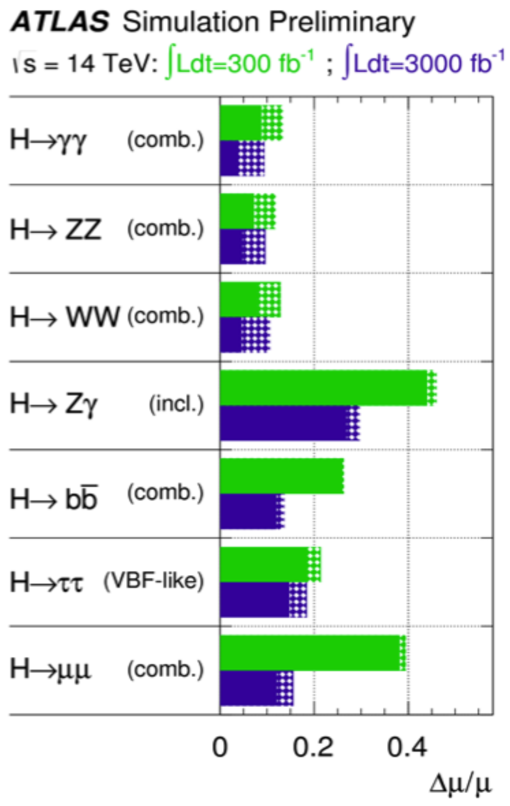






Combined categories	Value [MeV]	Stat. Unc.	Muon Unc.	Elec. Unc.	Recoil Unc.	Bckg. Unc.	QCD Unc.	EW Unc.	PDF Unc.	Total Unc.
m_T-p_T^ℓ, $W^\pm$, e-μ	80369.5	6.8	6.6	6.4	2.9	4.5	8.3	5.5	9.2	18.5

- **Higgs couplings** → need to model SM production precisely.

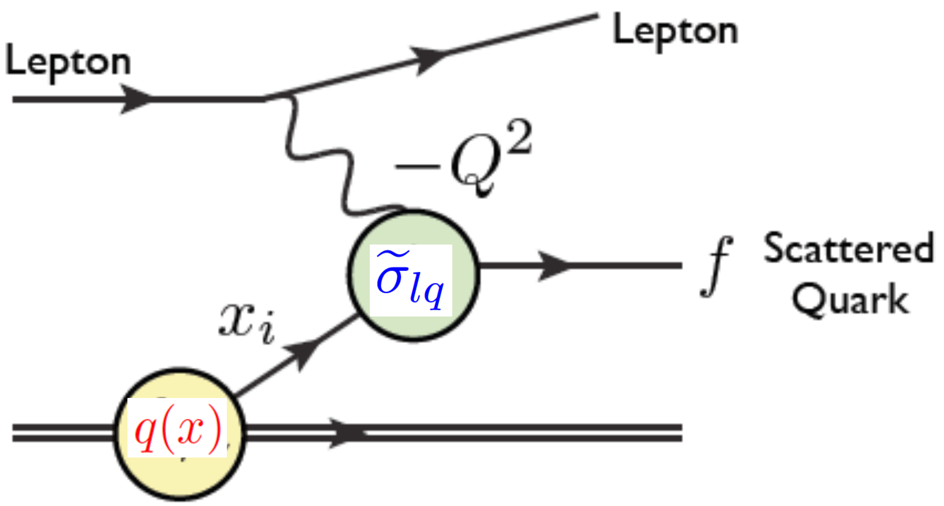


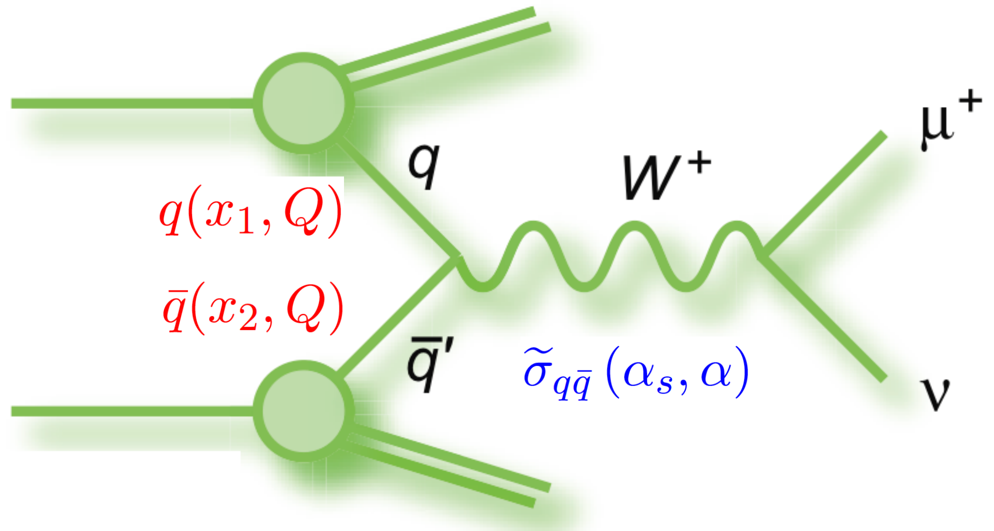
- **Precision SM** measurements - PDFs dominant uncertainty for e.g. W mass.

PDF Fits - Basic Idea

- Instead of trying to predict PDFs from first principles, **fit** to data we do understand/can rely on Standard Model predictions. Based on:
  - ★ QCD **factorization** theorems.
  - ★ **Precise** predictions for parton-level processes and reliability of pQCD.
  - ★ **Precise** experimental control of uncertainties.
  - ★ Understanding of all other sources of uncertainty (coupling, missing higher orders...).

$$\sigma_{lp} \simeq \widetilde{\sigma}_{lq}\,(\alpha_s, \alpha) \otimes q(x, Q) \qquad \sigma_{pp} \simeq \widetilde{\sigma}_{q\bar{q}}\,(\alpha_s, \alpha) \otimes q(x_1, Q) \otimes \bar{q}(x_2, Q)$$

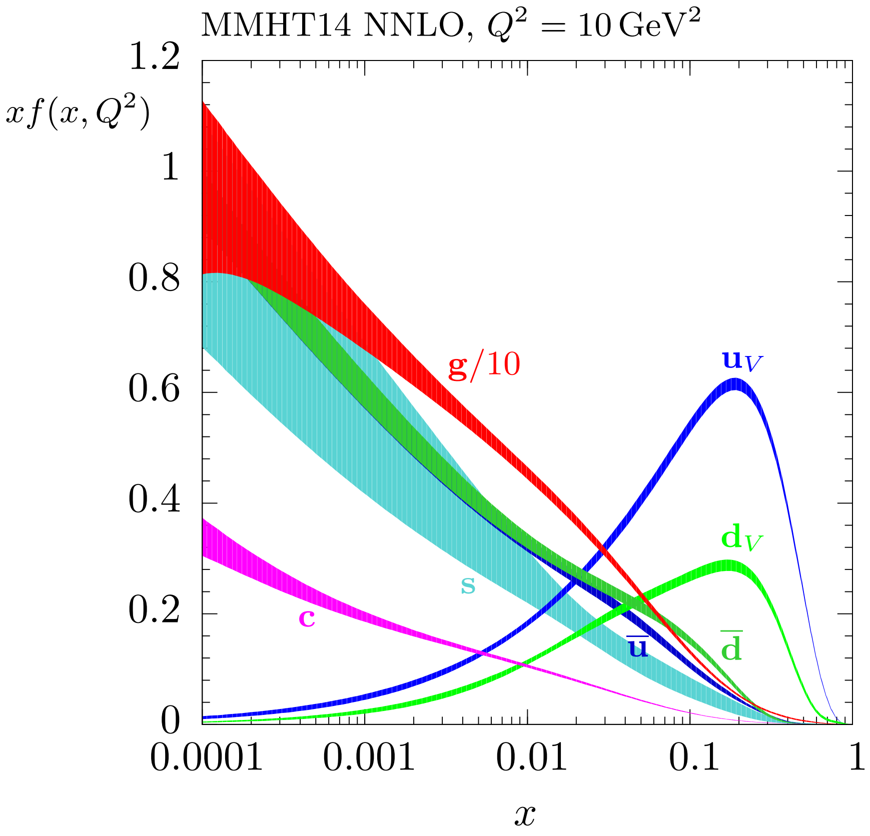


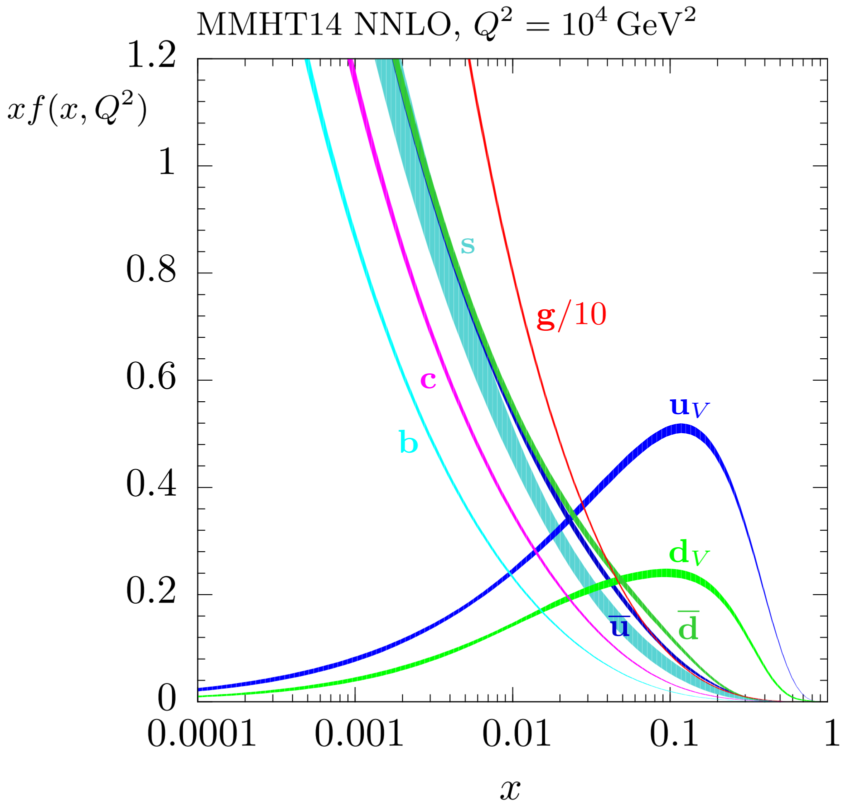


$$\text{Factorization} \Rightarrow q_{DIS}(x, Q^2) \equiv q_{DY}(x, Q^2)$$

PDF Fits

- For LHC (and elsewhere) aim to constrain PDFs to high precision for all flavours (q, $\overline{q}$, g ...) over a wide x region. To achieve this: performs **global PDF fits** to wide range of data.
- Various major global fitting collaboration (**ABM, CT, MMHT, NNPDF**), each taking different approach to this.
- Also various specialised PDF sets: **CJ** (focus on high x), **HERAPDF** (fit to HERA data alone), while **ATLAS/CMS** also performing fits to their data.





PDF Fits: Work Flow

Parameterise PDFs at low scale Q_0

$f_i(x, Q_0) : \; A_f x^{a_f} (1-x)^{b_f} \times$

→ $\sum_{i=1}^{n} \alpha_{f,i} P_i(y(x))$, **CT, MMHT…**

→ $\mathrm{NN}_i(x)$ **NNPDF**

↓

Select dataset

Global (**CT…**) ? Limited dataset (**HERAPDF…**)?

↓

Select theory

Treatment of heavy flavours, higher twists, perturbative order…..

↓

Perform fit

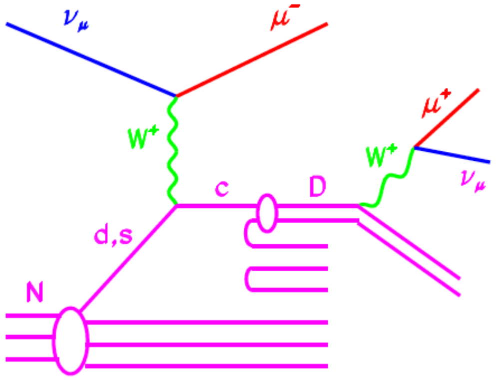
DGLAP: $f(x, Q_0) \to f(x, \mu_{\mathrm{data}})$

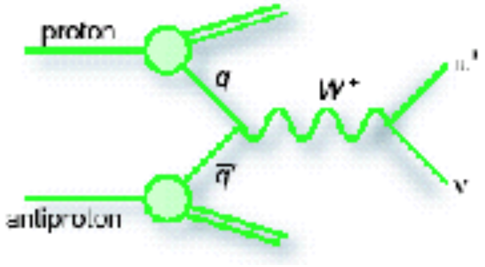
↓

Output PDFs

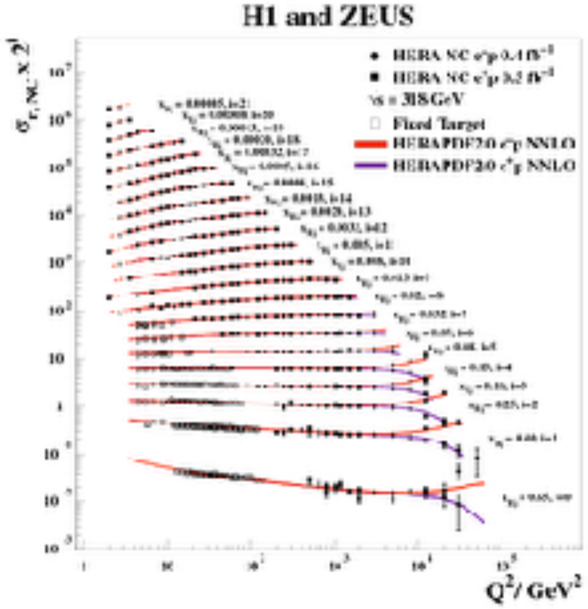
$$f(x, \mu) \pm \Delta(x, \mu)$$

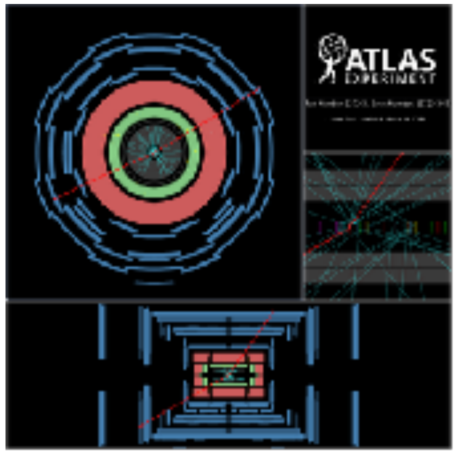
Global Fits: Datasets

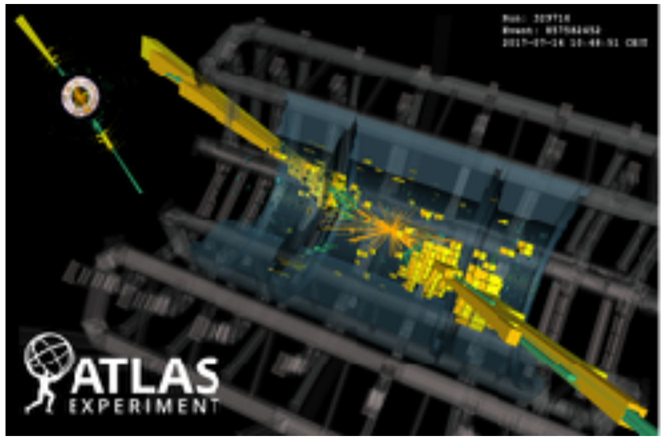




	Process	Subprocess	Partons	x range
Fixed Target	$\ell^\pm\{p,n\}\to\ell^\pm+X$	$\gamma^*q\to q$	$q,\bar{q},g$	$x\gtrsim 0.01$
	$\ell^\pm n/p\to\ell^\pm+X$	$\gamma^*d/u\to d/u$	d/u	$x\gtrsim 0.01$
	$pp\to\mu^+\mu^-+X$	$u\bar{u},d\bar{d}\to\gamma^*$	$\bar{q}$	$0.015\lesssim x\lesssim 0.35$
	$pn/pp\to\mu^+\mu^-+X$	$(u\bar{d})/(u\bar{u})\to\gamma^*$	$\bar{d}/\bar{u}$	$0.015\lesssim x\lesssim 0.35$
	$\nu(\bar{\nu})N\to\mu^-(\mu^+)+X$	$W^*q\to q'$	$q,\bar{q}$	$0.01\lesssim x\lesssim 0.5$
	$\nu N\to\mu^-\mu^++X$	$W^*s\to c$	s	$0.01\lesssim x\lesssim 0.2$
	$\bar{\nu}N\to\mu^+\mu^-+X$	$W^*\bar{s}\to\bar{c}$	$\bar{s}$	$0.01\lesssim x\lesssim 0.2$
Collider DIS	$e^\pm p\to e^\pm+X$	$\gamma^*q\to q$	$g,q,\bar{q}$	$0.0001\lesssim x\lesssim 0.1$
	$e^+p\to\bar{\nu}+X$	$W^+\{d,s\}\to\{u,c\}$	d,s	$x\gtrsim 0.01$
	$e^\pm p\to e^\pm c\bar{c}+X$	$\gamma^*c\to c,\gamma^*g\to c\bar{c}$	c,g	$10^{-4}\lesssim x\lesssim 0.01$
	$e^\pm p\to e^\pm b\bar{b}+X$	$\gamma^*b\to b,\gamma^*g\to b\bar{b}$	b,g	$10^{-4}\lesssim x\lesssim 0.01$
	$e^\pm p\to\text{jet}+X$	$\gamma^*g\to q\bar{q}$	g	$0.01\lesssim x\lesssim 0.1$
Tevatron	$p\bar{p}\to\text{jet}+X$	$gg,qg,qq\to 2j$	g,q	$0.01\lesssim x\lesssim 0.5$
	$p\bar{p}\to(W^\pm\to\ell^\pm\nu)+X$	$ud\to W^+,\bar{u}\bar{d}\to W^-$	$u,d,\bar{u},\bar{d}$	$x\gtrsim 0.05$
	$p\bar{p}\to(Z\to\ell^+\ell^-)+X$	$uu,dd\to Z$	u,d	$x\gtrsim 0.05$
	$p\bar{p}\to t\bar{t}+X$	$qq\to t\bar{t}$	q	$x\gtrsim 0.1$
LHC	$pp\to\text{jet}+X$	$gg,qg,q\bar{q}\to 2j$	g,q	$0.001\lesssim x\lesssim 0.5$
	$pp\to(W^\pm\to\ell^\pm\nu)+X$	$u\bar{d}\to W^+,d\bar{u}\to W^-$	$u,d,\bar{u},\bar{d},g$	$x\gtrsim 10^{-3}$
	$pp\to(Z\to\ell^+\ell^-)+X$	$q\bar{q}\to Z$	$q,\bar{q},g$	$x\gtrsim 10^{-3}$
	$pp\to(Z\to\ell^+\ell^-)+X,\ p_\perp$	$gq(\bar{q})\to Zq(\bar{q})$	$g,q,\bar{q}$	$x\gtrsim 0.01$
	$pp\to(\gamma^*\to\ell^+\ell^-)+X$, Low mass	$q\bar{q}\to\gamma^*$	$q,\bar{q},g$	$x\gtrsim 10^{-4}$
	$pp\to(\gamma^*\to\ell^+\ell^-)+X$, High mass	$q\bar{q}\to\gamma^*$	$\bar{q}$	$x\gtrsim 0.1$
	$pp\to W^+\bar{c},W^-c$	$sg\to W^+c,\bar{s}g\to W^-\bar{c}$	$s,\bar{s}$	$x\sim 0.01$
	$pp\to t\bar{t}+X$	$gg\to t\bar{t}$	g	$x\gtrsim 0.01$
	$pp\to D,B+X$	$gg\to c\bar{c},b\bar{b}$	g	$x\gtrsim 10^{-6},10^{-5}$
	$pp\to J/\psi,\Upsilon+pp$	$\gamma^*(gg)\to c\bar{c},b\bar{b}$	g	$x\gtrsim 10^{-6},10^{-5}$
	$pp\to\gamma+X$	$gq(\bar{q})\to\gamma q(\bar{q})$	g	$x\gtrsim 0.005$

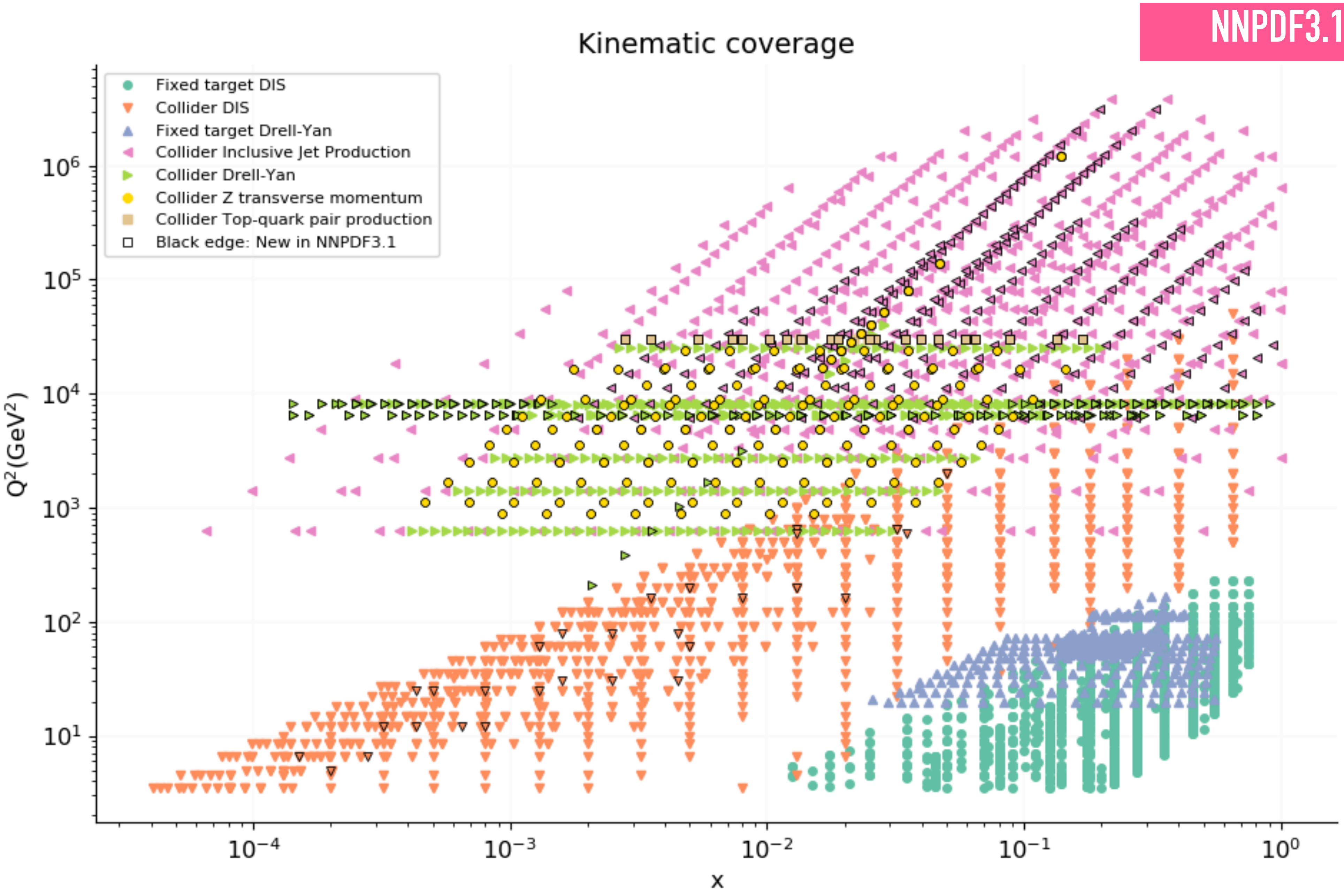






Global Fits: Kinematic Coverage

NNPDF3.1



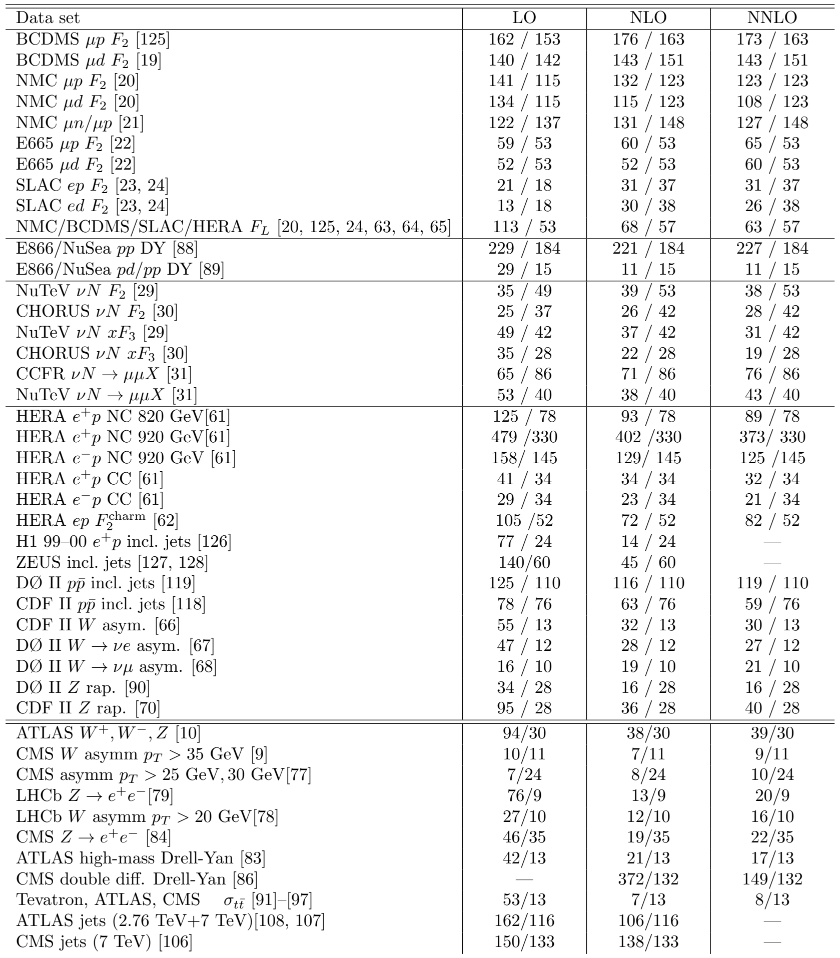
- Global fits achieve **broad coverage** from low to high x , and over many orders of magnitude in Q^2.

Fit Quality

- Fits to wide range of data from different colliders/experiments. Is a good/ reliable fit possible from this? **Yes**!

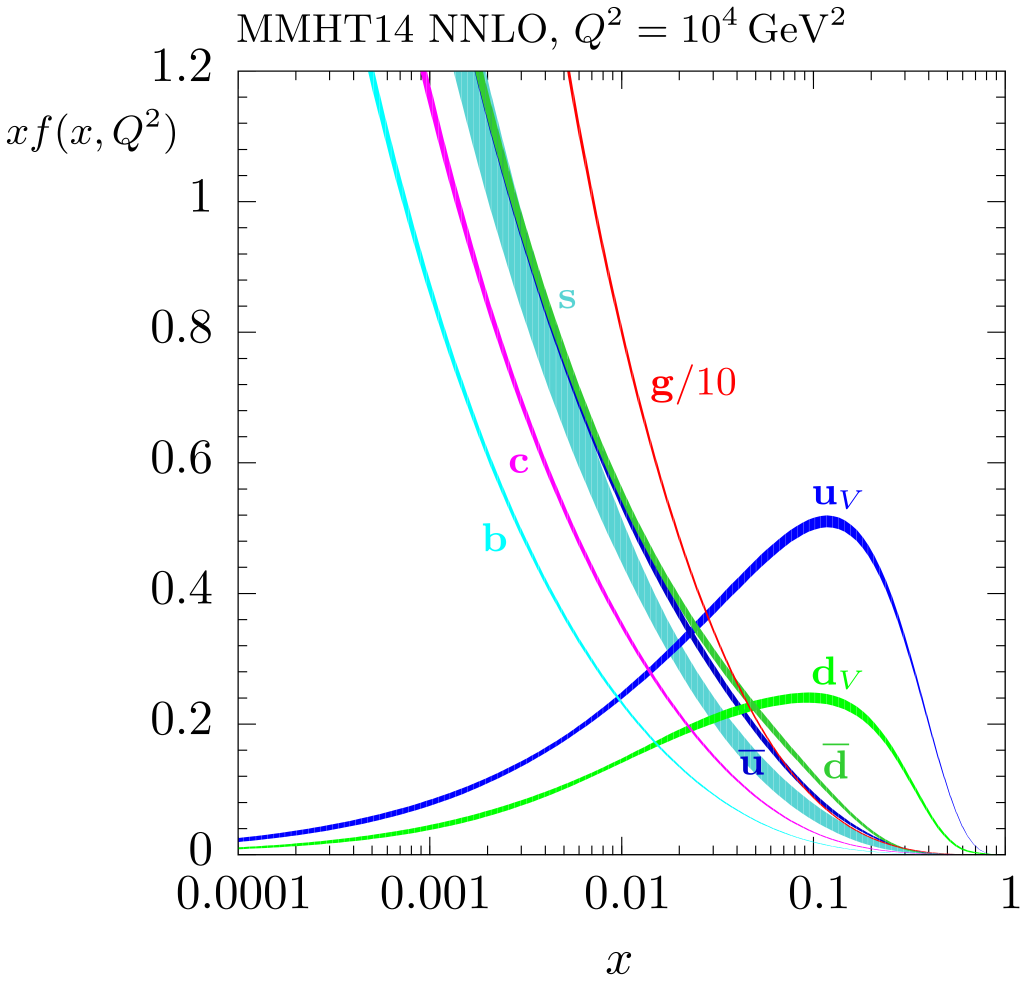
$\chi^2/\text{dof} \sim 1$

$\Rightarrow$ **Non-trivial check of QCD.**



Data set	LO	NLO	NNLO
BCDMS μp F_2 [125]	162 / 153	176 / 163	173 / 163
BCDMS μd F_2 [19]	140 / 142	143 / 151	143 / 151
NMC μp F_2 [20]	141 / 115	132 / 123	123 / 123
NMC μd F_2 [20]	134 / 115	115 / 123	108 / 123
NMC $\mu n/\mu p$ [21]	122 / 137	131 / 148	127 / 148
E665 μp F_2 [22]	59 / 53	60 / 53	65 / 53
E665 μd F_2 [22]	52 / 53	52 / 53	60 / 53
SLAC ep F_2 [23, 24]	21 / 18	31 / 37	31 / 37
SLAC ed F_2 [23, 24]	13 / 18	30 / 38	26 / 38
NMC/BCDMS/SLAC/HERA F_L [20, 125, 24, 63, 64, 65]	113 / 53	68 / 57	63 / 57
E866/NuSea pp DY [88]	229 / 184	221 / 184	227 / 184
E866/NuSea pd/pp DY [89]	29 / 15	11 / 15	11 / 15
NuTeV νN F_2 [29]	35 / 49	39 / 53	38 / 53
CHORUS νN F_2 [30]	25 / 37	26 / 42	28 / 42
NuTeV νN xF_3 [29]	49 / 42	37 / 42	31 / 42
CHORUS νN xF_3 [30]	35 / 28	22 / 28	19 / 28
CCFR $\nu N \to \mu\mu X$ [31]	65 / 86	71 / 86	76 / 86
NuTeV $\nu N \to \mu\mu X$ [31]	53 / 40	38 / 40	43 / 40
HERA e^+p NC 820 GeV[61]	125 / 78	93 / 78	89 / 78
HERA e^+p NC 920 GeV[61]	479 /330	402 /330	373/ 330
HERA e^-p NC 920 GeV [61]	158/ 145	129/ 145	125 /145
HERA e^+p CC [61]	41 / 34	34 / 34	32 / 34
HERA e^-p CC [61]	29 / 34	23 / 34	21 / 34
HERA ep F_2^{charm} [62]	105 /52	72 / 52	82 / 52
H1 99–00 e^+p incl. jets [126]	77 / 24	14 / 24	—
ZEUS incl. jets [127, 128]	140/60	45 / 60	—
DØ II $p\bar{p}$ incl. jets [119]	125 / 110	116 / 110	119 / 110
CDF II $p\bar{p}$ incl. jets [118]	78 / 76	63 / 76	59 / 76
CDF II W asym. [66]	55 / 13	32 / 13	30 / 13
DØ II $W \to \nu e$ asym. [67]	47 / 12	28 / 12	27 / 12
DØ II $W \to \nu\mu$ asym. [68]	16 / 10	19 / 10	21 / 10
DØ II Z rap. [90]	34 / 28	16 / 28	16 / 28
CDF II Z rap. [70]	95 / 28	36 / 28	40 / 28
ATLAS W^+, W^-, Z [10]	94/30	38/30	39/30
CMS W asymm $p_T > 35$ GeV [9]	10/11	7/11	9/11
CMS asymm $p_T > 25$ GeV, 30 GeV[77]	7/24	8/24	10/24
LHCb $Z \to e^+e^-$[79]	76/9	13/9	20/9
LHCb W asymm $p_T > 20$ GeV[78]	27/10	12/10	16/10
CMS $Z \to e^+e^-$ [84]	46/35	19/35	22/35
ATLAS high-mass Drell-Yan [83]	42/13	21/13	17/13
CMS double diff. Drell-Yan [86]	—	372/132	149/132
Tevatron, ATLAS, CMS $\sigma_{t\bar{t}}$ [91]–[97]	53/13	7/13	8/13
ATLAS jets (2.76 TeV+7 TeV)[108, 107]	162/116	106/116	—
CMS jets (7 TeV) [106]	150/133	138/133	—
All data sets	**3706 / 2763**	**3267 / 2996**	**2717 / 2663**

LO **NLO** **NNLO**



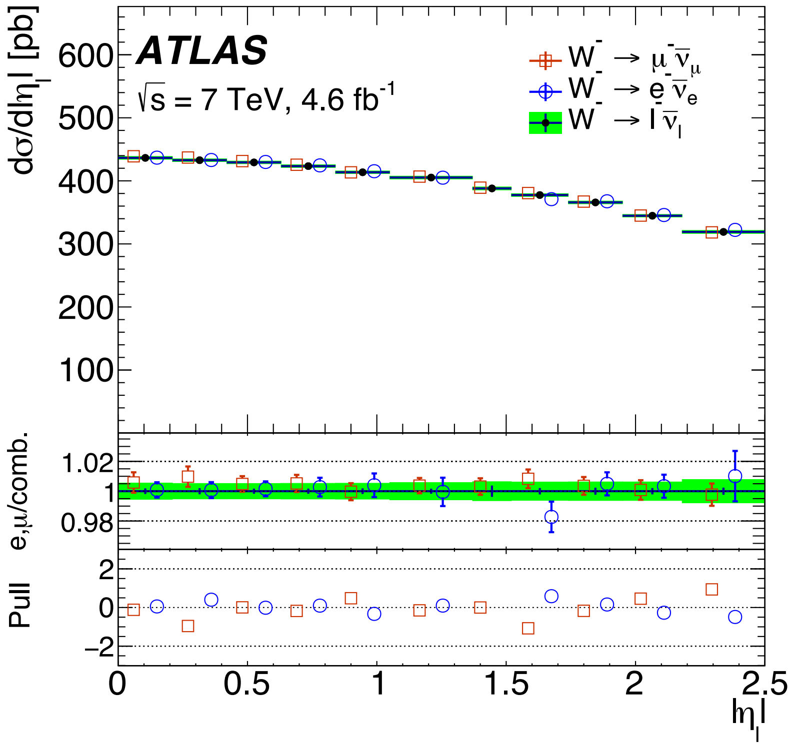
LHL et al., Eur. Phys. J. C75 (2015) no.5 204

Current Status

Fits Today

- Current fits very much aiming for (and in some cases achieving) high precision (~ **1% level**) PDF determination in some regions. Key ingredients:

Extremely precise LHC data



NNLO QCD calculations ‘standard’

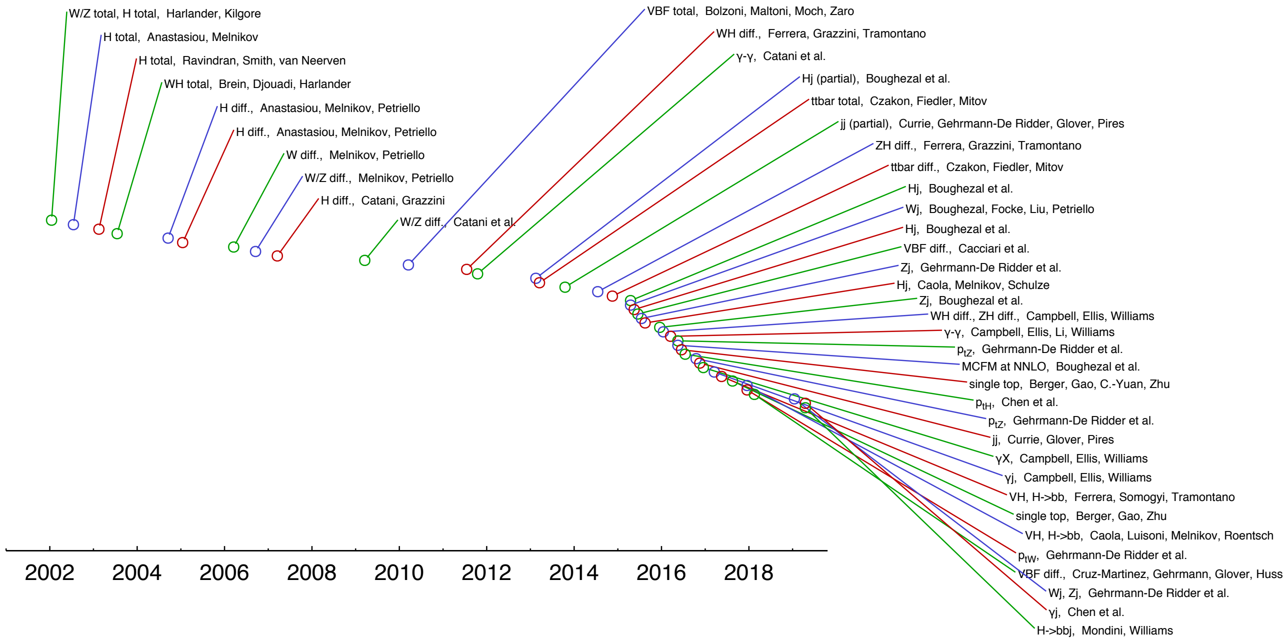
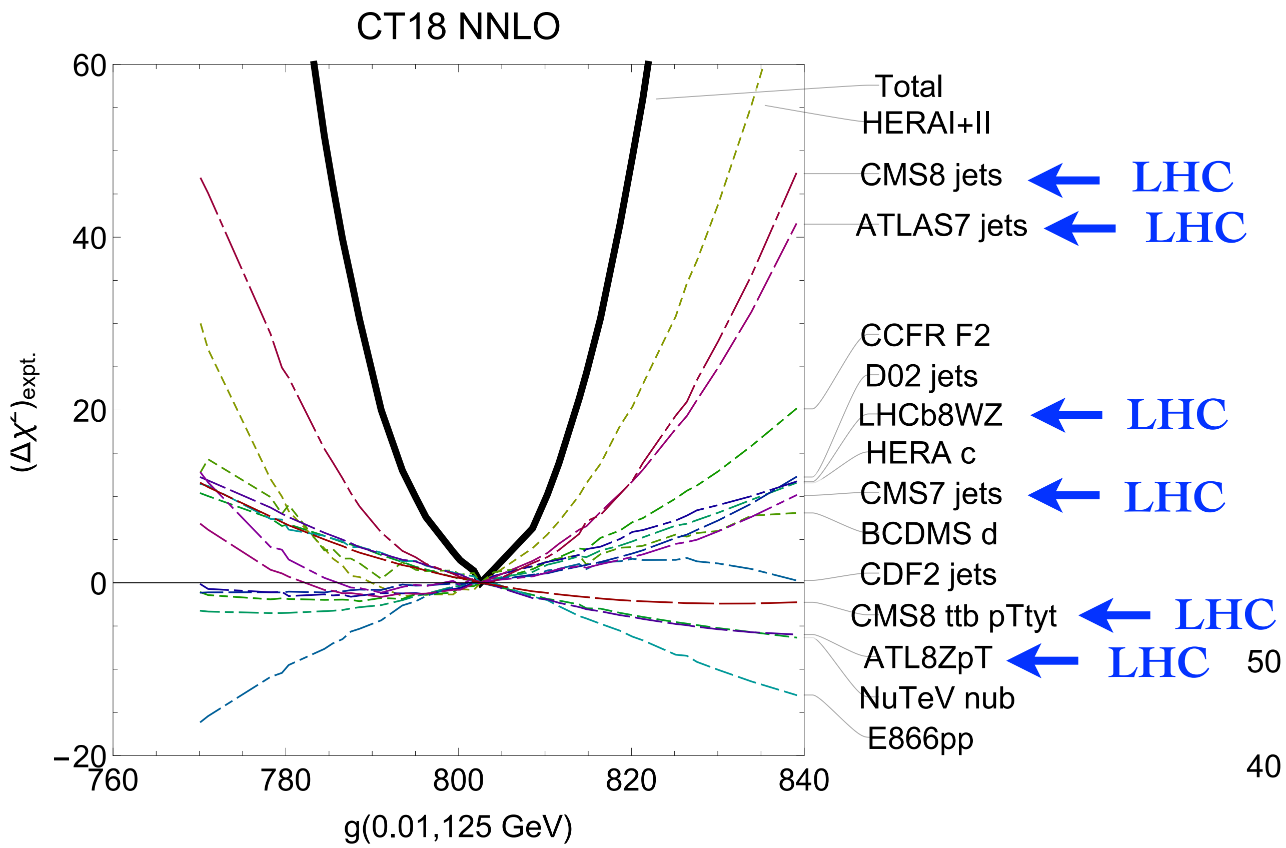
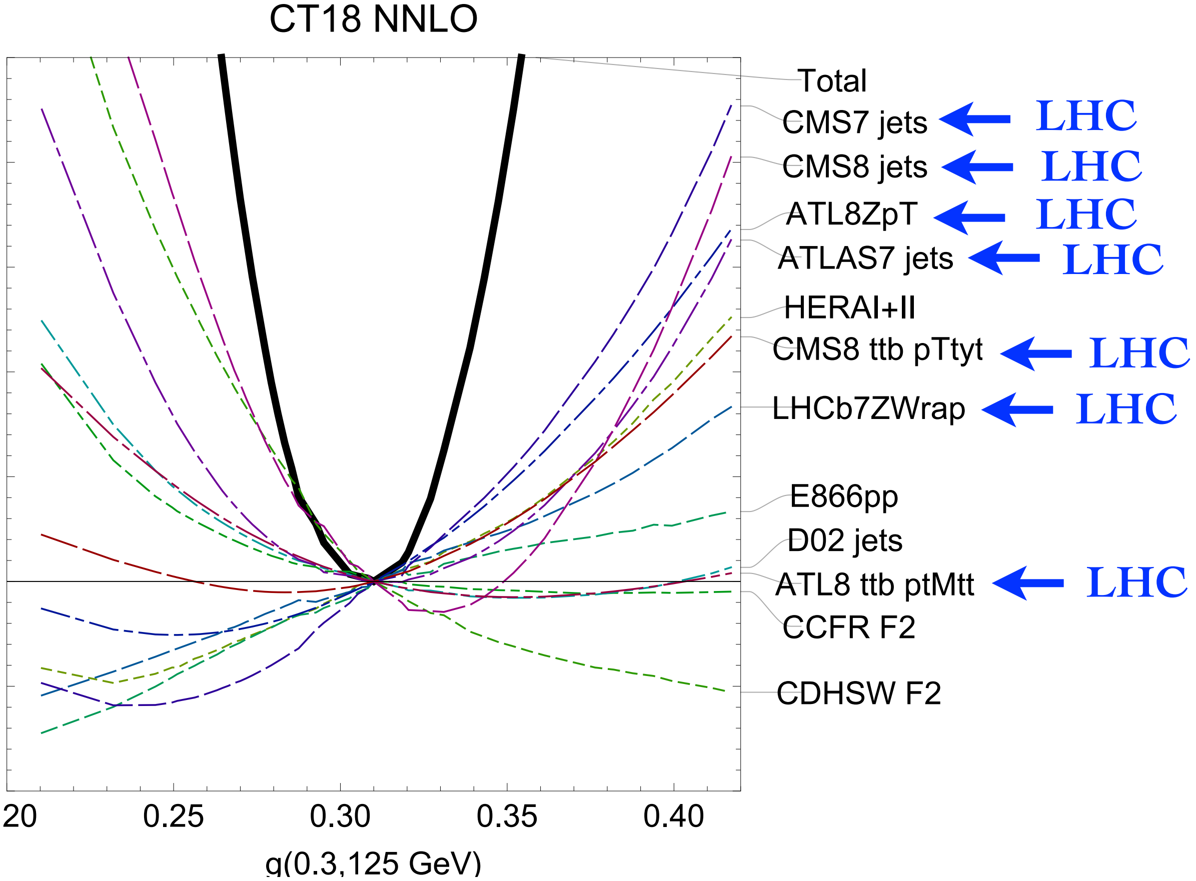


Image Credit: Gavin Salam

- **LHC data** now playing a key role in all fits.

- Example from recent **CT18** fit. Lagrange multiplier scans determining constraints on gluon at different χ^2 values:



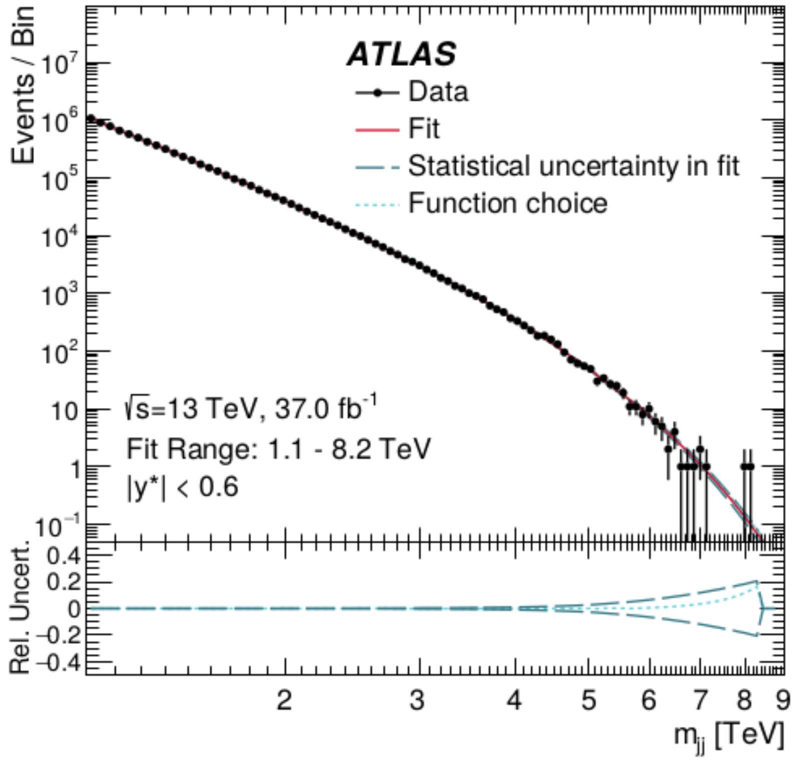


- Plenty of LHC data driving fits!

T-J Hou et al., arXiv:1908.11238

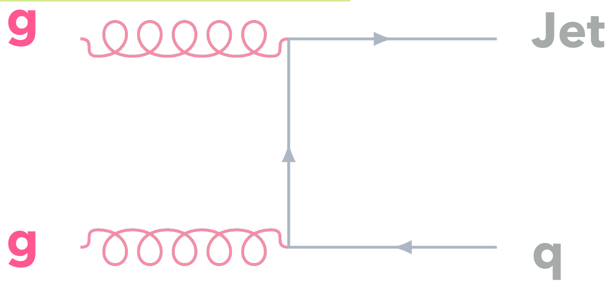
Example 1 - The Gluon

- Gluon at high x is both important for **BSM searches** and quite **poorly constrained** from DIS.
- LHC data such plays crucial role in constraining this.

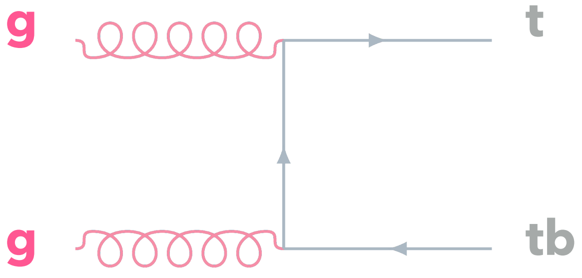


- Generically achieved by looking for gluon-initiated processes at high system transverse momentum/invariant mass/rapidity.
- Three textbook candidates at LHC:

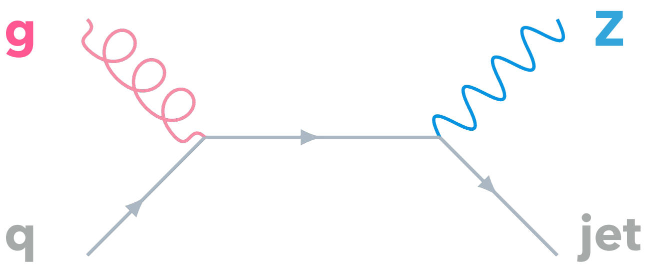
INCLUSIVE JETS



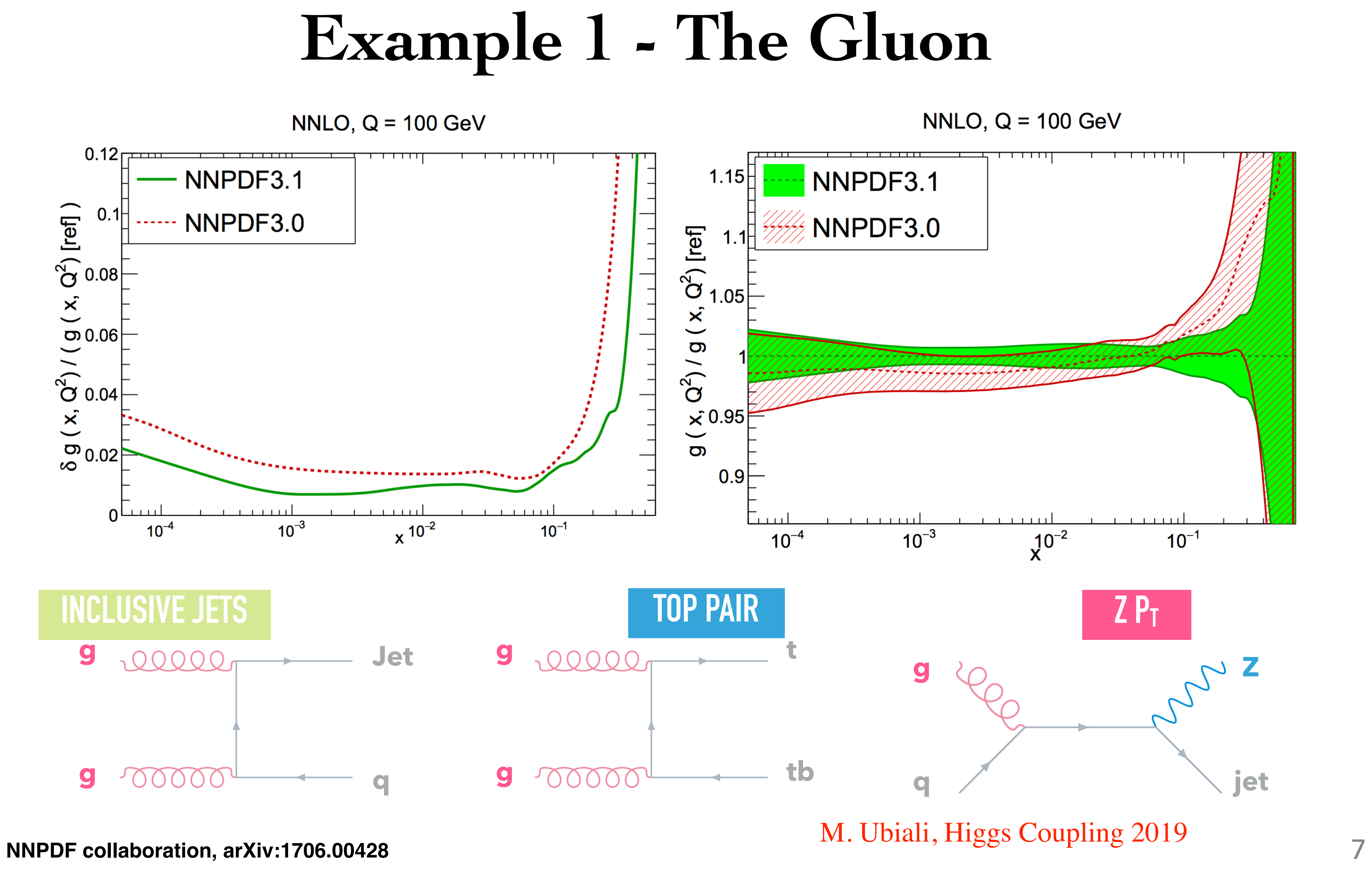
TOP PAIR



Z P_T



M. Ubiali, Higgs Coupling 2019



Example 1 - The Gluon

NNPDF collaboration, arXiv:1706.00428

M. Ubiali, Higgs Coupling 2019

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- Impact of most recent LHC data (red → green) **significant**, with percent level uncertainties across wide range of x .

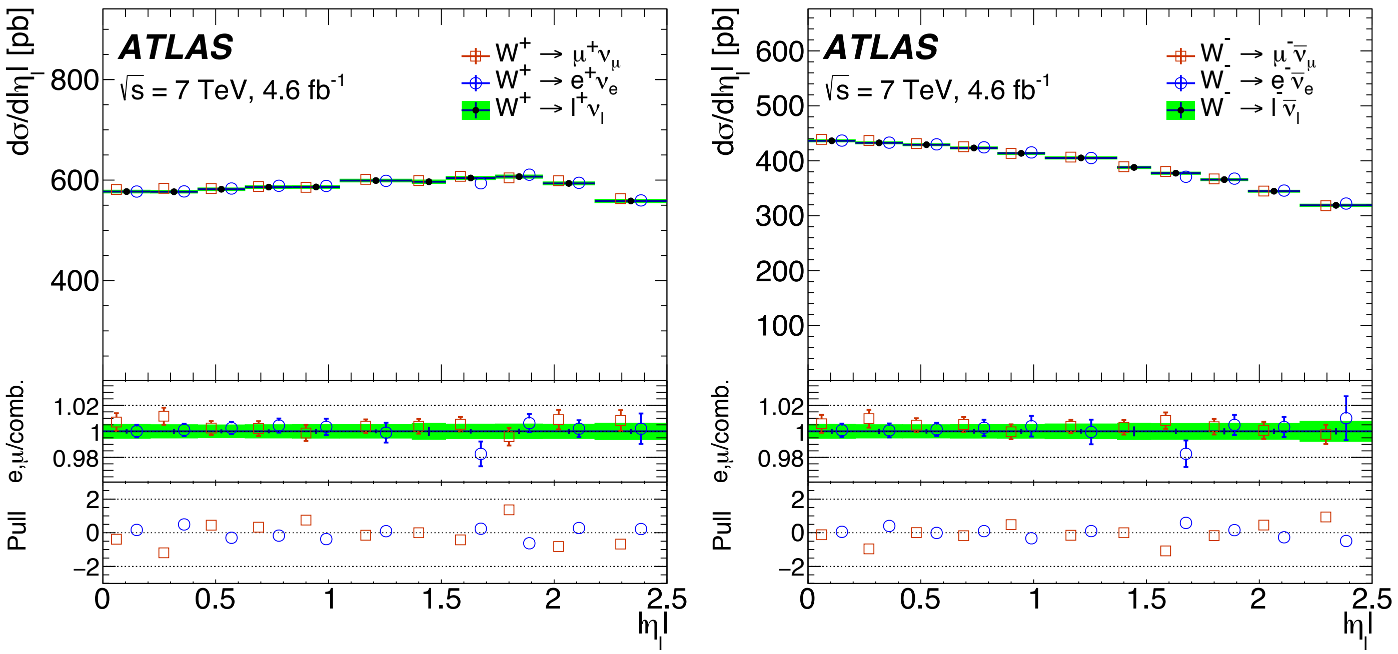
Example 2 - Proton Strangeness

- Vector boson (W, Z) production proceeds via range of channels.

$$
\begin{aligned}
u\bar{d},\, c\bar{s} \quad (u\bar{s},\, c\bar{d}) &\rightarrow W^+ \,, \\
d\bar{u},\, s\bar{c} \quad (s\bar{u},\, d\bar{c}) &\rightarrow W^- \,, \\
q\bar{q} &\rightarrow Z/\gamma^* \,,
\end{aligned}
$$

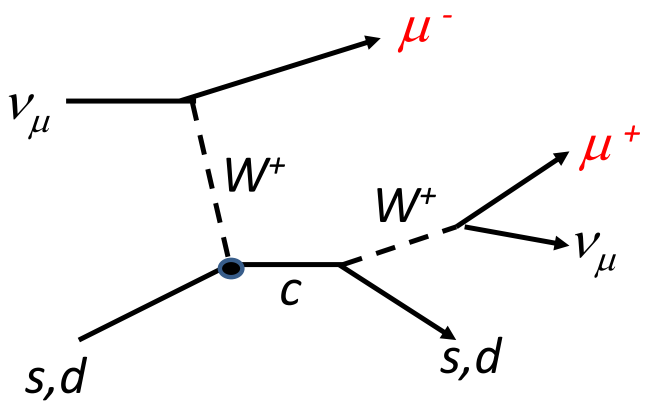
- Least constrained involves initial state s, $\bar{s}$ (no valence s) $\longrightarrow$ sensitive to **proton strangeness**.
- Only in principle: small contribution, requires **precise data** to pin down.

- Now available - highest ever precision measurement of W, Z production by **ATLAS.**
- Data uncertainties at the sub-% level. Statistical errors negligible completely dominated by systematics.



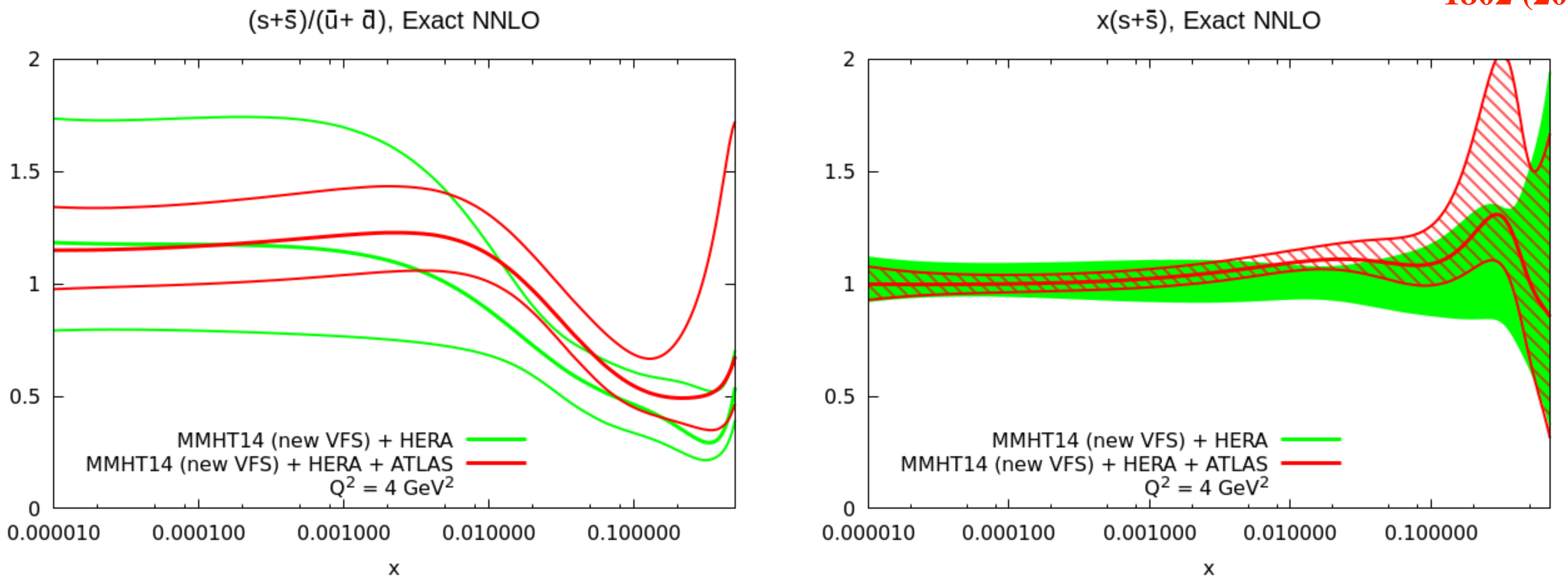
ATLAS collab., Eur. Phys. J C77 (2017) 367

Example 2 - Proton Strangeness



- Impact of ATLAS data significant. Most notably: prefers **larger** strangeness than global fits, where previous constraints from neutrino-induced DIS $(\overline{\nu}s \to lc)$.
- However global fits can safely accommodate both (rather distinct) datasets. Key ingredient: new **NNLO** calculation of DIS process.

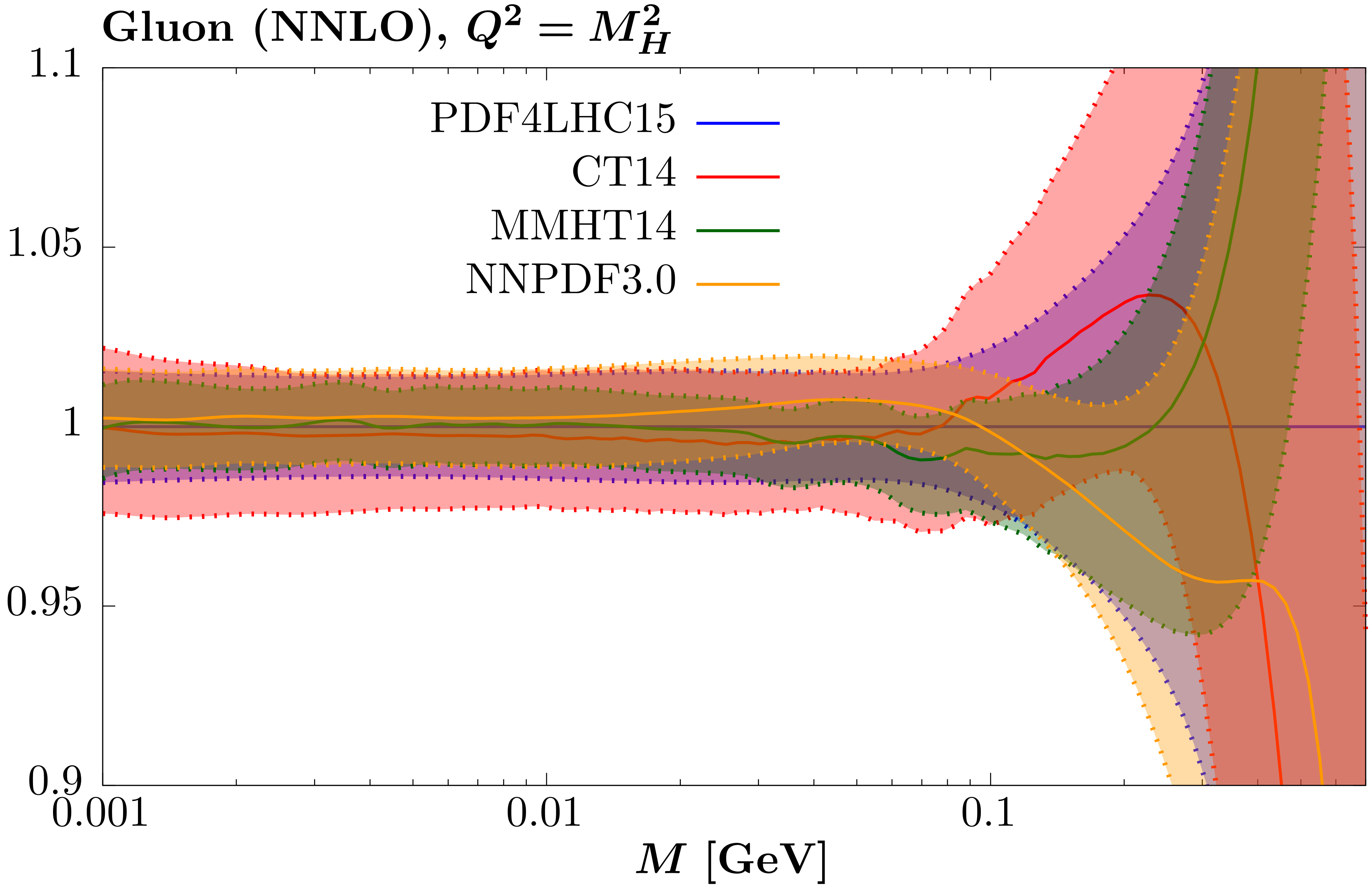
J. Gao, JHEP 1802 (2018) 026



R. Thorne, DIS19

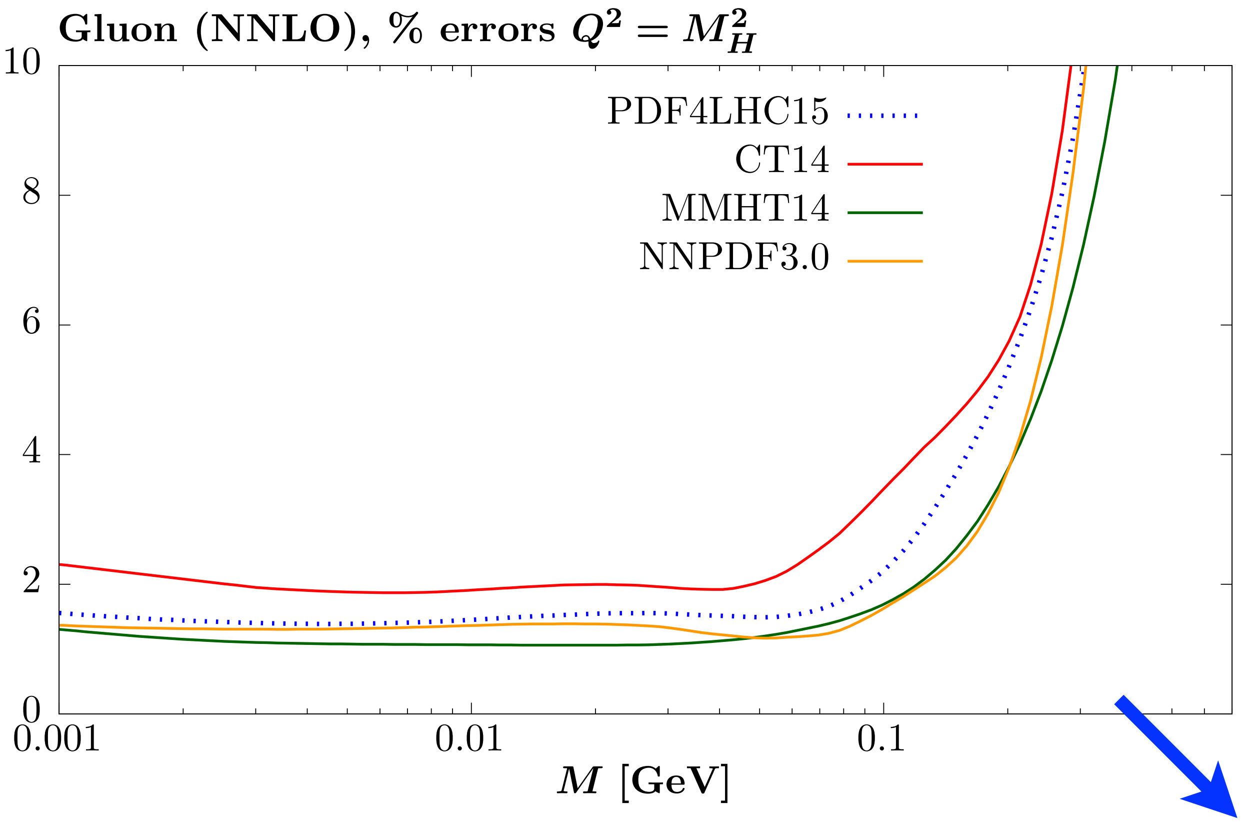
Status in 2015

- Typical to combine three major global fits into ‘**PDF4LHC**’ combination.
- Consider e.g. gluon PDF at scale relevant to Higgs production.
- Result in **2015** (already with some LHC data):

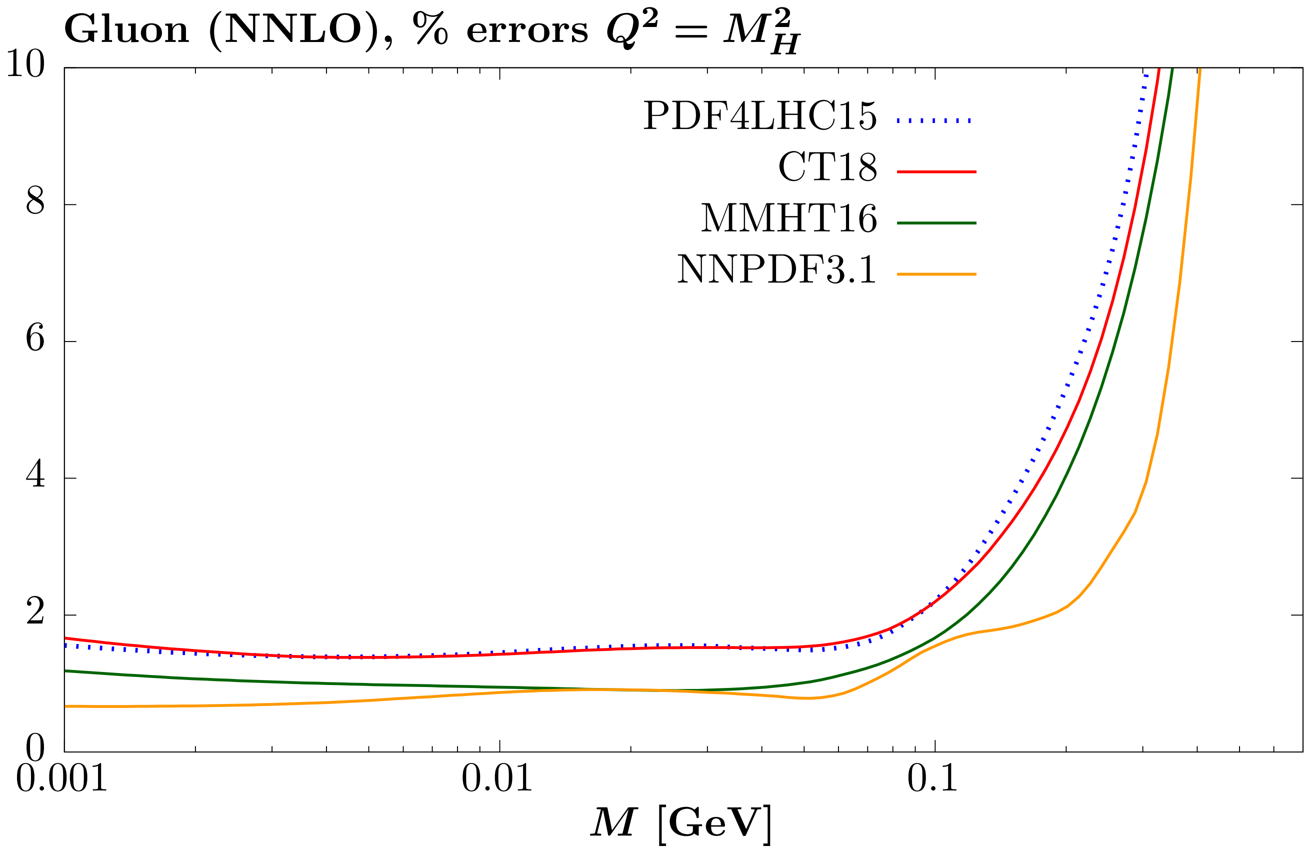


- How does this look 4 years (and much LHC data) later?

Status in 2019

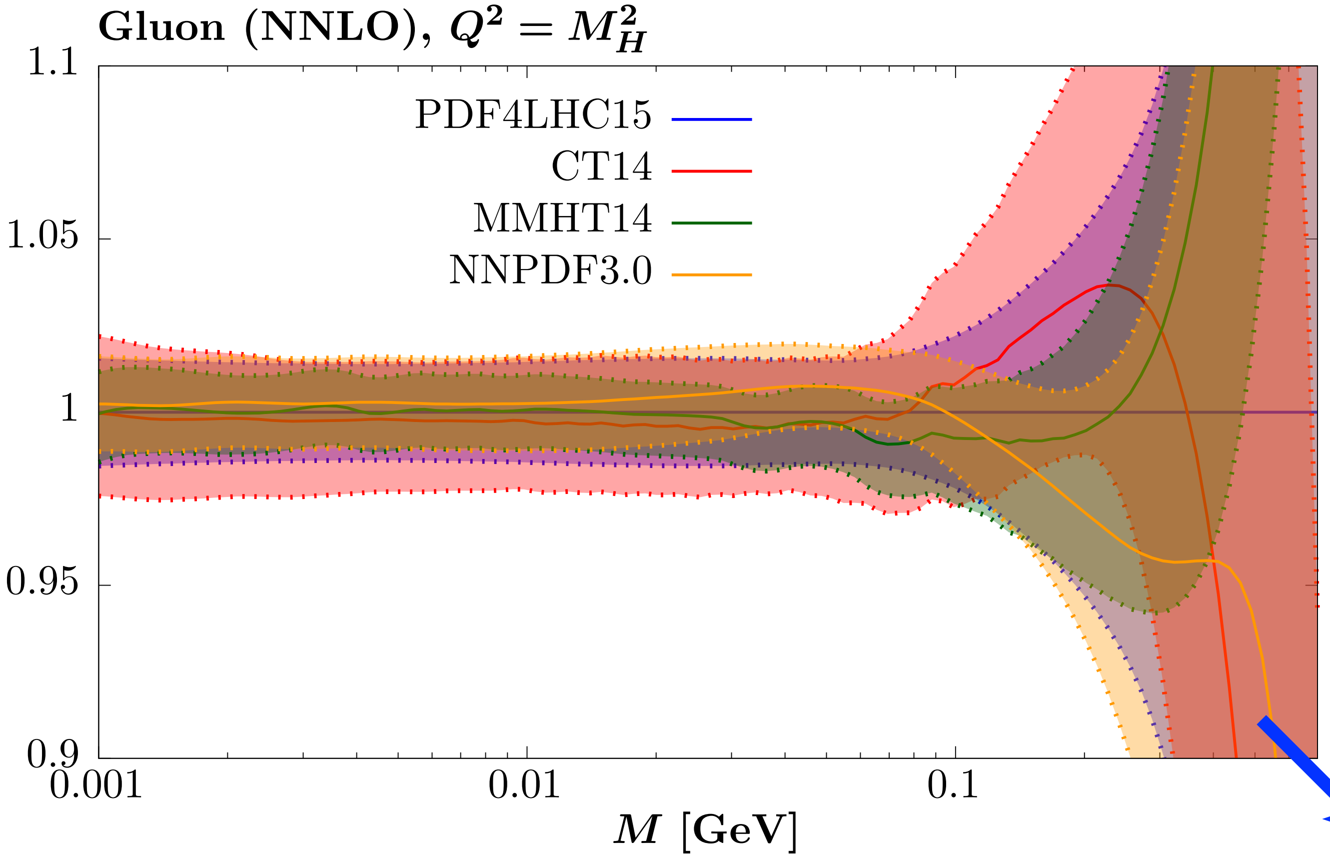


- Considering most recent fits. see clear reduction in **individual errors**.



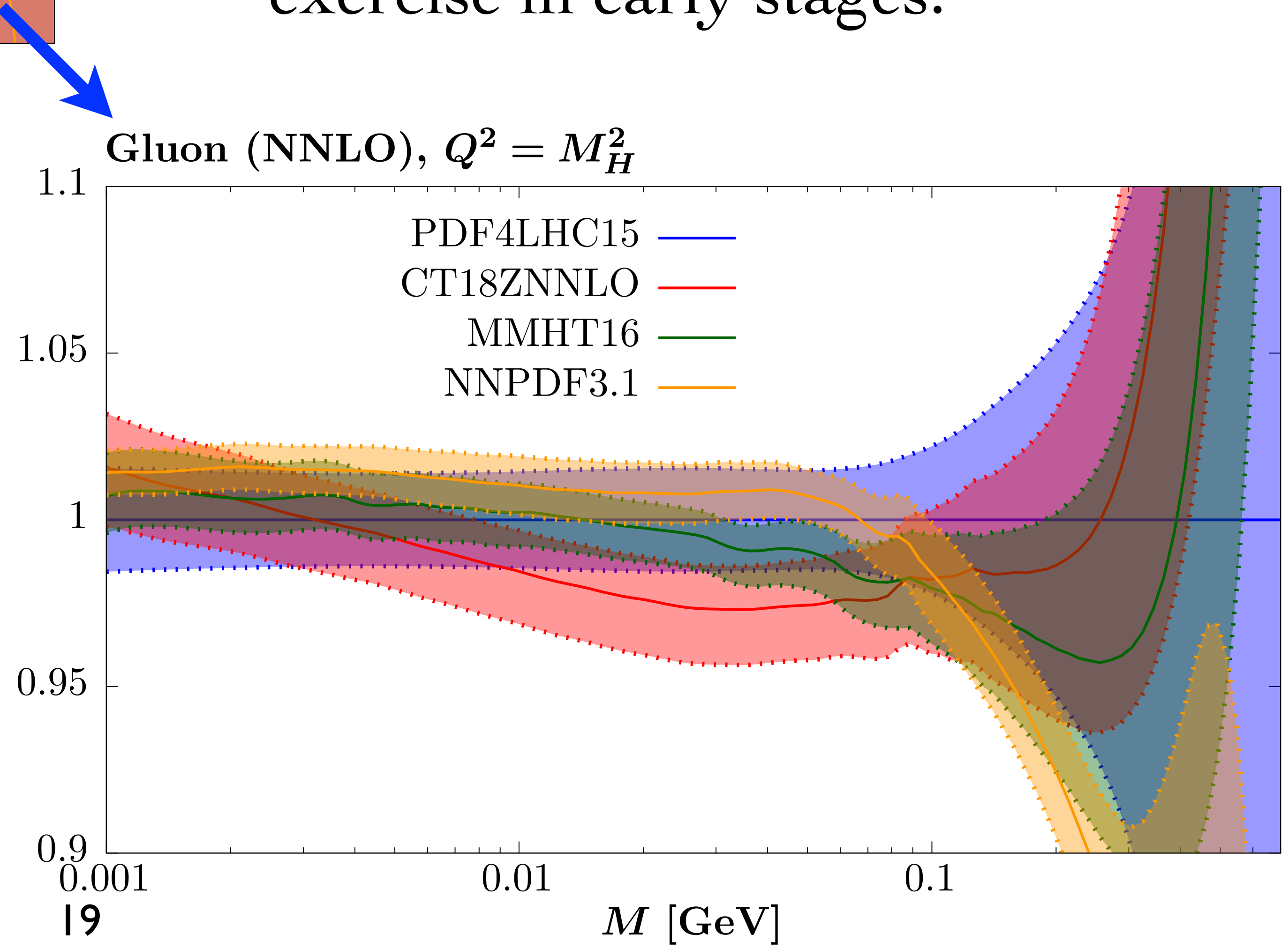
- However picture not as encouraging when looking closer…

Status in 2019



- Spread between groups has **increased**! Not always straightforward picture of ever decreasing PDF errors.
- To understand this: detailed **benchmarking** + combination exercise in early stages.

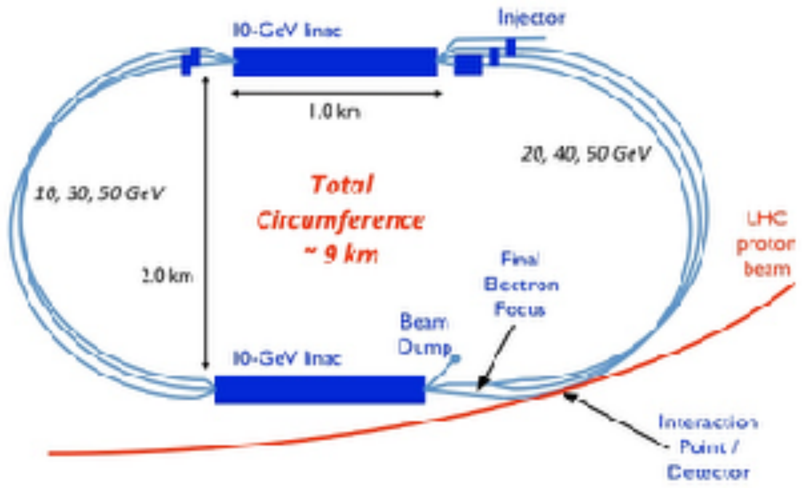
- Note preliminary: updated 'MMHT19' release coming soon.
- Similar situation for other partons (**backup**).

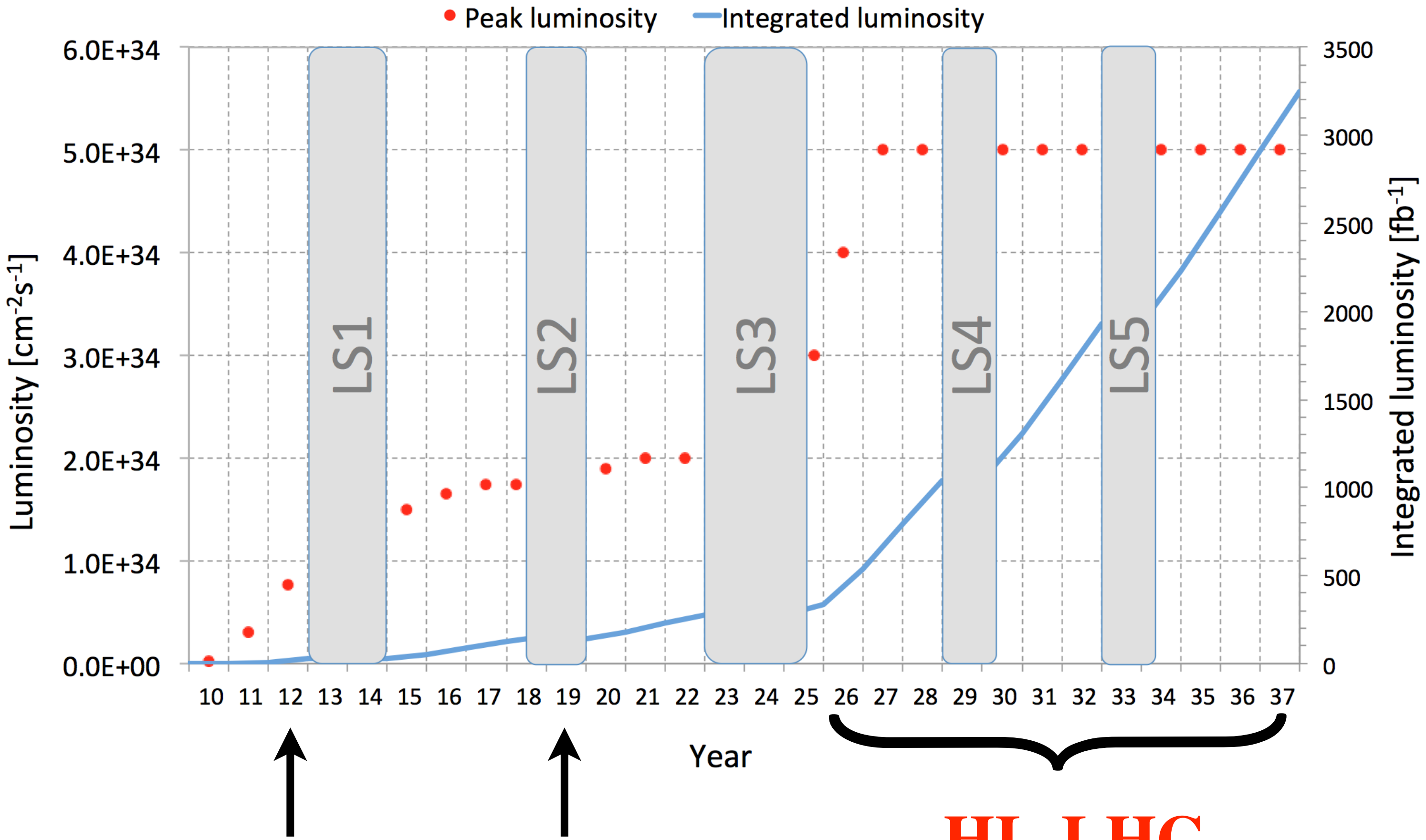


Looking to the Future

LHC: The Future

- At very **early stage** in LHC: so far only a few percent of the final projected data sample to be collected during High Luminosity **(HL)-LHC** running.





Higgs

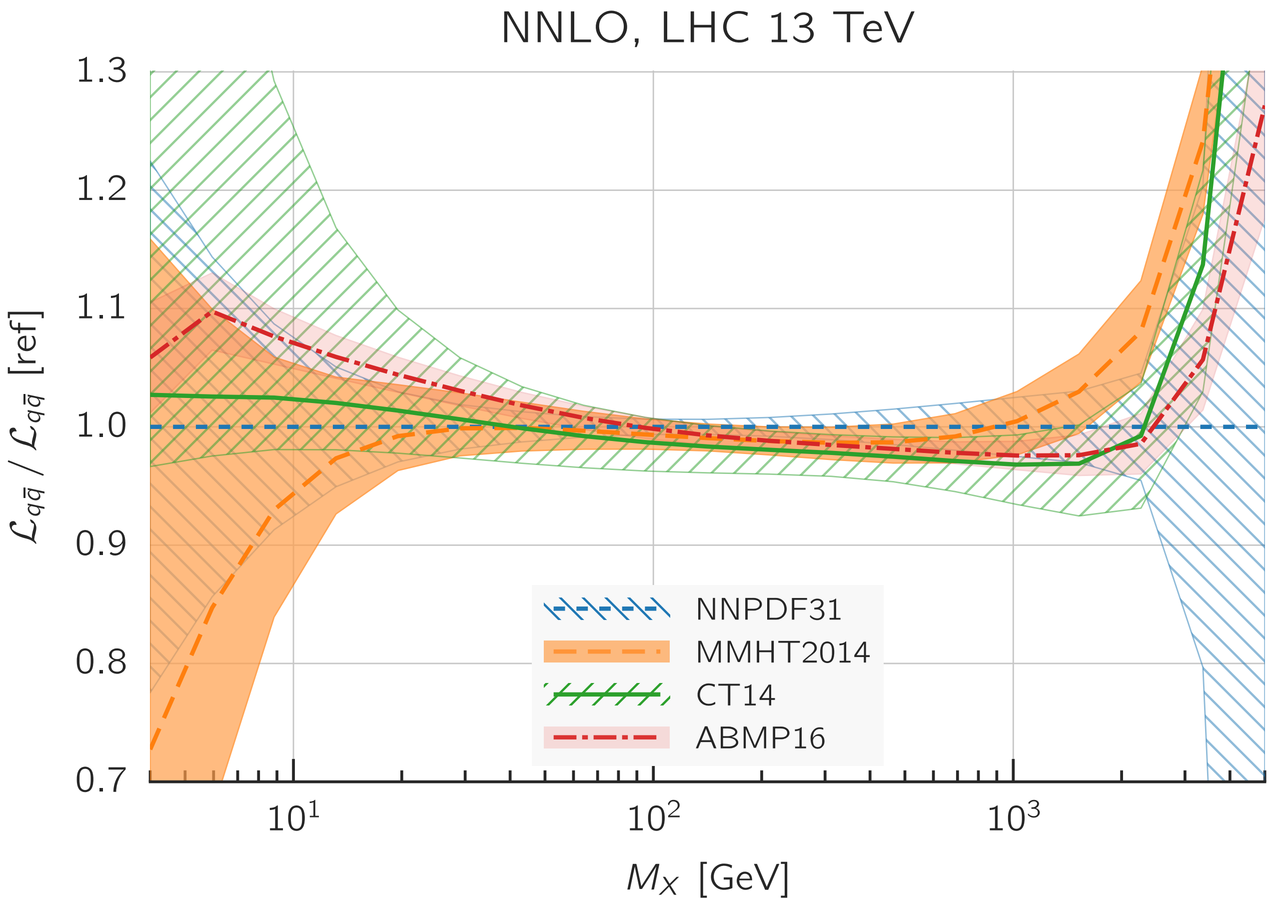
We are here

HL-LHC (LHeC?)

- In addition exciting upgrade possibility of Large Hadron Electron Collider (**LHeC**): colliding lepton beam with LHC protons. Providing unprecedented high precision DIS data on proton structure.

Ultimate PDFs - Motivation

- Both HL-LHC and LHeC (if approved) will provide a vast range of data with a direct impact on the PDFs.
- **Question:** what exactly can we expect that impact to be?
- Collaborative effort to produce '**Ultimate**' PDF set.



?

R. Abdul Khalek, S. Bailey, J. Gao, LHL, J. Rojo. Eur.Phys.J. C78 (2018) no.11, 962 & SciPost Phys. 7, 051 (2019)

- This ultimate expected precision from PDF fits sets the ultimate bar for any lattice determinations.

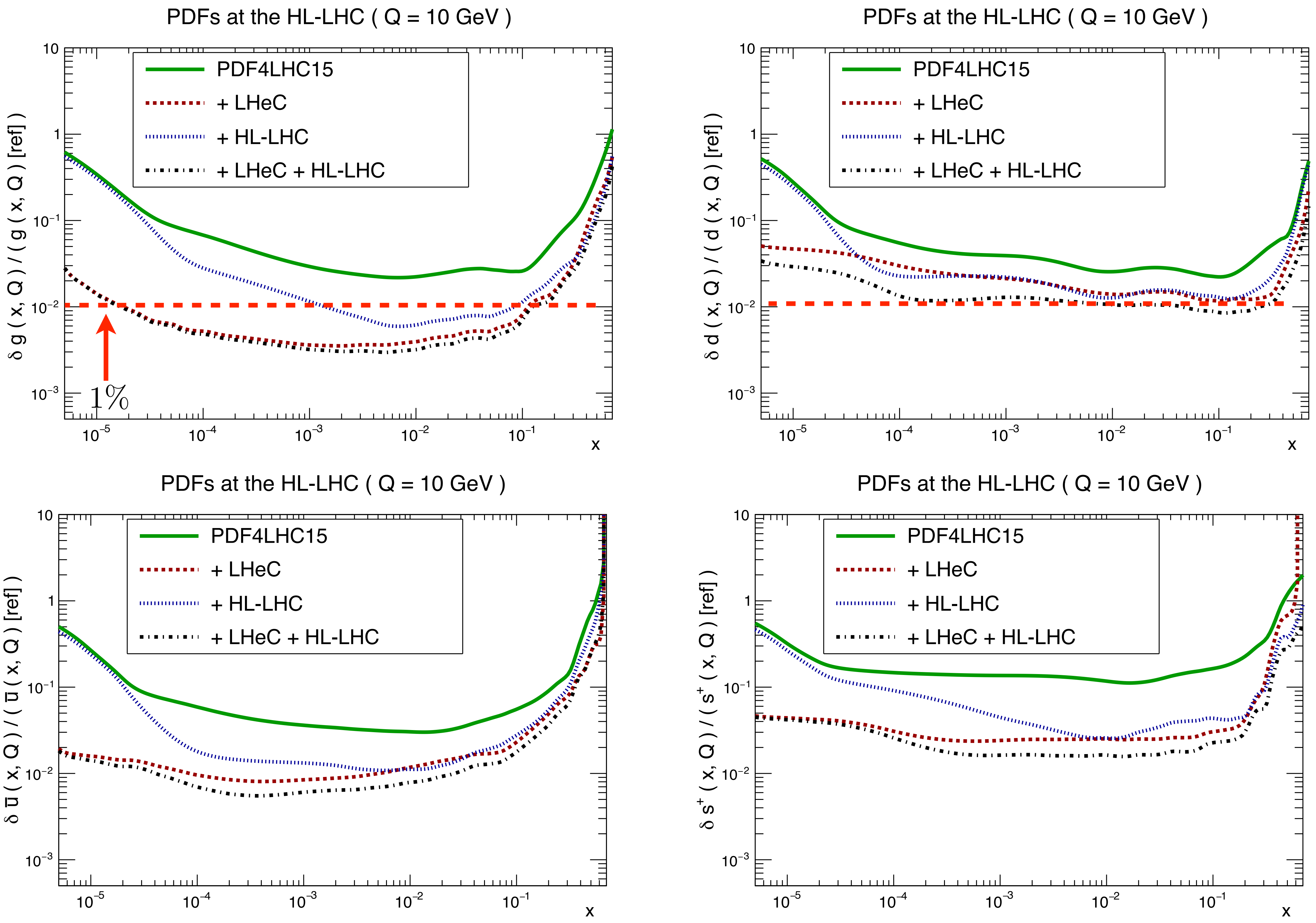
Basic Idea

Produce theory predictions for relevant processes, in kinematic region probed by HL-LHC and LHeC

Produce pseudodata - binned predictions, provided with corresponding statistical + systematic errors

Perform PDF fit to this pseudodata

Evaluate impact on PDF uncertainties

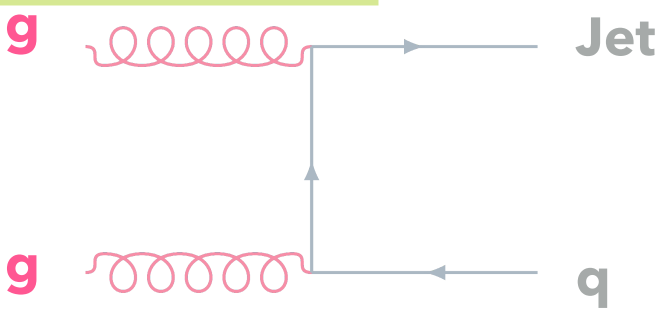


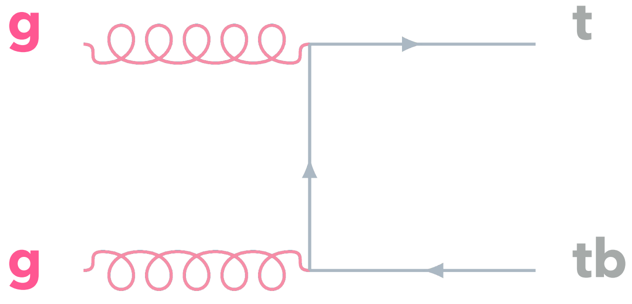
- **Sub percent level** uncertainty in e.g. gluon in some x regions. Impressive constraints out to rather high x in general.
- LHeC placing very clean constraints across x range.

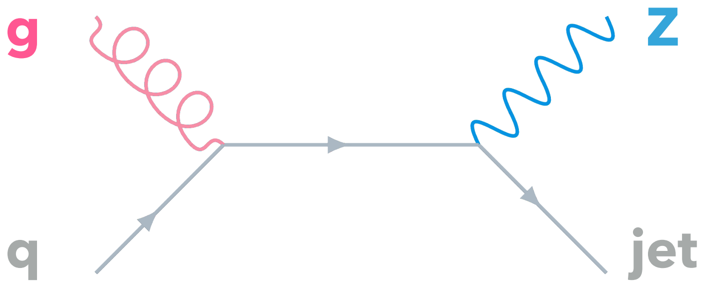
Challenges

Challenges

- Have so far covered the **good news**: the ongoing impact of LHC data on PDF errors ($\rightarrow$1% level) with encouraging outlook for future.
- However not the whole picture: as collider data becomes increasingly precise **cracks** starting to appear in data/theory comparison.
- In fact seen to occur in all three of the 'textbook' LHC processes for probing PDFs at high x :

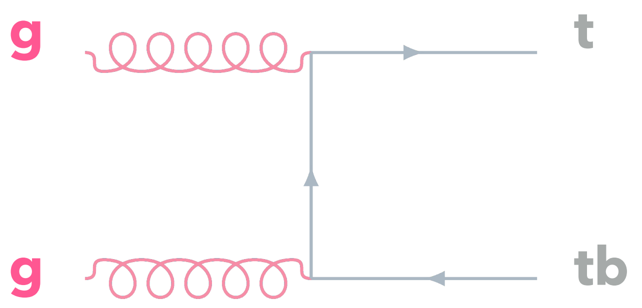

INCLUSIVE JETS


TOP PAIR


Z P_T

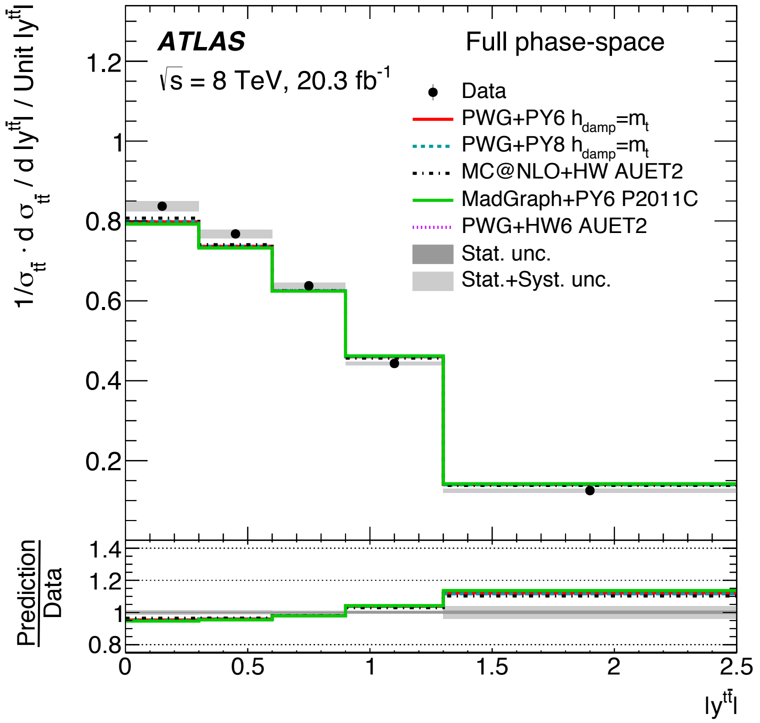
- In more detail…

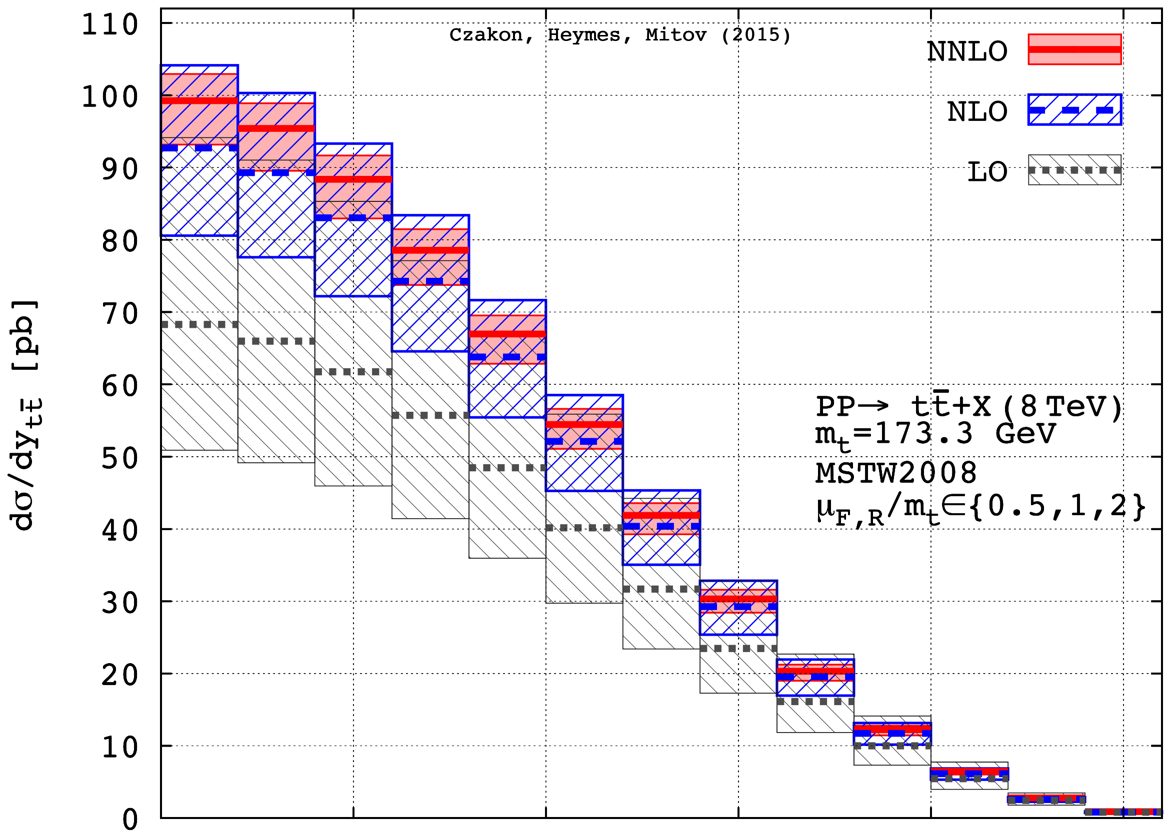
Top Quark Production



- **In principle** ideal candidate for precision PDF determination: parton-level theory known to **NNLO** in QCD, while precise data provided multi-differentially in various observables.

$$y_t,\ y_{t\bar{t}},\ p_\perp^t,\ M_{t\bar{t}}$$





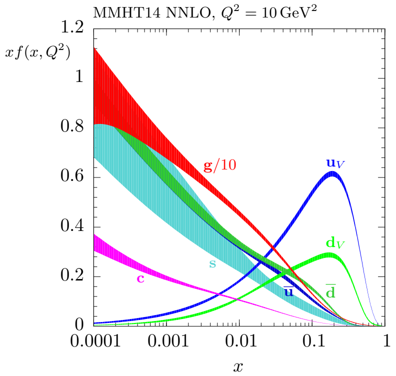
ATL-PHYS-PUB-2018-017

M. Czakon, D. Heymes, A. Mitov, PRL 116 (2016) no.8, 082003

- However when one tries to fit such data…

- …find **terrible fit** to full dataset! What is going on?

- Theory input: PDFs + parton-level theory:



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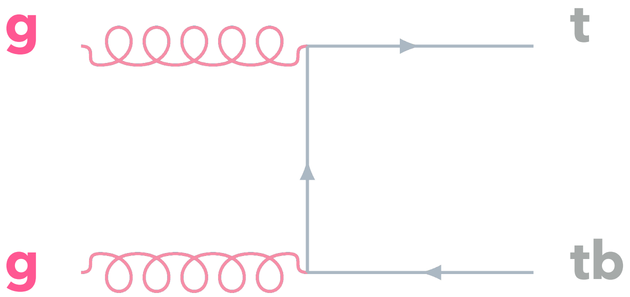
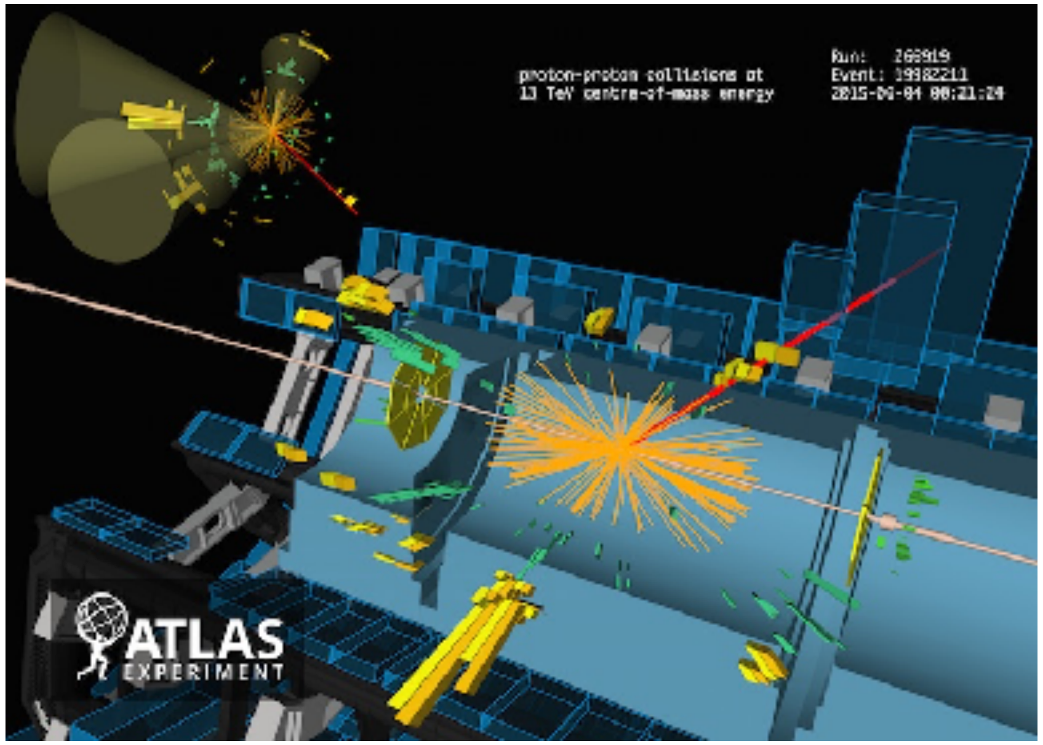


- Data itself: rather far from this!
- Procedure to ‘unfold’ data back to parton level distribution(s) is complex and introduces many new **systematic error** sources.

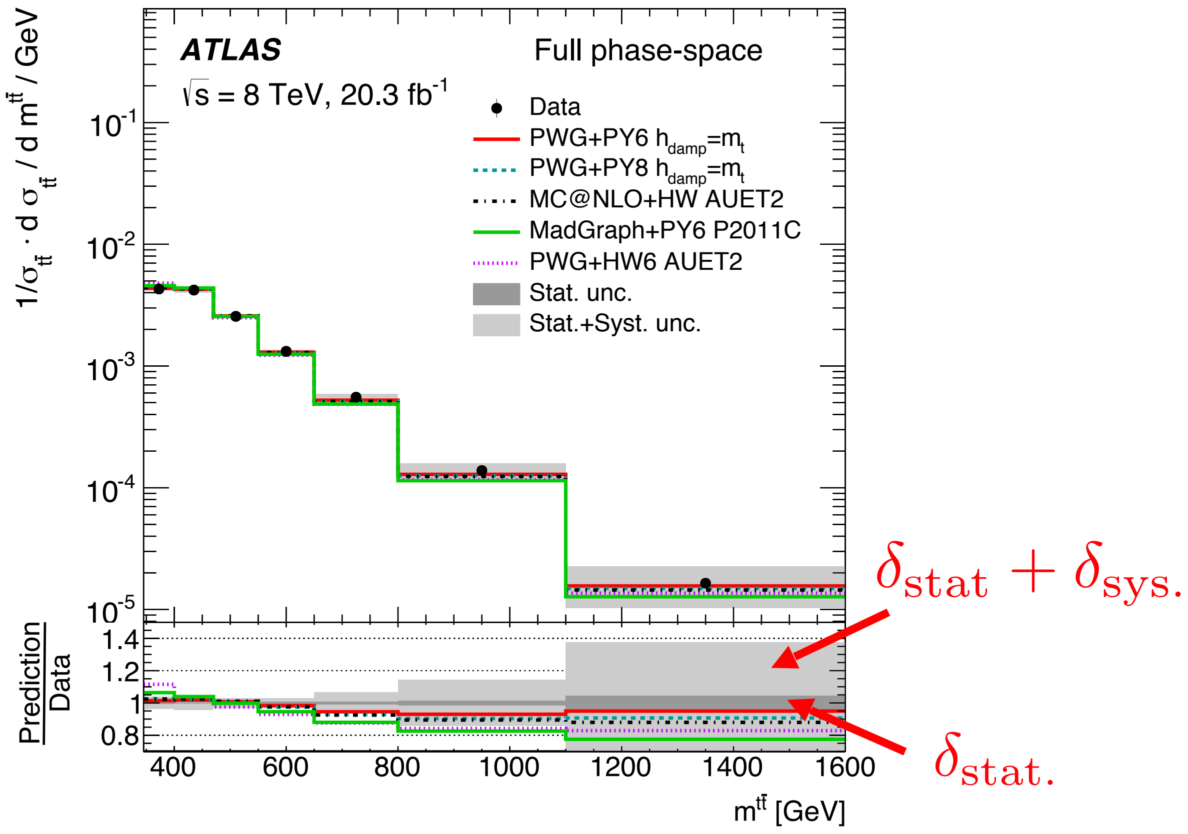
$$\chi^2/N_{\rm pts}\ (N_{\rm pts}^{\rm tot} = 25)$$

p_T	0.53
y_t	3.12
y_{tt}	3.51
M_{tt}	0.70
$p_T + M_{tt}$	5.73
Combined	$\underline{7.00}$

S. Bailey & LHL, arXiv:1909.10541



- Many such effects can be evaluated precisely in data-driven ways, but in some cases rely on further theory input.

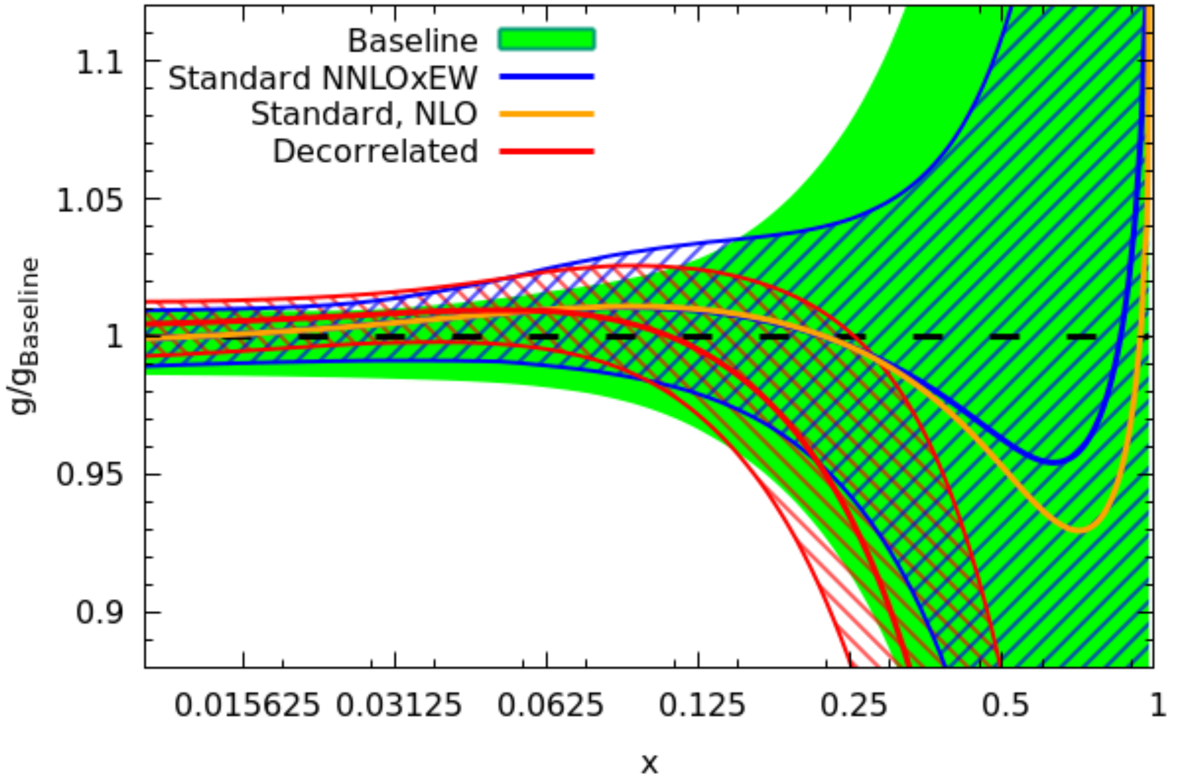


$\delta_{\text{stat}} + \delta_{\text{sys.}}$

$\delta_{\text{stat.}}$

- Correction from e.g. parton shower estimated by **Monte Carlo** simulation.
- Uncertainty on this: difference between results from two MCs.
- Increasingly such errors completely **dominate**!
- How to treat correlations in these error sources?
- Default correlation gives terrible χ^2. loosening this improves fit a lot.
- Unfortunately has rather large effect on extracted gluon PDF.
- **More work** needed to understand this.

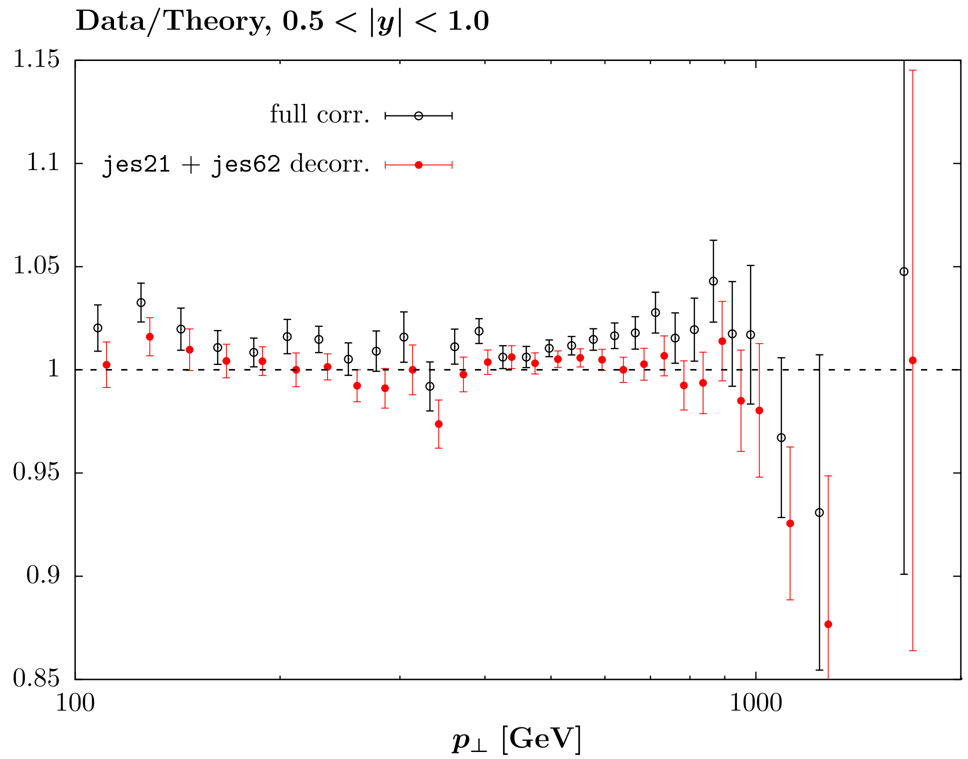
Comparing theoretical precision to decorrelation



Other Examples: Jets and Z pt

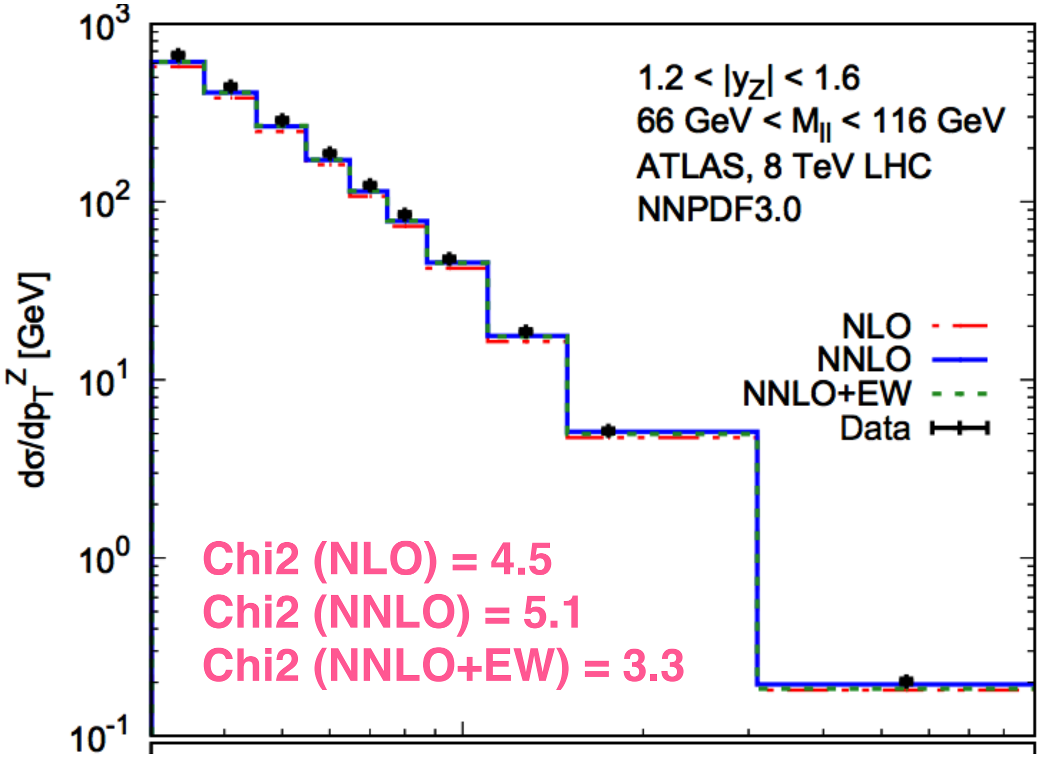
- ATLAS **Jet data**: again systematics dominated, and fit quality highly sensitive to correlations.
- Decorrelation improves things, as does including **theory uncertainty** from scale variations (= missing orders in pQCD)

LHL, R.S. Thorne, A.D. Martin, EPJC78 (2018) no.3, 248



- **Z boson** transverse momentum distribution: similar situation.
- Again can improve by adding in e.g. uncorrelated source of uncertainty (missing theory, underestimated experimental errors…) but motivation unclear.

R. Boughezal et al., JHEP 1707 (2017) 130



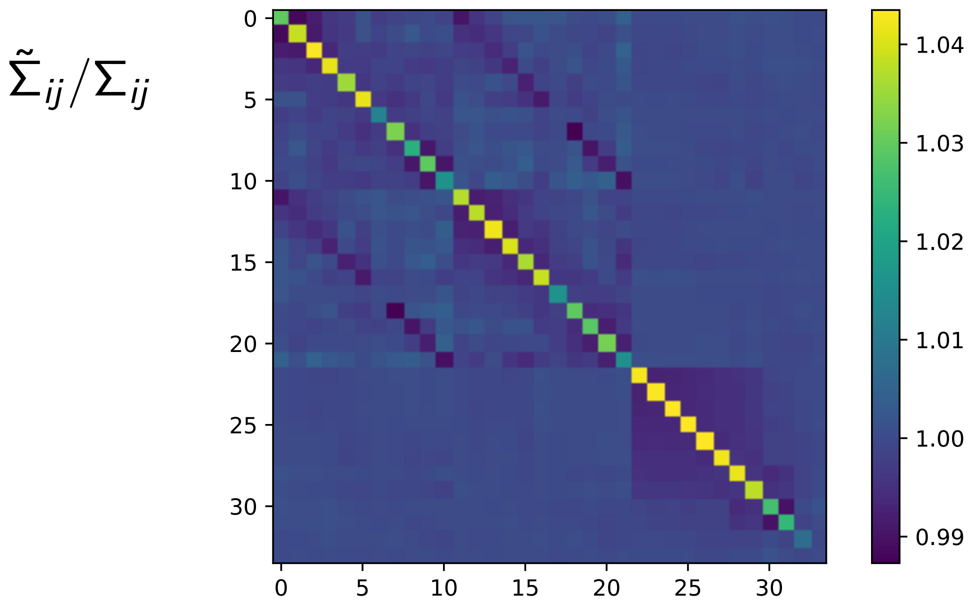
General Approach? What Next?

https://conference.ippp.dur.ac.uk/event/794/contributions/4605/attachments/3737/4229/michaelwilson_talk.pdf

M. Wilson, PDF4LHC September 2019

$$\Sigma \to \tilde{\Sigma}$$

Stat Estm.	Fit using $\tilde{\Sigma}\|_{k=500}$	fit using Σ
χ^2/N_{data}	1.00035	1.16328
$\langle\chi^2/N_{\text{data}}\rangle$	1.095 ± 0.038	1.253 ± 0.033



- One possible route: define a general procedure for dealing with case where fit quality bad simply because **correlations** difficult to estimate*.
- Recent approach attempts this by defining a modified 'stable' **covariance matrix**:
- One possibility, though need to be careful one keeps correlations where they are well established (luminosity (!) etc…).
- Basic conclusion/worry: these sort of issues *might* limit **eventual precision** we can achieve in LHC PDF fits. But no final word on this yet: work on this very much ongoing!

***N.B. It is not an established fact that this is the cause of the issues I have discussed!**

Other Topics of Interest

Theory Uncertainties

- PDF fit schematically given by inverting:

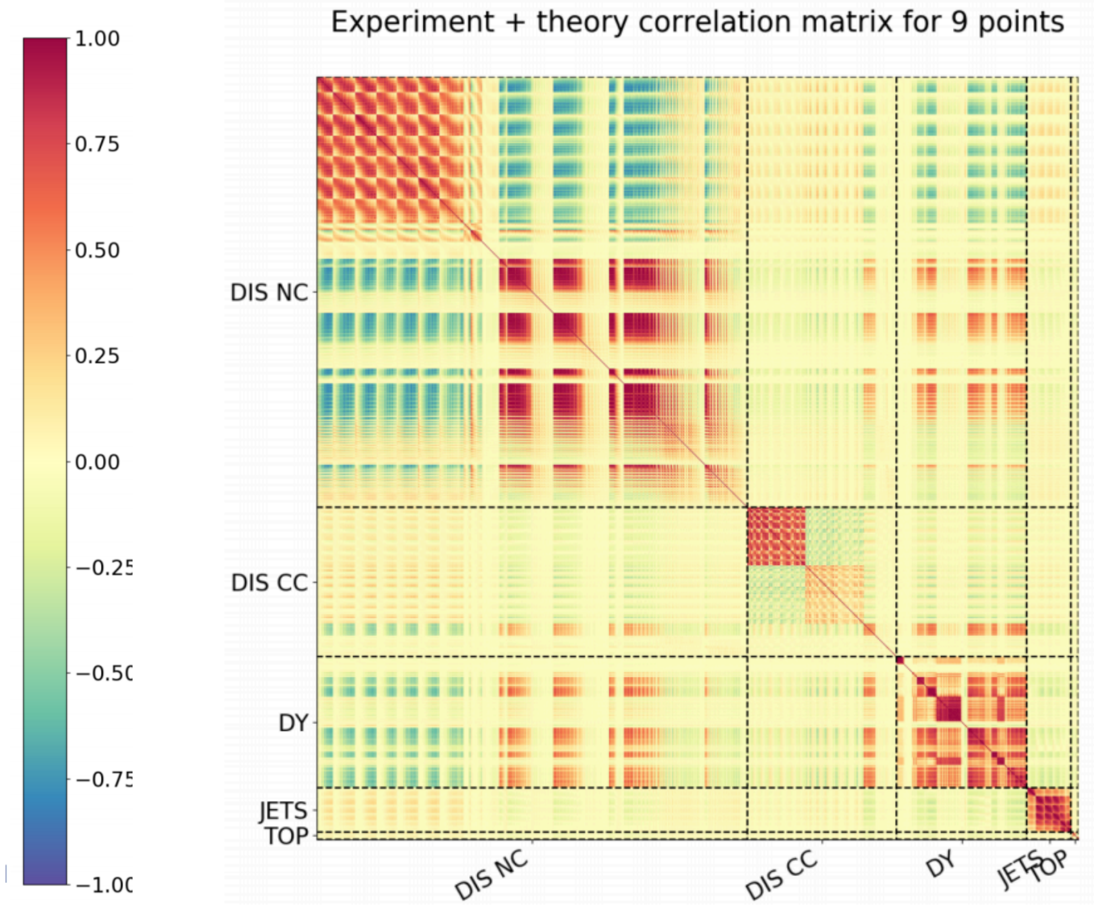
Dataset $$O \sim f \otimes \sigma \sim f \otimes \left(\sigma^{(0)} + \alpha_S \sigma^{(1)} + \cdots\right)$$

- Until recently only PDF errors corresponding to data errors in fit included.
- However in principle not only error source. Also that due to **missing higher orders** (the '…') in theory, from truncation of pert. expansion.

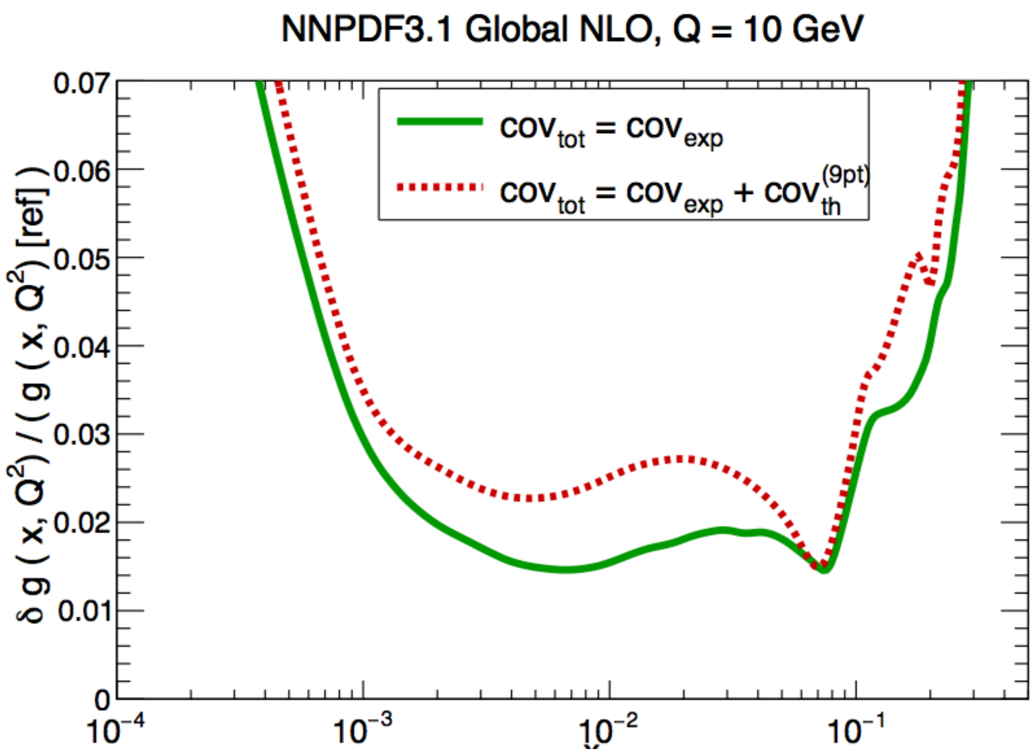
- Recent work on this: construct theory covariance matrix from **scale variations** (standard estimate of MHO uncertainty).

$$\delta O(\mu_F, \mu_R, \mu_0) : \quad \mu_{F,R} \in \left(k\mu_0, \frac{\mu_0}{k}\right)$$

R. Abdul Khalek et al., arXiv:1906.10698



- Impact on PDF uncertainties at NLO not negligible (will be less at NNLO).
- However not the end of the story. Important **open questions**:



★ Are scale variations the best way to estimate MHOs?

★ Risk of **double counting** with MHO uncertainty already accounted for when making predictions via PDFs?

$$\begin{tikzcd}
\textbf{Fit} & O_{\text{fit}} \sim f_i(\mu^2) \otimes \sigma_i(\mu^2) \arrow[dd, "\mathbf{A}"] & \sim f_i(\mu^2) \otimes \left(\sigma_i^{(0)}(\mu^2) + \alpha_S\, \sigma_i^{(1)}(\mu^2) + \cdots\right) \arrow[d, "\mathbf{B}"] \\
& & f_i \arrow[d, "\mathbf{C}"] \\
\textbf{Prediction} & O_{\text{pred}} \sim f_i(\mu^2) \otimes \sigma'_i(\mu^2) & \sim f_i(\mu^2) \otimes \left(\sigma_i^{(0)\prime}(\mu^2) + \alpha_S\, \sigma_i^{(1)\prime}(\mu^2) + \cdots\right)
\end{tikzcd}$$

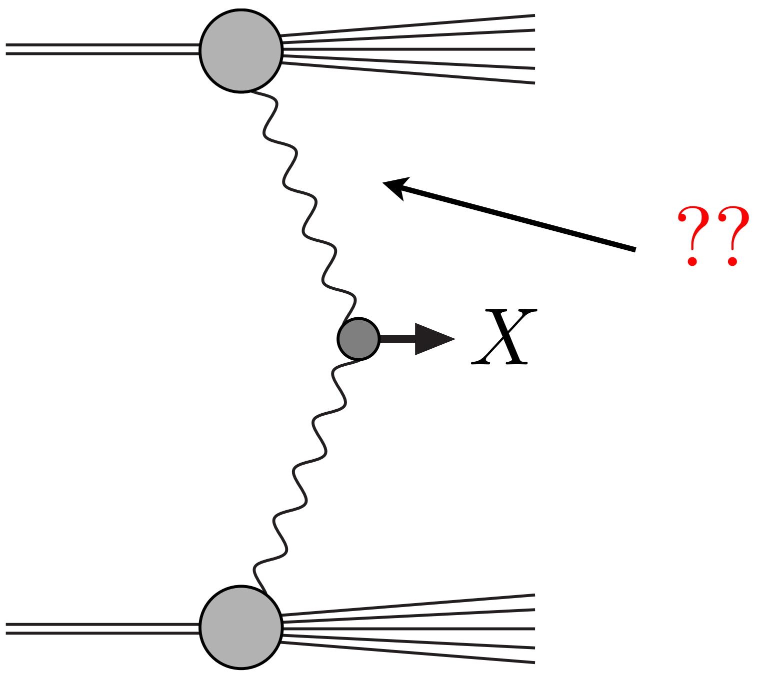
i : PDF type

LHL and R. S. Thorne, EPJC79 (2019), no.1, 39

- Recent work on this: yes!
- Basic idea: consider PDF fit as direct relationship between $O_{\text{fit}} \to O_{\text{pred}}$.
- **Work ongoing** on resolving these questions.

Photon PDF

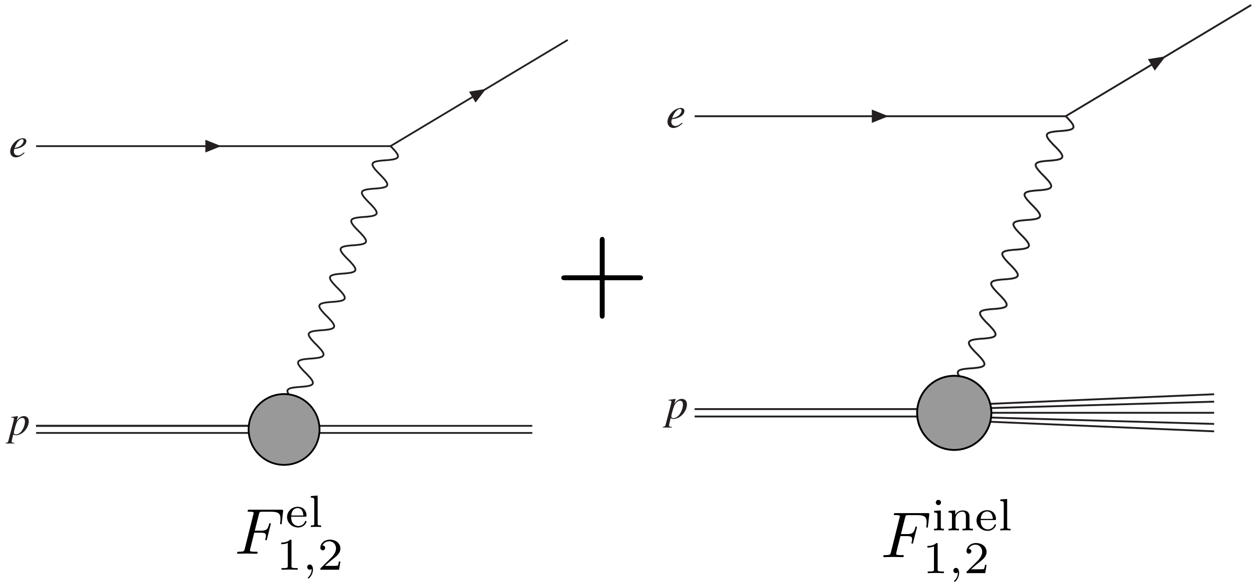
$\alpha_{\text{QED}}(M_Z) \sim \alpha_S^2(M_Z)$



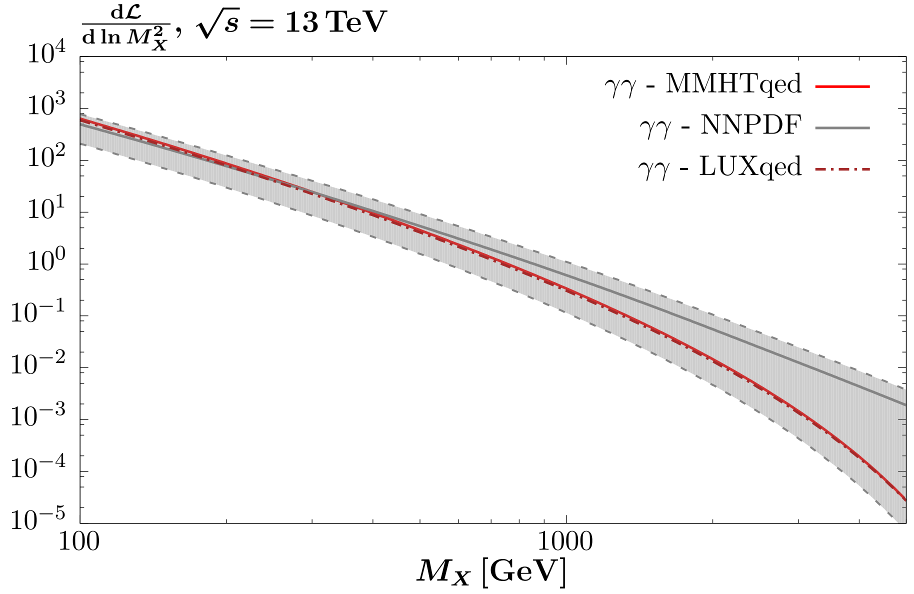
- In high precision LHC era, **photon-initiated** contributions of relevance to phenomenology.
- Nice example of parton that does not exactly fit into the same PDF paradigm as quark/gluons.

- First attempts at simple models/fitting freely replaced by '**LUXqed**': photon PDF directly related to (precisely measured) proton structure functions.

$\gamma(x, Q^2) \sim$ … $+$ …



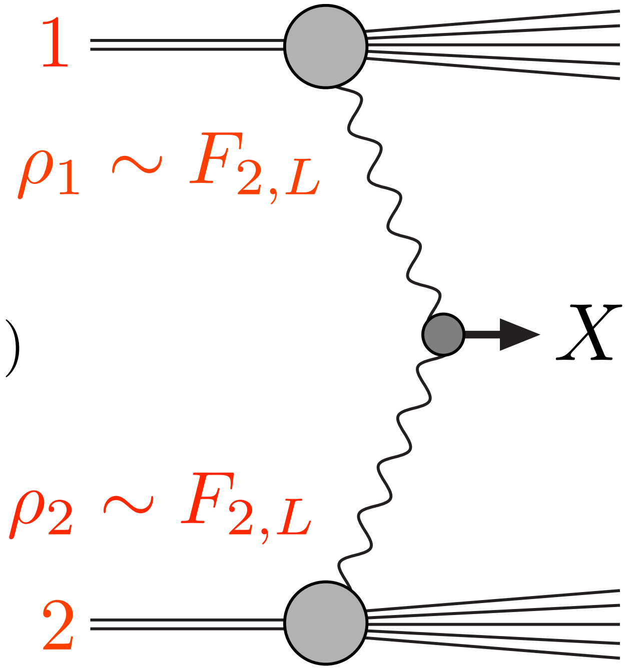
A. Manohar et al., JHEP 1712 (2017) 046



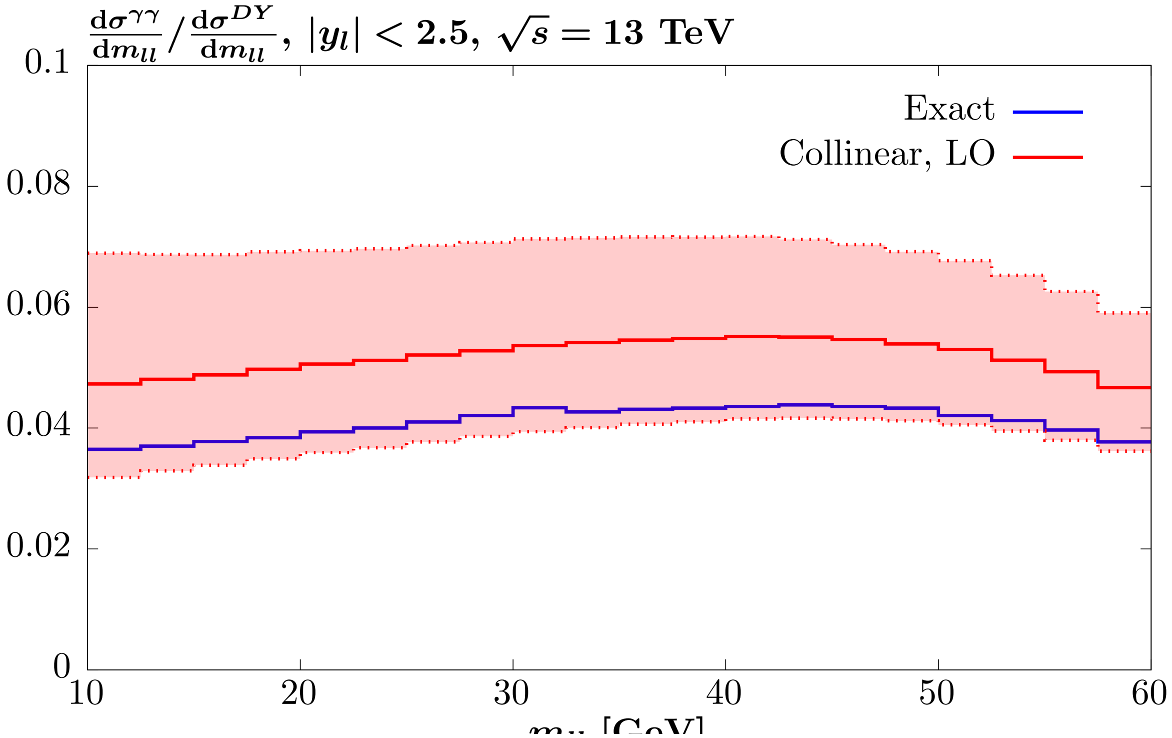
LHL eta al., Phys. Rev. D94 (2016) no.7, 074008

LHL, arXiv:1910.10178

- Very recent paper: not clear one needs to talk in terms of **photon PDF** to calculate photon-initiated production*, and more precise not to.
- Instead calculate cross section **directly** in terms of proton structure functions.



$$\sigma_{pp} = \frac{1}{2s}\int \frac{\mathrm{d}^3p_1\mathrm{d}^3p_2\mathrm{d}\Gamma}{E_1E_2}\alpha(Q_1^2)\alpha(Q_2^2)\frac{\rho_1^{\mu\mu'}\rho_2^{\nu\nu'}M^*_{\mu'\nu'}M_{\mu\nu}}{q_1^2q_2^2}\delta^{(4)}(q_1+q_2-k)$$



- Conventional PDF picture not the only input for LHC pheno.
- Though note input (proton structure functions) still fundamentally from **experiment**.

***For QED corrections to DGLAP - do need!**

Summary/Outlook

- ★ LHC phenomenology and PDF determination has entered high precision era. **Percent level** (and below) uncertainties possible.
- ★ **Encouraging** results from LHC already, and a very encouraging outlook from future High-Luminosity running (LHeC?).
- ★ However already **challenges** in accounting for this level precision in PDF fits appearing. Will be focus of much future work, but in such a context input from lattice could be invaluable.

Thank you for listening!

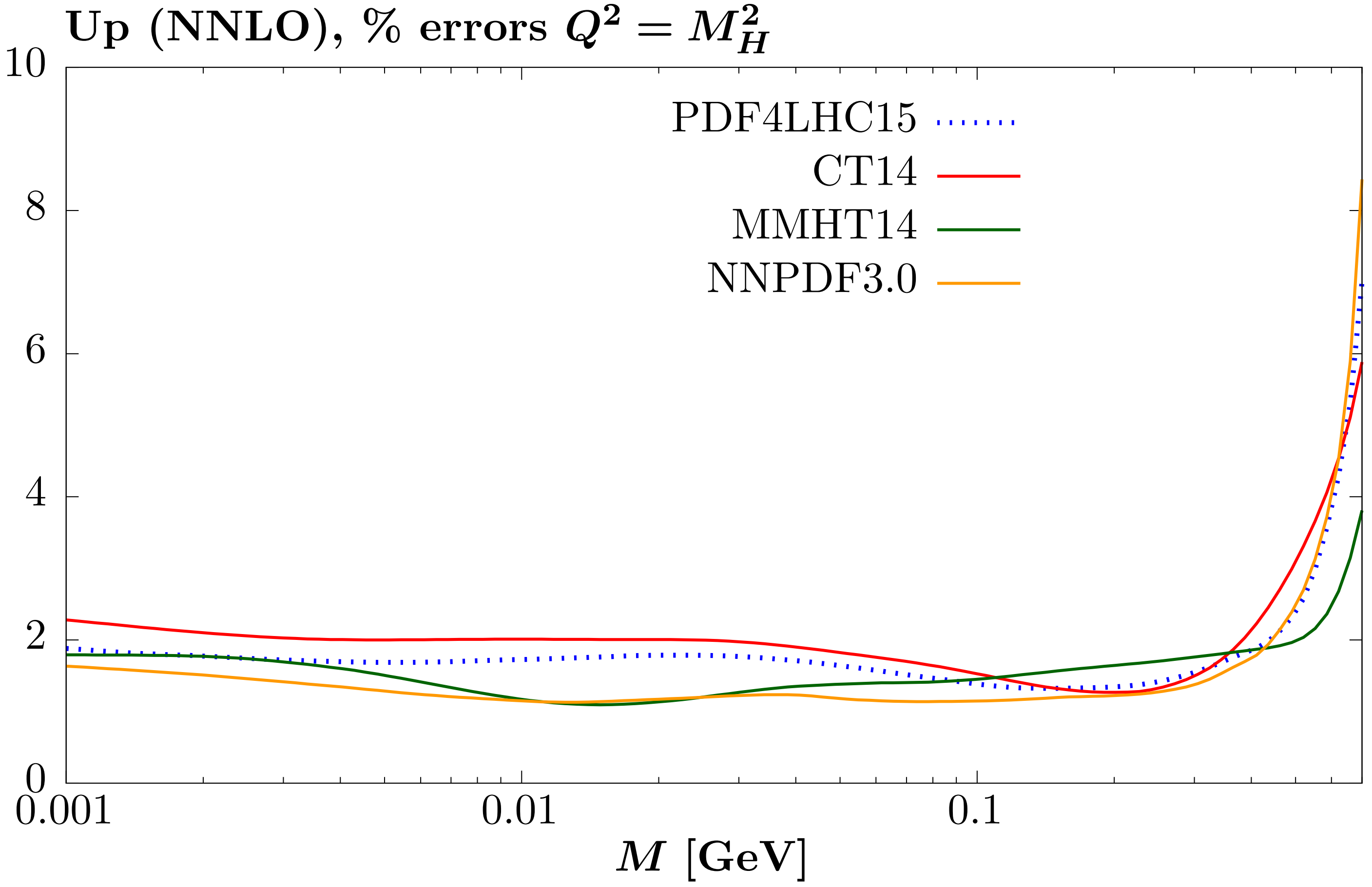
Backup

Modified Covariance Matrix

M. Wilson, PDF4LHC September 2019

- obtain correlation matrix from covariance matrix $c_{ij} = \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii}\Sigma_{jj}}}$
- perform eigenvalue decomposition on c giving Λ and U such that $c = U^t \Lambda U$.
- obtain new eigenvalues $\tilde{\Lambda}_{ij} = \delta_{ij} \min(\Lambda_{ij}, \hat{\lambda})$ where $\hat{\lambda} = \max(\Lambda_{ij})/k$ where k is an input parameter specifying a threshold condition number
- construct $\tilde{c} = U^t \tilde{\Lambda} U$ and use to obtain new, regularized covariance matrix $\tilde{\Sigma}_{ij} = \tilde{c}\sqrt{\Sigma_{ii}\Sigma_{jj}}$

This is our regulariation procedure!


Up (NNLO), % errors $Q^2 = M_H^2$
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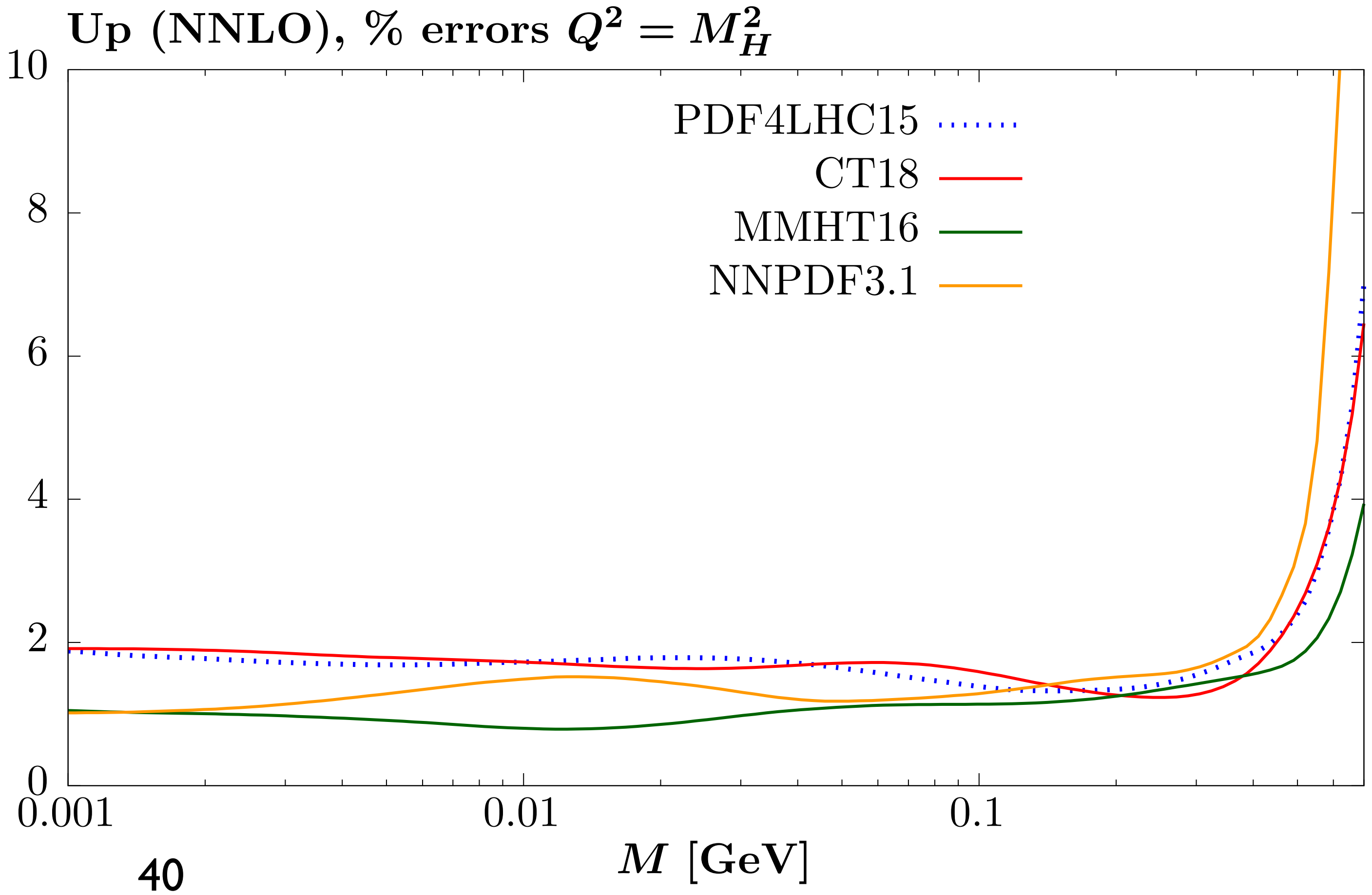
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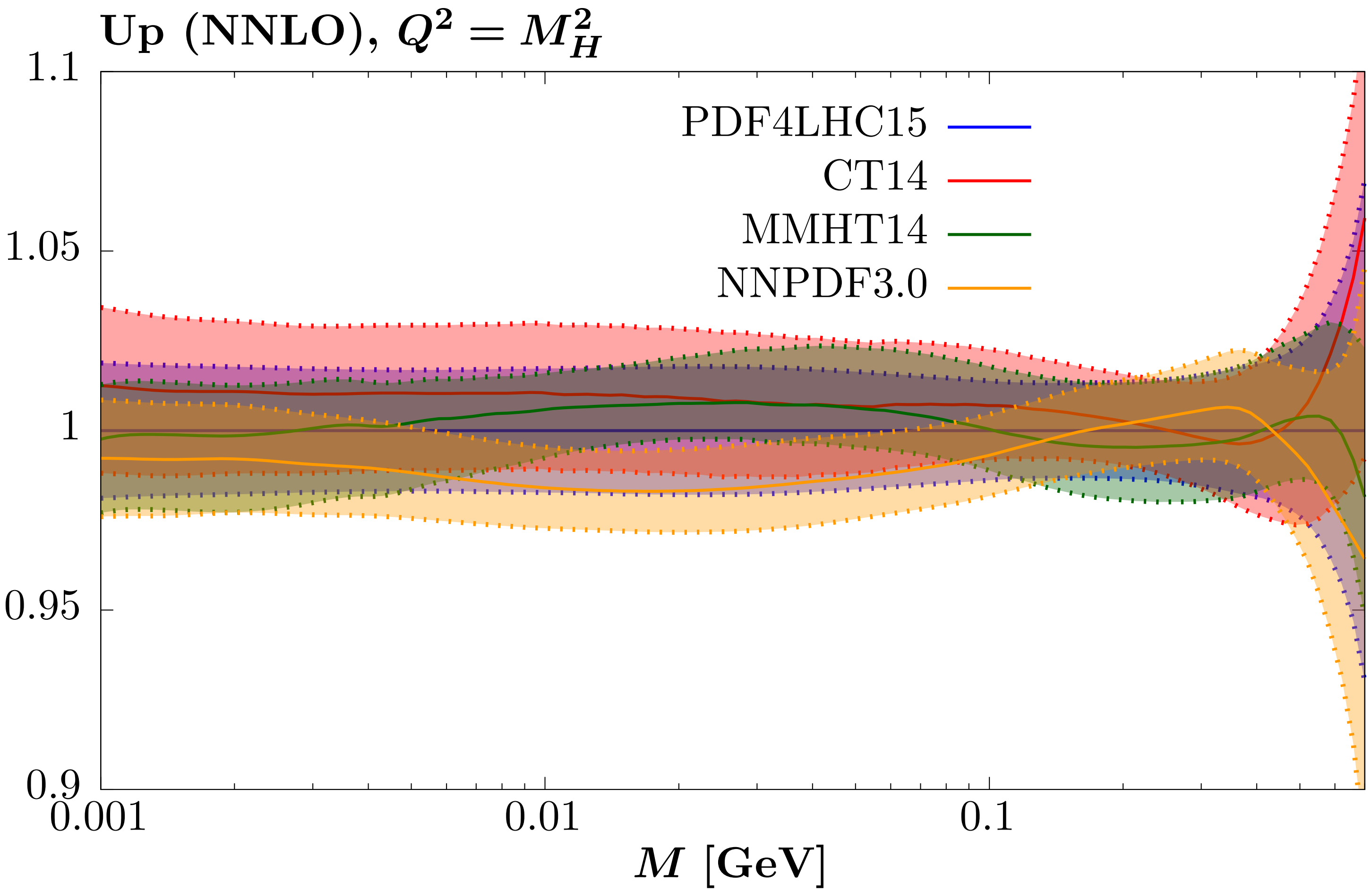
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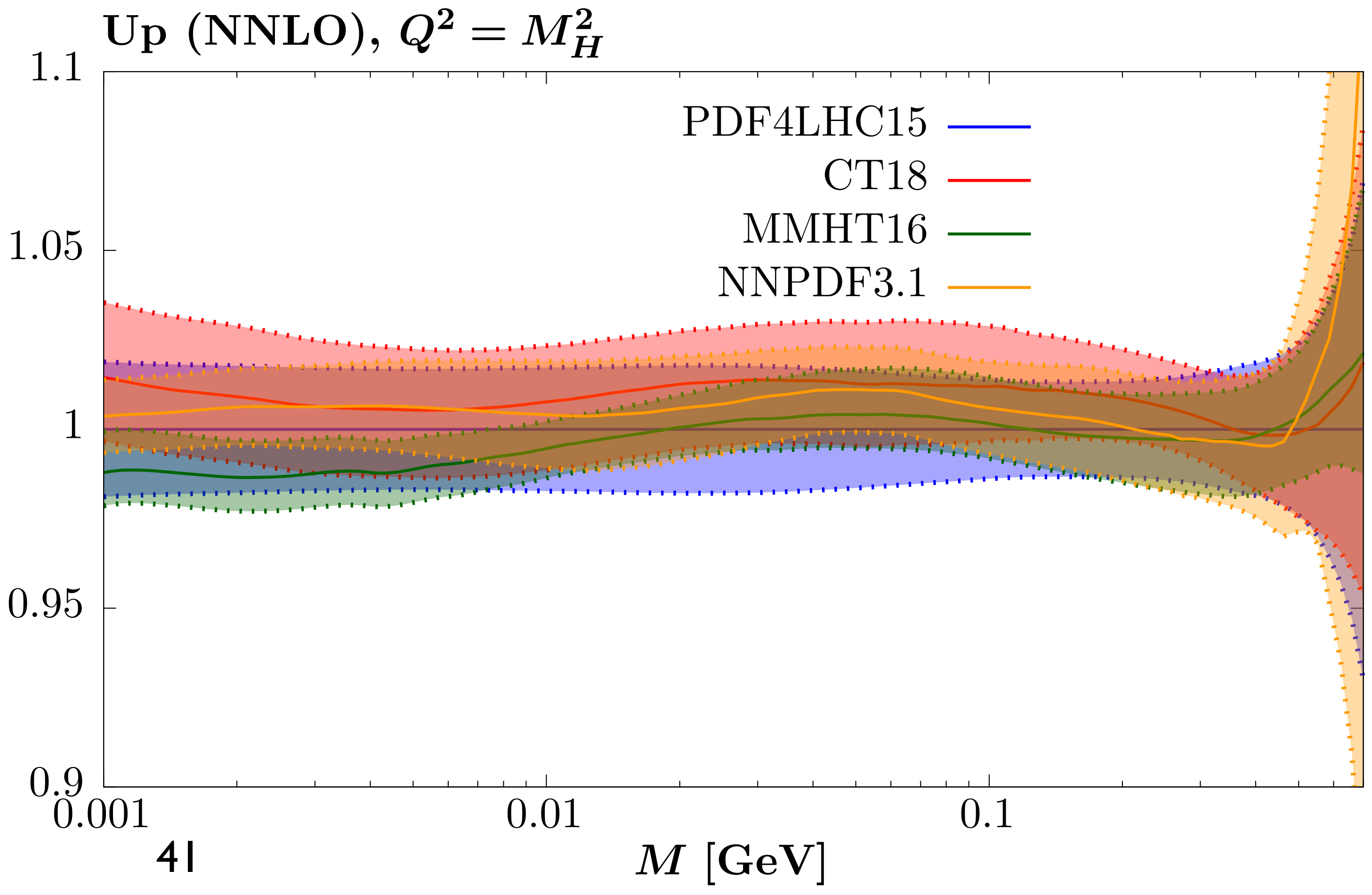
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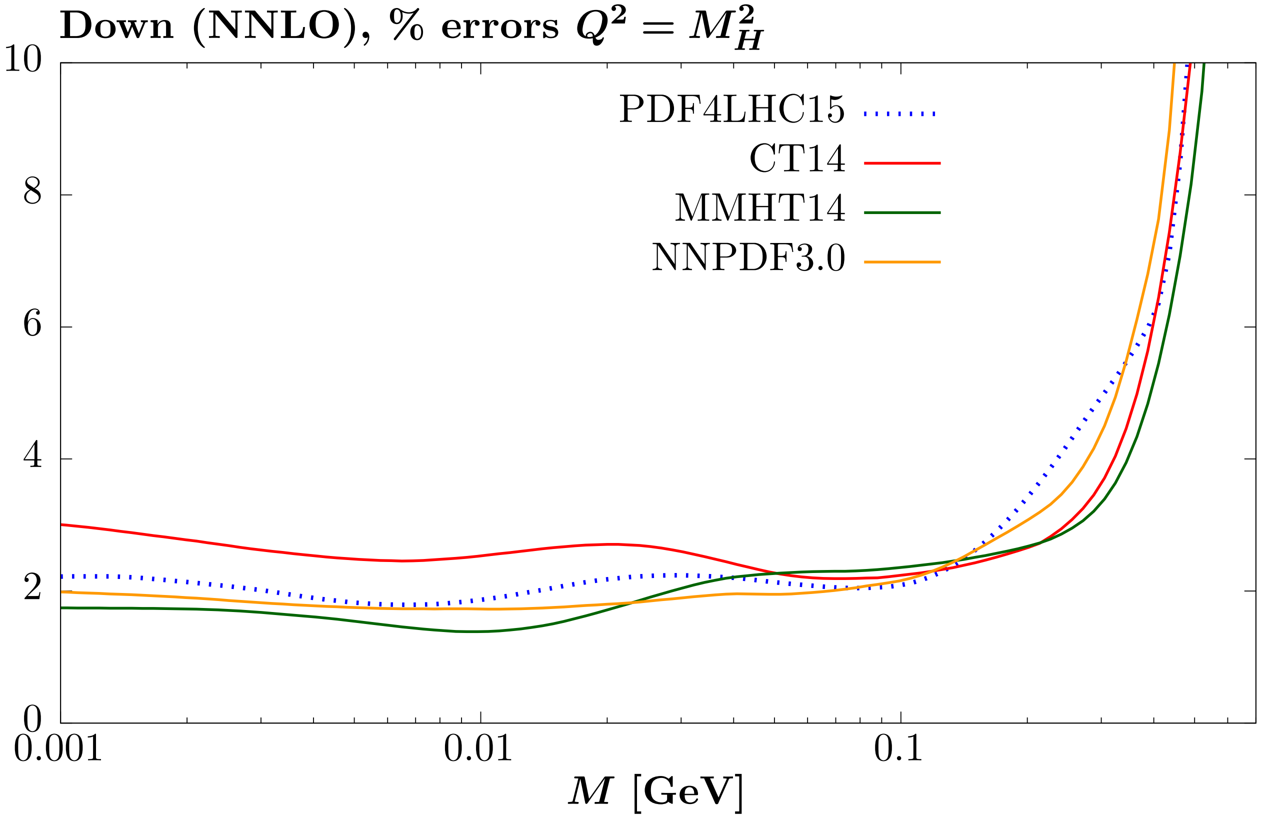
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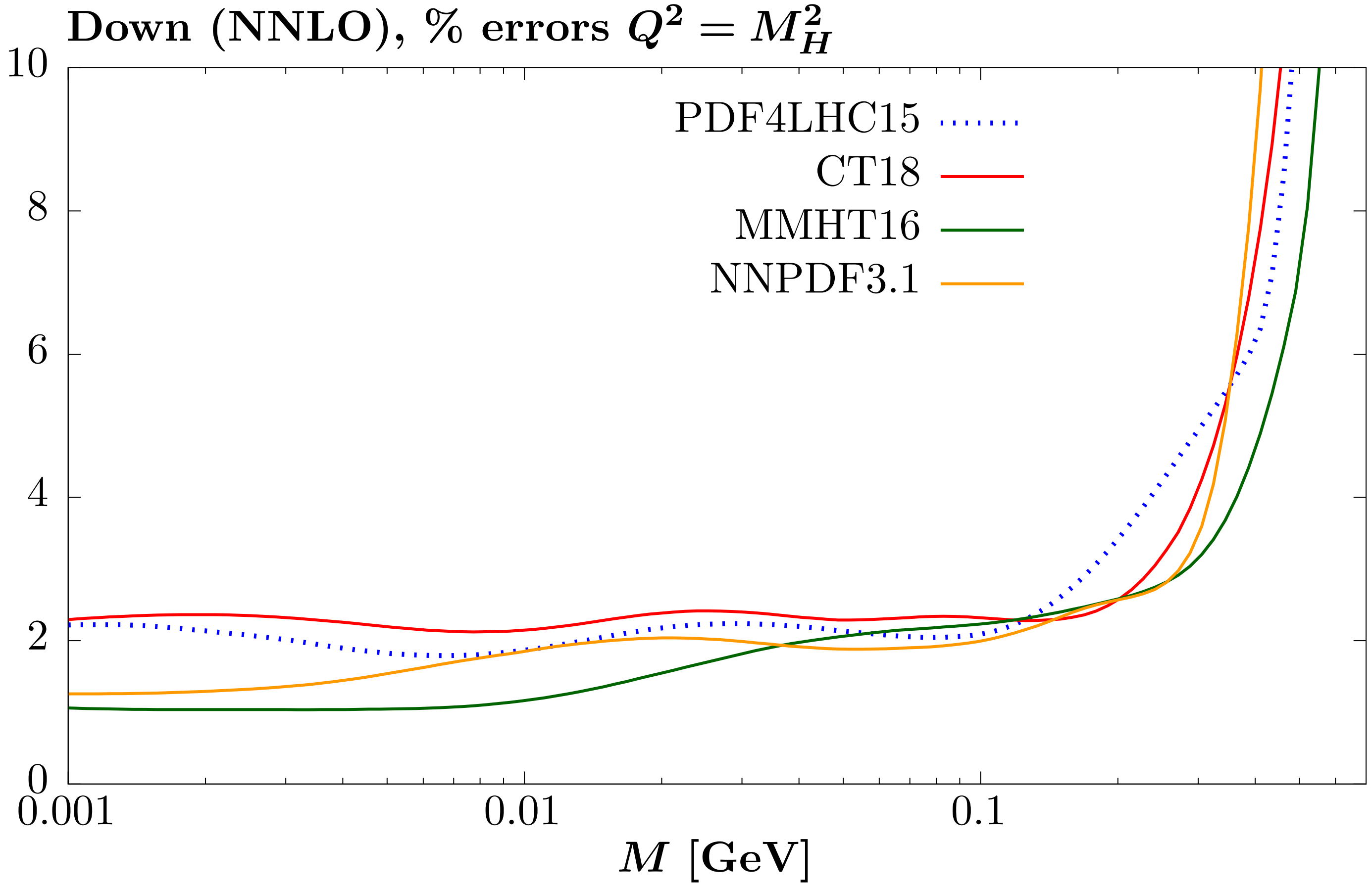
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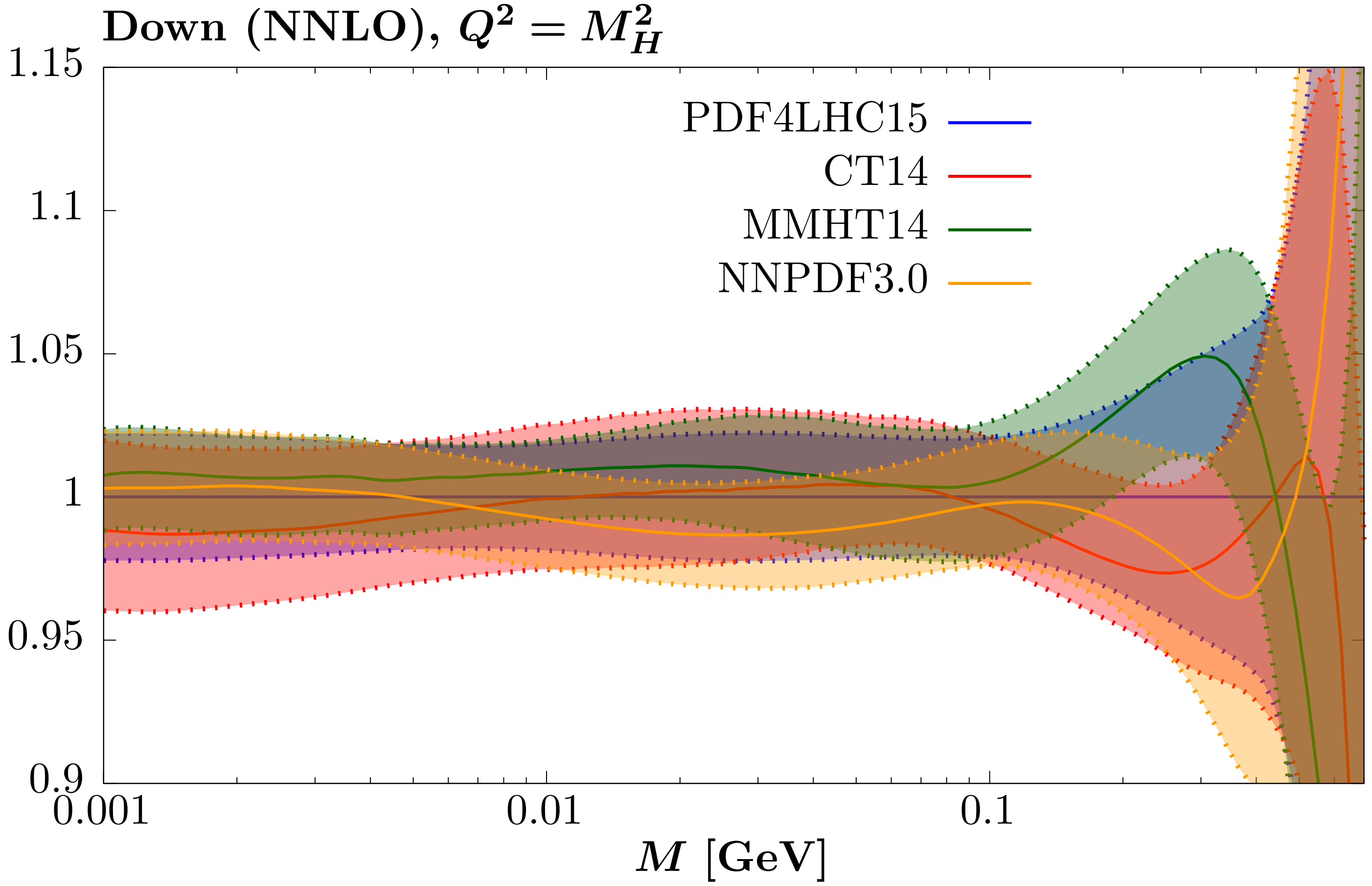
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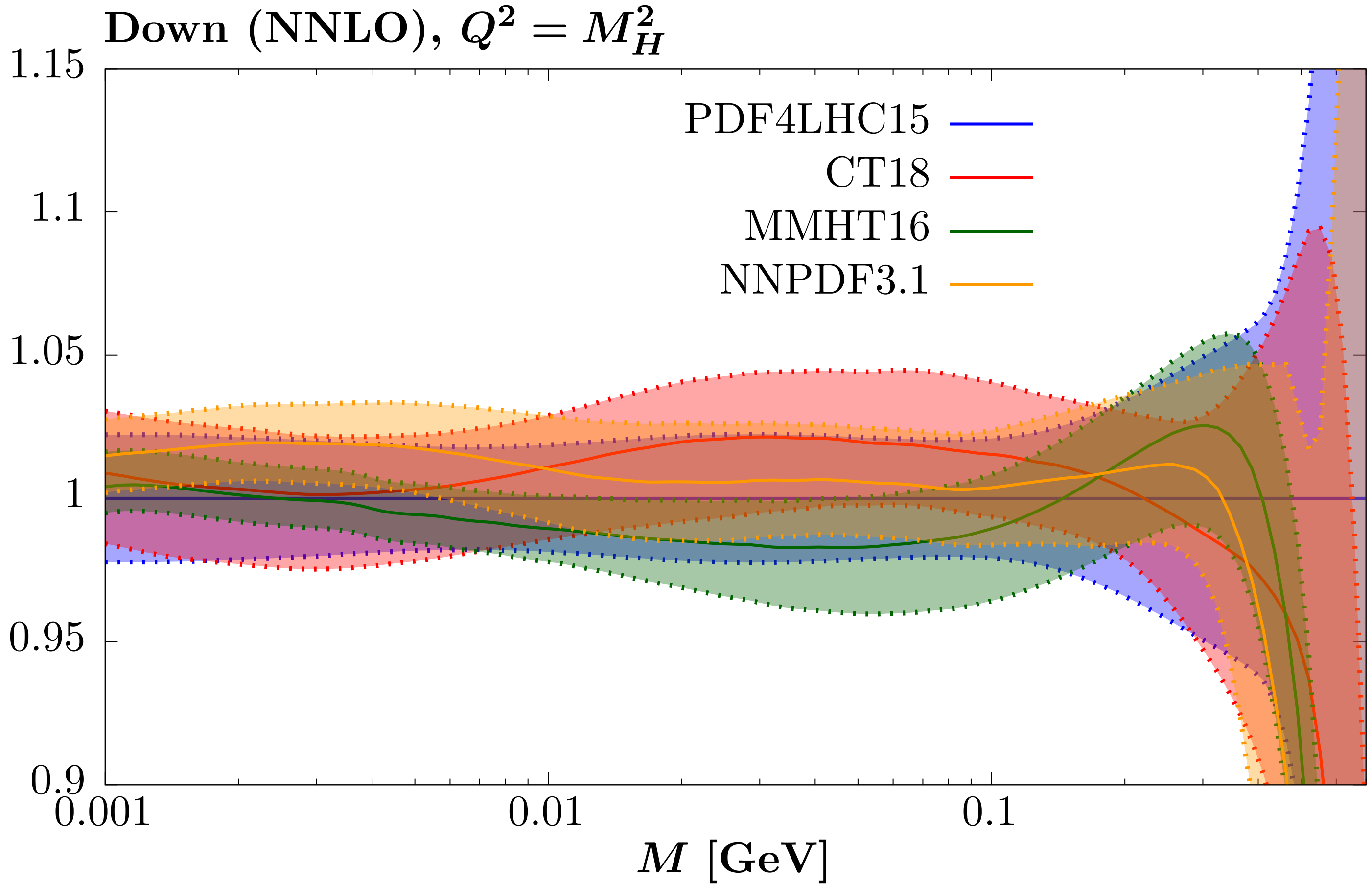
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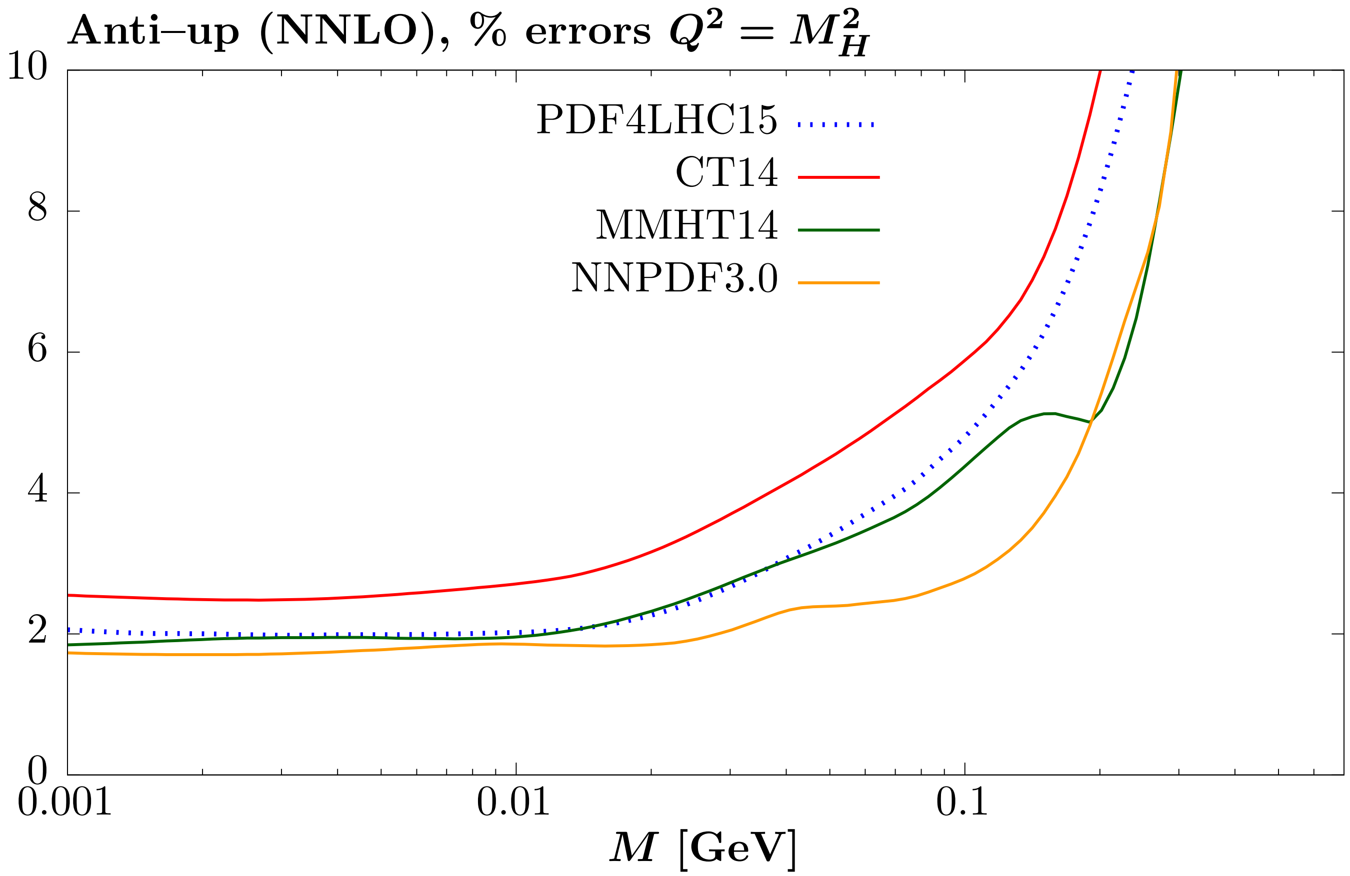
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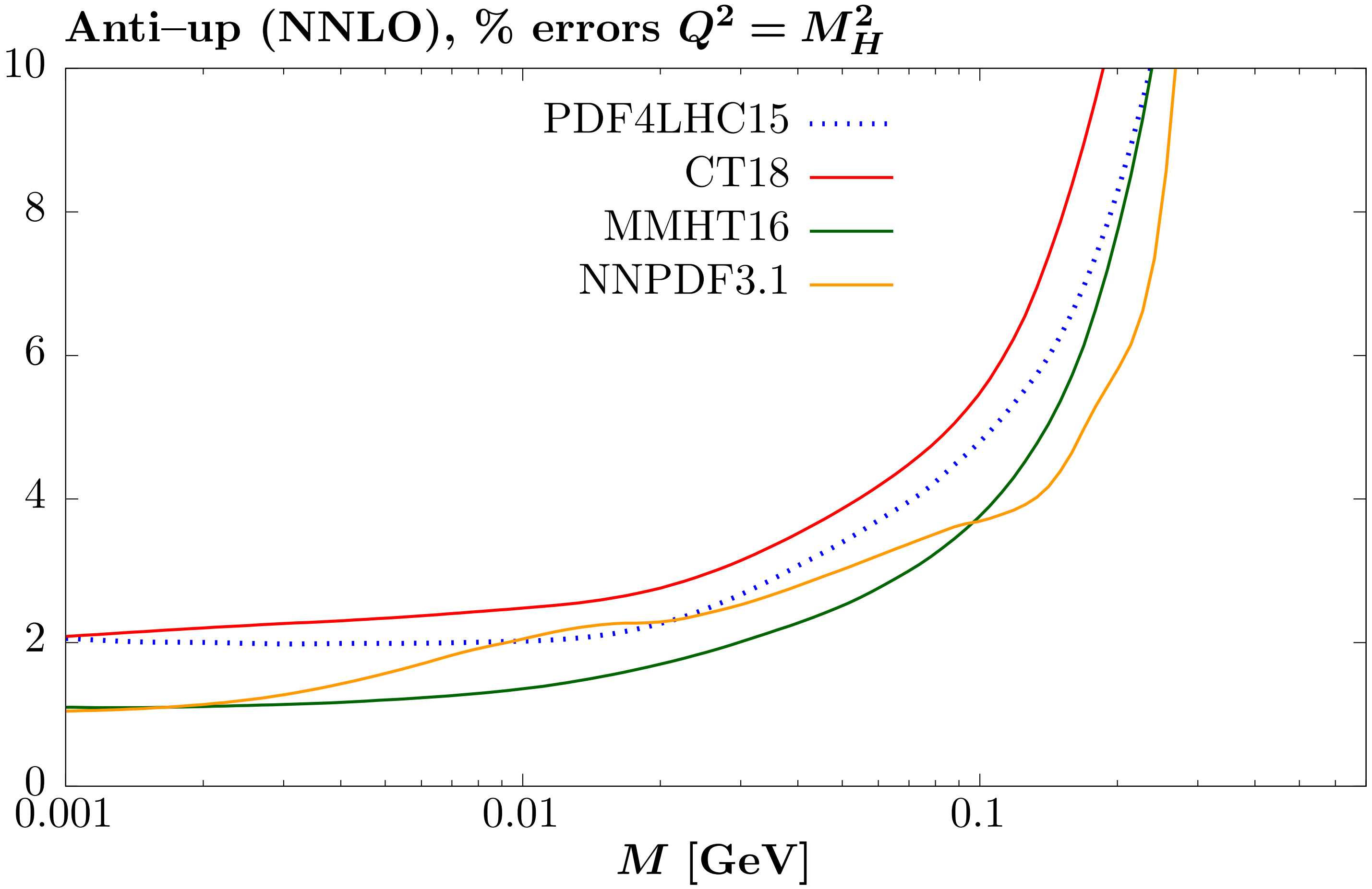
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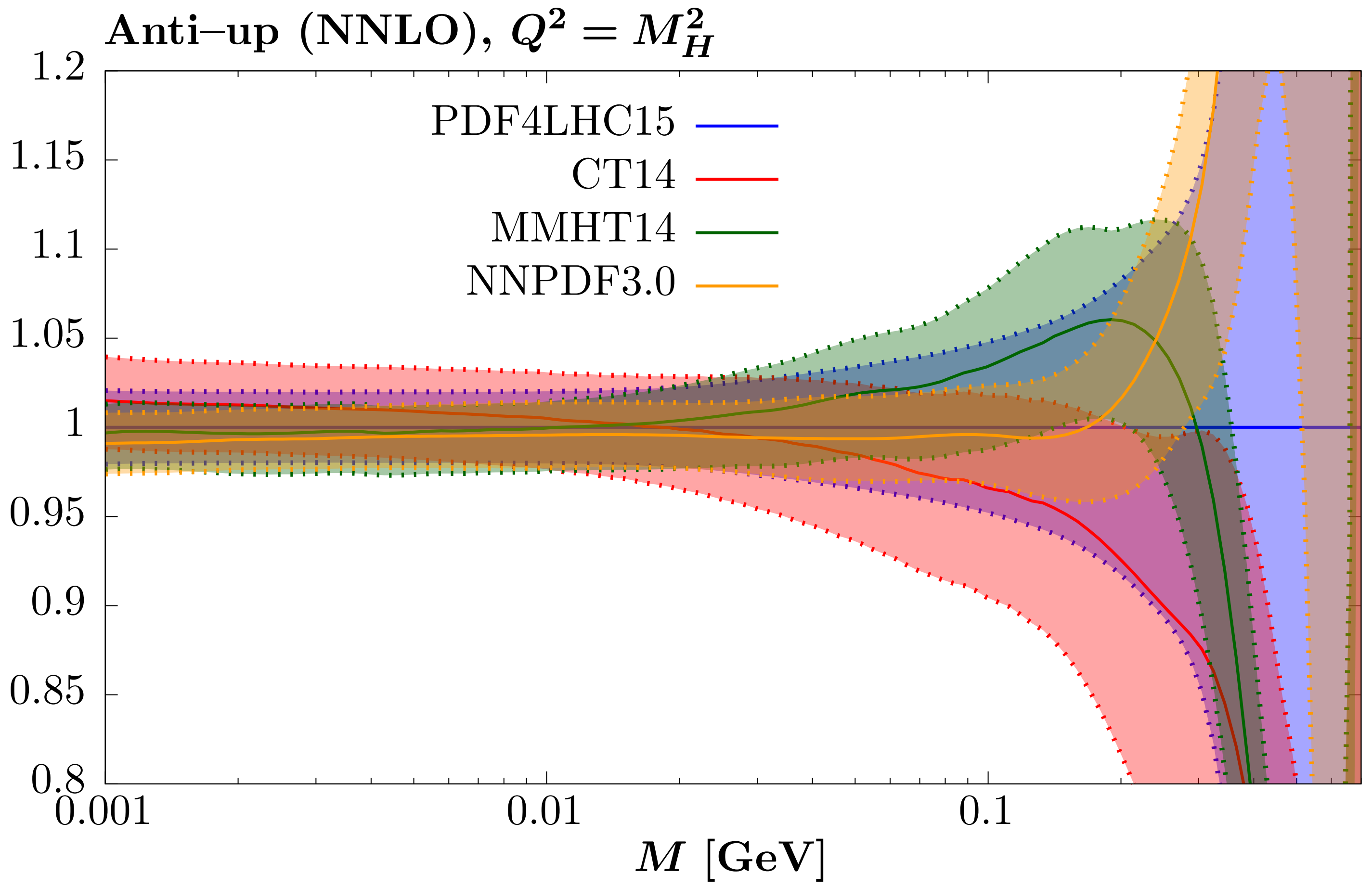
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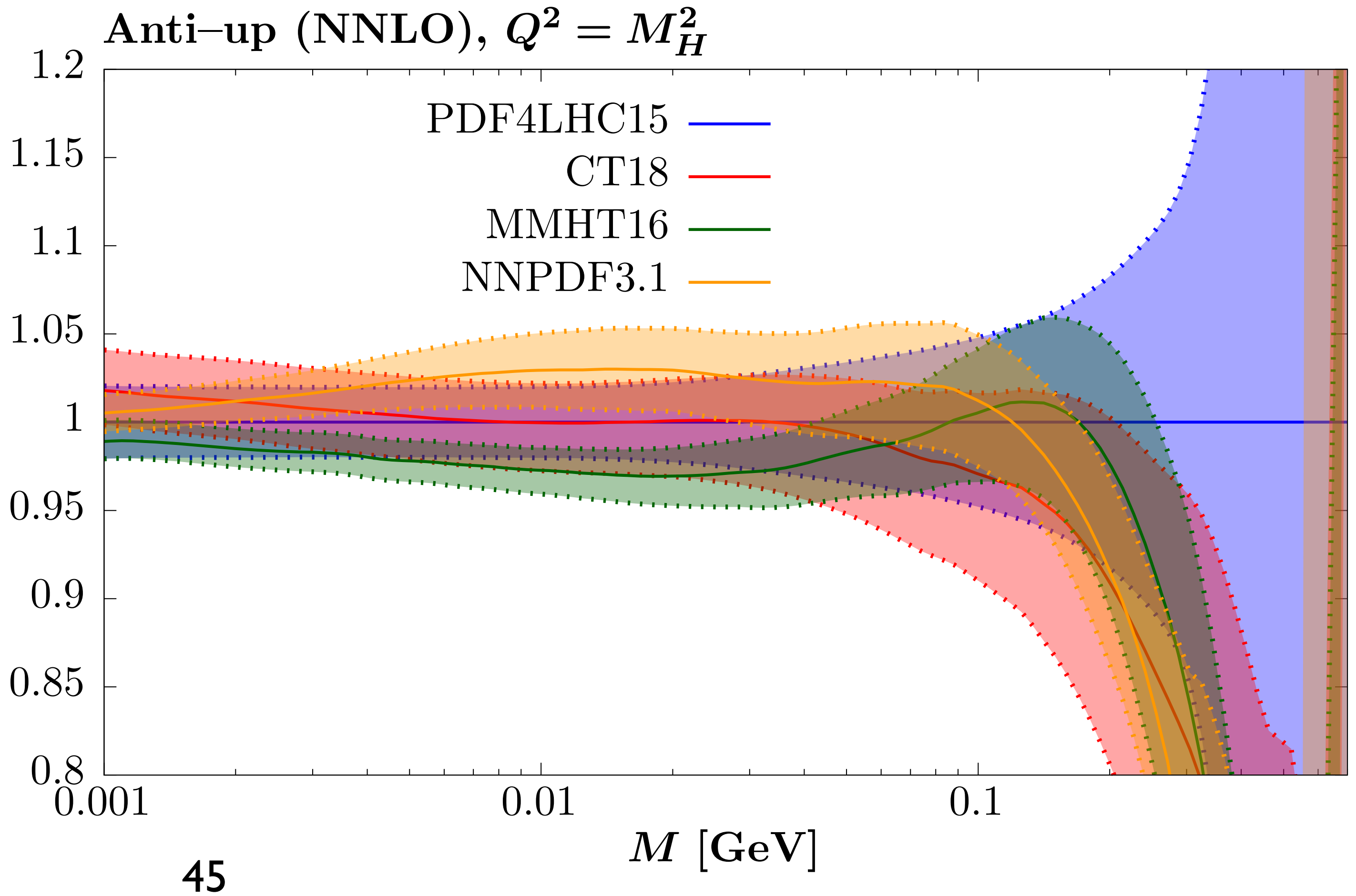
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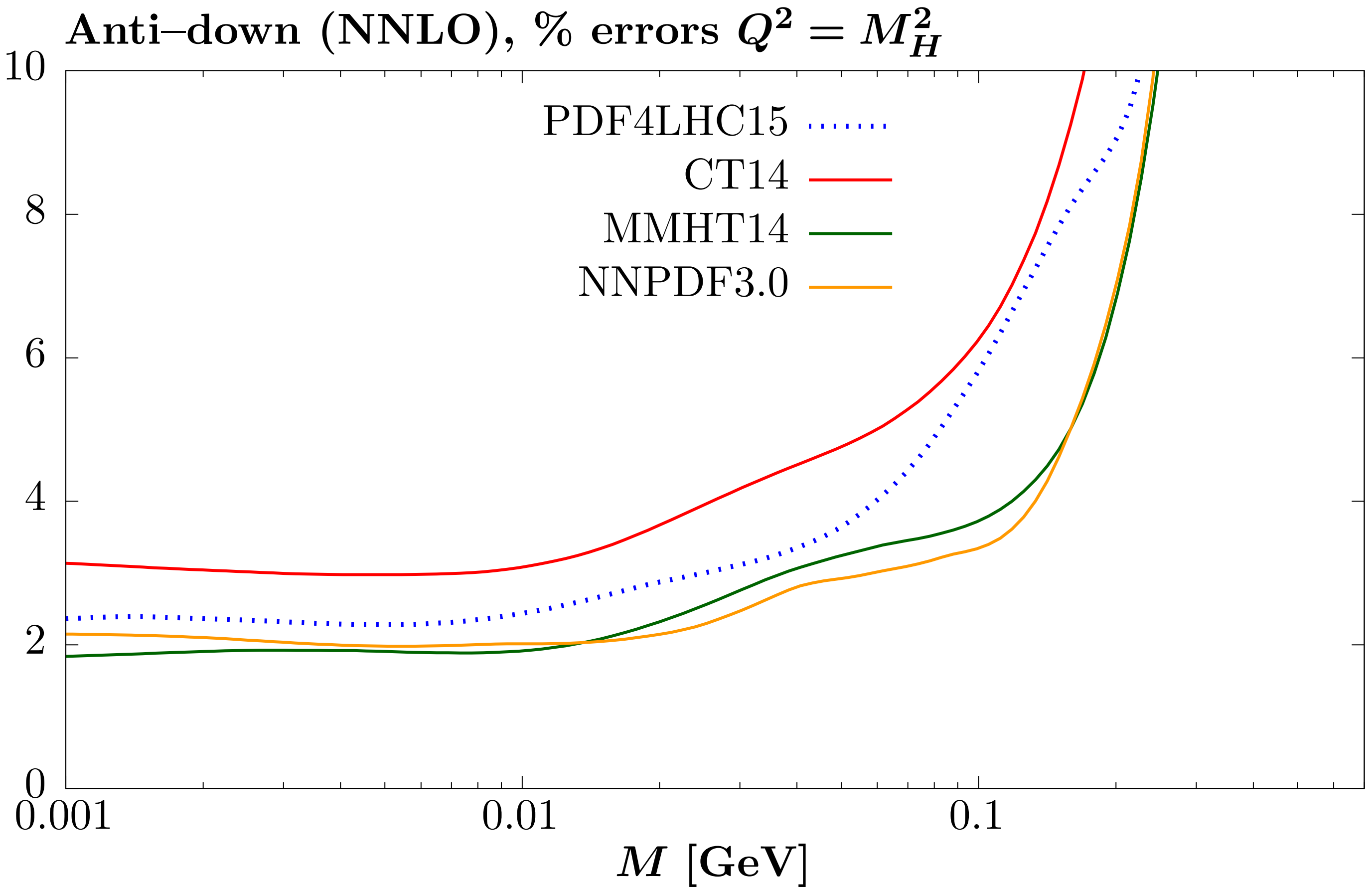
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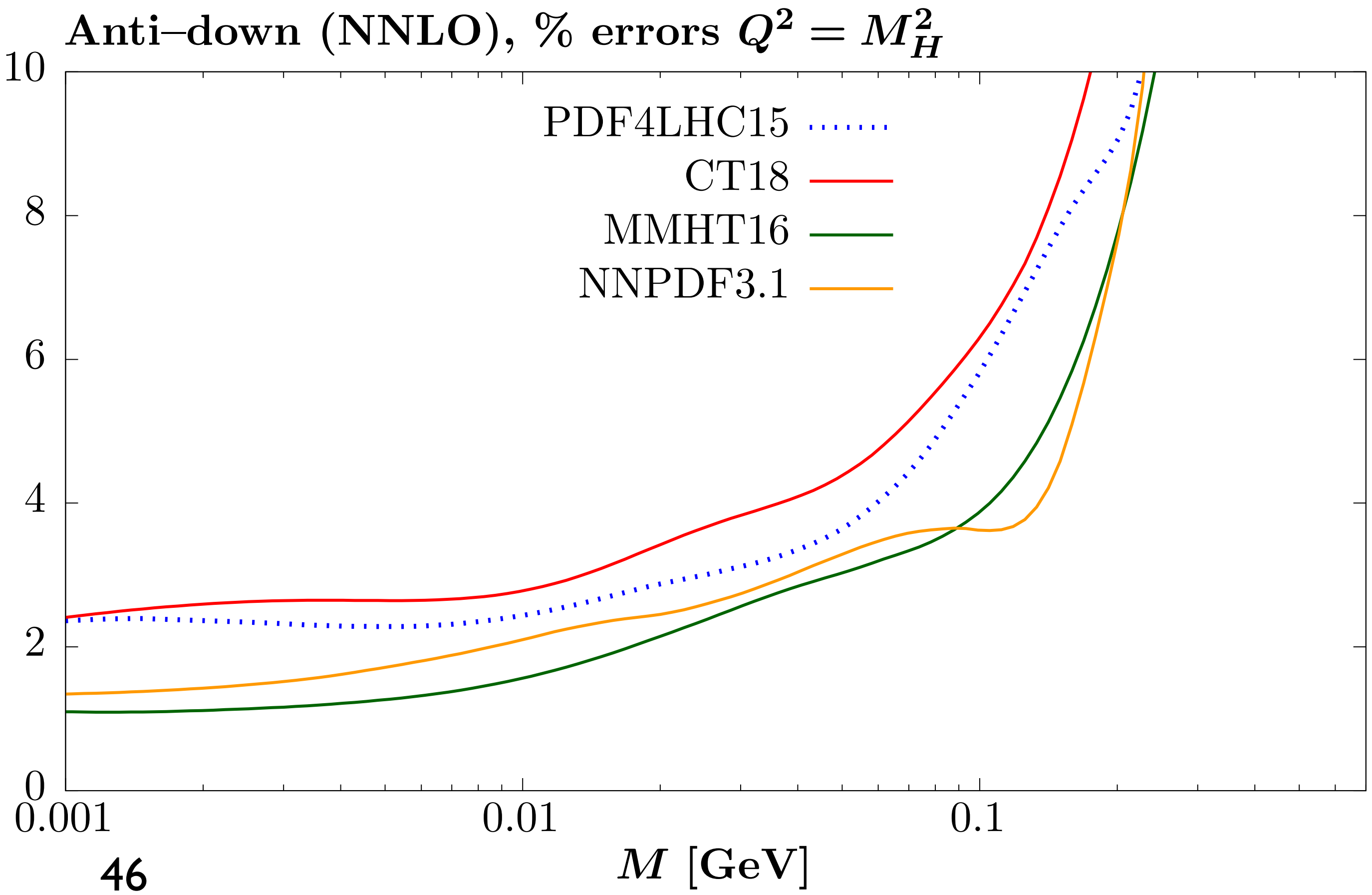
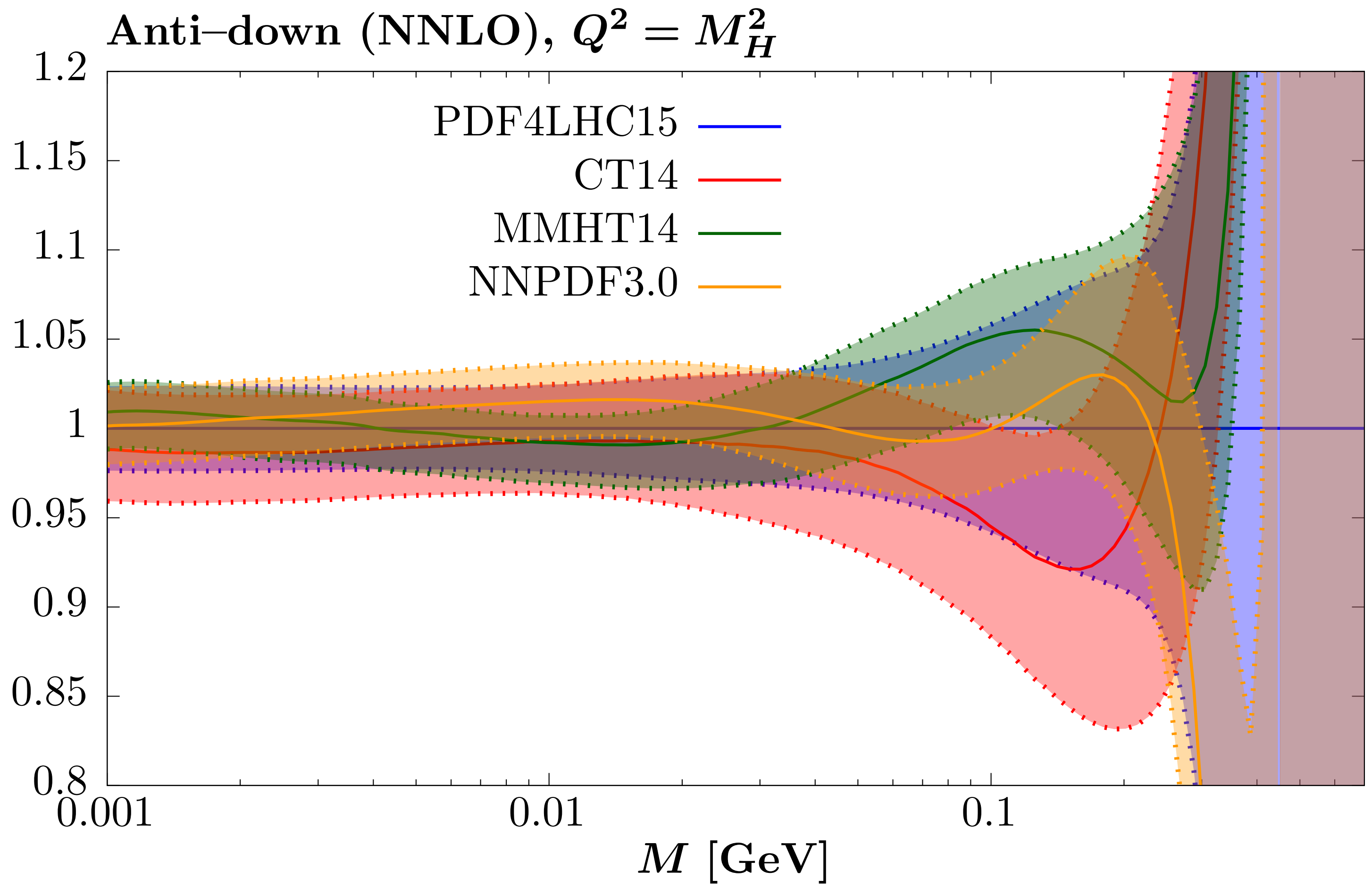
2019

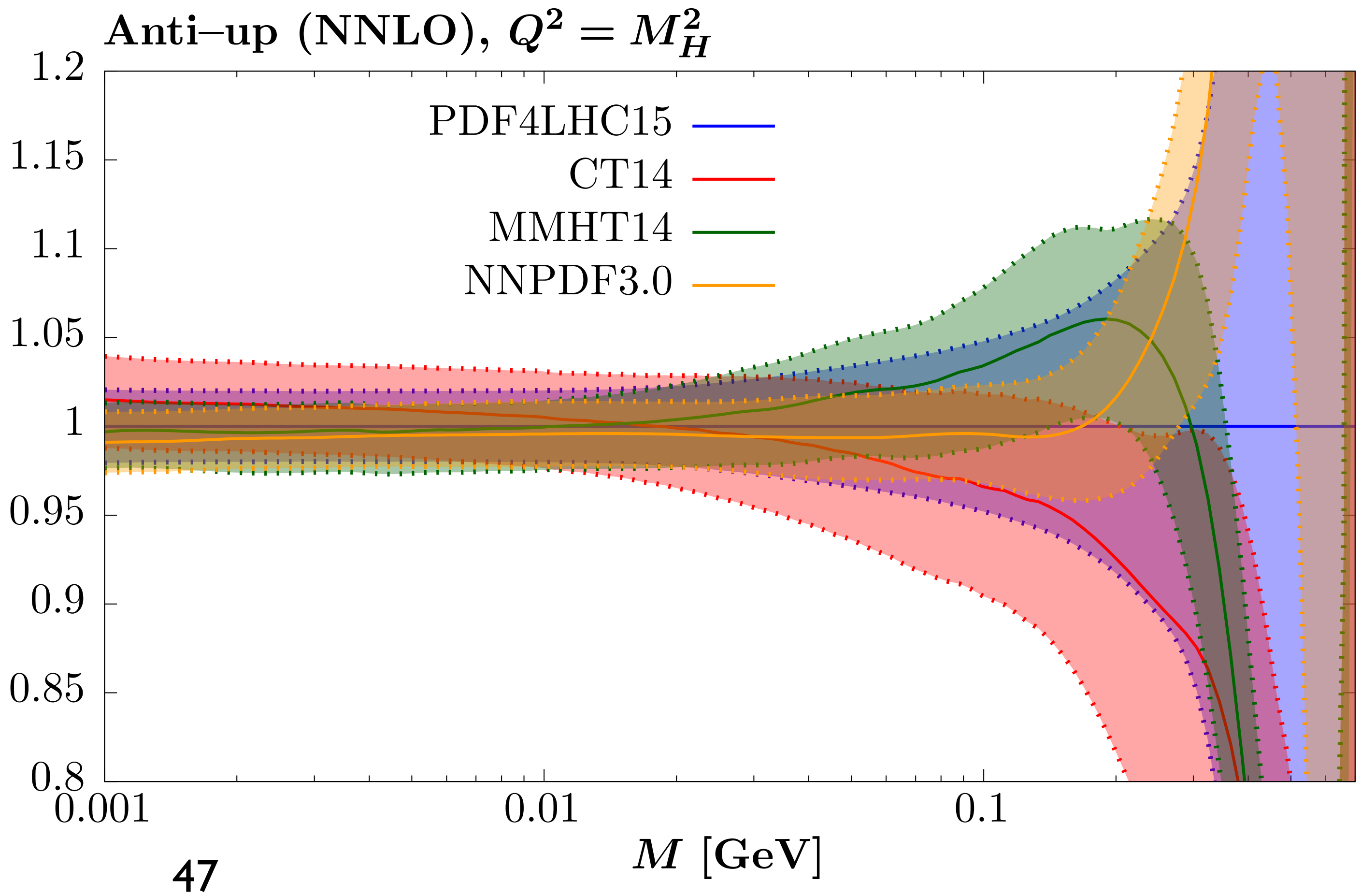

Anti–down (NNLO), % errors $Q^2 = M_H^2$
PDF4LHC15
CT18
MMHT16
NNPDF3.1
10
8
6
4
2
0
0.001
0.01
0.1
M [GeV]



Anti–down (NNLO), $Q^2 = M_H^2$

2015

2019



Anti–up (NNLO), $Q^2 = M_H^2$

HL-LHC: Datasets

Process	Kinematics	N_{dat}	f_{corr}	f_{red}	Baseline
$Z\ p_T$	$20\,\text{GeV} \le p_T^{ll} \le 3.5$ TeV $12\,\text{GeV} \le m_{ll} \le 150$ GeV $\lvert y_{ll}\rvert \le 2.4$	338	0.5	$(0.4, 1)$	[52] (8 TeV)
high-mass Drell-Yan	$p_T^{l1(2)} \ge 40(30)\,\text{GeV}$ $\lvert\eta^l\rvert \le 2.5,\ m_{ll} \ge 116\,\text{GeV}$	32	0.5	$(0.4, 1)$	[47] (8 TeV)
top quark pair	$m_{t\bar{t}} \simeq 5$ TeV, $\lvert y_t\rvert \le 2.5$	110	0.5	$(0.4, 1)$	[50] (8 TeV)
W+charm (central)	$p_T^{\mu} \ge 26\,\text{GeV},\ p_T^{c} \ge 5\,\text{GeV}$ $\lvert\eta^{\mu}\rvert \le 2.4$	12	0.5	$(0.2, 0.5)$	[24] (13 TeV)
W+charm (forward)	$p_T^{\mu} \ge 20\,\text{GeV},\ p_T^{c} \ge 20\,\text{GeV}$ $p_T^{\mu+c} \ge 20\,\text{GeV}$ $2 \le \eta^{\mu} \le 4.5,\ 2.2 \le \eta^{c} \le 4.2$	10	0.5	$(0.4, 1)$	LHCb projection
Direct photon	$E_T^{\gamma} \lesssim 3$ TeV, $\lvert\eta_{\gamma}\rvert \le 2.5$	118	0.5	$(0.2, 0.5)$	[55] (13 TeV)
Forward W, Z	$p_T^{l} \ge 20\,\text{GeV},\ 2.0 \le \eta^{l} \le 4.5$ $60\,\text{GeV} \le m_{ll} \le 120\,\text{GeV}$	90	0.5	$(0.4, 1)$	[49] (8 TeV)
Inclusive jets	$\lvert y\rvert \le 3,\ R = 0.4$	58	0.5	$(0.2, 0.5)$	[61] (13 TeV)
Total		768			

LHeC: Datasets

Observable	E_p	Kinematics	$N_{\rm dat}$	$\mathcal{L}_{\rm int}$ [ab^{-1}]
$\tilde{\sigma}^{\rm NC}$ (e^-p)	7 TeV	$5 \times 10^{-6} \le x \le 0.8$, $5 \le Q^2 \le 10^6$ GeV^2	150	1.0
$\tilde{\sigma}^{\rm CC}$ (e^-p)	7 TeV	$8.5 \times 10^{-5} \le x \le 0.8$, $10^2 \le Q^2 \le 10^6$ GeV^2	114	1.0
$\tilde{\sigma}^{\rm NC}$ (e^+p)	7 TeV	$5 \times 10^{-6} \le x \le 0.8$, $5 \le Q^2 \le 5 \times 10^5$ GeV^2	148	0.1
$\tilde{\sigma}^{\rm CC}$ (e^+p)	7 TeV	$8.5 \times 10^{-5} \le x \le 0.7$, $10^2 \le Q^2 \le 5 \times 10^5$ GeV^2	109	0.1
$\tilde{\sigma}^{\rm NC}$ (e^-p)	1 TeV	$5 \times 10^{-5} \le x \le 0.8$, $2.2 \le Q^2 \le 10^5$ GeV^2	128	0.1
$\tilde{\sigma}^{\rm CC}$ (e^-p)	1 TeV	$5 \times 10^{-4} \le x \le 0.8$, $10^2 \le Q^2 \le 10^5$ GeV^2	94	0.1
$F_2^{c,{\rm NC}}$ (e^-p)	7 TeV	$7 \times 10^{-6} \le x \le 0.3$, $4 \le Q^2 \le 2 \times 10^5$ GeV^2	111	1.0
$F_2^{b,{\rm NC}}$ (e^-p)	7 TeV	$3 \times 10^{-5} \le x \le 0.3$, $32 \le Q^2 \le 2 \times 10^5$ GeV^2	77	1.0
$F_2^{c,{\rm CC}}$ (e^-p)	7 TeV	$10^{-4} \le x \le 0.25$, $10^2 \le Q^2 \le 10^5$ GeV^2	14	1.0
Total			945