Computing x-dependent PDFs on the lattice

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

This project is supported by the National Science Center of Poland SONATA BIS grant No 2016/22/E/ST2/00013 (2017-2022).

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 642069.

Outline of the talk

1. PDFs on the lattice
2. Quasi-PDFs
3. Other approaches
4. Selected results
5. New directions
6. Conclusions and prospects

Collaborators:

- C. Alexandrou (Cyprus)
- M. Constantinou (Temple)
- L. Del Debbio (Edinburgh)
- T. Giani (Edinburgh)
- K. Hadjiyiannakou (Cyprus)
- K. Jansen (DESY)
- A. Scapellato (Poznań)
- F. Steffens (Bonn)

Based on:

- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. D99 (2019) 114504
- K. Cichy, L. Del Debbio, T. Giani, "Parton distributions from lattice data: the nonsinglet case", JHEP 10 (2019) 137
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, "Quasi-PDFs with twisted mass fermions", arXiv:1910.13229, LATTICE19 proceedings
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Light-Cone Parton Distribution Functions from Lattice QCD", Phys. Rev. Lett. 121 (2018) 112001
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Transversity parton distribution functions from lattice QCD", Phys. Rev. D98 (2018) 091503 (Rapid Communications)
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)

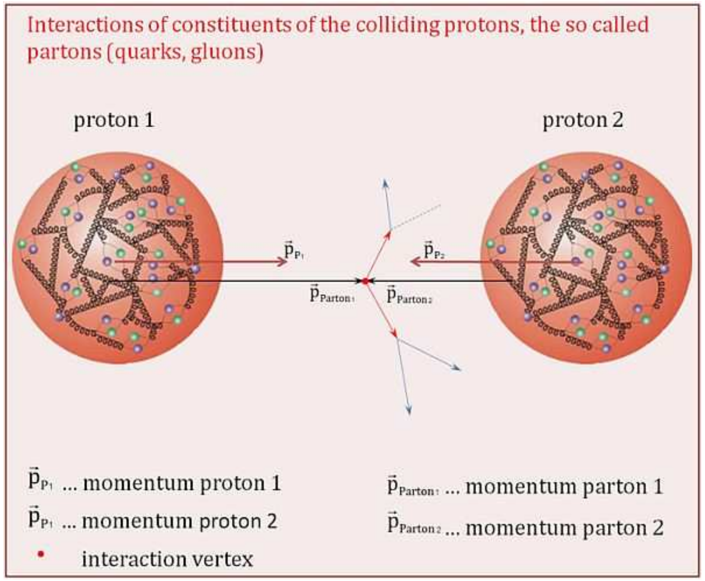
Review of the field:

- K. Cichy, M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results", invited review article for a special issue of Advances in High Energy Physics, Adv. High Energy Phys. 2019 (2019) 3036904, arXiv: 1811.07248 [hep-lat]

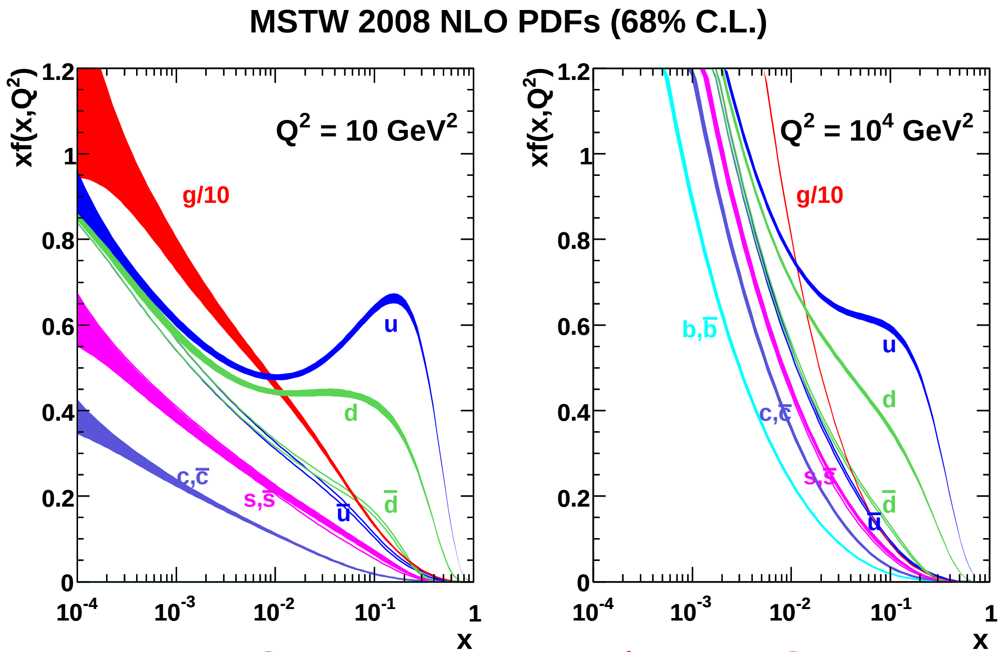
Parton distribution functions

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider experiments.

$$\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$$



Source: LHC, CERN



MSTW2008, Eur. Phys. J. C63, 189

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.
- Hence, desirable to have them also from first principles.

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.
- Hence, desirable to have them also from first principles.
- PDFs have non-perturbative nature $\Rightarrow$ LATTICE?

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.
- Hence, desirable to have them also from first principles.
- PDFs have non-perturbative nature ⇒ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.
- Hence, desirable to have them also from first principles.
- PDFs have non-perturbative nature ⇒ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0 - \xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-, 0)$ is the Wilson line from 0 to ξ^-.

- ⋆ inaccessible on the lattice...

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.
- Hence, desirable to have them also from first principles.
- PDFs have non-perturbative nature ⇒ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

  - ⋆ inaccessible on the lattice...
- On the other hand, PDFs moments given in terms of local matrix elements, but only lowest 2-3 accessible.

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.
- Hence, desirable to have them also from first principles.
- PDFs have non-perturbative nature ⇒ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

  - ⋆ inaccessible on the lattice...
- On the other hand, PDFs moments given in terms of local matrix elements, but only lowest 2-3 accessible. See talks by G. Koutsou, C. Urbach

PDFs and the lattice

- PDFs can be obtained from fits to experimental data:
  - ⋆ good knowledge of unpolarized and helicity PDFs,
  - ⋆ transversity PDFs – not much constrained by experiment,
  - ⋆ some ambiguities for certain kinematical regions.
- Hence, desirable to have them also from first principles.
- PDFs have non-perturbative nature ⇒ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

  - ⋆ inaccessible on the lattice...
- On the other hand, PDFs moments given in terms of local matrix elements, but only lowest 2-3 accessible. See talks by G. Koutsou, C. Urbach
- Recently: new **direct** approaches to get x-dependence.

Approaches to light-cone PDFs

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$Q(x,\mu_R) = \int_{-1}^{1} \frac{dy}{y}\, C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

some lattice observable

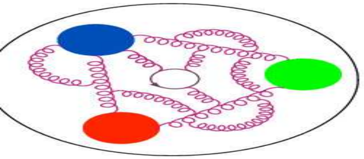
Approaches to light-cone PDFs

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$Q(x,\mu_R) = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

some lattice observable

- Two classes of approaches:
  - ⋆ generalizations of light-cone functions; direct x-dependence,
  - ⋆ hadronic tensor; decomposition into structure functions.



Approaches to light-cone PDFs

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$Q(x,\mu_R) = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

some lattice observable

- Two classes of approaches:
  - ⋆ generalizations of light-cone functions; direct x-dependence,
  - ⋆ hadronic tensor; decomposition into structure functions.

- Matrix elements: $\langle N|\bar\psi(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ,Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Approaches to light-cone PDFs

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

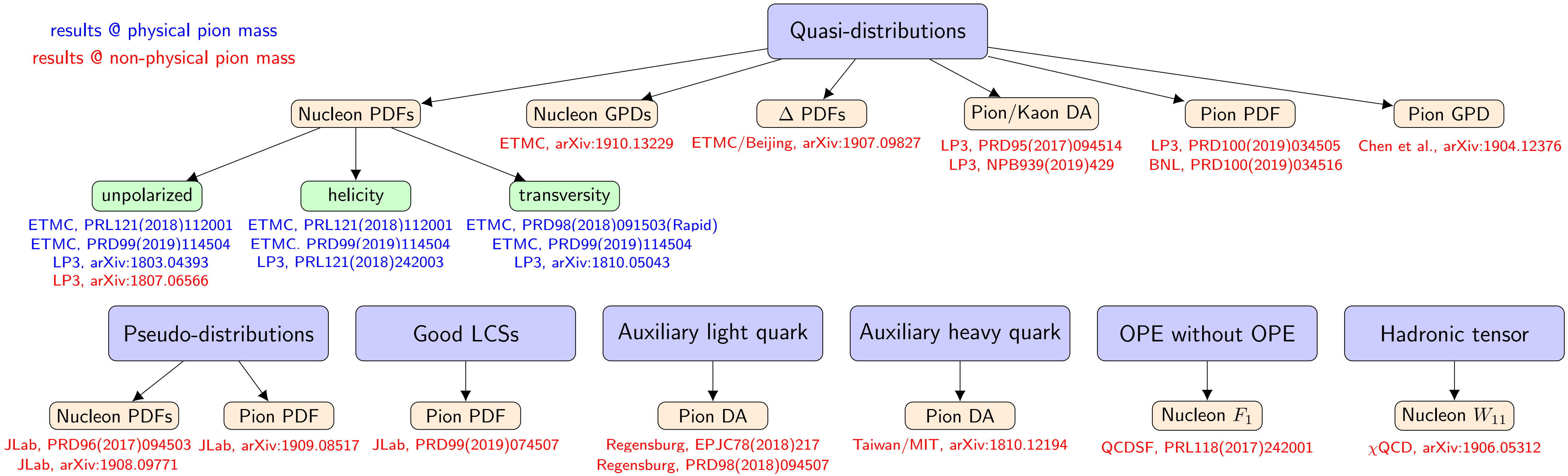
$$\underbrace{Q(x,\mu_R)}_{\text{some lattice observable}} = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

- Two classes of approaches:
  - ⋆ generalizations of light-cone functions; direct x-dependence,
  - ⋆ hadronic tensor; decomposition into structure functions.

- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ, Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

See talks by:
K. Jansen, X. Ji, J. Qiu
D. Richards, A. Scapellato
S. Zafeiropoulos, J. Zhang

Overview of results from different approaches



Review of lattice partonic functions

Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy [1] **and Martha Constantinou** [2]

[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

Adv. High Energy Phys. 2019 (2019) 3036904, arXiv:1811.07248

Special issue *Transverse Momentum Dependent Observables from Low to High Energy: Factorization, Evolution, and Global Analyses*,

- discusses in detail quasi-distributions:
  nucleon: **non-singlet quark qPDFs**, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
  hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, “OPE without OPE”, lattice cross sections

Quasi-PDFs

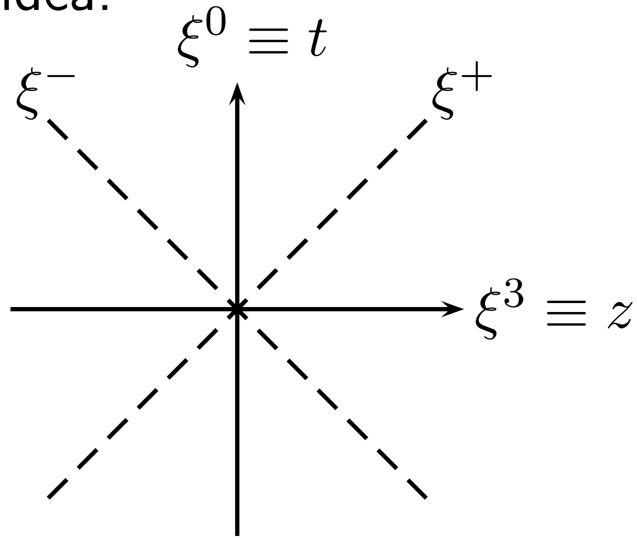
Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

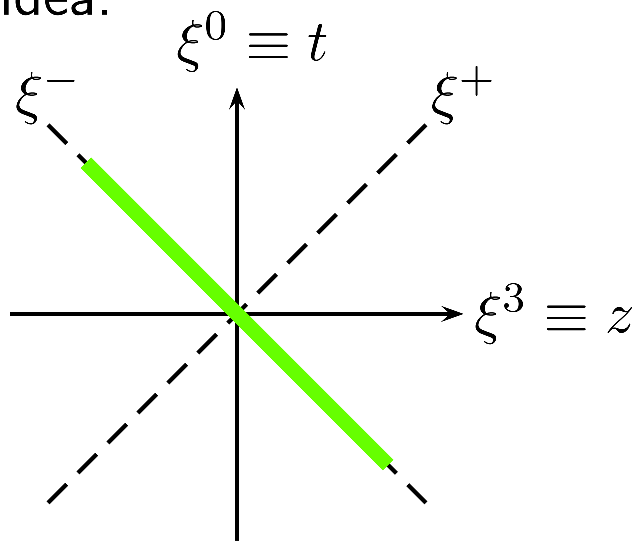
Main idea:



Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

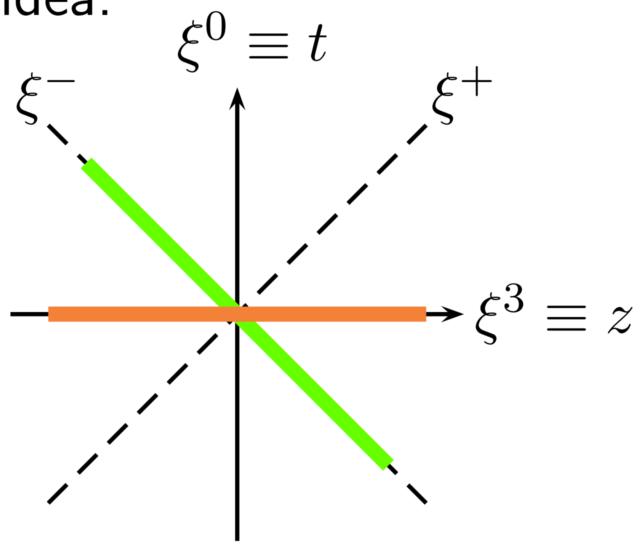
$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma \mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

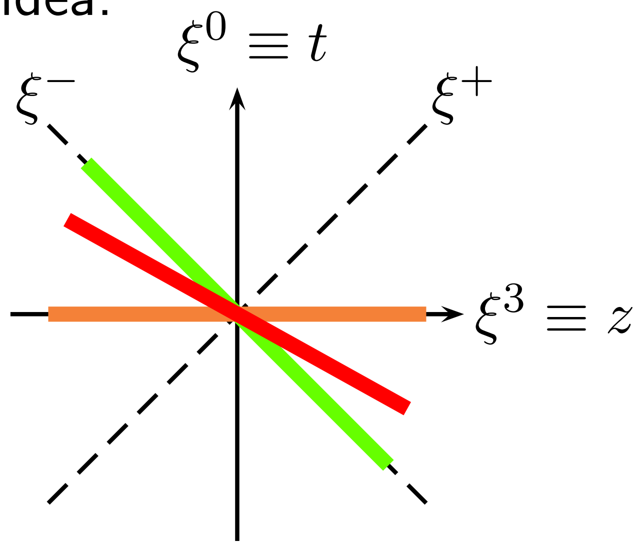
$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

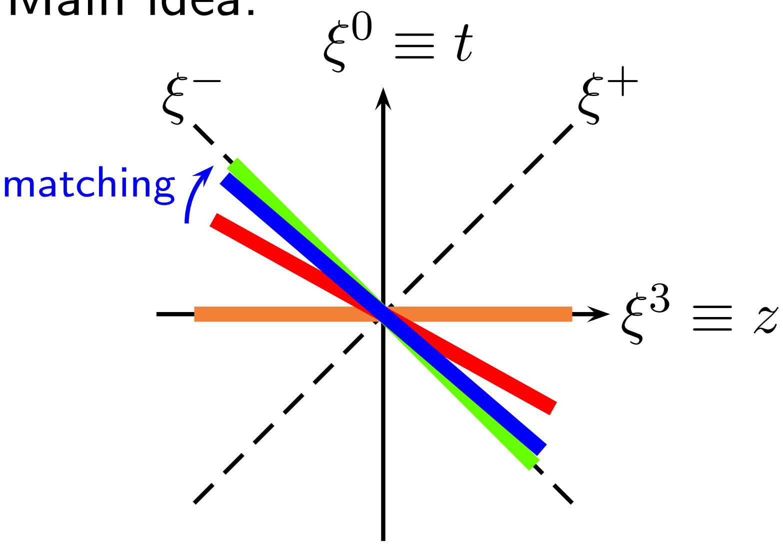
$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

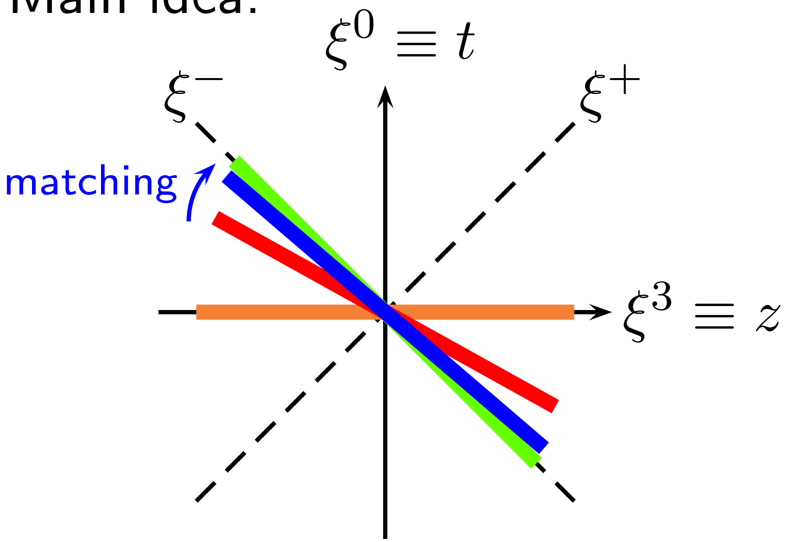
$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1}\frac{dy}{|y|}\,C\left(\frac{x}{y},\frac{\mu}{P_3}\right)q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma \mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma \mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle P|\overline{\psi}(z)\Gamma \mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

→ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

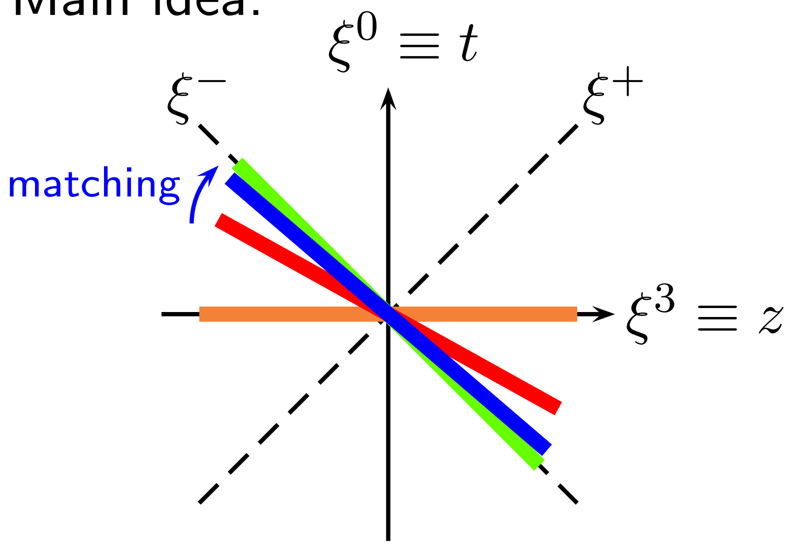
quasi-PDF

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi} \int dz\, e^{ixP_3 z} \langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

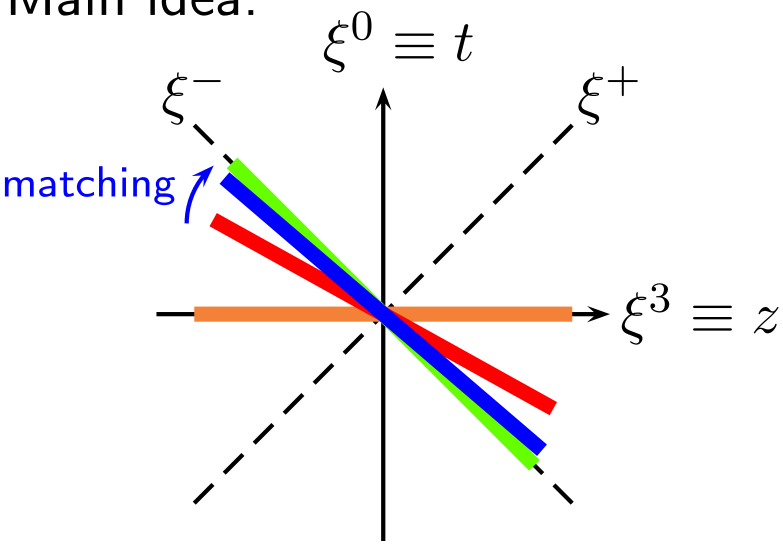
$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF PDF

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3 z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

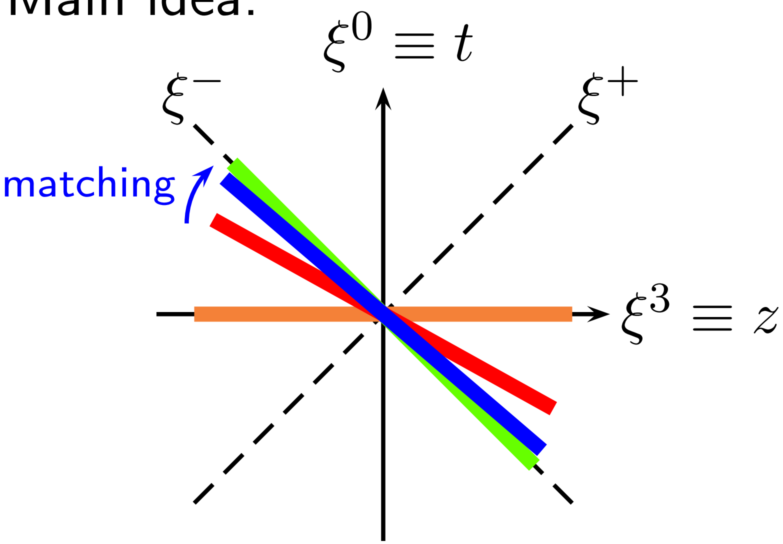
$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1}\frac{dy}{|y|}\,C\left(\frac{x}{y},\frac{\mu}{P_3}\right)q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF　pert.kernel　PDF

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

Main idea:



Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

Correlation along the $\xi^3 \equiv z$-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the standard frame

Correlation along the ξ^3-direction:

$$\tilde{q}(x) = \frac{1}{2\pi}\int dz\, e^{ixP_3z}\langle P|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P\rangle$$

$|P\rangle$ – boosted nucleon

Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1}\frac{dy}{|y|}\underbrace{C\left(\frac{x}{y},\frac{\mu}{P_3}\right)}_{\text{pert.kernel}}\underbrace{q(y,\mu)}_{\text{PDF}} + \underbrace{\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)}_{\text{higher-twist effects}}$$

$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

- **quasi distribution** $\tilde{q}$ – probes purely spatial correlations and uses nucleons with finite momentum.

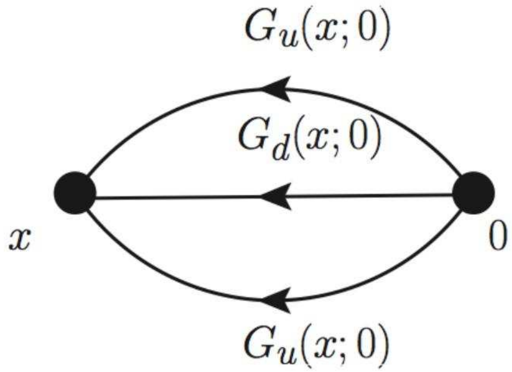
$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

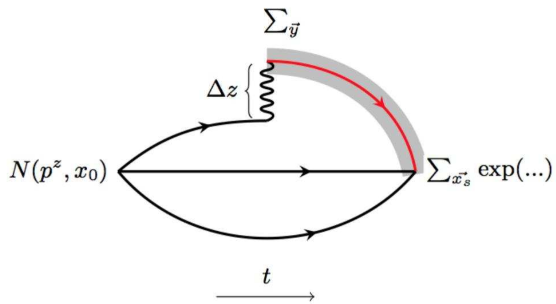
- **quasi distribution** $\tilde{q}$ – probes purely spatial correlations and uses nucleons with finite momentum.
- The Dirac matrix Γ gives access to different kinds of PDFs:
  - ⋆ $\Gamma = \gamma_0,\ \gamma_3$ – unpolarized,
  - ⋆ $\Gamma = \gamma_5\gamma_3$ – helicity,
  - ⋆ $\Gamma = \sigma_{31},\ \sigma_{32}$ – transversity

Quasi-PDFs

$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

- **quasi distribution** $\tilde{q}$ – probes purely spatial correlations and uses nucleons with finite momentum.
- The Dirac matrix Γ gives access to different kinds of PDFs:
  - ⋆ $\Gamma=\gamma_0,\ \gamma_3$ – unpolarized,
  - ⋆ $\Gamma=\gamma_5\gamma_3$ – helicity,
  - ⋆ $\Gamma=\sigma_{31},\ \sigma_{32}$ – transversity
- On the lattice, one needs to compute 2-pt and 3-pt functions:





Pseudo-PDFs

The same matrix elements that are the basis for the quasi-distribution approach can also be used to define pseudo-distributions. [A. Radyushkin, Phys. Rev. D96 (2017) 034025]

The same matrix elements that are the basis for the quasi-distribution approach can also be used to define pseudo-distributions. [A. Radyushkin, Phys. Rev. D96 (2017) 034025]

- Difference w.r.t. quasi: Fourier transform taken in $\nu \equiv P_3 z$ ("Ioffe time"), instead of in z.

Pseudo-PDFs

The same matrix elements that are the basis for the quasi-distribution approach can also be used to define pseudo-distributions. [A. Radyushkin, Phys. Rev. D96 (2017) 034025]

- Difference w.r.t. quasi: Fourier transform taken in $\nu \equiv P_3 z$ ("Ioffe time "), instead of in z.
- This has far-reaching consequences and makes this approach inequivalent to quasi.

Pseudo-PDFs

The same matrix elements that are the basis for the quasi-distribution approach can also be used to define pseudo-distributions. [A. Radyushkin, Phys. Rev. D96 (2017) 034025]

- Difference w.r.t. quasi: Fourier transform taken in $\nu \equiv P_3 z$ ("Ioffe time "), instead of in z.
- This has far-reaching consequences and makes this approach inequivalent to quasi.
- Renormalization of divergences by taking suitable ratios at $P_3 = 0$ and $z = 0$.

Pseudo-PDFs

The same matrix elements that are the basis for the quasi-distribution approach can also be used to define pseudo-distributions. [A. Radyushkin, Phys. Rev. D96 (2017) 034025]

- Difference w.r.t. quasi: Fourier transform taken in $\nu \equiv P_3 z$ ("Ioffe time"), instead of in z.
- This has far-reaching consequences and makes this approach inequivalent to quasi.
- Renormalization of divergences by taking suitable ratios at $P_3 = 0$ and $z = 0$.
- Factorization does **not** require large momentum (hence, **not** LaMET).

Pseudo-PDFs

The same matrix elements that are the basis for the quasi-distribution approach can also be used to define pseudo-distributions. [A. Radyushkin, Phys. Rev. D96 (2017) 034025]

- Difference w.r.t. quasi: Fourier transform taken in $\nu \equiv P_3 z$ ("Ioffe time"), instead of in z.
- This has far-reaching consequences and makes this approach inequivalent to quasi.
- Renormalization of divergences by taking suitable ratios at $P_3 = 0$ and $z = 0$.
- Factorization does **not** require large momentum (hence, **not** LaMET).
- Pseudo-ITD can be matched to light-cone $\overline{\mathrm{MS}}$ ITD – the latter is a **physical object**.
  [A. Radyushkin, Phys. Rev. D98 (2018) 014019]
  [J.-H. Zhang, J.-W. Chen, C. Monahan, Phys. Rev. D97 (2018) 074508]
  [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

Pseudo-PDFs

The same matrix elements that are the basis for the quasi-distribution approach can also be used to define pseudo-distributions. [A. Radyushkin, Phys. Rev. D96 (2017) 034025]

- Difference w.r.t. quasi: Fourier transform taken in $\nu \equiv P_3 z$ ("Ioffe time"), instead of in z.
- This has far-reaching consequences and makes this approach inequivalent to quasi.
- Renormalization of divergences by taking suitable ratios at $P_3 = 0$ and $z = 0$.
- Factorization does **not** require large momentum (hence, **not** LaMET).
- Pseudo-ITD can be matched to light-cone $\overline{\mathrm{MS}}$ ITD – the latter is a **physical object**.
  [A. Radyushkin, Phys. Rev. D98 (2018) 014019]
  [J.-H. Zhang, J.-W. Chen, C. Monahan, Phys. Rev. D97 (2018) 074508]
  [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]
- $\overline{\mathrm{MS}}$ ITD can be Fourier-transformed to obtain $\overline{\mathrm{MS}}$ PDF (here one needs large Ioffe times and hence, in practice, large momentum).

See talks by D. Richards, S. Zafeiropoulos

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).
- The LCSs are good if they are:

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).
- The LCSs are good if they are:
  - ⋆ computable on the lattice,

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).
- The LCSs are good if they are:
  - ⋆ computable on the lattice,
  - ⋆ have a well-defined continuum limit (renormalizable),

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).
- The LCSs are good if they are:
  - ⋆ computable on the lattice,
  - ⋆ have a well-defined continuum limit (renormalizable),
  - ⋆ are perturbatively factorizable into PDFs.

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).
- The LCSs are good if they are:
  - ⋆ computable on the lattice,
  - ⋆ have a well-defined continuum limit (renormalizable),
  - ⋆ are perturbatively factorizable into PDFs.
- Examples of good LCSs:

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).
- The LCSs are good if they are:
  - ⋆ computable on the lattice,
  - ⋆ have a well-defined continuum limit (renormalizable),
  - ⋆ are perturbatively factorizable into PDFs.
- Examples of good LCSs:
  quasi-PDFs, pseudo-PDFs, "OPE without OPE"

Good lattice cross-sections

- Y.-Q. Ma and J.-W. Qiu proposed to extract PDFs from a global fit of lattice data, in full analogy to phenomenological fits of experimental data.
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]
  [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. D98 (2018) 074021]
- The analogy makes it natural to call lattice data **lattice cross sections** (LCSs).
- The LCSs are good if they are:
  - ⋆ computable on the lattice,
  - ⋆ have a well-defined continuum limit (renormalizable),
  - ⋆ are perturbatively factorizable into PDFs.
- Examples of good LCSs:
  quasi-PDFs, pseudo-PDFs, "OPE without OPE"
- Another class: current-current correlators
  related idea in [V. Braun and D. Müller, EPJC 55 (2008) 349]

$$\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle; \qquad \omega = P\cdot\xi$$

with (as one possibility):

$$\mathcal{O}_{j_1 j_2}(\xi) = \xi^{d_{j_1}+d_{j_2}-2} Z_{j_1} Z_{j_2} j_1(\xi) j_2(0).$$

See talks by J. Qiu, D. Richards

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M}\overline{\mathrm{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \rightarrow Z^{\mathrm{M}\overline{\mathrm{MS}}}(z,\bar{\mu})$.

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M}\overline{\mathrm{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M}\overline{\mathrm{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M}\overline{\mathrm{MS}}$ scheme.

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:

$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M}\overline{\mathrm{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M}\overline{\mathrm{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M}\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M}\overline{\mathrm{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M}\overline{\mathrm{MS}}}.$$
6. Relate $\mathrm{M}\overline{\mathrm{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M}\overline{\mathrm{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.

Quasi-PDFs procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

Lattice setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$
- gauge field configurations generated by ETMC

β=2.10, $c_{\rm SW}$=1.57751, a=0.0938(3)(2) fm	
$48^3 \times 96$	$a\mu = 0.0009$ $\quad m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $\quad m_\pi L = 2.98(1)$

	$P_3 = \frac{6\pi}{L}$		$P_3 = \frac{8\pi}{L}$		$P_3 = \frac{10\pi}{L}$	
Insertion	$N_{\rm conf}$	$N_{\rm meas}$	$N_{\rm conf}$	$N_{\rm meas}$	$N_{\rm conf}$	$N_{\rm meas}$
γ^0	50	4800	425	38250	811	72990
$\gamma^5\gamma^3$	65	6240	425	38250	811	72990
σ^{3j}	50	9600	425	38250	811	72990

Step 1

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3 z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

What momentum should be used to obtain reliable light-cone PDFs?

What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

- we have finite lattice spacing $\rightarrow$ UV cut-off of ≈ 2 GeV.

What momentum should be used to obtain reliable light-cone PDFs?

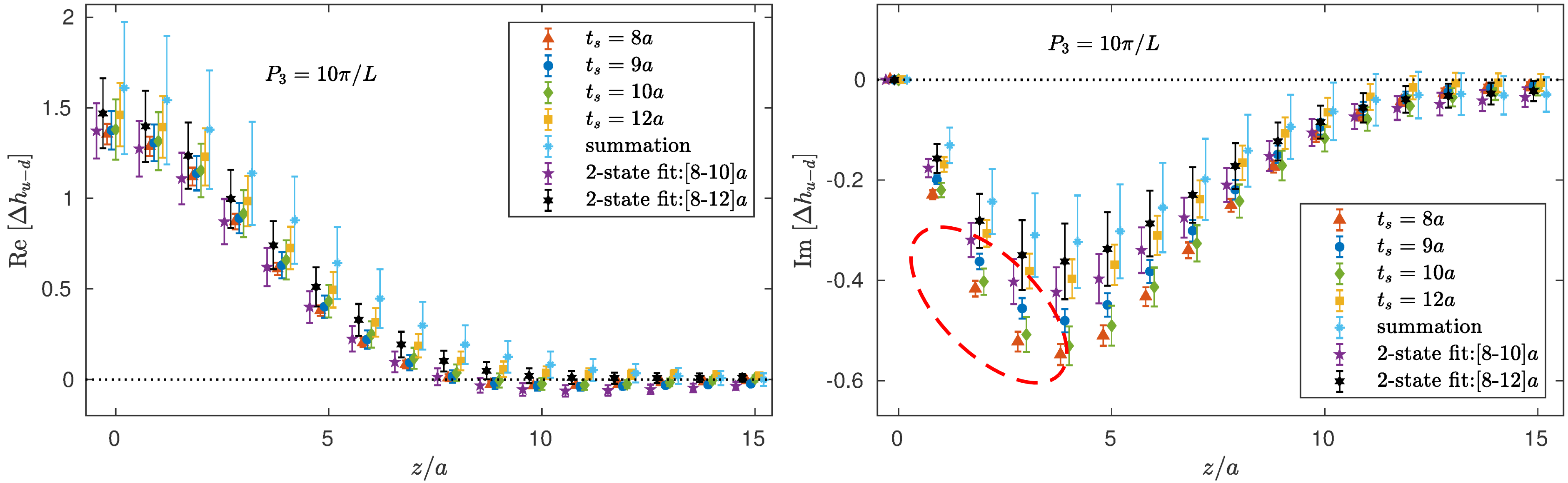
The answer is seemingly simple – **large** momentum, but:

- we have finite lattice spacing $\rightarrow$ UV cut-off of ≈ 2 GeV.
- large momentum means it is very difficult to isolate the ground state $\rightarrow$ excessive excited states contamination $\rightarrow$ one needs to go to large enough source-sink separation $t_s \Rightarrow$ COSTLY!

What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

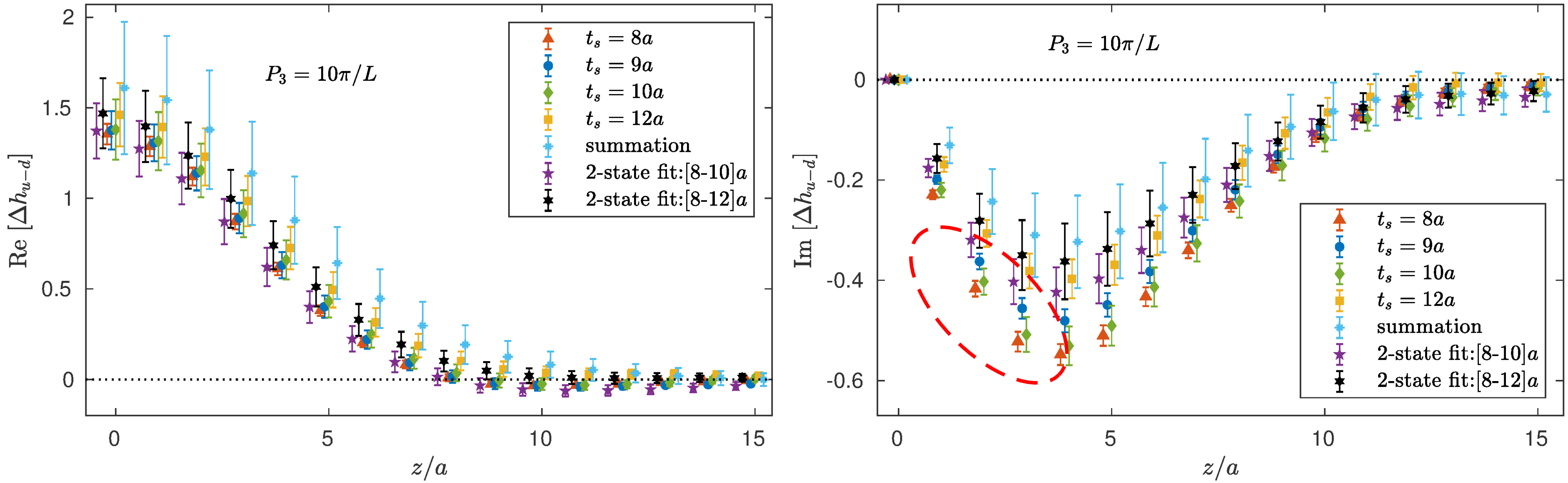
- we have finite lattice spacing $\rightarrow$ UV cut-off of ≈ 2 GeV.
- large momentum means it is very difficult to isolate the ground state $\rightarrow$ excessive excited states contamination $\rightarrow$ one needs to go to large enough source-sink separation t_s $\Rightarrow$ COSTLY!



What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

- we have finite lattice spacing → UV cut-off of ≈ 2 GeV.
- large momentum means it is very difficult to isolate the ground state → excessive excited states contamination → one needs to go to large enough source-sink separation t_s ⇒ COSTLY!



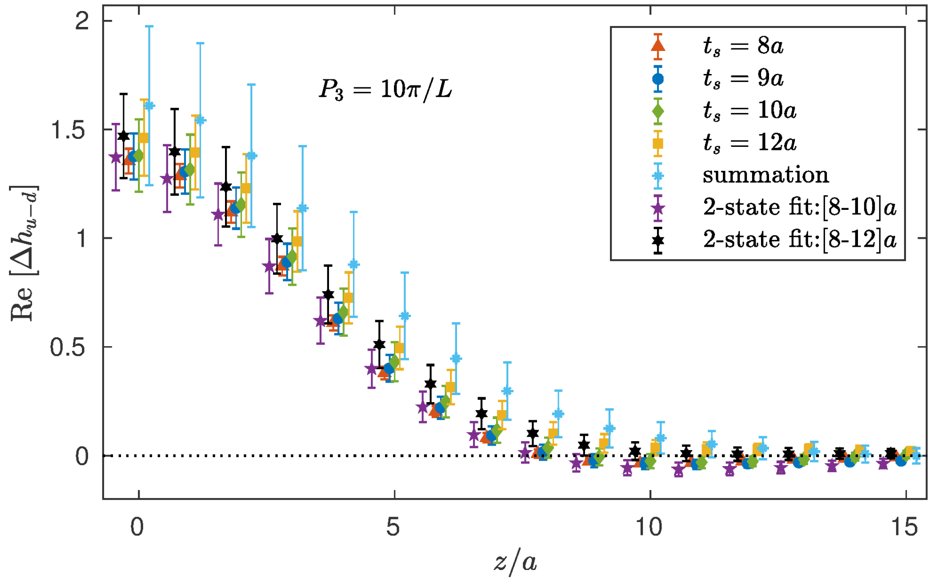
- **Robust statements about excited states only when checking a few analysis methods.**

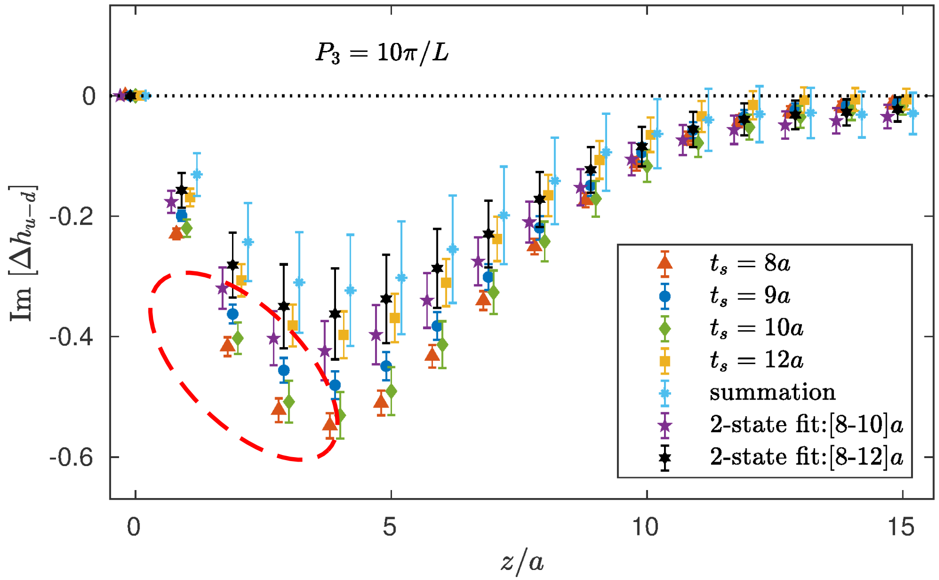
Choice of nucleon momentum

What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

- we have finite lattice spacing → UV cut-off of ≈ 2 GeV.
- large momentum means it is very difficult to isolate the ground state → excessive excited states contamination → one needs to go to large enough source-sink separation t_s ⇒ COSTLY!





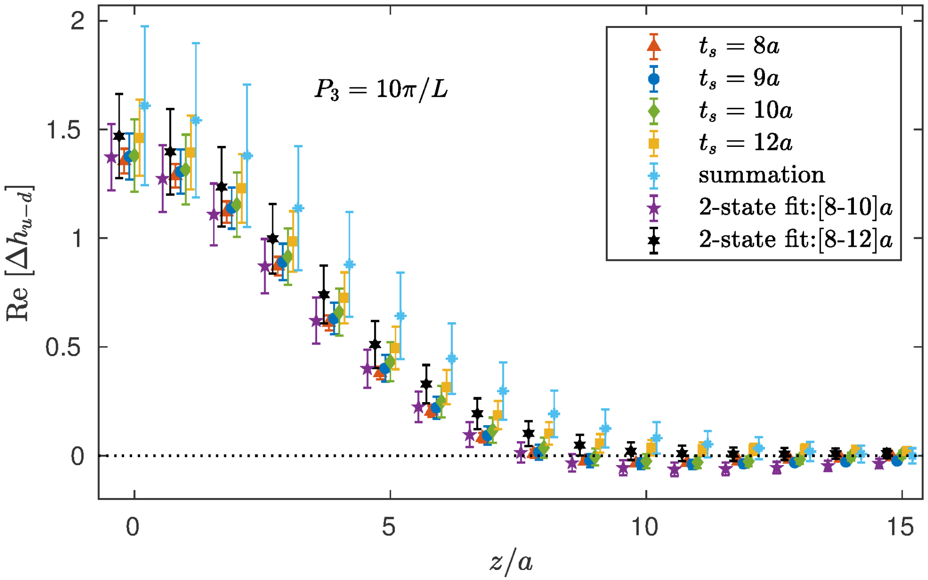
- **Robust statements about excited states only when checking a few analysis methods.** here: 2-state fit with $t_s/a = 8, 9, 10, 12$ shows full consistency with the 1-state fit at $t_s = 12a$.

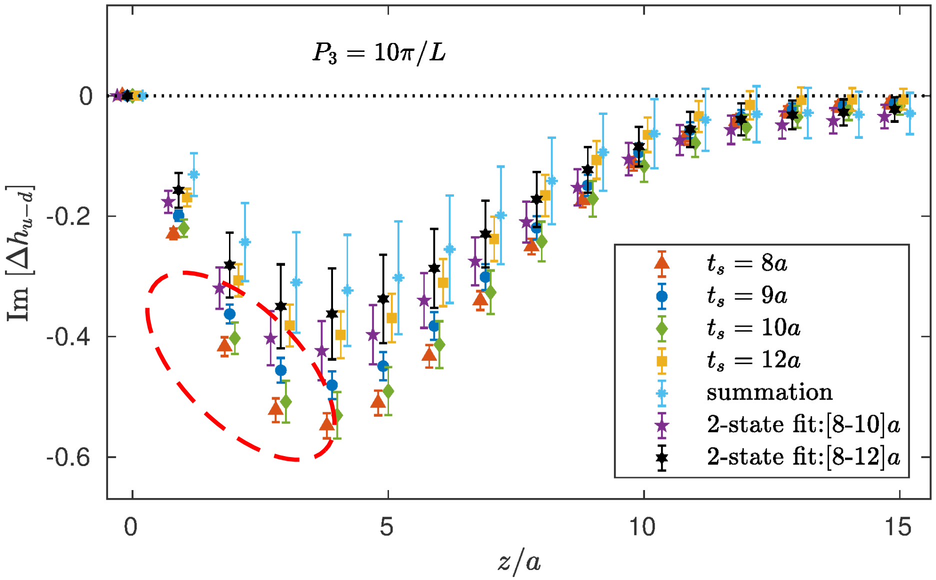
Choice of nucleon momentum

What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

- we have finite lattice spacing → UV cut-off of ≈ 2 GeV.
- large momentum means it is very difficult to isolate the ground state → excessive excited states contamination → one needs to go to large enough source-sink separation t_s ⇒ COSTLY!





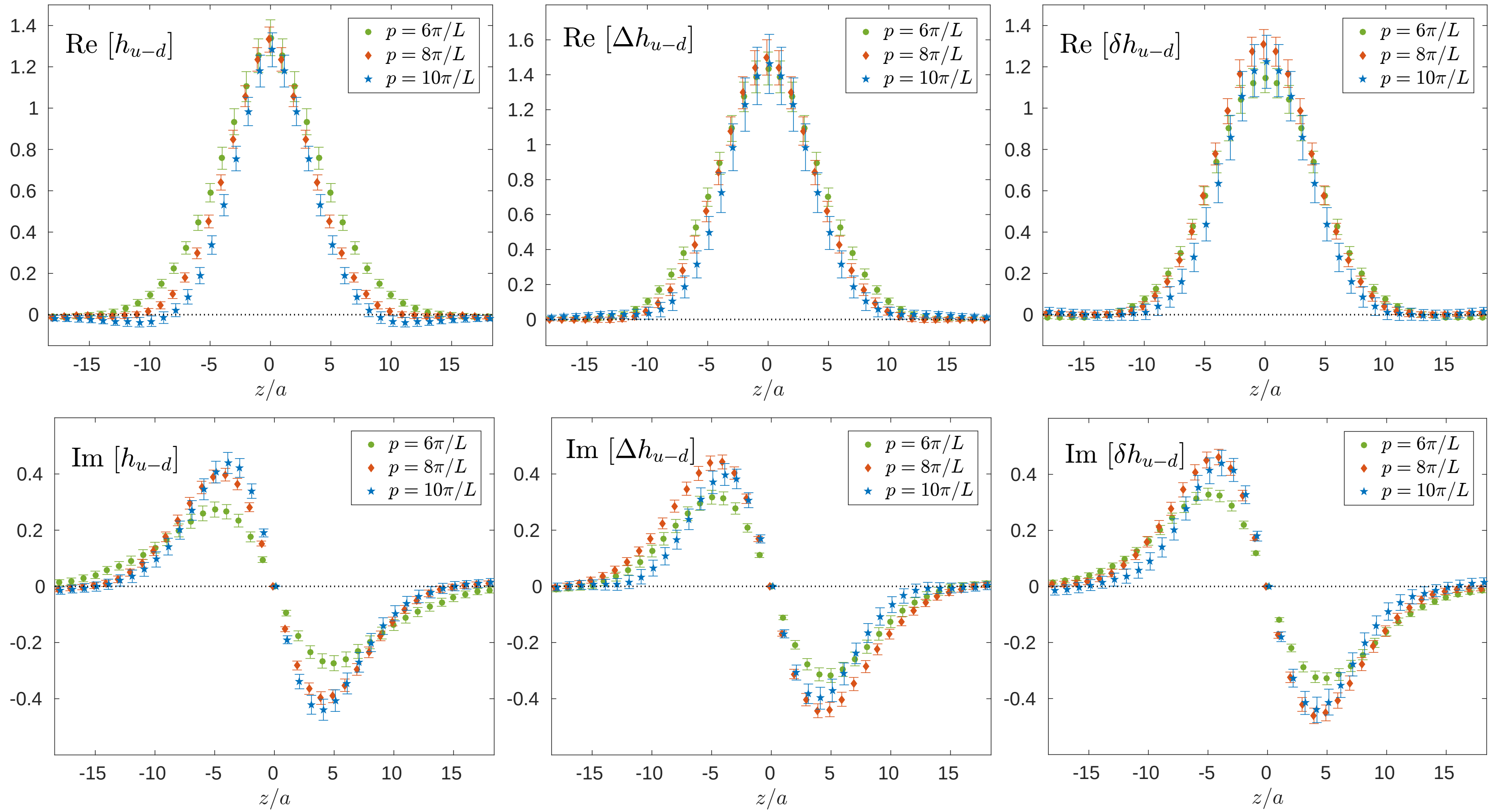
- **Robust statements about excited states only when checking a few analysis methods.** here: 2-state fit with $t_s/a = 8, 9, 10, 12$ shows full consistency with the 1-state fit at $t_s = 12a$.

Our largest momentum: ≈ 1.4 GeV

- safely below UV cut-off,
- excited states contamination shown to be smaller than statistical errors.

Bare matrix elements at $t_s = 12a$

unpolarized (γ_0) helicity $(\gamma_5\gamma_3)$ transversity (σ_{3i})



STATISTICS:

unpolarized	helicity	transversity
$P_3 = \frac{6\pi}{L}$ – 4800 meas.	$P_3 = \frac{6\pi}{L}$ – 6240 meas.	$P_3 = \frac{6\pi}{L}$ – 9600 meas.
$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.
$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.

Steps 2-4

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

Renormalization

Historical remarks:

Historical remarks:

1. it was very important to clarify the issue of renormalizability of the quasi-PDFs:

   T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, Phys. Rev. D96 (2017) 094019

   X. Ji, J.-H. Zhang, Y. Zhao, Phys. Rev. Lett. 120 (2018) 112001

Renormalization

Historical remarks:

1. it was very important to clarify the issue of renormalizability of the quasi-PDFs:

   T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, Phys. Rev. D96 (2017) 094019

   X. Ji, J.-H. Zhang, Y. Zhao, Phys. Rev. Lett. 120 (2018) 112001

2. and also to find practical renormalization prescription:
   RI-MOM-type scheme C. Alexandrou et al., Nucl. Phys. B923 (2017) 394 (Frontiers Article)
   auxiliary field scheme J. Green, K. Jansen, F. Steffens, Phys. Rev. Lett. 121 (2018) 022004

Renormalization

Historical remarks:

1. it was very important to clarify the issue of renormalizability of the quasi-PDFs:

   T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, Phys. Rev. D96 (2017) 094019

   X. Ji, J.-H. Zhang, Y. Zhao, Phys. Rev. Lett. 120 (2018) 112001

2. and also to find practical renormalization prescription:
   RI-MOM-type scheme C. Alexandrou et al., Nucl. Phys. B923 (2017) 394 (Frontiers Article)
   auxiliary field scheme J. Green, K. Jansen, F. Steffens, Phys. Rev. Lett. 121 (2018) 022004

3. important insights also from lattice perturbation theory: mixing for certain Dirac structures $\rightarrow$ led to abandoning $\Gamma = \gamma_3$ for unpolarized PDFs, in favor of $\Gamma = \gamma_0$ M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506

Renormalization

Historical remarks:

1. it was very important to clarify the issue of renormalizability of the quasi-PDFs:

   T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, Phys. Rev. D96 (2017) 094019

   X. Ji, J.-H. Zhang, Y. Zhao, Phys. Rev. Lett. 120 (2018) 112001

2. and also to find practical renormalization prescription:
   RI-MOM-type scheme C. Alexandrou et al., Nucl. Phys. B923 (2017) 394 (Frontiers Article)
   auxiliary field scheme J. Green, K. Jansen, F. Steffens, Phys. Rev. Lett. 121 (2018) 022004

3. important insights also from lattice perturbation theory: mixing for certain Dirac structures $\rightarrow$ led to abandoning $\Gamma = \gamma_3$ for unpolarized PDFs, in favor of $\Gamma = \gamma_0$ M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506

Two types of divergences that need to be removed:

Renormalization

Historical remarks:

1. it was very important to clarify the issue of renormalizability of the quasi-PDFs:

   T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, Phys. Rev. D96 (2017) 094019

   X. Ji, J.-H. Zhang, Y. Zhao, Phys. Rev. Lett. 120 (2018) 112001

2. and also to find practical renormalization prescription:
   RI-MOM-type scheme C. Alexandrou et al., Nucl. Phys. B923 (2017) 394 (Frontiers Article)
   auxiliary field scheme J. Green, K. Jansen, F. Steffens, Phys. Rev. Lett. 121 (2018) 022004

3. important insights also from lattice perturbation theory: mixing for certain Dirac structures → led to abandoning $\Gamma = \gamma_3$ for unpolarized PDFs, in favor of $\Gamma = \gamma_0$ M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506

Two types of divergences that need to be removed:

- standard logarithmic divergence w.r.t. the regulator, $\log(a\mu)$,

Renormalization

Historical remarks:

1. it was very important to clarify the issue of renormalizability of the quasi-PDFs:

   T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, Phys. Rev. D96 (2017) 094019

   X. Ji, J.-H. Zhang, Y. Zhao, Phys. Rev. Lett. 120 (2018) 112001

2. and also to find practical renormalization prescription:
   RI-MOM-type scheme C. Alexandrou et al., Nucl. Phys. B923 (2017) 394 (Frontiers Article)
   auxiliary field scheme J. Green, K. Jansen, F. Steffens, Phys. Rev. Lett. 121 (2018) 022004

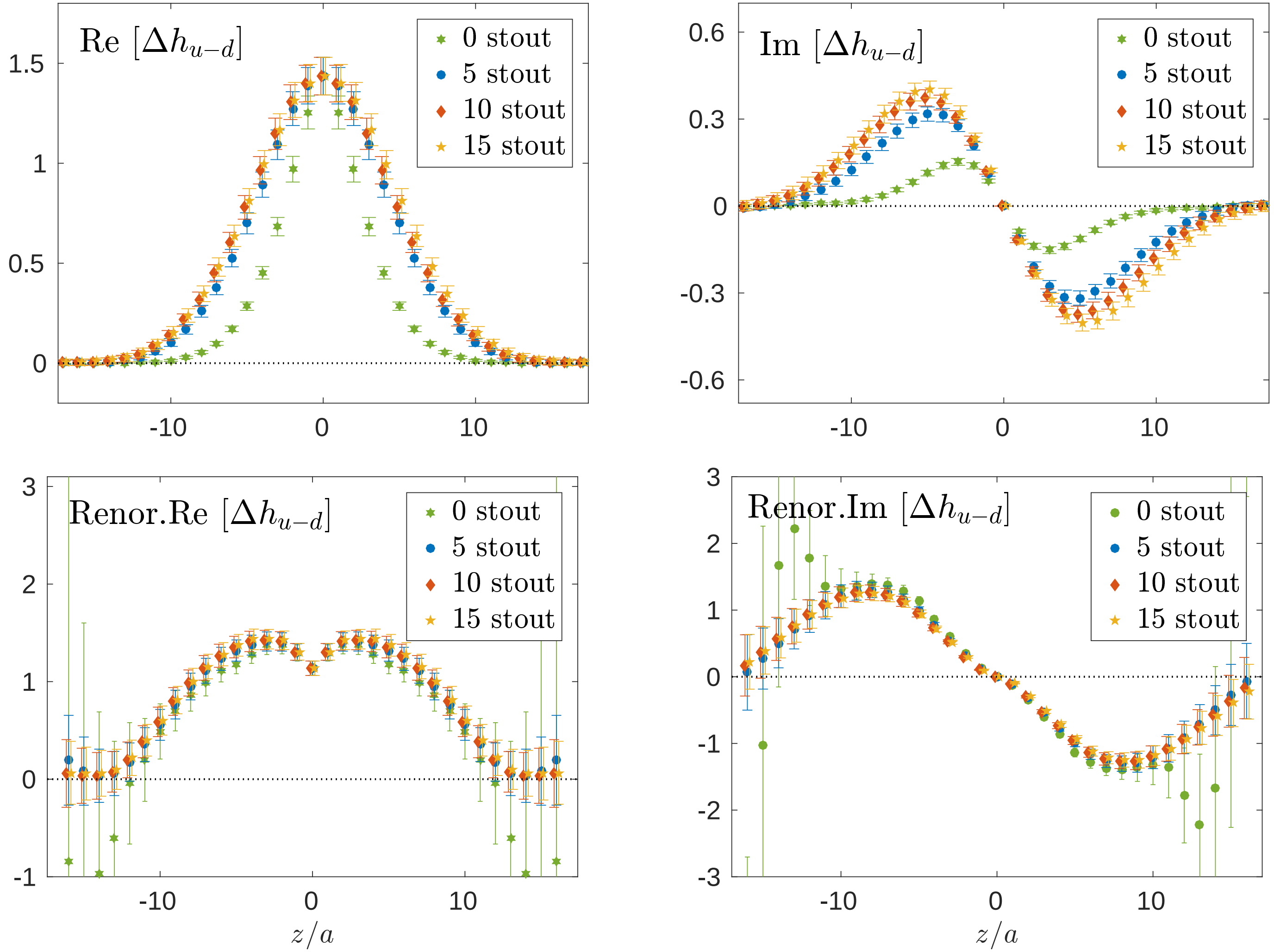
3. important insights also from lattice perturbation theory: mixing for certain Dirac structures $\rightarrow$ led to abandoning $\Gamma = \gamma_3$ for unpolarized PDFs, in favor of $\Gamma = \gamma_0$ M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506

Two types of divergences that need to be removed:

- standard logarithmic divergence w.r.t. the regulator, $\log(a\mu)$,
- power divergence related to the Wilson line; resums into a multiplicative exponential factor, $\exp\left(-\delta m|z|/a + c|z|\right)$
  δm – strength of the divergence, operator independent,
  c – arbitrary scale (fixed by the renormalization prescription).

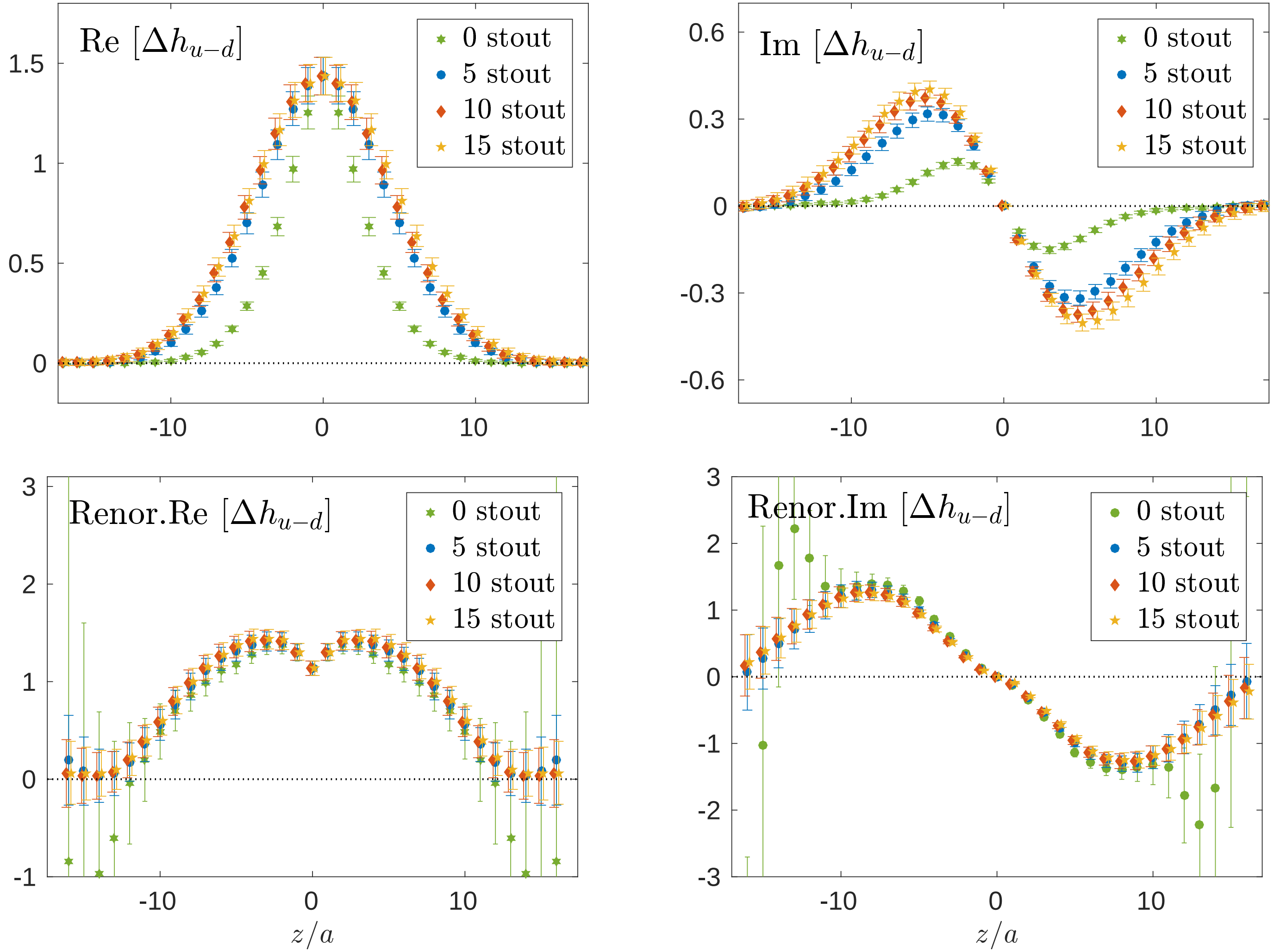
Renormalized matrix elements for helicity PDFs

Nucleon momentum $\frac{6\pi}{48}$



Renormalized matrix elements for helicity PDFs

Nucleon momentum $\frac{6\pi}{48}$



Important self-consistency check for the renormalization procedure!

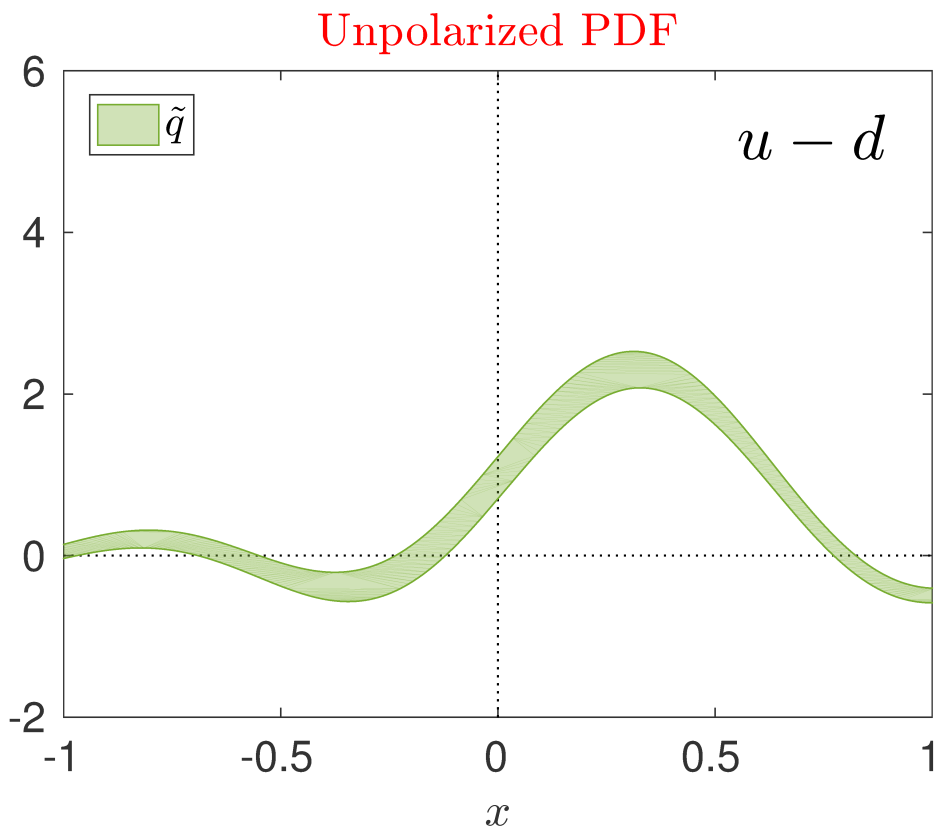
Step 5

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

Fourier transform

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2

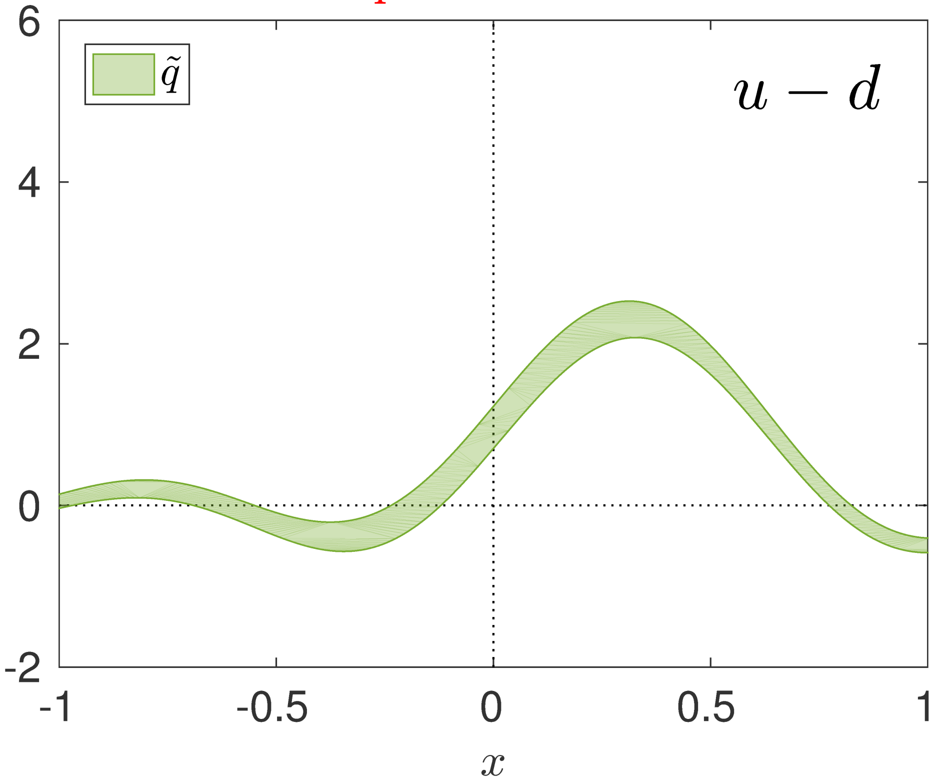


C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

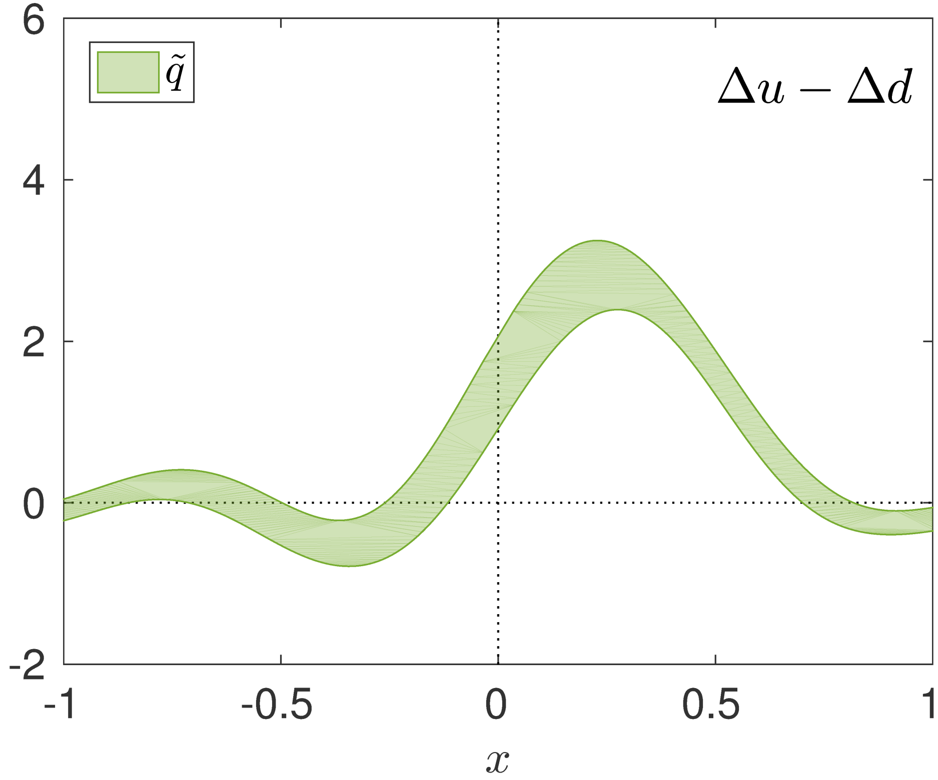
Fourier transform

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2

Unpolarized PDF

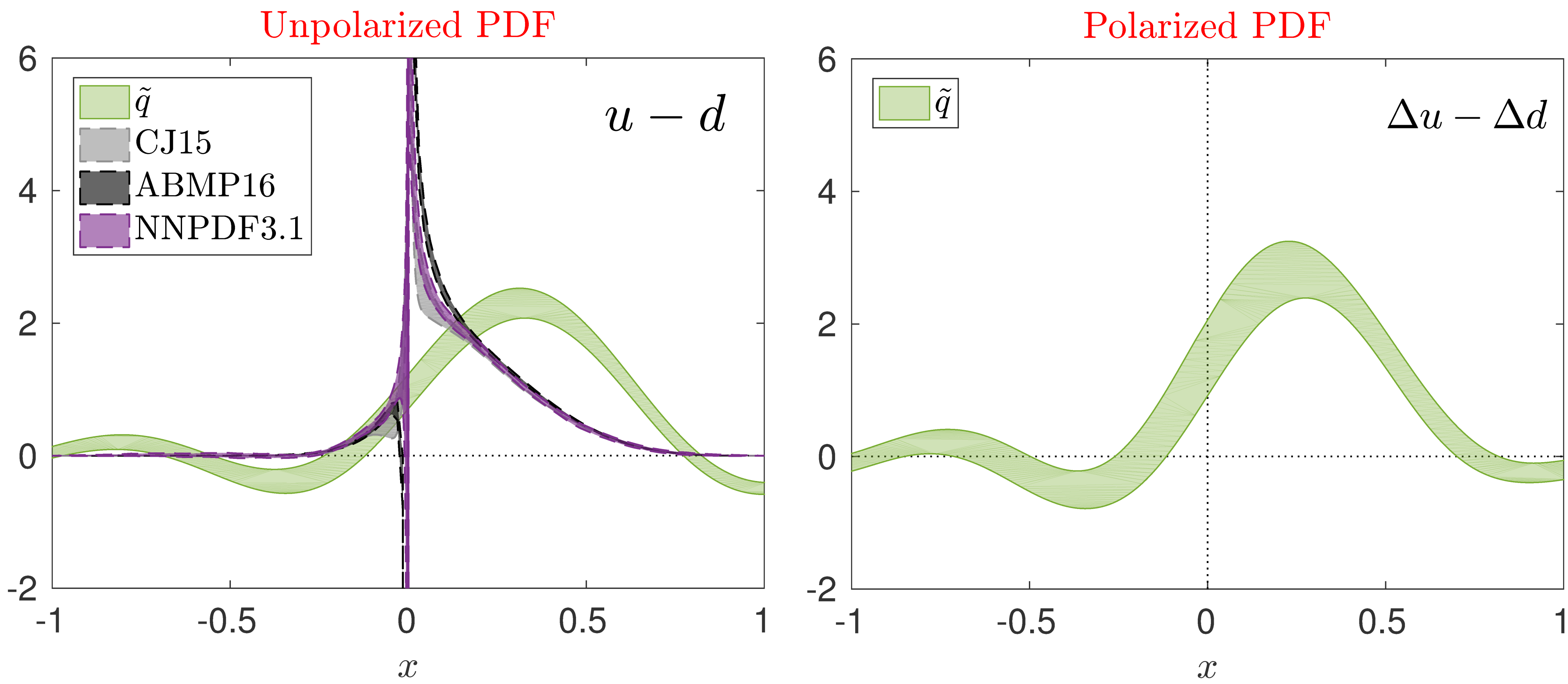


Polarized PDF



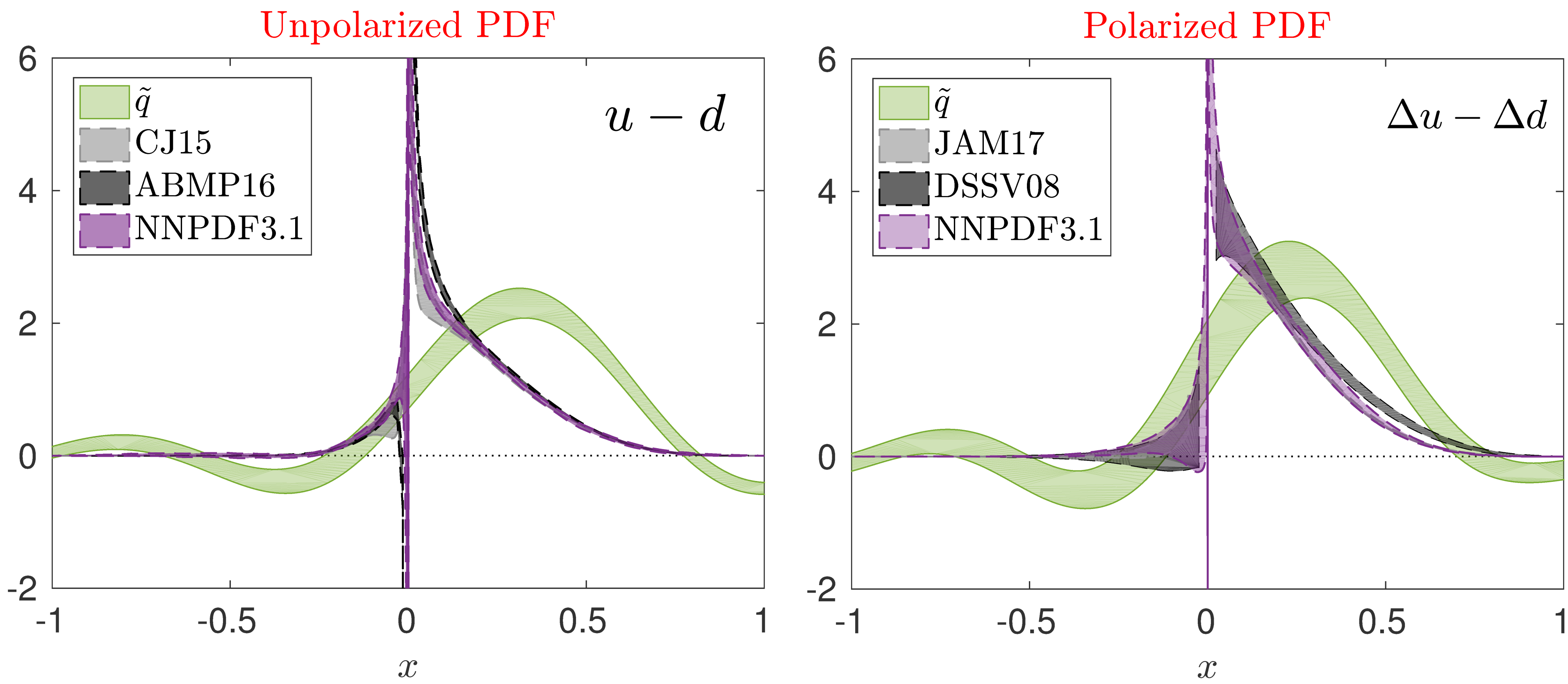
C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Step 6

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

Strategies for matching

Matching is the essence of LaMET and was subject to important developments over the years:

- transverse momentum cut-off scheme PDFs [X. Xiong et al., Phys. Rev. D90 (2014) 014051]
- same for unpolarized and helicity GPDs [X. Ji et al., Phys. Rev. D92 (2015) 014039]
- same for transversity GPDs [X. Xiong, J. Zhang, Phys. Rev. D92 (2015) 054037]
- $\overline{\rm MS} \to \overline{\rm MS}$, non-singlet and singlet PDFs [W. Wang, S. Zhao, R. Zhu, EPJC 78 (2018) 147]
- RI $\to \overline{\rm MS}$, unpolarized PDFs (γ_3) [I.W. Stewart, Y. Zhao, Phys. Rev. D97 (2018) 054512]
- $\overline{\rm MS} \to \overline{\rm MS}$, treatment of UV log divergence in wave function corrections (but: violates vector current conservation) [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]
- $\rm M\overline{MS} \to \overline{\rm MS}$, treatment of UV log divergence in wave function corrections (preserves vector current conservation) [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]
- ratio $\to \overline{\rm MS}$, treatment of UV log divergence in wave function corrections (preserves vector current conservation) [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]
- RI $\to \overline{\rm MS}$, unpolarized PDFs (γ_0) [Y.-S. Liu et al., arXiv:1807.06566]
- RI $\to \overline{\rm MS}$, transversity PDFs [Y.-S. Liu et al., arXiv:1810.05043]
- RI $\to \overline{\rm MS}$, non-singlet GPDs [Y.-S. Liu et al., Phys. Rev. D100 (2019) 034006]
- RI $\to \overline{\rm MS}$, non-singlet and singlet PDFs [W. Wang et al., arXiv:1904.00978]

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2P_3^2}{\xi^2\mu^2}\left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\mathrm{M}\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2P_3^2}{\xi^2\mu^2}\left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

- Additional subtractions with respect to $\overline{\mathrm{MS}}$ – made outside the physical region of the unintegrated vertex corrections.

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\mathrm{M}\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln \frac{x^2 P_3^2}{\xi^2\mu^2} (4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

ι=0 for γ_0 and ι=1 for $\gamma_3/\gamma_5\gamma_3$.

- Additional subtractions with respect to $\overline{\mathrm{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF $\to$ $\mathrm{M}\overline{\mathrm{MS}}$ scheme.

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\mathrm{M}\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

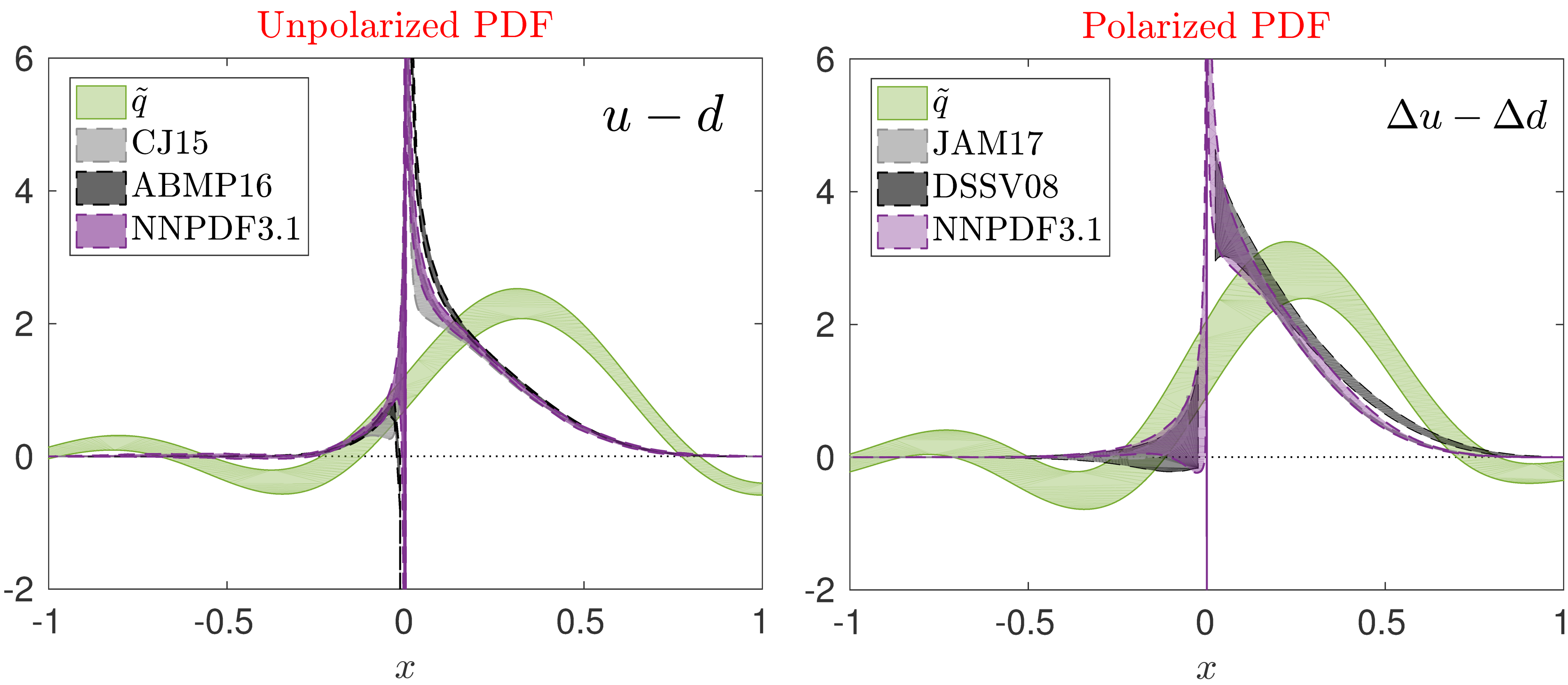
$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{x^2P_3^2}{\xi^2\mu^2}\left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

- Additional subtractions with respect to $\overline{\mathrm{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF $\to$ $\mathrm{M}\overline{\mathrm{MS}}$ scheme.
- In this procedure, vector current is **conserved**.

Matched PDFs

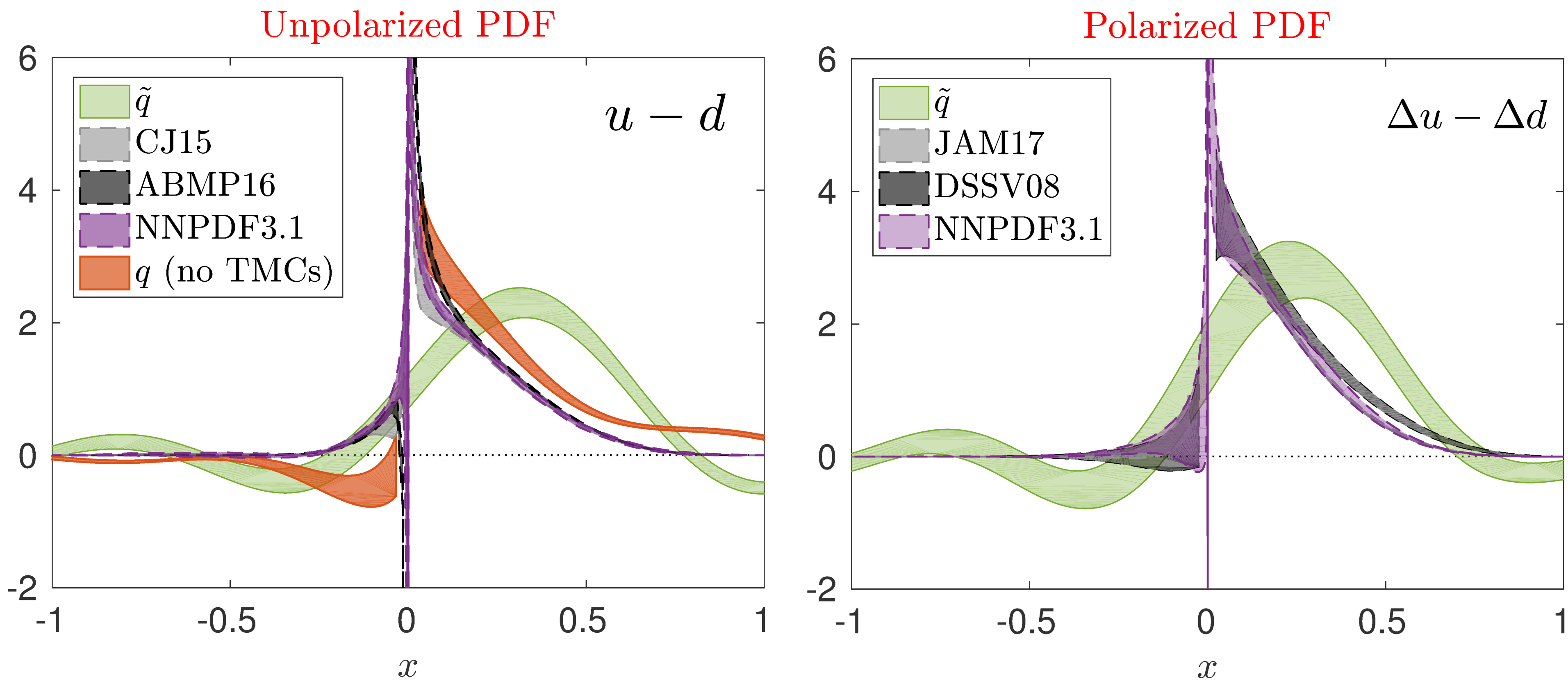
Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

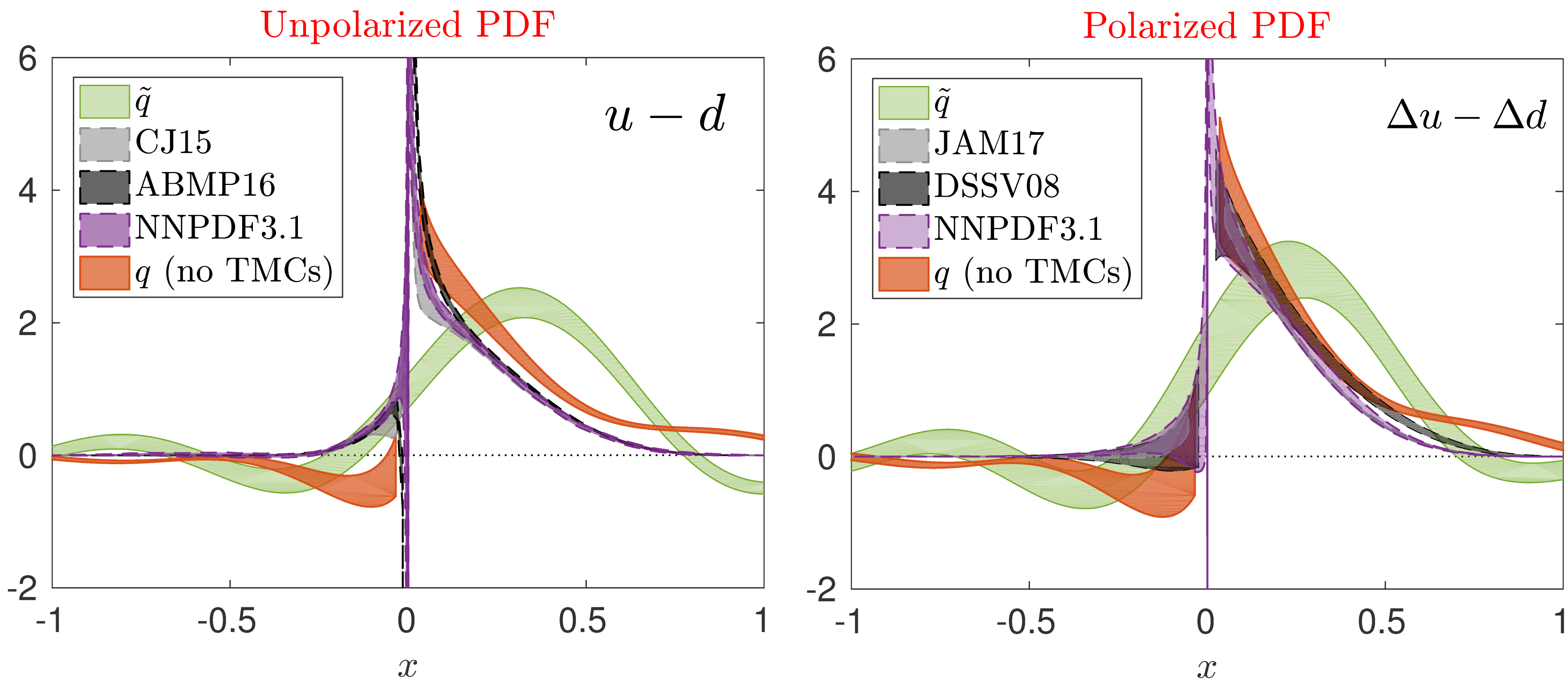
Matched PDFs

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Step 7

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

Nucleon mass corrections

In the infinite momentum frame, nucleon mass does not matter, i.e. $m_N/P_3 = 0$.

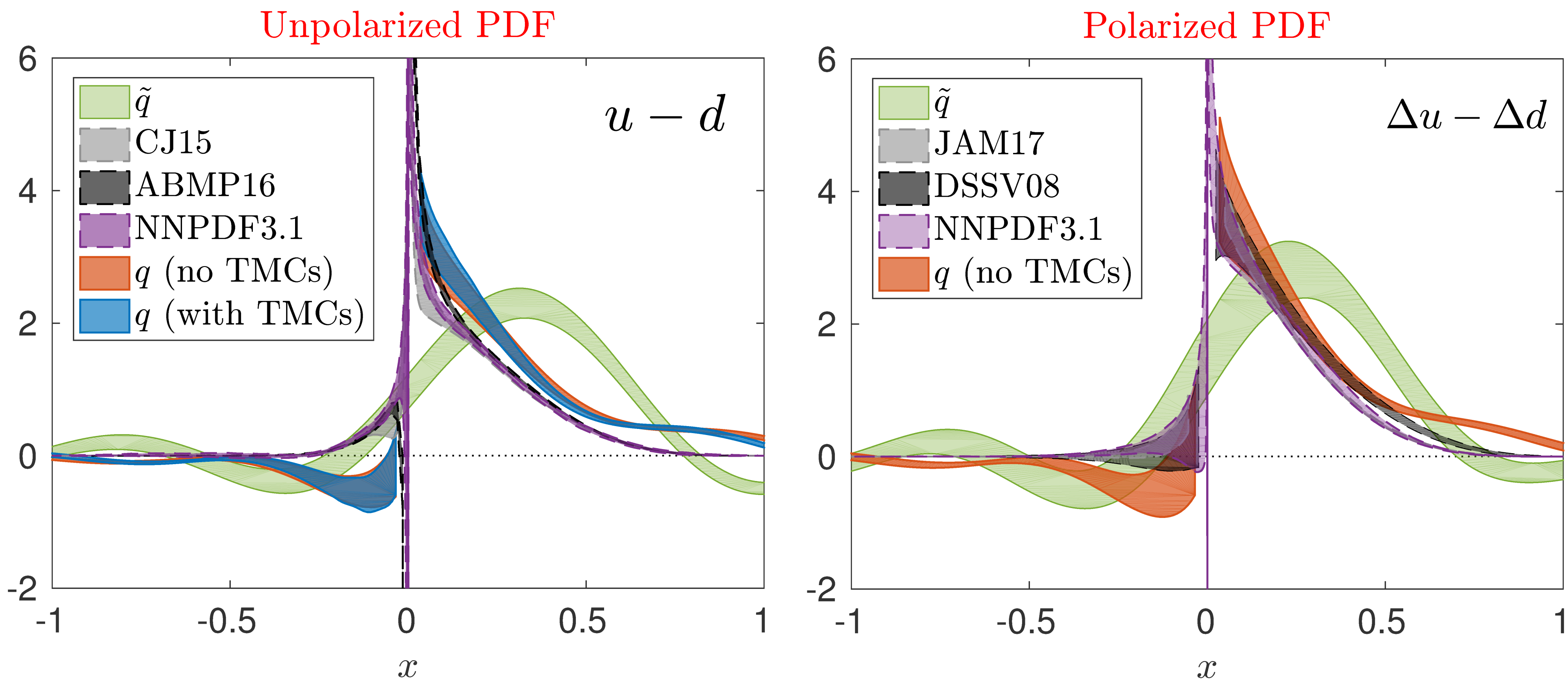
Here, we work with nucleon boosted to finite momentum P_3 and we need to correct for $m_N/P_3 \neq 0$.

These corrections were derived in:
[J.W. Chen et al., Nucl.Phys. B911 (2016) 246-273, arXiv:1603.06664 [hep-ph]]

Important feature: particle number is conserved in nucleon mass corrections.

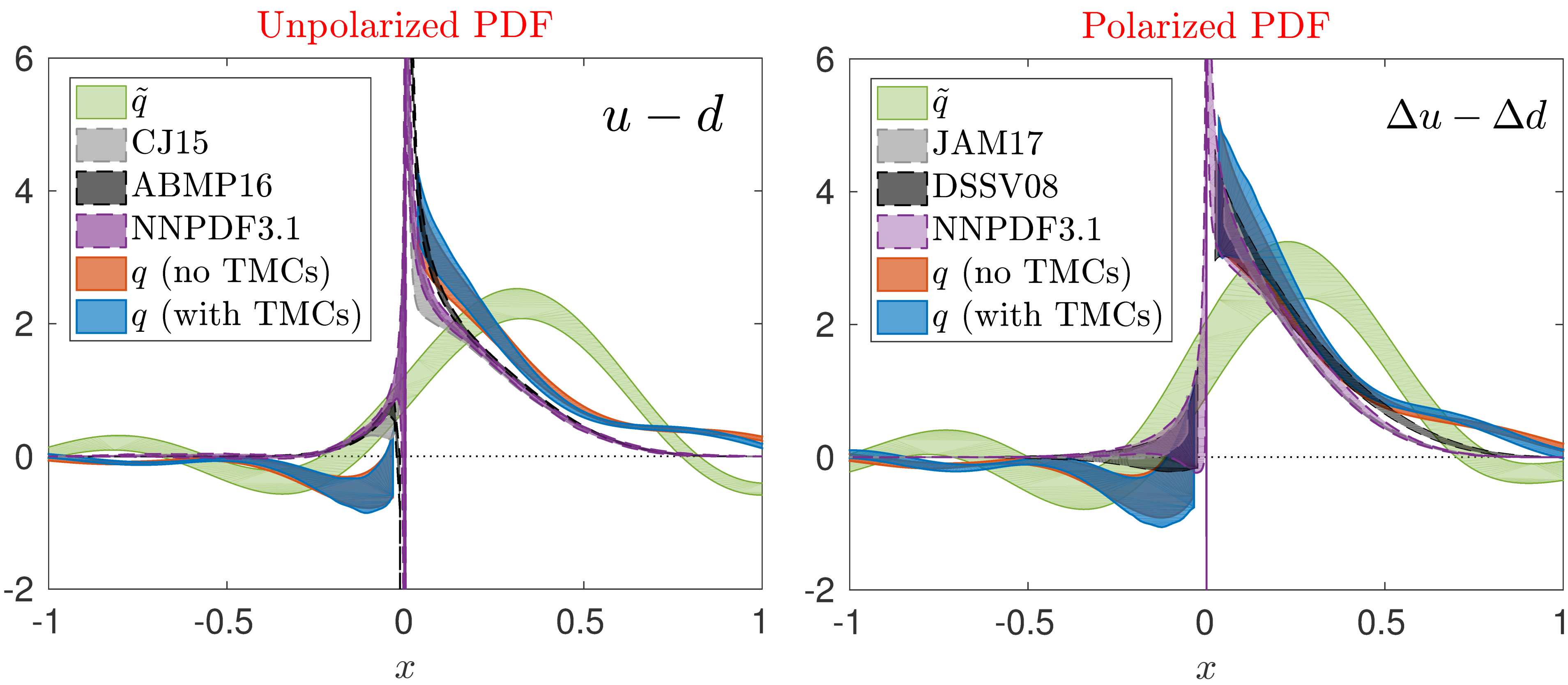
Matched PDF + TMCs

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

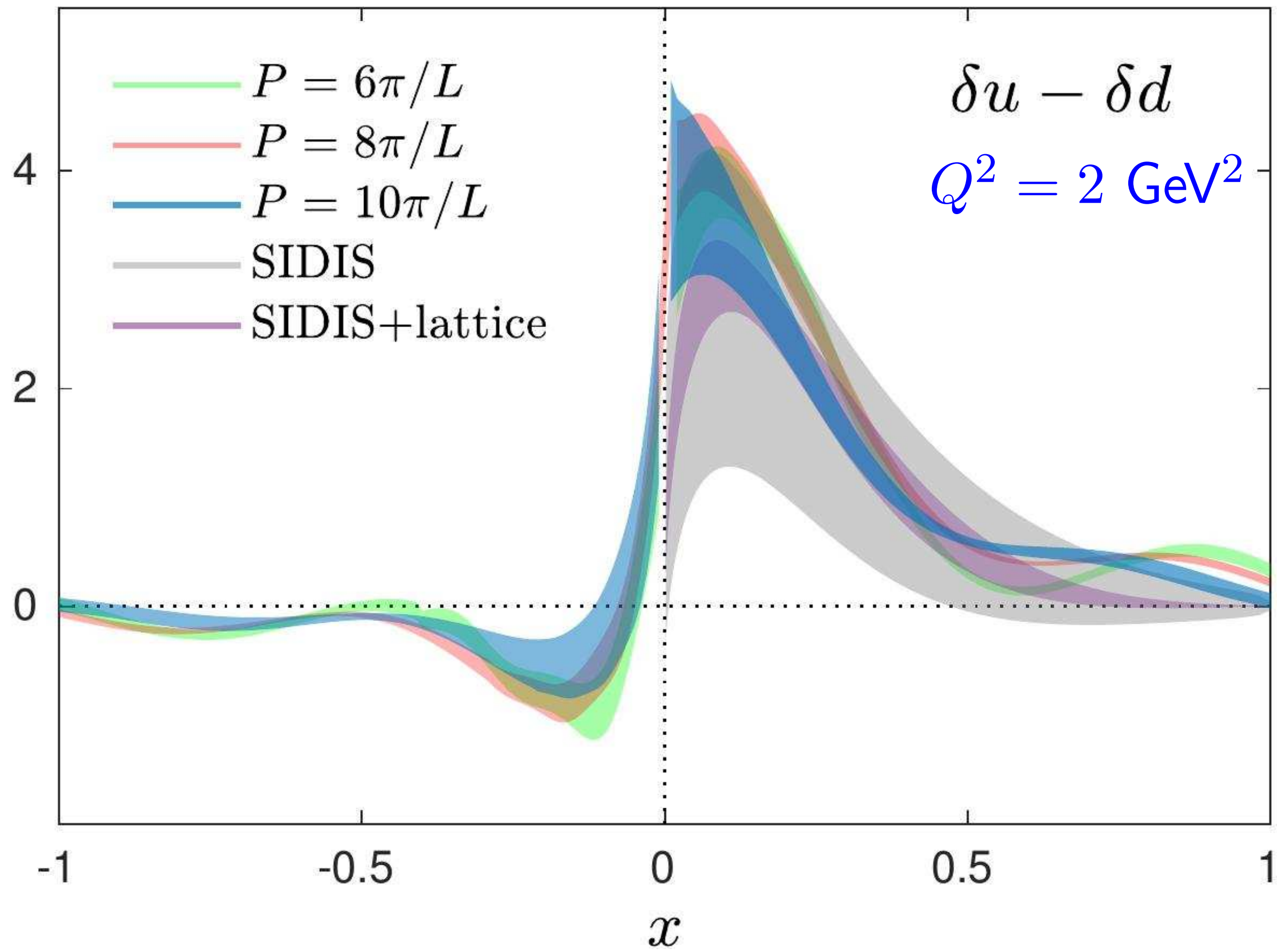
Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Transversity PDF

C. Alexandrou et al., Phys. Rev. D98 (2018) 091503 (Rapid Communications)



Statistical precision already much better than the precision of phenomenological fits from SIDIS: JAM Collaboration, Phys. Rev. Lett. 120 (2018) 152502

Different systematic effects:

Systematics

Different systematic effects:

- pion mass

Different systematic effects:

- pion mass ✓

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)
- Fourier transform reconstruction

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)
- Fourier transform reconstruction ✓✗

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)
- Fourier transform reconstruction ✓✗
  J. Karpie et al., JHEP 1904 (2019) 057

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)
- Fourier transform reconstruction ✓✗
  J. Karpie et al., JHEP 1904 (2019) 057
- truncation of conversion, evolution and matching

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)
- Fourier transform reconstruction ✓✗
  J. Karpie et al., JHEP 1904 (2019) 057
- truncation of conversion, evolution and matching ✗
  (uncontrolled, can be sizable)

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)
- Fourier transform reconstruction ✓✗
  J. Karpie et al., JHEP 1904 (2019) 057
- truncation of conversion, evolution and matching ✗
  (uncontrolled, can be sizable)
- ......

Systematics

Different systematic effects:

- pion mass ✓
- contamination by excited states ✓ (suppressed below stat. error)
- cut-off effects ✗ (continuum dispersion relation holds, but $h(z)$???)
- finite volume effects ✓✗ (typically small in lattice HS)
- lattice artifacts in renormalization functions ✓✗ (typical $\mathcal{O}(\lesssim 1\%)$)
- higher-twist effects ✓✗
  (expected to be $\mathcal{O}(\Lambda^2_{\rm QCD}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV)
- Fourier transform reconstruction ✓✗
  J. Karpie et al., JHEP 1904 (2019) 057
- truncation of conversion, evolution and matching ✗
  (uncontrolled, can be sizable)
- ......

Investigation of several of these systematics in:
C. Alexandrou et al. [ETM Collaboration], "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point",
Phys. Rev. D99 (2019) 114504.

GPDs – can be accessed with the same type of matrix elements as PDFs:

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink momentum transfer source

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink momentum transfer source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum momentum transfer

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink momentum transfer source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum momentum transfer

$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P_3'' - P_3'}{P_3'' + P_3'} = -\frac{Q_3}{2P_3}$$

momentum transfer quasi-skewness

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink   momentum transfer   source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum   momentum transfer

$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P''_3 - P'_3}{P''_3 + P'_3} = -\frac{Q_3}{2P_3}$$

momentum transfer   quasi-skewness

GPDs: $H(x,t,\xi,\mu^2)$
$E(x,t,\xi,\mu^2)$

Preliminary new results – qGPDs $N_f = 2+1+1$

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink   momentum transfer   source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum   momentum transfer

$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P_3'' - P_3'}{P_3'' + P_3'} = -\frac{Q_3}{2P_3}$$

momentum transfer   quasi-skewness

GPDs: $H(x,t,\xi,\mu^2)$
$E(x,t,\xi,\mu^2)$

Aim: to cover a wide range of t, ξ parameter space.

Preliminary new results – qGPDs $N_f = 2+1+1$

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink momentum transfer source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum momentum transfer

$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P''_3 - P'_3}{P''_3 + P'_3} = -\frac{Q_3}{2P_3}$$

momentum transfer quasi-skewness

GPDs: $H(x,t,\xi,\mu^2)$
$E(x,t,\xi,\mu^2)$

Aim: to cover a wide range of t,ξ parameter space.

- fermions: $N_f = 2+1+1$
  TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$,
- a=0.081 fm,
- $m_\pi L \approx 270$ MeV.

ETMC, arXiv:1910.13229

Preliminary new results – qGPDs $N_f = 2+1+1$

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink momentum transfer source

$$P = \frac{P'+P''}{2}, \quad Q = P''-P'$$

avg.momentum momentum transfer

$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P_3''-P_3'}{P_3''+P_3'} = -\frac{Q_3}{2P_3}$$

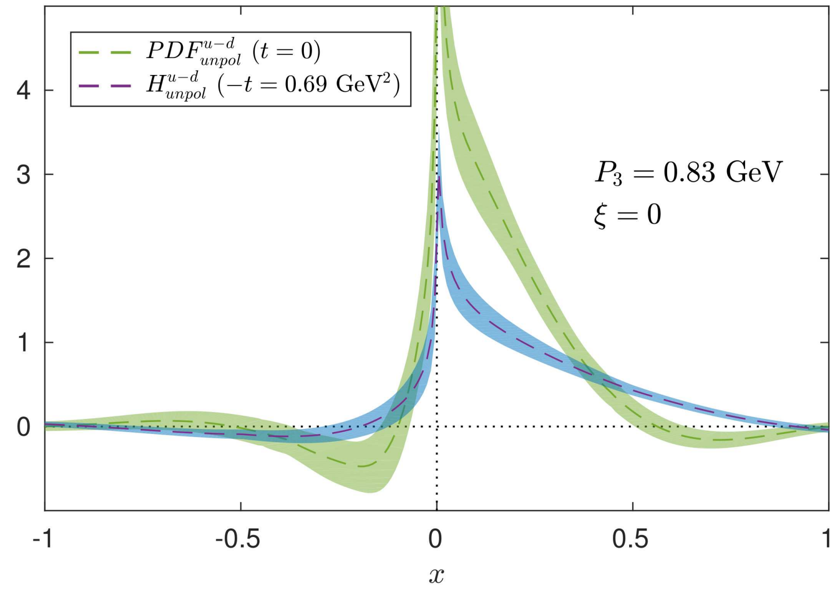
momentum transfer quasi-skewness

GPDs: $H(x,t,\xi,\mu^2)$
$E(x,t,\xi,\mu^2)$

Aim: to cover a wide range of t,ξ parameter space.

- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $32^3 \times 64$, $L=3$ fm, $m_\pi L = 4$,
- a=0.081 fm,
- $m_\pi L \approx 270$ MeV.

ETMC, arXiv:1910.13229



GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink   momentum transfer   source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum   momentum transfer

$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P_3''-P_3'}{P_3''+P_3'} = -\frac{Q_3}{2P_3}$$

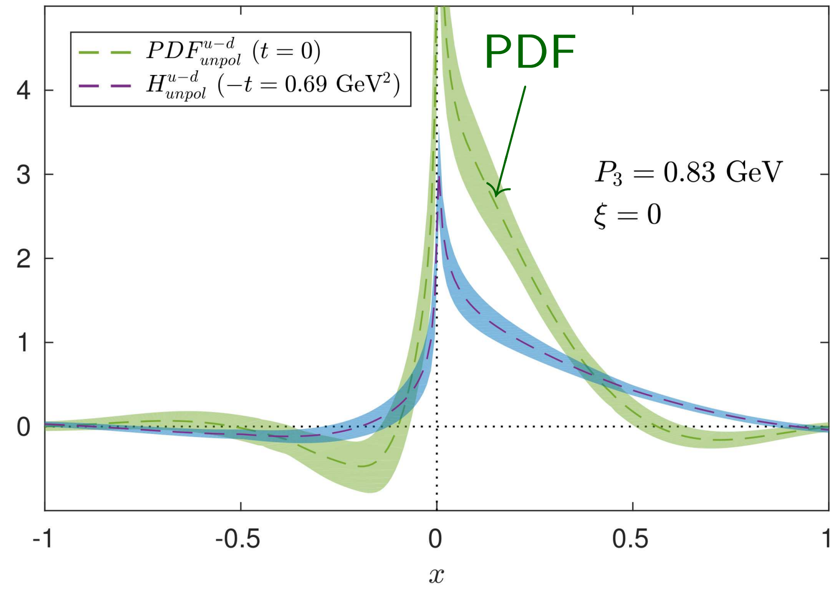
momentum transfer   quasi-skewness

- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$,
- a=0.081 fm,
- $m_\pi L \approx 270$ MeV.

ETMC, arXiv:1910.13229

GPDs: $H(x,t,\xi,\mu^2)$
$E(x,t,\xi,\mu^2)$

Aim: to cover a wide range of t,ξ parameter space.



GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink   momentum transfer   source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum   momentum transfer

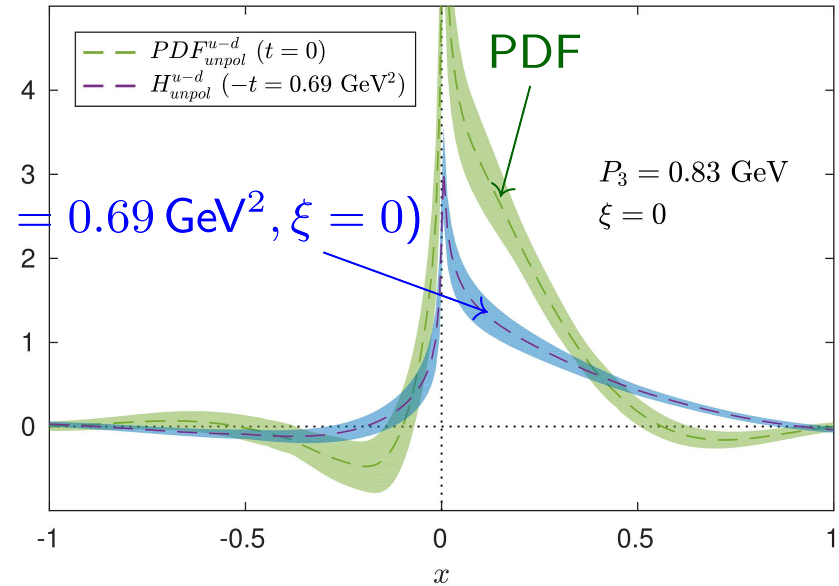
$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P_3''-P_3'}{P_3''+P_3'} = -\frac{Q_3}{2P_3}$$

momentum transfer   quasi-skewness

- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$,
- a=0.081 fm,
- $m_\pi L \approx 270$ MeV.

ETMC, arXiv:1910.13229

GPDs: $H(x,t,\xi,\mu^2)$
$E(x,t,\xi,\mu^2)$



Aim: to cover a wide range of t, ξ parameter space.

GPDs – can be accessed with the same type of matrix elements as PDFs:

$$\langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

sink   momentum transfer   source

$$P = \frac{P'+P''}{2}, \quad Q = P'' - P'$$

avg.momentum   momentum transfer

$$t = -Q^2, \qquad \tilde{\xi} = -\frac{P_3'' - P_3'}{P_3'' + P_3'} = -\frac{Q_3}{2P_3}$$

momentum transfer   quasi-skewness

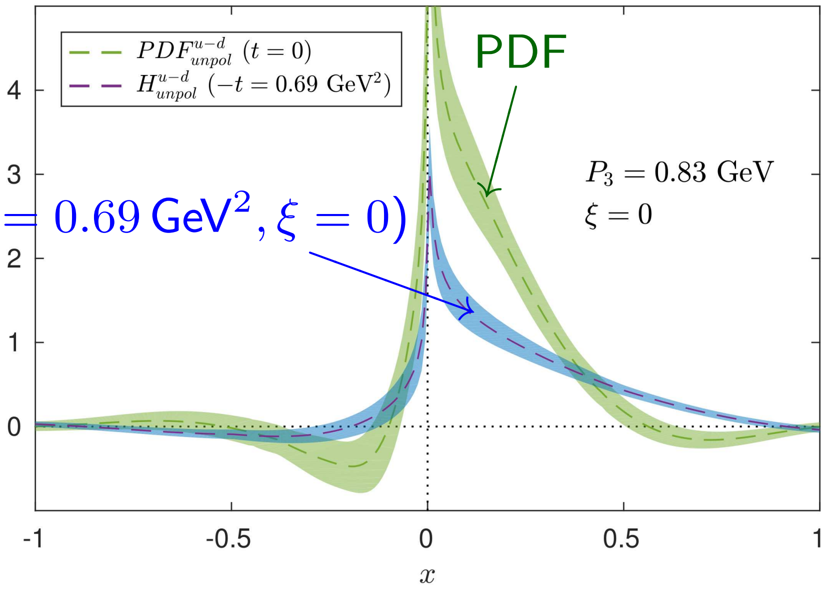
GPDs: $H(x,t,\xi,\mu^2)$
$E(x,t,\xi,\mu^2)$

Aim: to cover a wide range of t,ξ parameter space.

See talk by A. Scapellato

- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$,
- a=0.081 fm,
- $m_\pi L \approx 270$ MeV.

ETMC, arXiv:1910.13229



- Factorization relates experimental cross sections to PDFs.

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\,(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\,(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

- Question: can we treat lattice observables similarly to cross sections?

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\,(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

- Question: can we treat lattice observables similarly to cross sections?
- Recent attempt to learn something about this question:
  K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\left(x,\mu,P_3\right) = \int_{-1}^{1} \frac{d\xi}{|\xi|} C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

- Question: can we treat lattice observables similarly to cross sections?
- Recent attempt to learn something about this question:
  K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137
- Using the robust NNPDF framework for fitting.

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\,(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

- Question: can we treat lattice observables similarly to cross sections?
- Recent attempt to learn something about this question:
  K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137
- Using the robust NNPDF framework for fitting.
- Observables: non-singlet distributions V_3 and T_3 (unpolarized):
  $V_3 = u - \bar{u} - (d - \bar{d}) = u_V - d_V$
  $T_3 = u + \bar{u} - (d + \bar{d}) = u_V - d_V + 2(u_S - d_S)$

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\,(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

- Question: can we treat lattice observables similarly to cross sections?
- Recent attempt to learn something about this question:
  K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137
- Using the robust NNPDF framework for fitting.
- Observables: non-singlet distributions V_3 and T_3 (unpolarized):
  $V_3 = u - \bar{u} - (d - \bar{d}) = u_V - d_V$
  $T_3 = u + \bar{u} - (d + \bar{d}) = u_V - d_V + 2(u_S - d_S)$
- We have:

$$\mathcal{O}_{\gamma^0}^{\text{Re/Im}}(z,\mu) = \int_0^1 dx\, \mathcal{C}_3^{\text{Re/Im}}\left(x, z, \frac{\mu}{P_z}\right) V_3/T_3\,(x,\mu) = \mathcal{C}_3^{\text{Re/Im}}\left(z, \frac{\mu}{P_z}\right) \circledast V_3/T_3\,(\mu)\,,$$

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\,(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

- Question: can we treat lattice observables similarly to cross sections?
- Recent attempt to learn something about this question:
  K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137
- Using the robust NNPDF framework for fitting.
- Observables: non-singlet distributions V_3 and T_3 (unpolarized):
  $V_3 = u - \bar{u} - (d - \bar{d}) = u_V - d_V$
  $T_3 = u + \bar{u} - (d + \bar{d}) = u_V - d_V + 2(u_S - d_S)$
- We have:
  $\mathcal{O}^{\mathrm{Re/Im}}_{\gamma^0}(z,\mu) = \int_0^1 dx\, \mathcal{C}_3^{\mathrm{Re/Im}}\left(x, z, \frac{\mu}{P_z}\right) V_3/T_3\,(x,\mu) = \mathcal{C}_3^{\mathrm{Re/Im}}\left(z, \frac{\mu}{P_z}\right) \circledast V_3/T_3\,(\mu)\,,$
- The above equations implemented using FastKernel tables that combine the matching and DGLAP evolution.

Impact of lattice data on phenomenology?

- Factorization relates experimental cross sections to PDFs.
- Similarly: factorization relates lattice observables to PDFs, e.g.:

$$\tilde{q}\,(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C\left(\frac{x}{\xi},\mu,P_3\right) q(x,\mu)$$

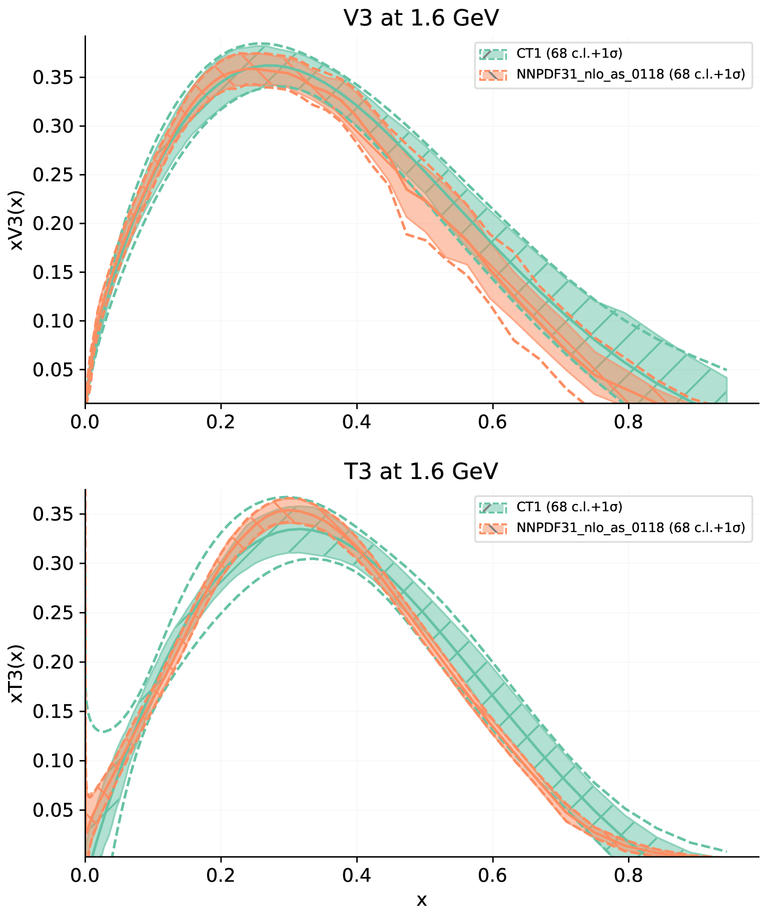
- Question: can we treat lattice observables similarly to cross sections?
- Recent attempt to learn something about this question:
  K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137
- Using the robust NNPDF framework for fitting.
- Observables: non-singlet distributions V_3 and T_3 (unpolarized):
  $V_3 = u - \bar{u} - (d - \bar{d}) = u_V - d_V$
  $T_3 = u + \bar{u} - (d + \bar{d}) = u_V - d_V + 2(u_S - d_S)$
- We have:
  $\mathcal{O}^{\text{Re/Im}}_{\gamma^0}(z,\mu) = \int_0^1 dx\, \mathcal{C}_3^{\text{Re/Im}}\left(x, z, \frac{\mu}{P_z}\right) V_3/T_3\,(x,\mu) = \mathcal{C}_3^{\text{Re/Im}}\left(z, \frac{\mu}{P_z}\right) \circledast V_3/T_3\,(\mu)\,,$
- The above equations implemented using FastKernel tables that combine the matching and DGLAP evolution.
- NN parametrization: $V_3/T_3\,(x,\mu) \propto x^{\alpha_{V/T}}\,(1-x)^{\beta_{V/T}}\, \text{NN}_{V/T}\,(x)$.

Closure tests

- Exercise: generate pseudo data from a selected NNPDF and run fitting code over them.
- 16 “lattice points“ generated (16 real, 15 imaginary)

Closure tests

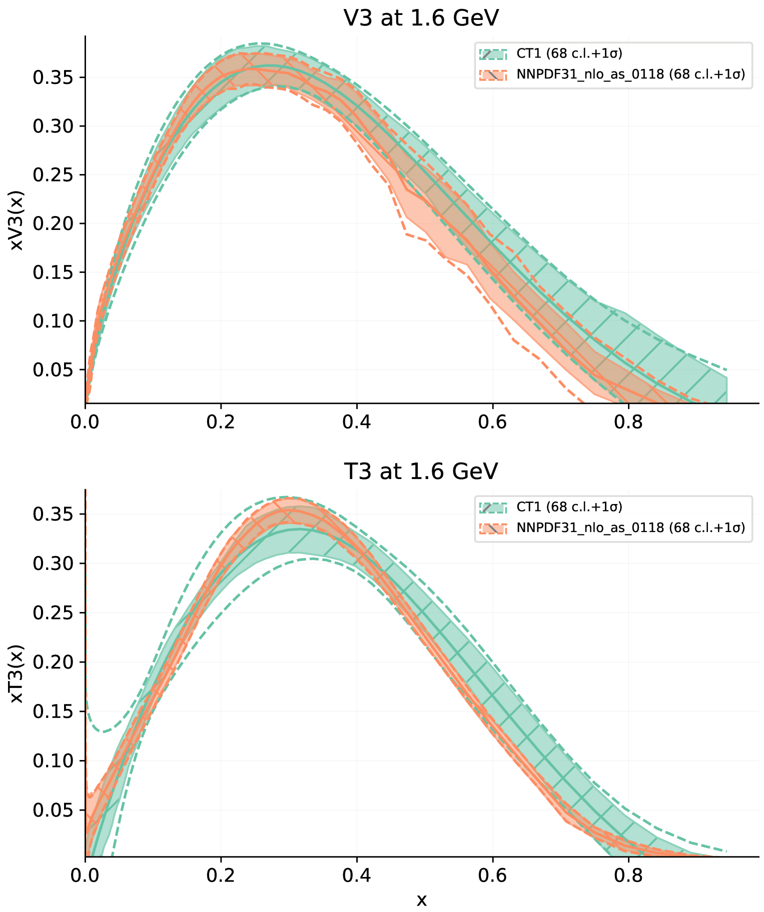
- Exercise: generate pseudo data from a selected NNPDF and run fitting code over them.
- 16 "lattice points" generated (16 real, 15 imaginary)
- Test different scenarios: K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137



only error of NNPDF

Closure tests

- Exercise: generate pseudo data from a selected NNPDF and run fitting code over them.
- 16 "lattice points" generated (16 real, 15 imaginary)
- Test different scenarios: K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137



only error of NNPDF

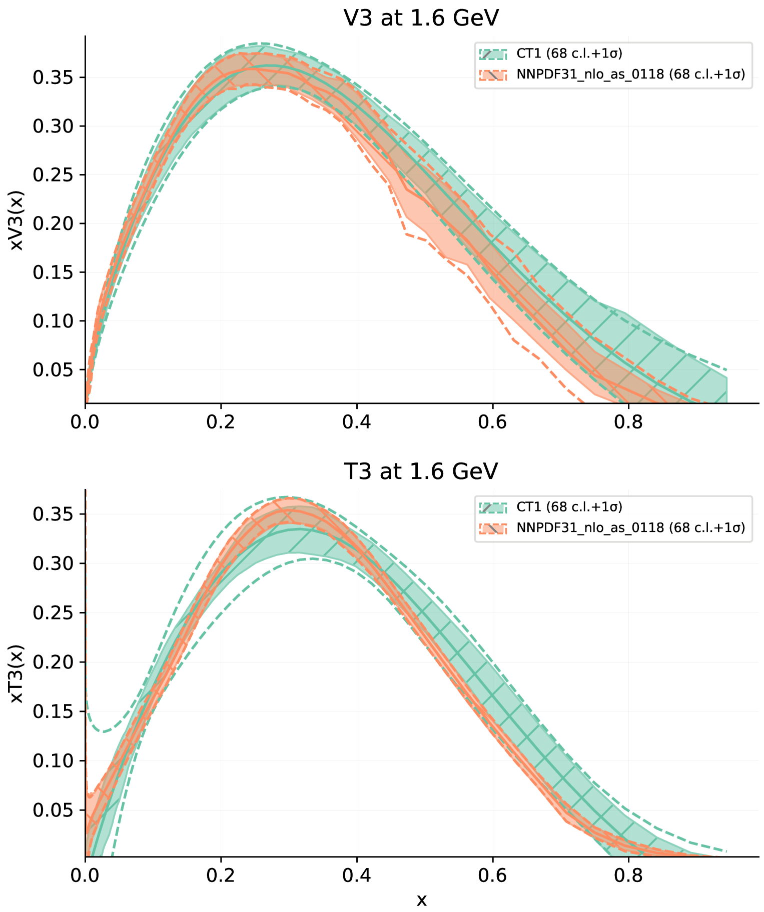
Very robust result!

pseudo data:
1. DGLAP evolution 1.65→2 GeV
2. inverse matching
3. inverse Fourier

reconstruction:
1. NN fit
2. matching
3. DGLAP evolution 2→1.65 GeV

Closure tests

- Exercise: generate pseudo data from a selected NNPDF and run fitting code over them.
- 16 “lattice points“ generated (16 real, 15 imaginary)
- Test different scenarios: K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137



only error of NNPDF

Very robust result!

pseudo data:
1. DGLAP evolution 1.65→2 GeV
2. inverse matching
3. inverse Fourier

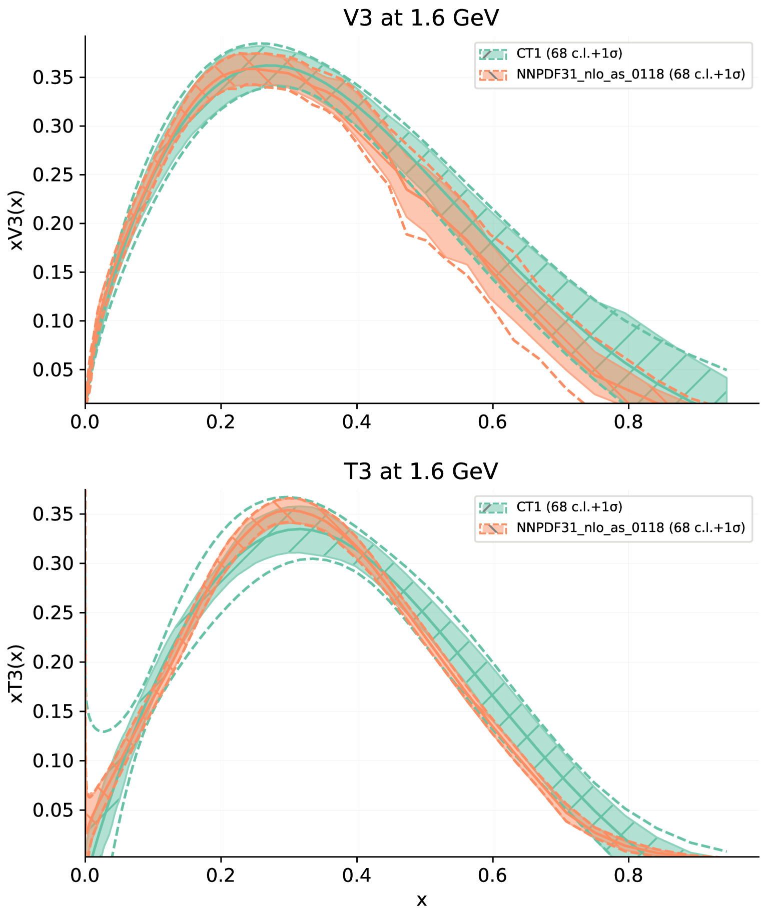
reconstruction:
1. NN fit
2. matching
3. DGLAP evolution 2→1.65 GeV

Shows the power of the convolution ⊛ in constraining PDFs! (only 16 lat. points!)

See also:
J.Karpie et al., JHEP04(2019)057

Closure tests

- Exercise: generate pseudo data from a selected NNPDF and run fitting code over them.
- 16 "lattice points" generated (16 real, 15 imaginary)
- Test different scenarios: K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137



only error of NNPDF

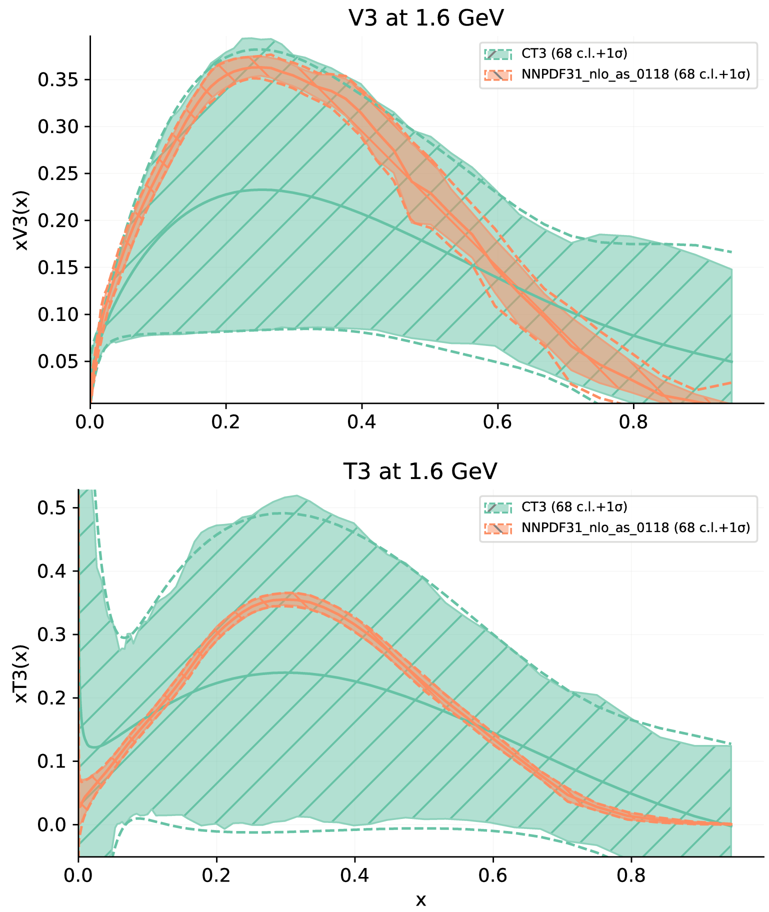
Very robust result!

pseudo data:
1. DGLAP evolution 1.65→2 GeV
2. inverse matching
3. inverse Fourier

reconstruction:
1. NN fit
2. matching
3. DGLAP evolution 2→1.65 GeV

Shows the power of the convolution ⊛ in constraining PDFs! (only 16 lat. points!)

See also:
J.Karpie et al., JHEP04(2019)057



stat.error of ETMC lattice data + a scenario for systematics

Fitting actual lattice data

- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.

Fitting actual lattice data

- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.
- We took actual statistical errors and considered different scenarios for systematics:

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}$%	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

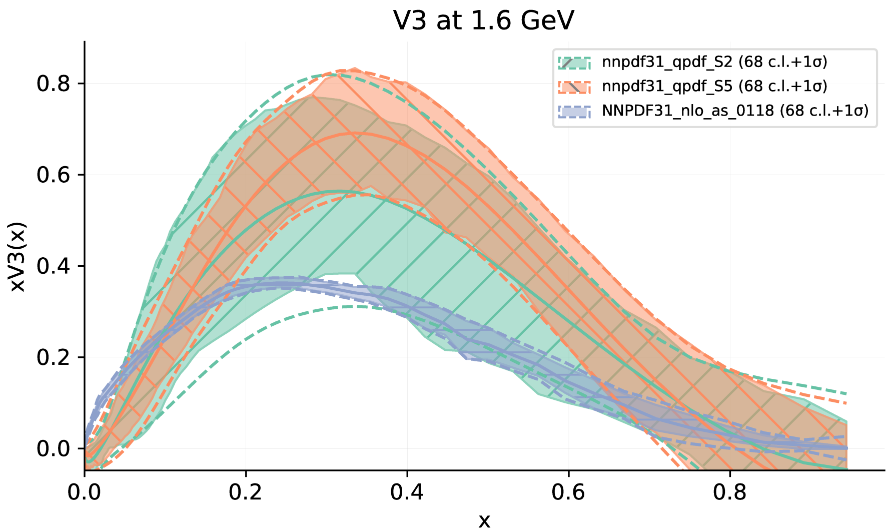
Fitting actual lattice data

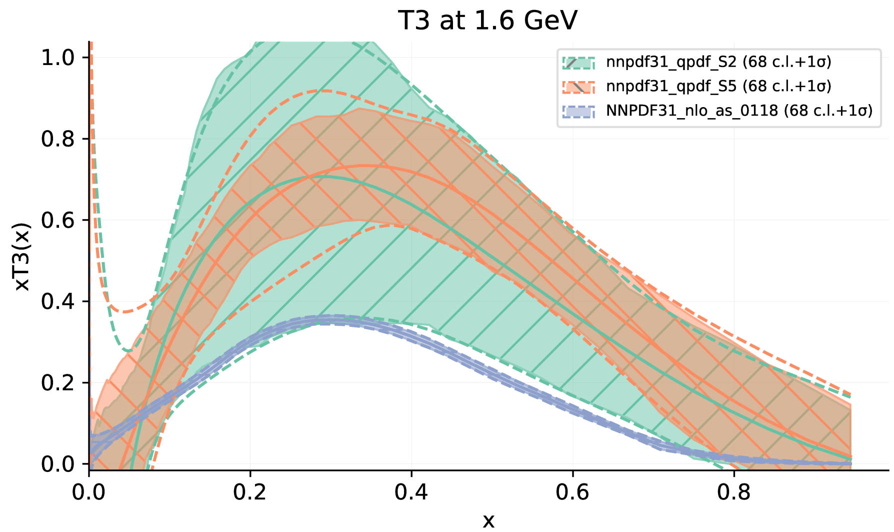
- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.
- We took actual statistical errors and considered different scenarios for systematics:

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

Results from scenarios S2 and S5 ("realistic"):

K.C., L. Del Debbio, T. Giani
JHEP 10 (2019) 137





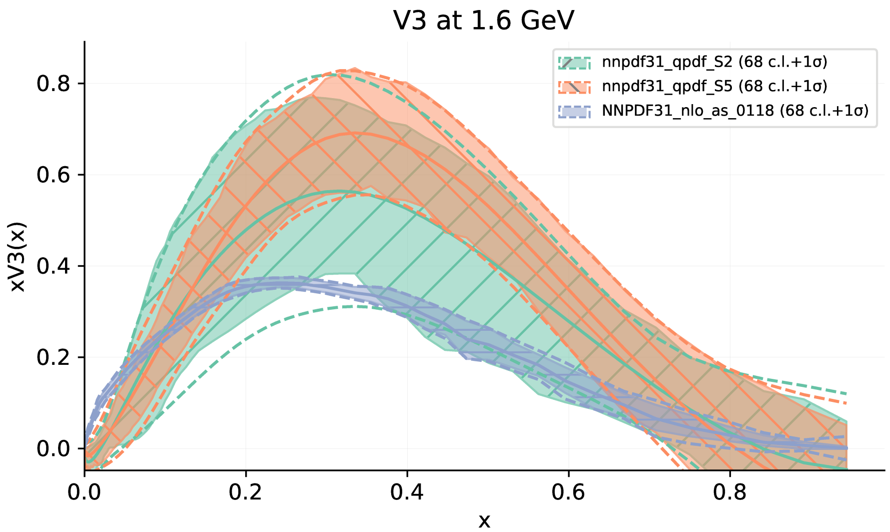
Fitting actual lattice data

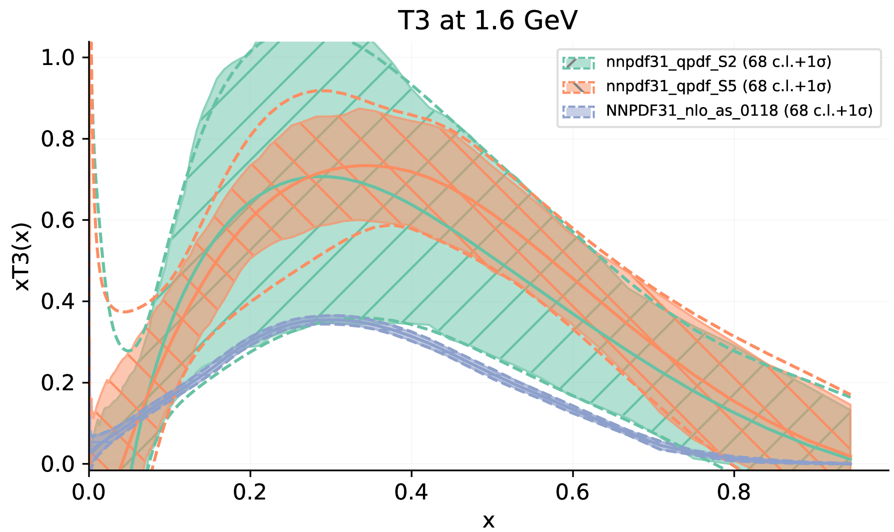
- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.
- We took actual statistical errors and considered different scenarios for systematics:

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}$%	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

K.C., L. Del Debbio, T. Giani
JHEP 10 (2019) 137

Results from scenarios S2 and S5 ("realistic"):





Reasonable agreement, but a lot of work for the lattice to reduce uncertainties!

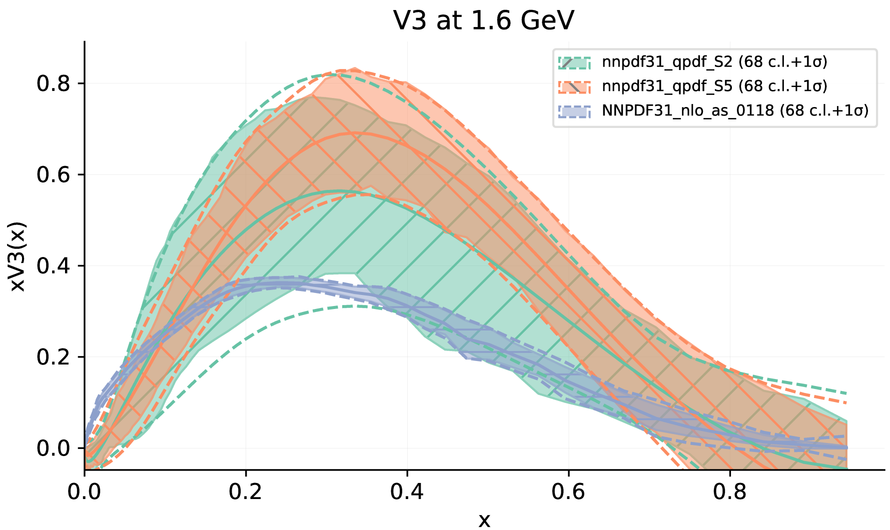
Fitting actual lattice data

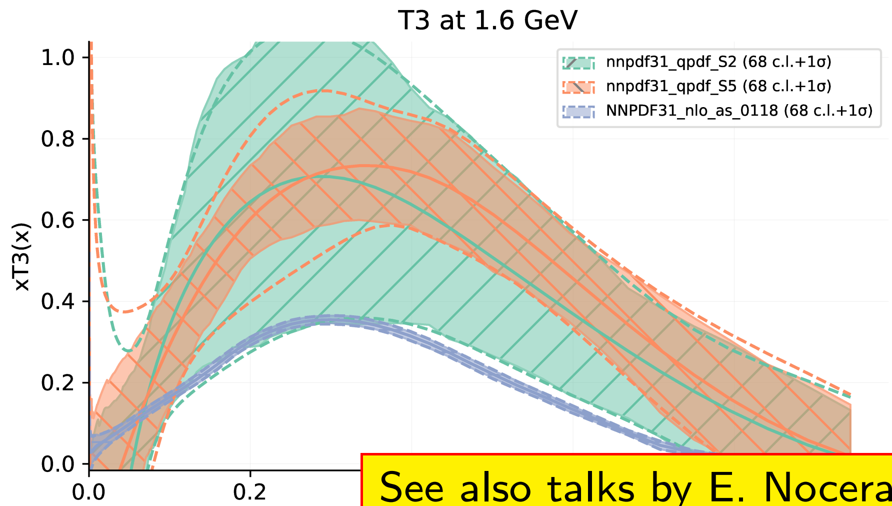
- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.
- We took actual statistical errors and considered different scenarios for systematics:

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

Results from scenarios S2 and S5 ("realistic"):

K.C., L. Del Debbio, T. Giani
JHEP 10 (2019) 137

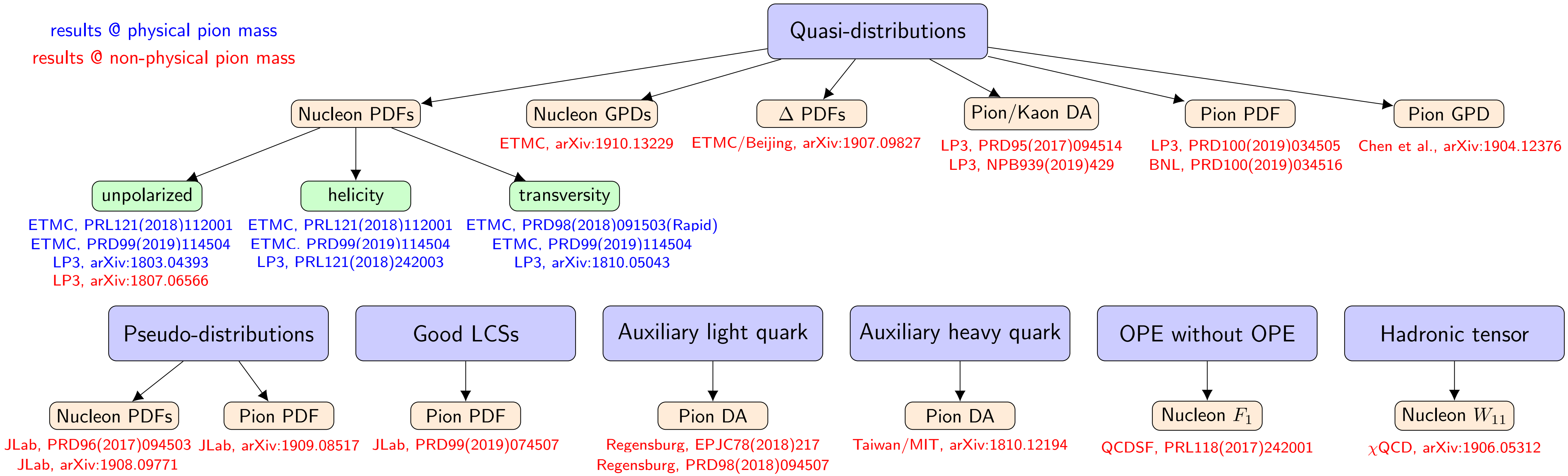




See also talks by E. Nocera, N. Sato

Reasonable agreement, but a lot of work for the lattice to reduce uncertainties!

Overview of results from different approaches

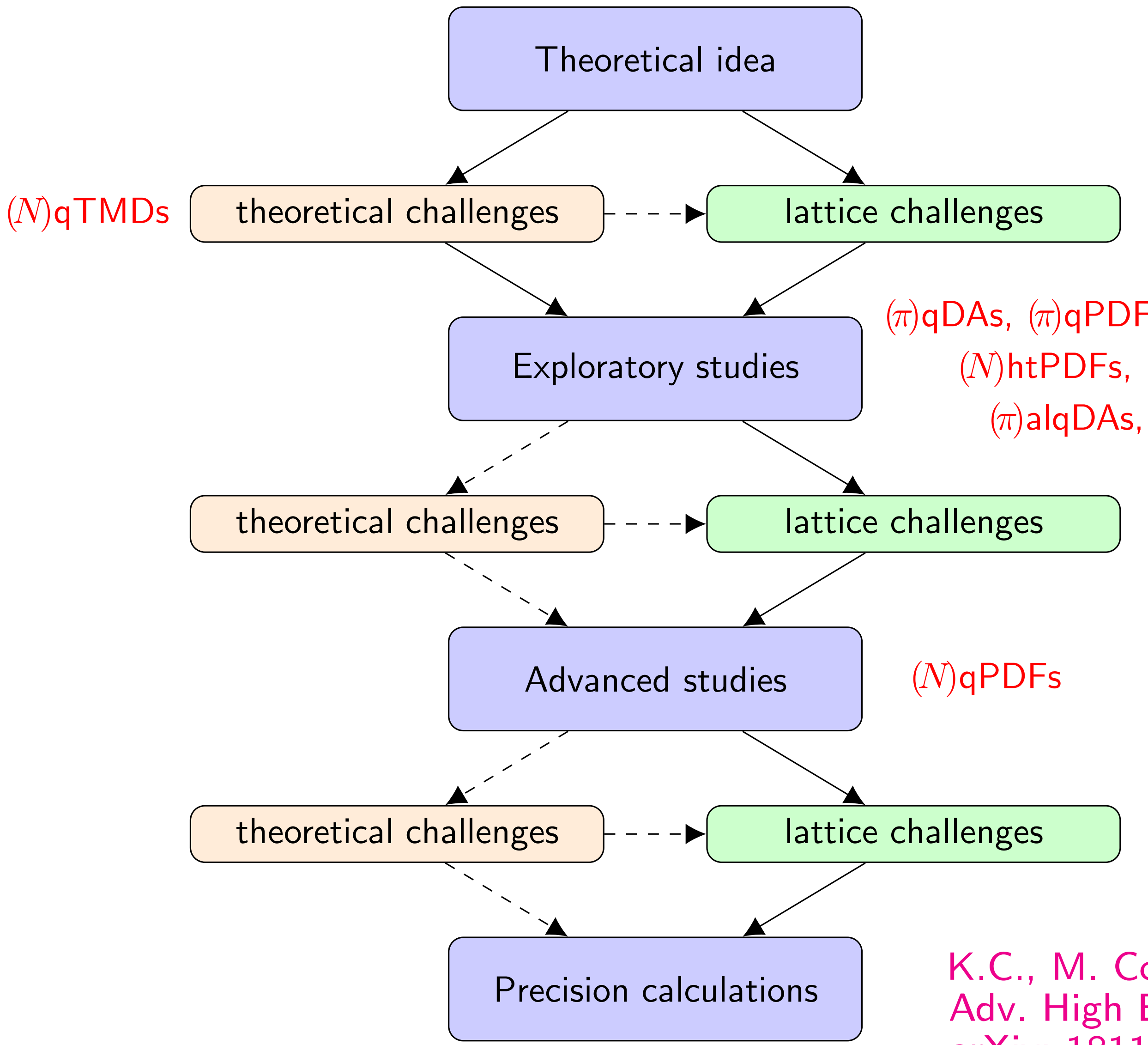


See talks in the parallel workshop:

Distribution functions: Lattice QCD meets phenomenology

Approaches to light-cone PDFs

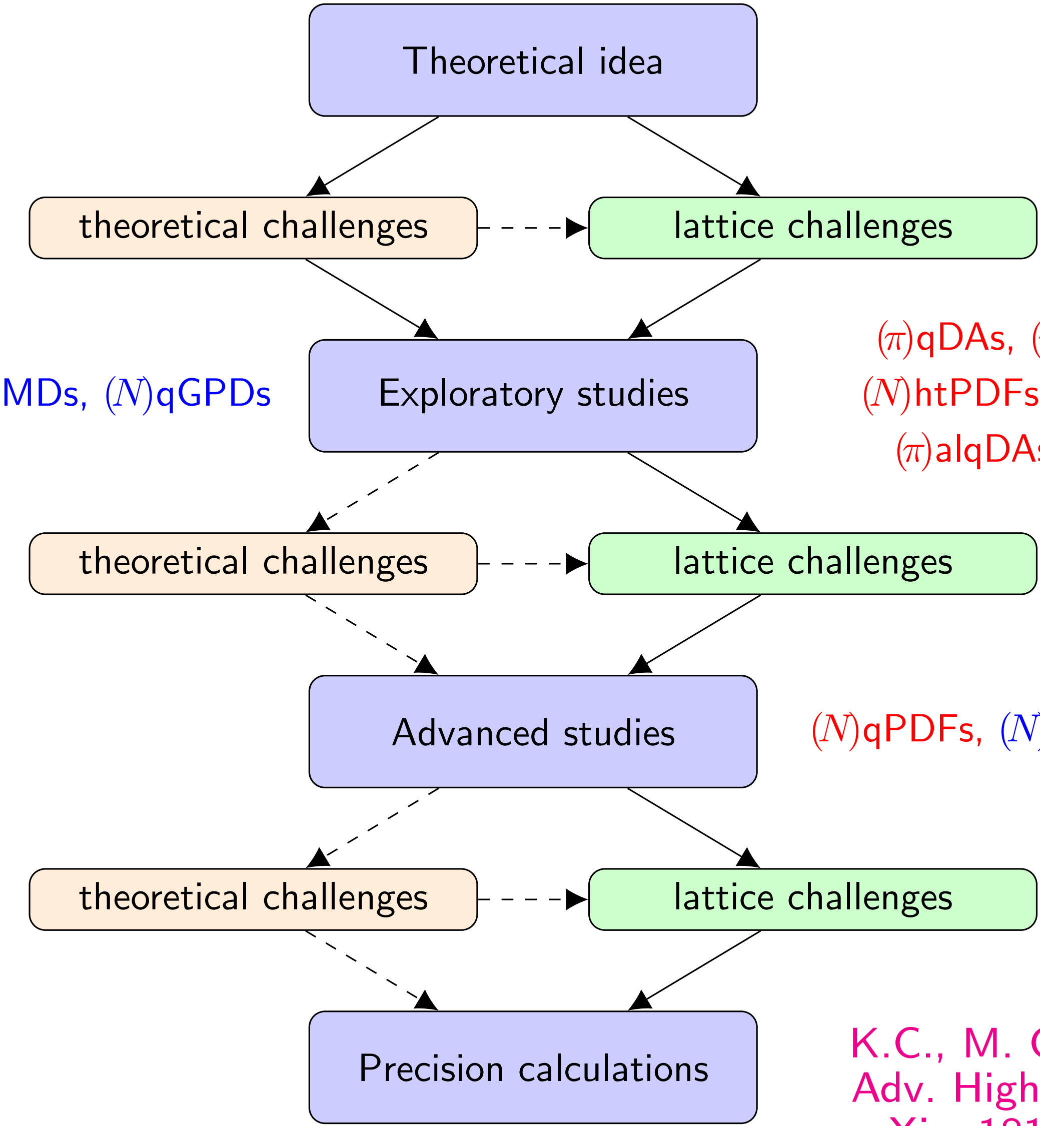
2018



K.C., M. Constantinou
Adv. High Energy Phys. 2019, 3036904
arXiv: 1811.07248 [hep-lat]

Approaches to light-cone PDFs

2019 (update)



Exploratory studies: (N)qTMDs, (N)qGPDs

(π)qDAs, (π)qPDFs, (N,π)qPDFs(g)
(N)htPDFs, (N)opePDFs, (π)ahqDAs
(π)alqDAs, (π)lcsDAs, (π)lcsPDFs

Advanced studies: (N)qPDFs, (N)pPDFs

K.C., M. Constantinou
Adv. High Energy Phys. 2019, 3036904
arXiv: 1811.07248 [hep-lat]

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.
- Very encouraging results and already agreement with pheno for a range of x values.

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.
- Very encouraging results and already agreement with pheno for a range of x values.
- But: still a long way to go to control all systematics.

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.
- Very encouraging results and already agreement with pheno for a range of x values.
- But: still a long way to go to control all systematics.
- Reliable checks necessary for all kinds of effects.

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.
- Very encouraging results and already agreement with pheno for a range of x values.
- But: still a long way to go to control all systematics.
- Reliable checks necessary for all kinds of effects.
- In the future, lattice results can have important impact on phenomenology.

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.
- Very encouraging results and already agreement with pheno for a range of x values.
- But: still a long way to go to control all systematics.
- Reliable checks necessary for all kinds of effects.
- In the future, lattice results can have important impact on phenomenology.
- Also exploratory results for quasi-GPDs.

Conclusions and prospects

- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.
- Very encouraging results and already agreement with pheno for a range of x values.
- But: still a long way to go to control all systematics.
- Reliable checks necessary for all kinds of effects.
- In the future, lattice results can have important impact on phenomenology.
- Also exploratory results for quasi-GPDs.
- Several directions for the future – also other kinds of structure functions: singlet quark PDFs, TMDs, gluon PDFs etc.

Conclusions and prospects

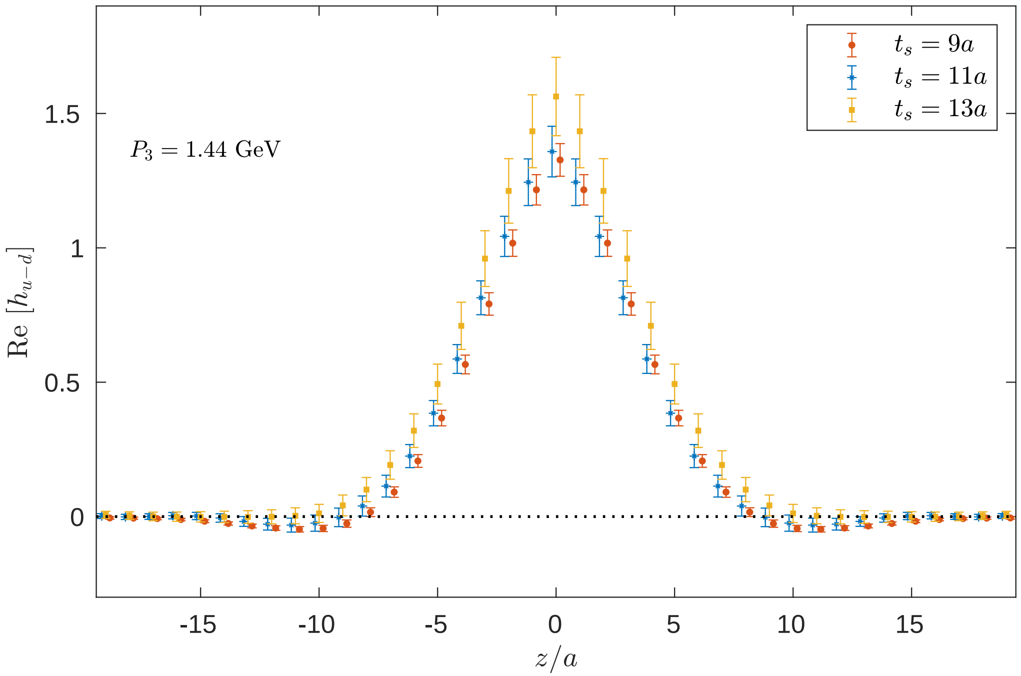
- Message of the talk: **enormous progress in lattice calculations of x-dependence of partonic functions!**
- Lattice QCD can now get the full Bjorken-x dependence of non-singlet PDFs from first principles at a physical pion mass, with proper non-perturbative renormalization.
- Very encouraging results and already agreement with pheno for a range of x values.
- But: still a long way to go to control all systematics.
- Reliable checks necessary for all kinds of effects.
- In the future, lattice results can have important impact on phenomenology.
- Also exploratory results for quasi-GPDs.
- Several directions for the future – also other kinds of structure functions: singlet quark PDFs, TMDs, gluon PDFs etc.

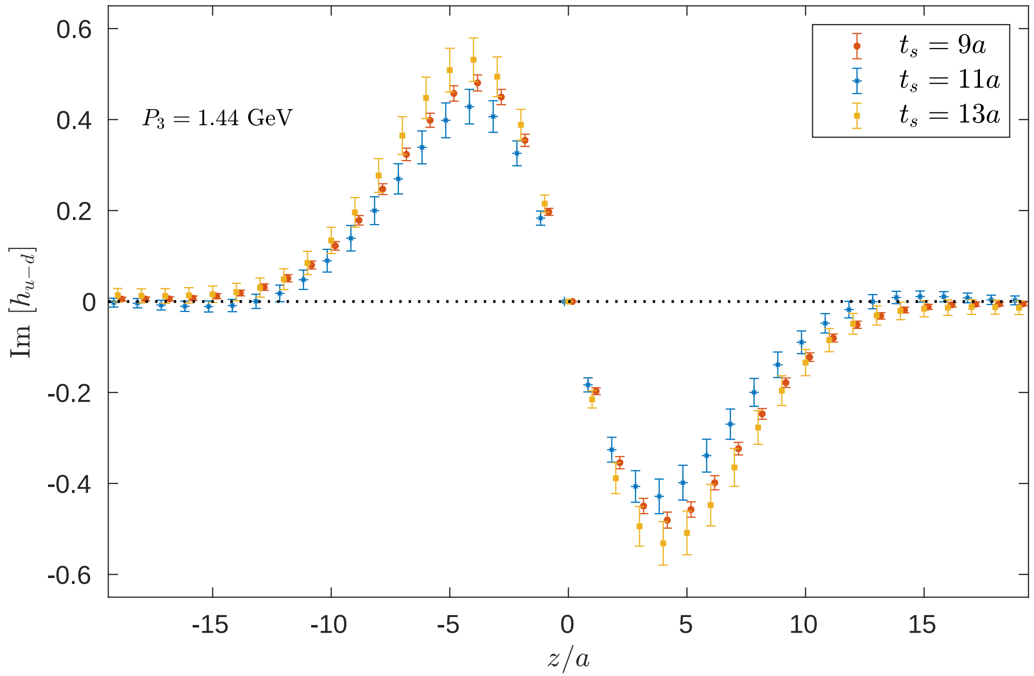
Thank you for your attention!

Backup slides

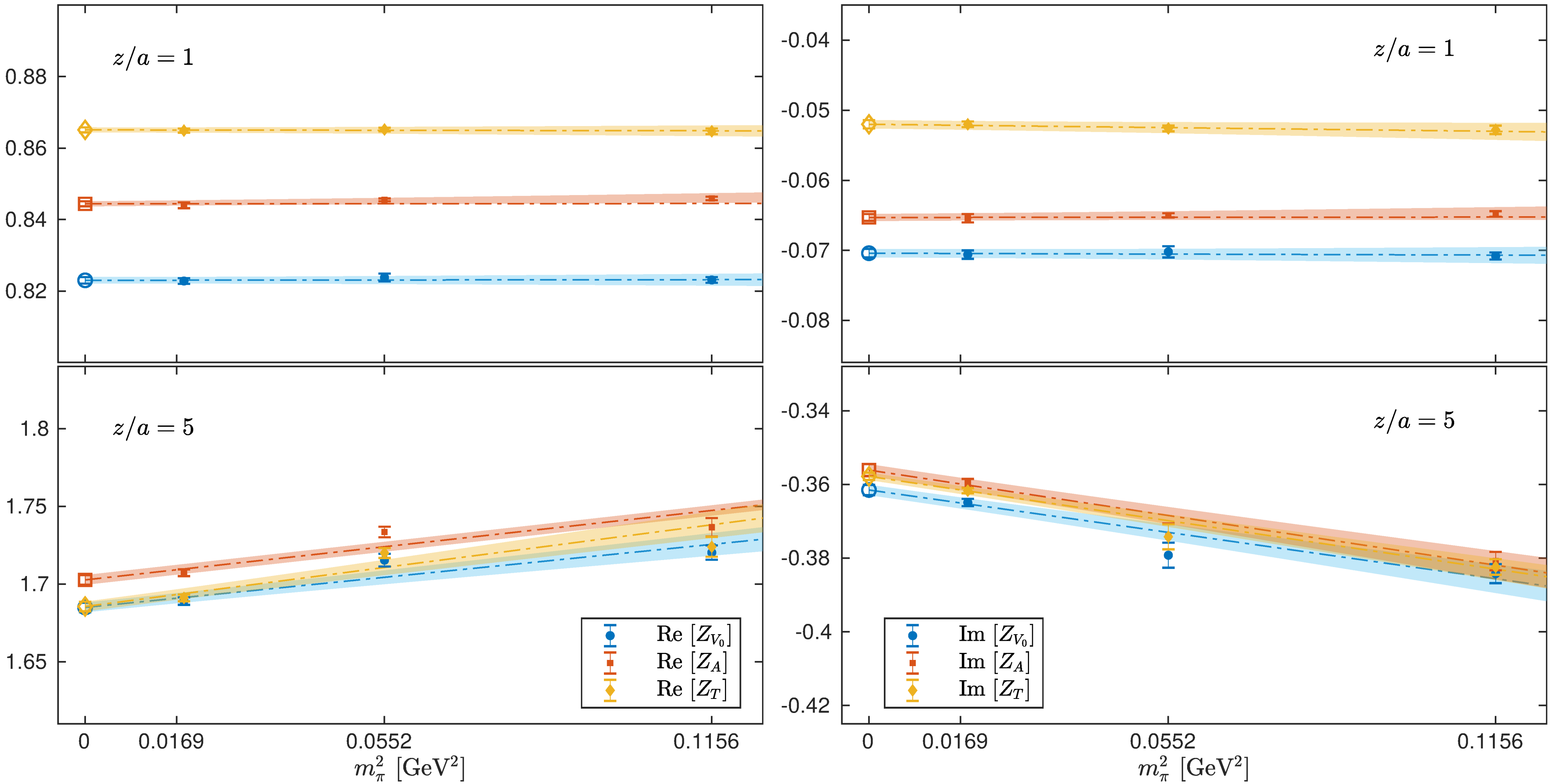
- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $64^3 \times 128$, $L = 5.2$ fm, $m_\pi L = 3.55$,
- a=0.081 fm
- physical pion mass,
- around 30000 measurements and increasing.

ETMC, arXiv:1910.13229





Pion mass dependence of Z-factors



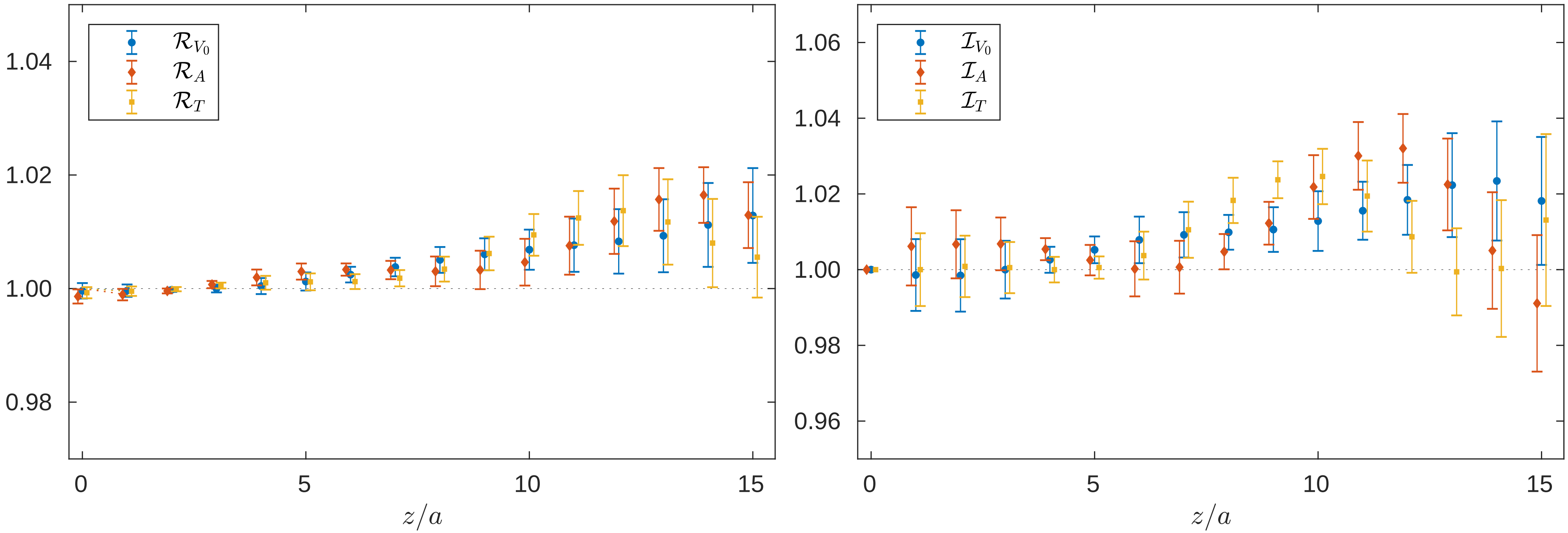
C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

FVE in Z-factors

Possibly enhanced FVE in non-local operators suggested in:

R. Briceño, J. Guerrero, M. Hansen, C. Monahan, Phys. Rev. D98 (2018) 014511

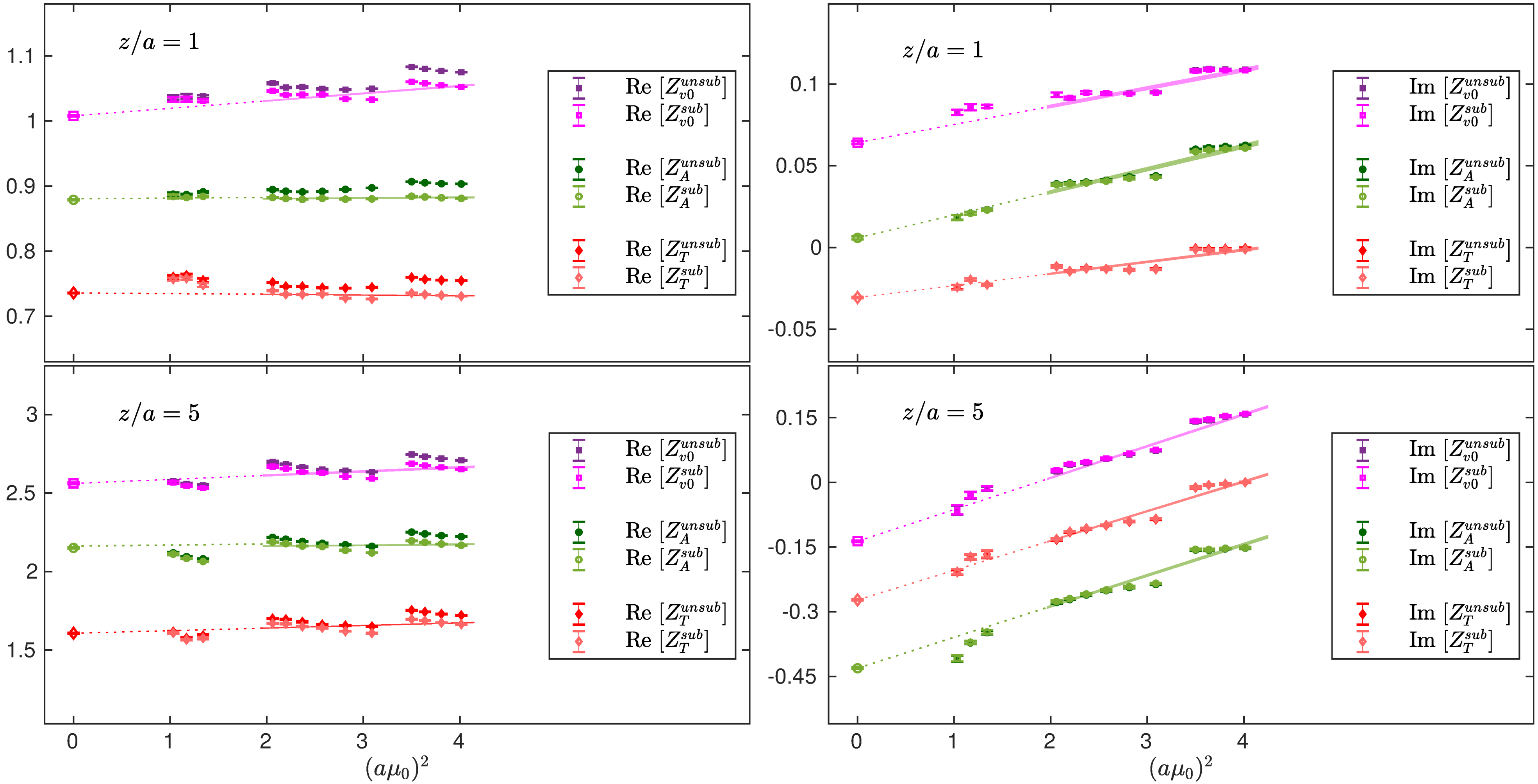
$$\mathcal{R}_{\mathcal{O}}(z) \equiv \frac{\mathrm{Re}[Z^{\mathrm{RI}'}_{\mathcal{O},64}(z,\mu_0,m_\pi)]}{\mathrm{Re}[Z^{\mathrm{RI}'}_{\mathcal{O},48}(z,\mu_0,m_\pi)]}\,, \qquad \mathcal{I}_{\mathcal{O}}(z) \equiv \frac{\mathrm{Im}[Z^{\mathrm{RI}'}_{\mathcal{O},64}(z,\mu_0,m_\pi)]}{\mathrm{Im}[Z^{\mathrm{RI}'}_{\mathcal{O},48}\mathcal{O}(z,\mu_0,m_\pi)]}$$



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Z-factors can have $\mathcal{O}(g^2 a^\infty)$ artefacts perturbatively subtracted

By: M. Constantinou, H. Panagopoulos, e.g. Phys. Rev. D95 (2017) 034505



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$: [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_{+(1)}^{[1,\infty]} - \frac{3}{2\xi} & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi}\ln\frac{x^2P_3^2}{\xi^2\mu^2}\left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_{+(1)}^{[0,1]} & 0 < \xi < 1 \\ \left[-\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_{+(1)}^{[-\infty,0]} - \frac{3}{2(1-\xi)} & \xi < 0, \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{\mu^2}{4y^2P_3^2} + \frac{5}{2}\right), \quad \iota{=}0 \text{ for } \gamma_0 \text{ and } \iota{=}1 \text{ for } \gamma_3/\gamma_5\gamma_3.$$

Problem: violates vector current conservation:

$$\int_{-\infty}^{\infty} dx\, q(x,\mu) \neq \int_{-\infty}^{\infty} dx\, \tilde{q}(x,\mu,P_3) \qquad \text{and} \qquad \int_{-\infty}^{\infty} d\xi\, C(\xi, \xi\mu/xP_3) \neq 1,$$

which **increases** with growing P_3 (around 8% at $P_3 = 10\pi/48$).

Matching to light-front PDFs

Alternative matching: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2 P_3^2}{\xi^2\mu^2}(4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

- In this procedure, vector current is **conserved**.
- Additional subtractions with respect to $\overline{\mathrm{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF.
- However, modification **decreases** with growing P_3.

Modification of the $\overline{\text{MS}}$ scheme

We introduce a modified $\overline{\text{MS}}$ scheme ($\text{M}\overline{\text{MS}}$) with an extra subtraction made outside the physical region of the unintegrated vertex corrections. [C. Alexandrou et al., Phys. Rev. D99 (2019) 114504]
This renormalizes the ξ-dependence for $\xi > 1$ and $\xi < 0$.

$$\tilde{Z}^{M\overline{MS}}_{\Gamma_{\gamma^0}}(\xi) = 1 - \frac{\alpha_s}{2\pi} C_F \frac{3}{2}\left(-\frac{1}{\xi}\theta(\xi-1) - \frac{1}{1-\xi}\theta(-\xi)\right) - \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{1}{4} + \frac{5}{2}\right)$$

In z-space:

$$\begin{aligned} Z^{M\overline{MS}}_{\Gamma_{\gamma^0}}(z\mu) &= 1 - \frac{\alpha_s}{2\pi} C_F \left(\frac{3}{2}\ln\left(\frac{1}{4}\right) + \frac{5}{2}\right) \\ &+ \frac{3}{2}\frac{\alpha_s}{2\pi} C_F \left(i\pi\frac{|z\mu|}{2z\mu} - Ci(z\mu) + \ln(z\mu) - \ln(|z\mu|) - iSi(z\mu)\right) \\ &- \frac{3}{2}\frac{\alpha_s}{2\pi} C_F e^{iz\mu}\left(\frac{2Ei(-iz\mu) - \ln(-iz\mu) + \ln(iz\mu) + i\pi Sign(z\mu)}{2}\right). \end{aligned}$$

The above has to modify the conversion factor, i.e. the conversion will be RI$\to \overline{\text{MS}} \to \text{M}\overline{\text{MS}}$.
Consistency check: $z \to 0$ limit:

$$Z^{M\overline{MS}}_{\Gamma_{\gamma^0}}(z\to 0) = 1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\left(\frac{\mu^2 z^2 e^{2\gamma_E}}{4}\right) + \frac{5}{2}\right) = Z^{Ratio}_{\Gamma_{\gamma^0}}(z\mu)$$

Exactly cancels the divergence in $\ln(z)$ present in $\overline{\text{MS}}$!
(consistency with: M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506
and with the "Ratio" scheme of T. Izubuchi et al., Phys. Rev. D98 (2018) 056004)

Matching to light-front PDFs

Another alternative matching ("ratio" scheme):

[T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

$$C\left(\xi, \frac{\mu}{|y|P_3}\right) = \delta(1-\xi) + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 - \frac{3}{2(1-\xi)}\right)_{+(1)} & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi}\left[\ln\frac{y^2 P_3^2}{\mu^2}(4\xi(1-\xi)) - 1\right] + 1 + 2\iota(1-\xi) + \frac{3}{2(1-\xi)}\right)_{+(1)} & 0<\xi<1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi} - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)} & \xi < 0 \end{cases}$$

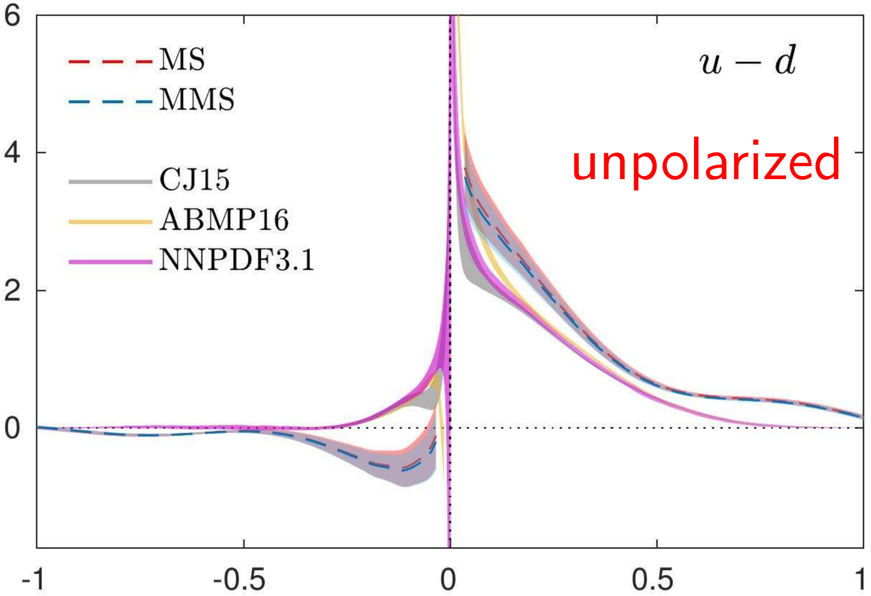
In this scheme, all regions in the ξ-integration of the plus functions (including the "physical" one) contain the same $3/2(1-\xi)$ term and no additional term appears.

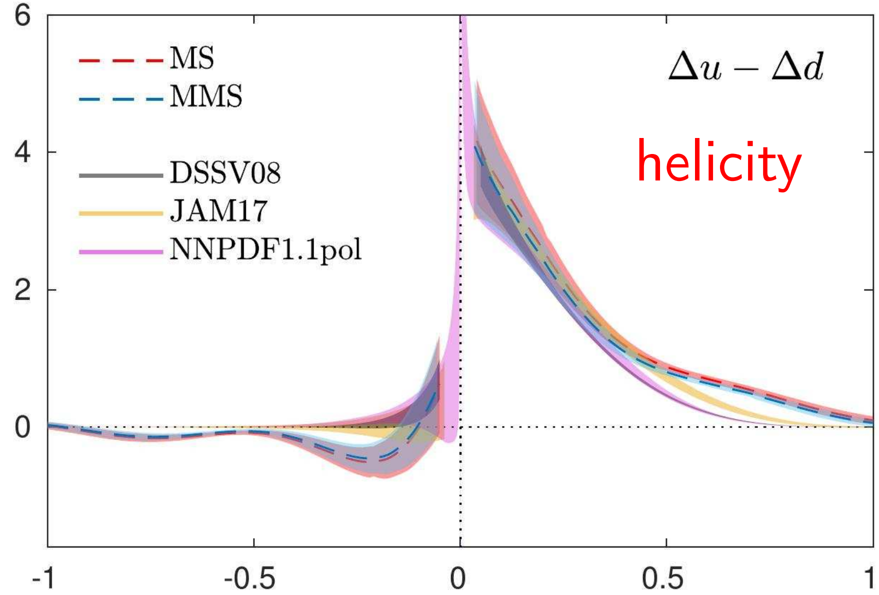
Modification of the perturbative conversion from the intermediate renormalization scheme to $\overline{\mathrm{MS}}$:

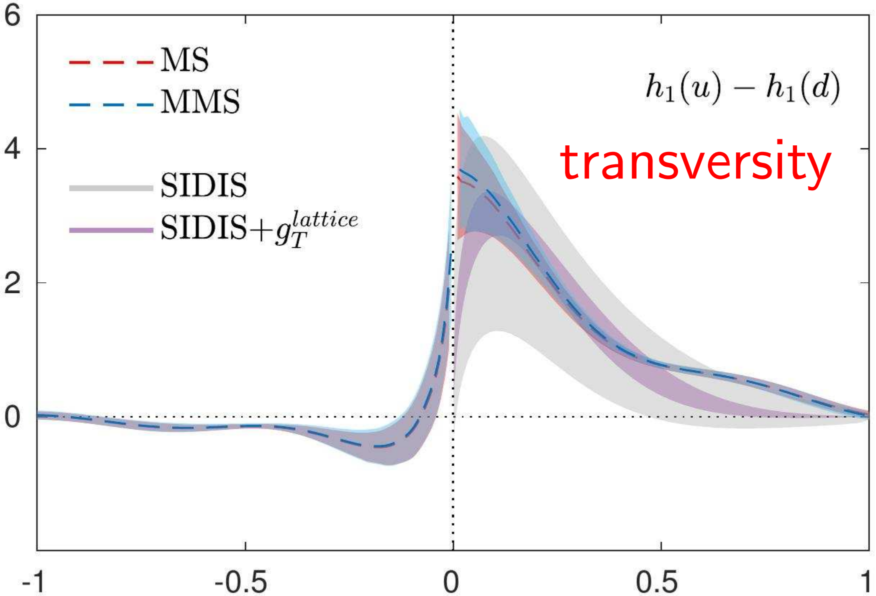
$$C_0(\mu^2 z^2) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{3}{2}\ln(\mu^2 z^2 e^{2\gamma_E}/4) + \frac{5}{2}\right]$$

Caveat: modification of the *physical* ξ-region – potentially large numerical effect.

Effect from M$\overline{\mathrm{MS}}$







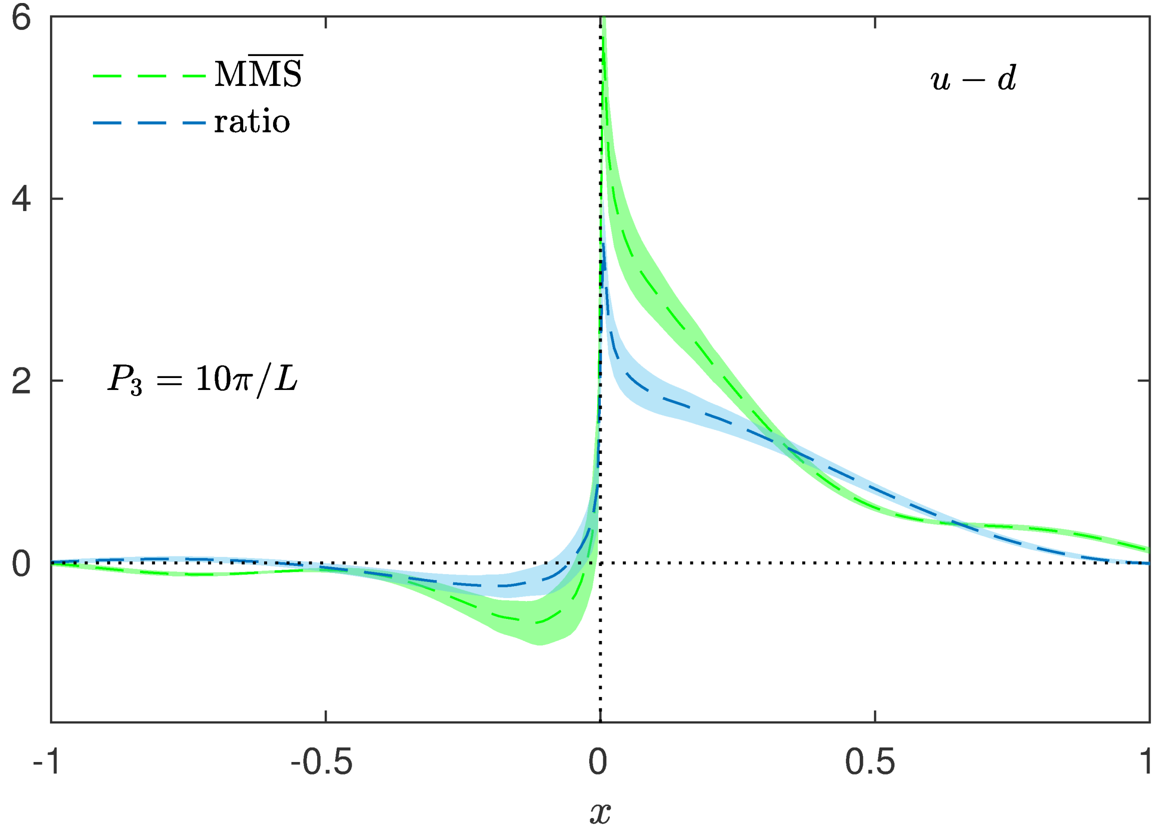
Nucleon momentum $\frac{10\pi}{48}$

As expected, the effect is very small (modification of $\overline{\mathrm{MS}}$ only in unphysical regions)

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

$\mathrm{M\overline{MS}}$ vs. "ratio" scheme

$\mathrm{M\overline{MS}}$ – modification only of the "non-physical" regions $\xi < 0$, $\xi > 1$.
"ratio" – modification also of the "physical" region $0 < \xi < 1$.

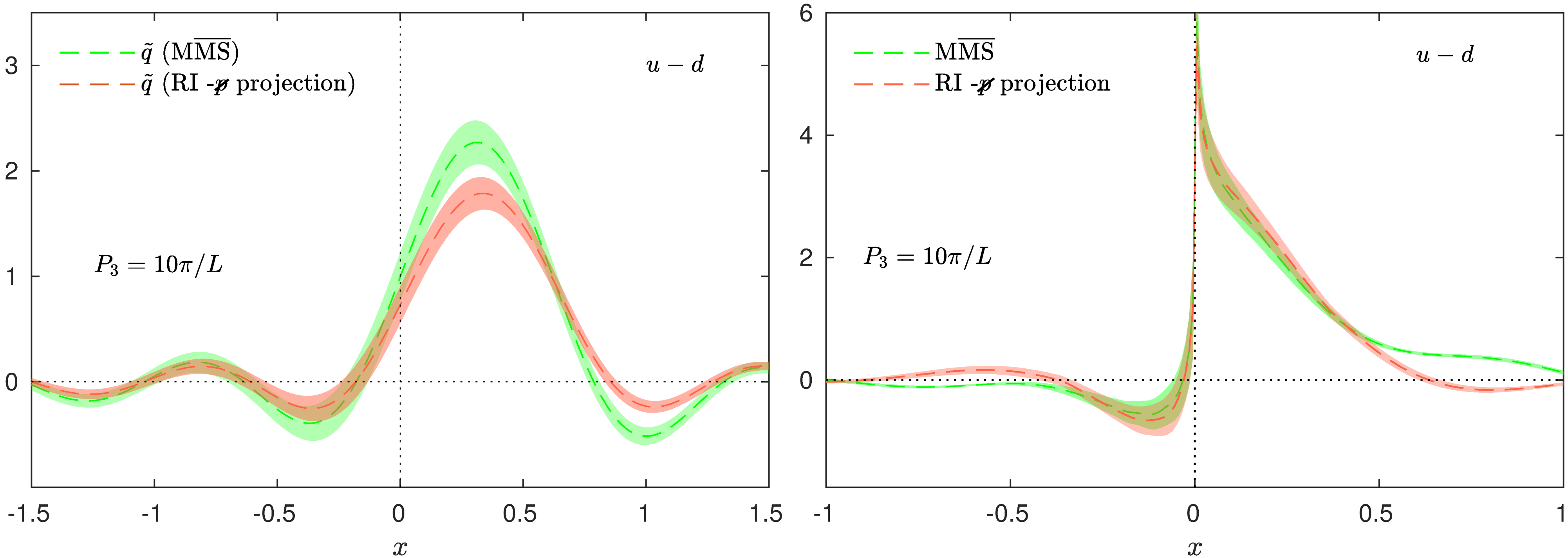


C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$ vs. RI$\to$ $\overline{\mathrm{MS}}$ matching

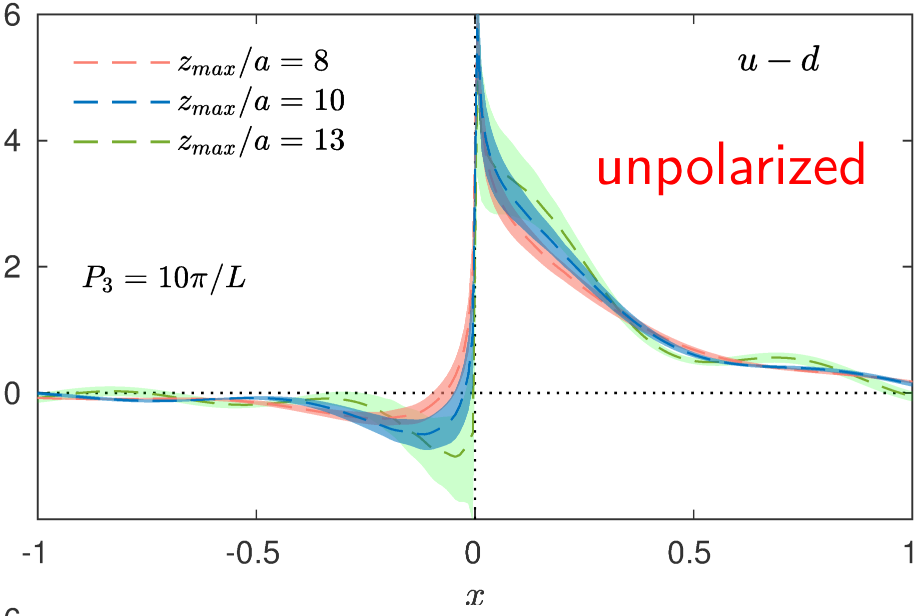
Matching can also be performed directly from the RI scheme to $\overline{\mathrm{MS}}$

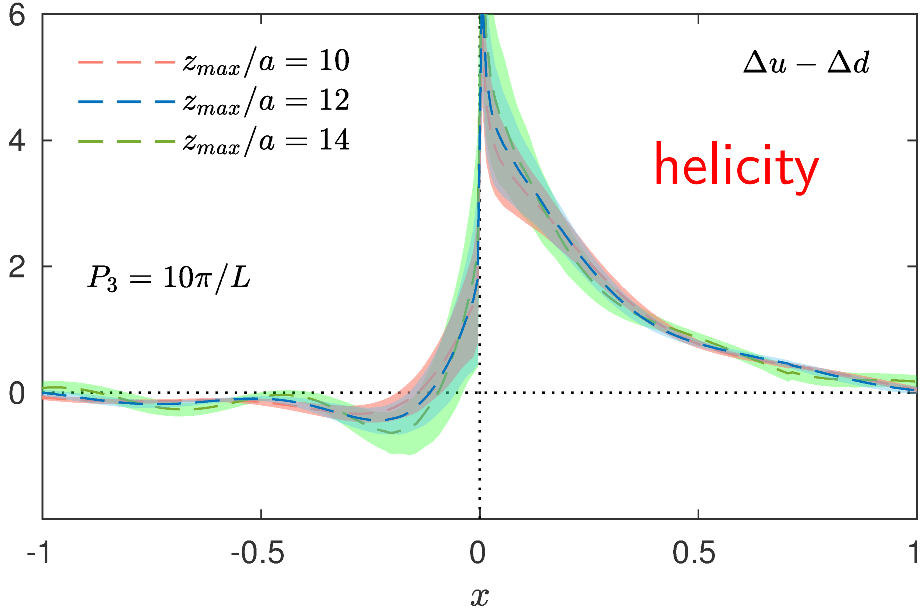
I.W. Stewart, Y. Zhao, Phys. Rev. D97 (2018) 054512

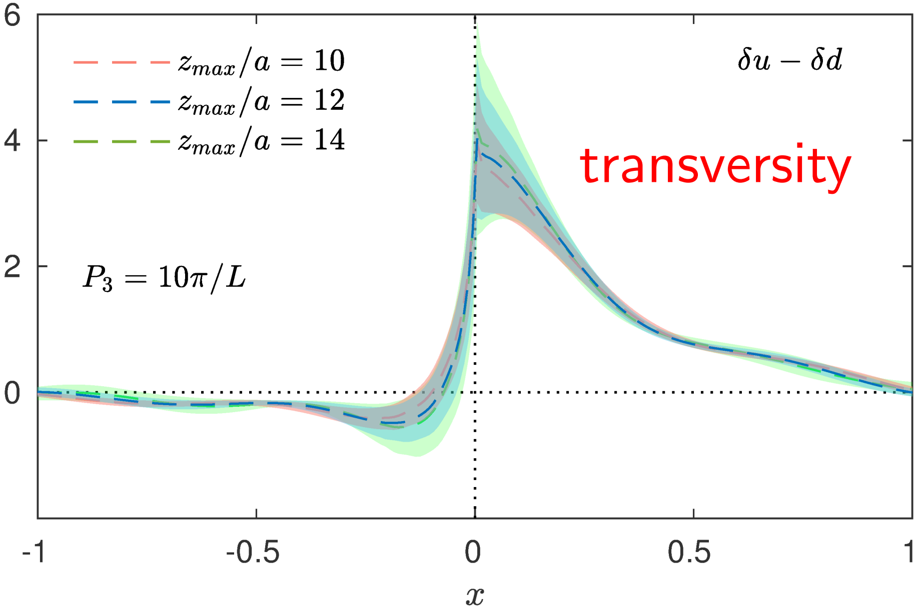


C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Truncation of Fourier transform







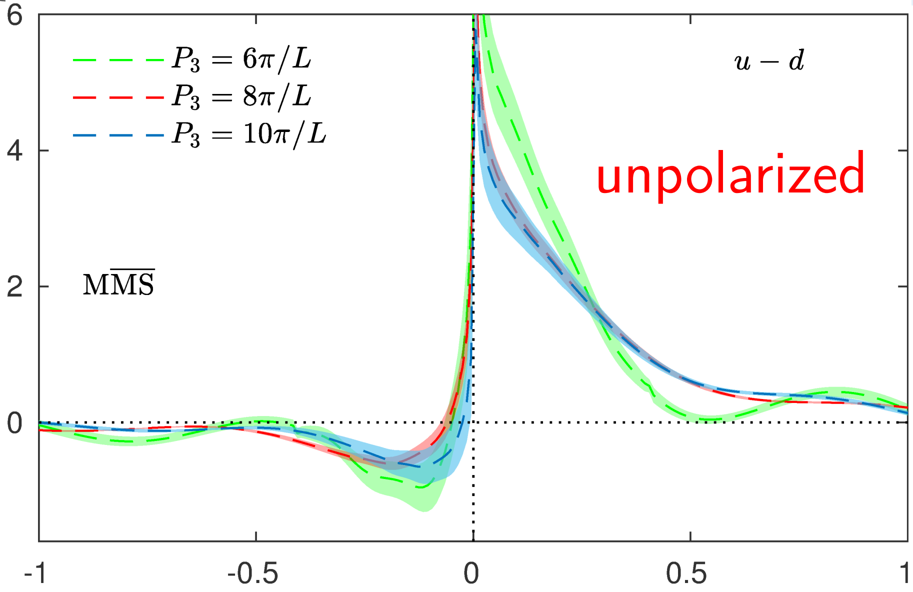
Nucleon momentum $\frac{10\pi}{48}$

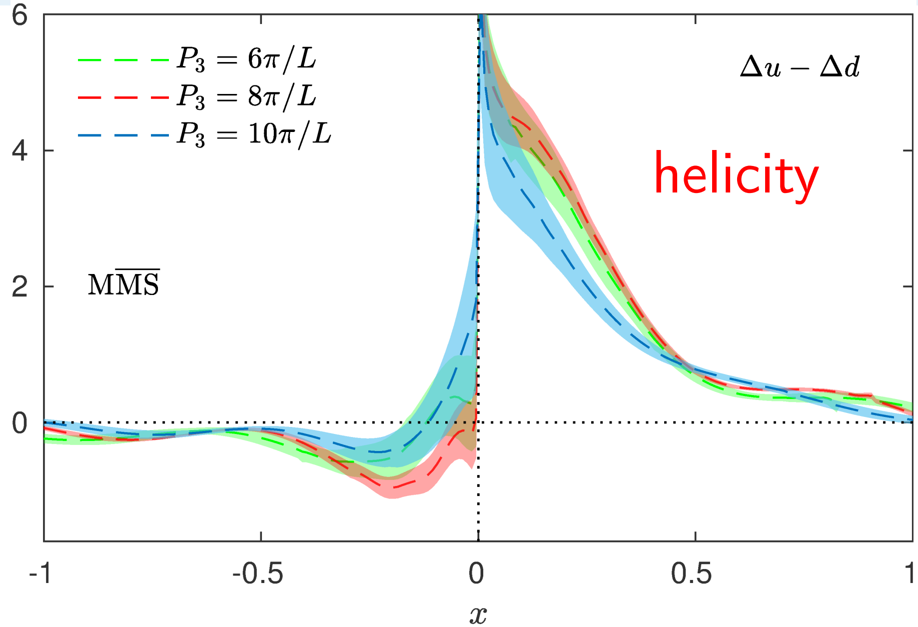
Needs the use of advanced reconstruction techniques

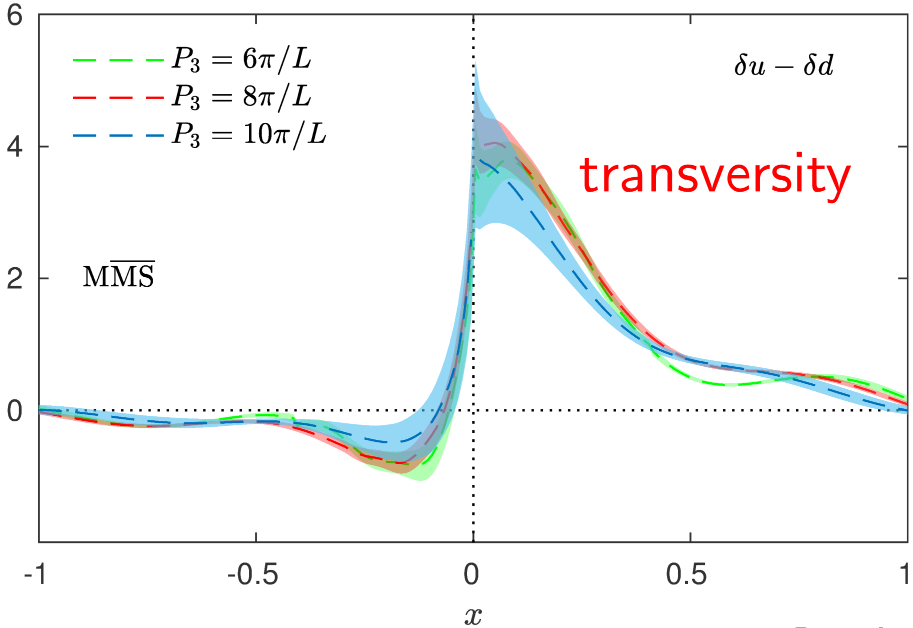
J. Karpie et al., JHEP 1904 (2019) 057

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Momentum dependence of final PDFs







Nucleon momenta $\frac{6\pi}{48}, \frac{8\pi}{48}, \frac{10\pi}{48}$

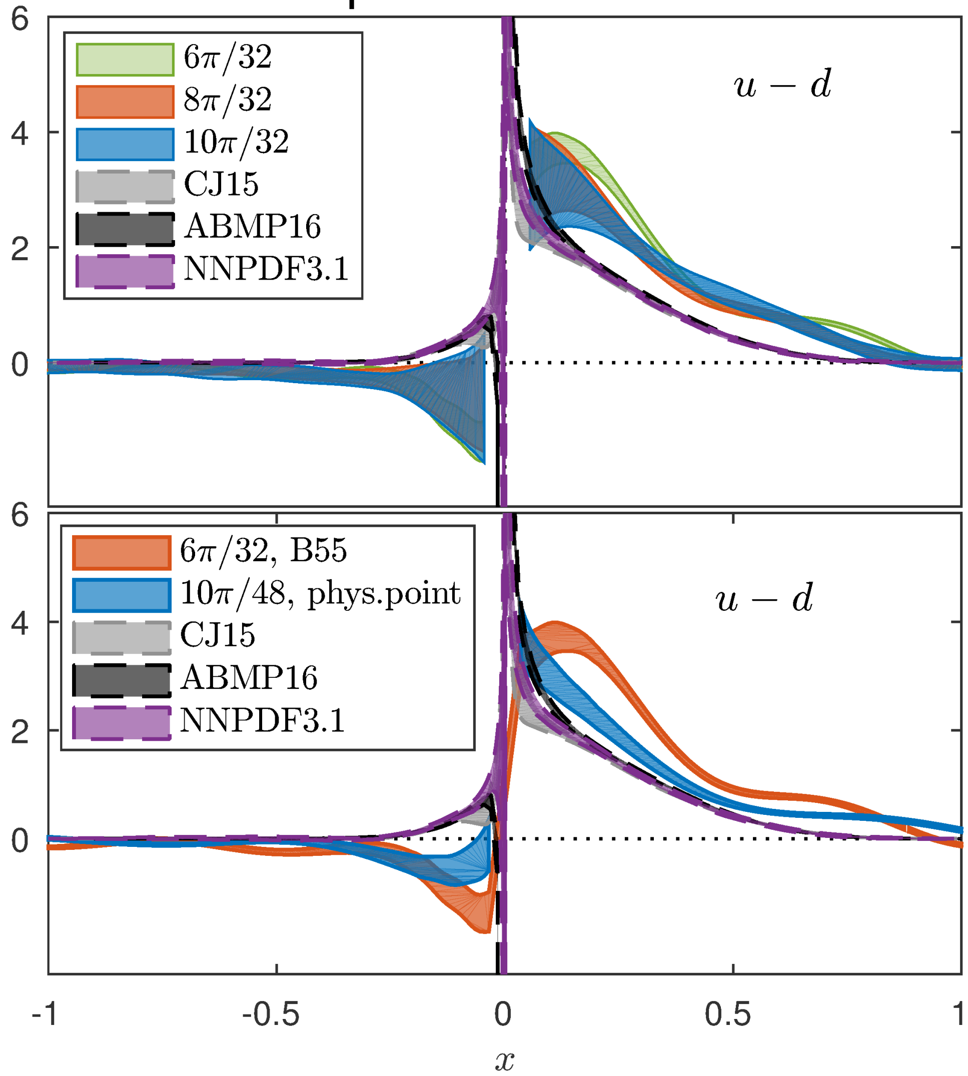
Results seem to indicate convergence in nucleon boost

Expected HTE:
$\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Comparison with non-physical pion mass

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001