Hadronic contributions to the muon anomalous magnetic moment from lattice QCD

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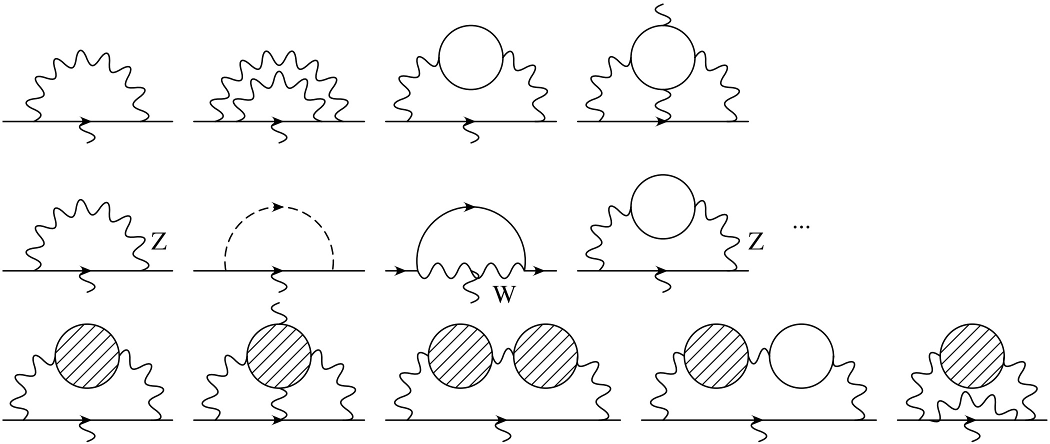
and the RBC/UKQCD collaborations

Outline I

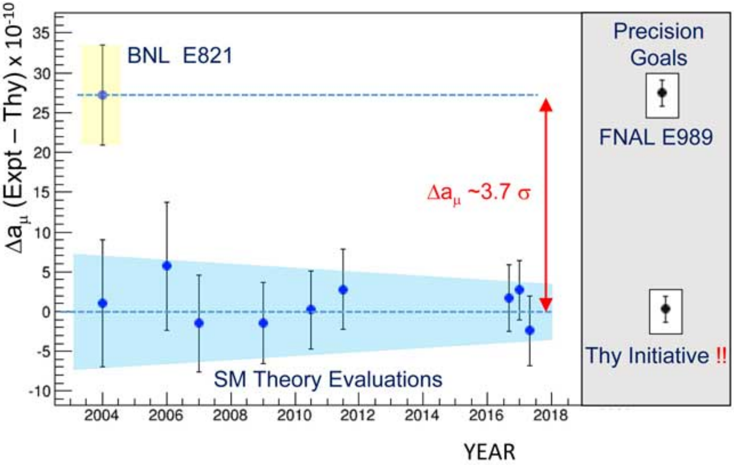
Standard Model Theory: QED+EW+QCD

$$\langle\mu(\vec{p}')|J_\nu(0)|\mu(\vec{p})\rangle = -e\bar{u}(\vec{p}')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right)u(\vec{p})$$

$$a_\mu \equiv (g-2)/2 = F_2(0) \qquad (q = p' - p)$$



Longstanding discrepancy, new experiment, new theory



INT Workshop on Hadronic Contributions to g-2, Seattle, September 2019

Experiment - Theory

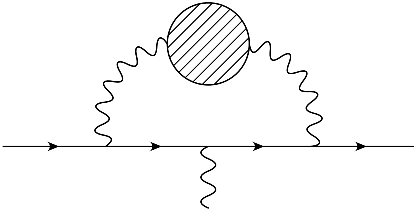
SM Contribution	Value $\pm$ Error $(\times 10^{11})$	Ref	notes
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]	
HVP LO	6931 ± 34	[Davier et al., 2017]	2019 $\rightarrow 3.3\sigma$
	6932.6 ± 24.6	[Keshavarzi et al., 2018]	$\rightarrow 3.7\sigma$
	6925 ± 27	[Blum et al., 2018]	lattice+R-ratio (J17), $\rightarrow 3.7\sigma$
HVP NLO	-98.2 ± 0.4	[Keshavarzi et al., 2018]	
		[Kurz et al., 2014]	
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]	
HLbL	105 ± 26	[Prades et al., 2009]	Glasgow Consensus
HLbL (NLO)	3 ± 2	[Colangelo et al., 2014]	
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]	
SM Tot	116591820.5 ± 35.6	[Keshavarzi et al., 2018]	
Exp (0.54 ppm)	116592080 ± 63	[Bennett et al., 2006]	
Diff (Exp − SM)	259.5 ± 72	[Keshavarzi et al., 2018]	$\rightarrow 3.7\sigma$

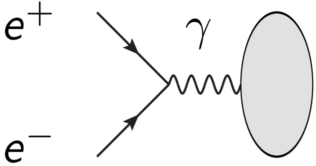
QCD errors dominate, Δ HLbL $\sim$ Δ HVP,
Discrepancy is large

Outline I

HVP contribution to muon g-2 from dispersion relation/data

Use dispersion relation + $e^+e^- \to$ hadrons cross section [Bouchiat and Michel, 1961]





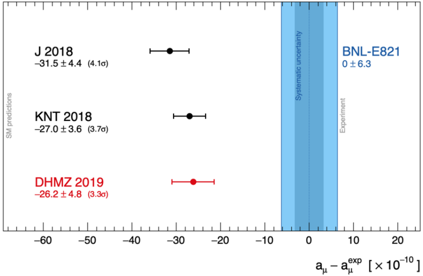
Blob: all possible hadronic states

$$\Im\hat{\Pi}(s) \quad \propto \quad R(s) = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$a_\mu^{\text{HVP}} \quad = \quad \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

HVP contribution to muon g-2 from dispersion relation/data

- DHMZ 2017: $693.1 \pm 3.4 \times 10^{-10}$
- FJ 2018: $688.07 \pm 4.14 \times 10^{-10}$
- DHMZ 2019: $693.9 \pm 4.0 \times 10^{-10}$
- KNT 2018: $693.3 \pm 2.5 \times 10^{-10}$
- BaBar and KLOE dominate $\pi\pi$ channel
- Longstanding discrepancy

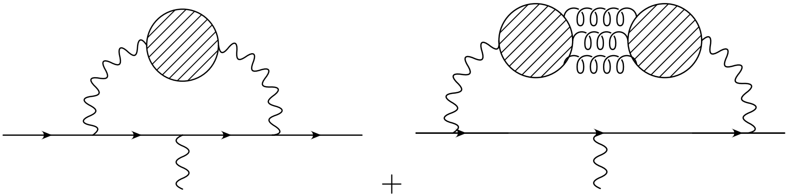


Zhang, *et al.*, EPS 19

$\sqrt{s}$ range [GeV]	a_μ^{had} [10^{-10}] All data	a_μ^{had} [10^{-10}] All but BABAR	a_μ^{had} [10^{-10}] All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{total}$	$505.0 \pm 2.1_{total}$	$510.6 \pm 2.2_{total}$

HVP contribution to muon g-2 from lattice QCD

Use lattice QCD [Blum, 2003, Lautrup et al., 1971]



Blobs: Non-Perturbative quark loops

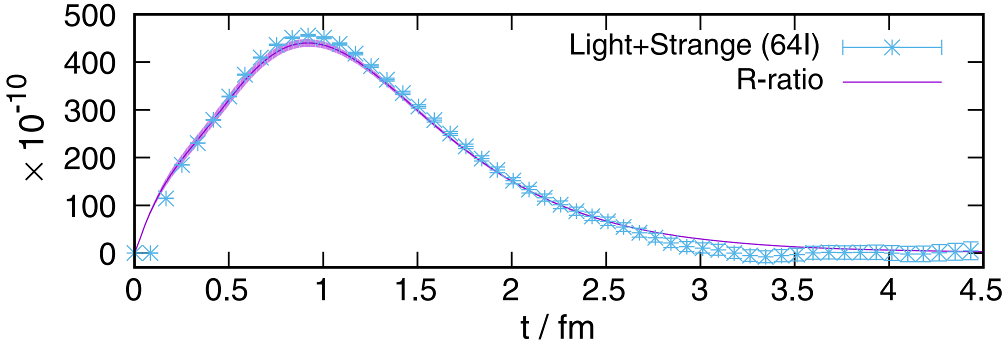
$$\Pi^{\mu\nu}(q) = \int d^4x\, e^{iqx} \langle j_\mu(x) j_\nu(0)\rangle = \hat{\Pi}(q^2)\left(q_\mu q_\nu - q^2\delta_{\mu\nu}\right)$$

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

Time Momentum Representation (TMR) [Bernecker and Meyer, 2011]

$$a_\mu^{\mathrm{HVP}} = \sum_t w(t) C(t), \qquad C(t) = \frac{1}{3}\sum_{i,\vec{x}} \langle j_i(\vec{x}, t) j_i(0)\rangle$$

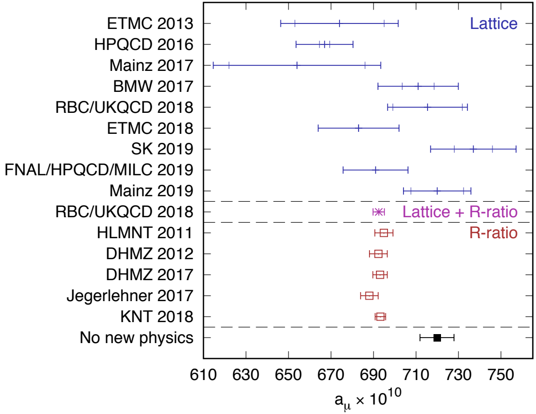
HVP contribution to muon g-2 from lattice QCD



[Blum et al., 2018]

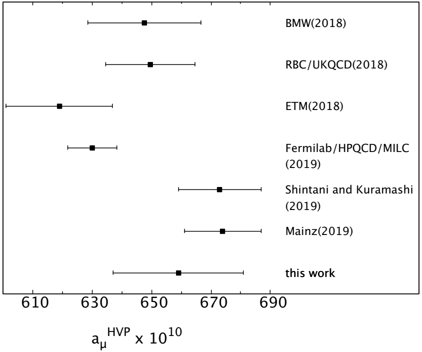
- $a_\mu^{\mathrm{HVP}} = 715.4(16.3)_S(7.8)_V(3.0)_C(1.9)_A(3.2)_{\mathrm{other}} \times 10^{-10} = 715.4(18.7) \times 10^{-10}$
- $O(5-6) \times 10^{-10}$ error by end of year

Summary of HVP theory results



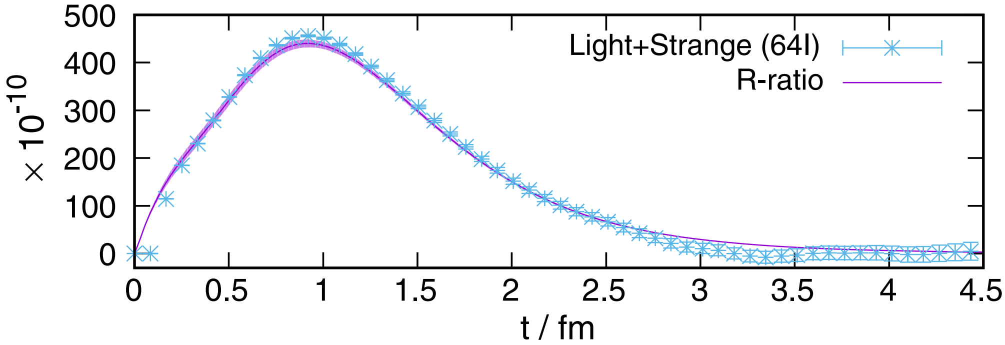
(C. Lehner, Lattice 2019)

u+d conn. contribution. [Aubin et al., 2019]



Comparing and Combing HVP theory results

$$C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t},$$
$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had}).$$



[Blum et al., 2018]

Comparing and Combing HVP theory results

RBC/UKQCD Window Method [Blum et al., 2018]

$$a_\mu = \sum_t w(t)C(t) = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}, \quad (1)$$

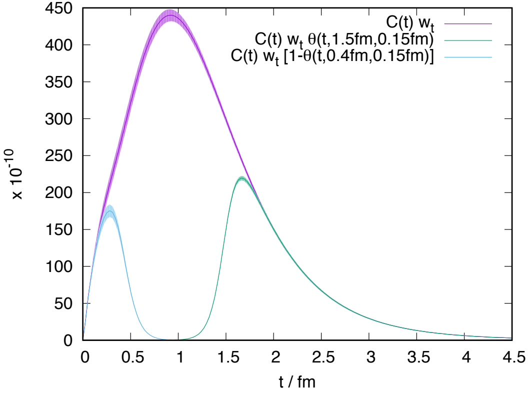
$$a_\mu^{\mathrm{SD}} = \sum_t w(t)C(t)[1 - \Theta(t, t_0, \Delta)], \quad (2)$$

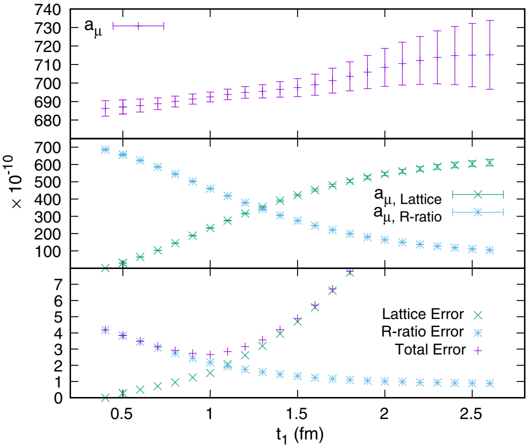
$$a_\mu^{\mathrm{W}} = \sum_t w(t)C(t)[\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)], \quad (3)$$

$$a_\mu^{\mathrm{LD}} = \sum_t w(t)C(t)\Theta(t, t_1, \Delta) \quad (4)$$

$$\Theta(t, t', \Delta) = [1 + \tanh[(t - t')/\Delta]]/2 \quad (5)$$

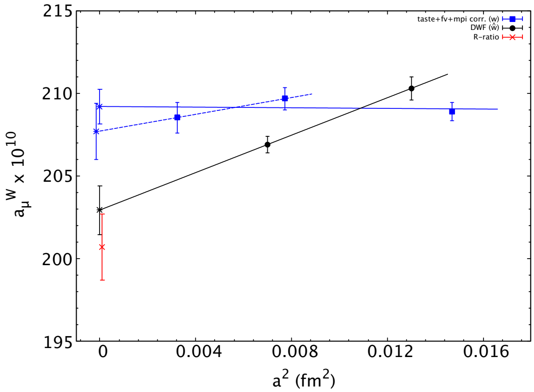
Comparing and Combining HVP theory results





Lattice+R ratio: $a_\mu^{\text{HVP}} = 692.5(2.7) \times 10^{-10}$ [Blum et al., 2018]

Comparing and Combining HVP theory results

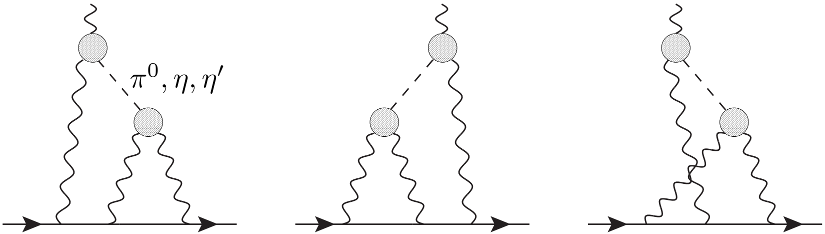


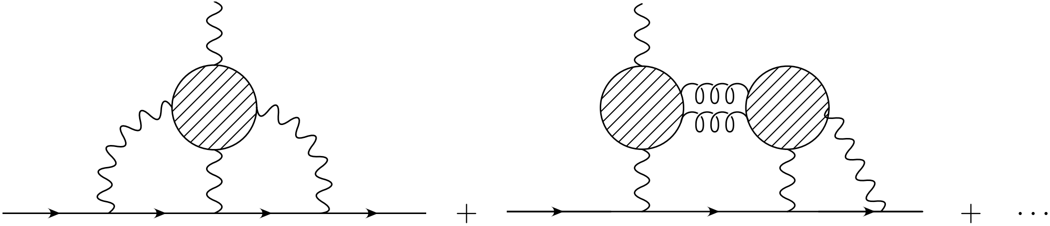
HISQ, DWF, R ratio comparison [Aubin et al., 2019]

$t_0 = 0.4,\ t_1 = 1.0,\ \Delta = 0.15$ fm

Outline I

Hadronic Light-by-Light Scattering





Analytic/Data Approach to HLbL Summary

Contribution	PdRV(09)	N/JN(09)	J(17)	White Paper
π^0, η, η'-poles	114 ± 13	99 ± 16	95.45 ± 12.40	93.8 ± 4.0
π, K-loop/box	-19 ± 19	-19 ± 13	-20 ± 5	-16.4 ± 0.2
S-wave $\pi\pi$	–	–	–	-8 ± 1
scalars	-7 ± 7	-7 ± 2	-5.98 ± 1.20	$\}-2 \pm 3$
tensors	–	–	1.1 ± 0.1	
axials	15 ± 10	22 ± 5	7.55 ± 2.71	8 ± 8
q-loops / SD	2.3	21 ± 3	22.3 ± 5.0	10 ± 10
total	105 ± 26	116 ± 39	100.4 ± 28.2	$85 \pm XX$

HLbL in units of 10^{-11}.

PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N = Nyffeler;

J = Jegerlehner

HLbL contribution to g-2 from lattice QCD+QED

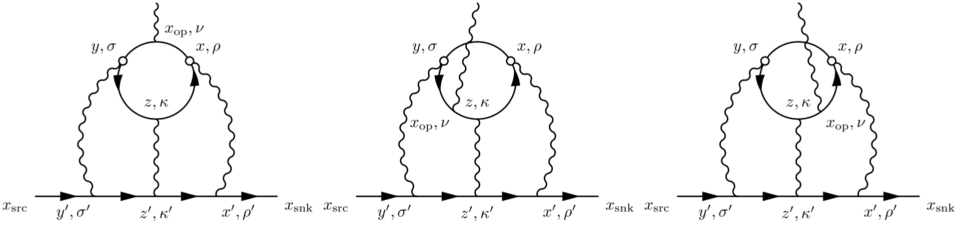
- QED treated in finite volume: QED_L [Blum et al., 2015]
- QED treated in ∞ volume, continuum: QED_∞ [Asmussen et al., 2016]

- RBC results at physical mass, $V \to \infty$, $a \to 0$, QED_L; prelim results for QED_∞
- Mainz and RBC cross-checked at heavy mass, QED_∞

- Mainz computed pion TFF, pion-pole contribution, $V \to \infty$, $a \to 0$
- RBC preliminary results for pion-pole contribution

Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]

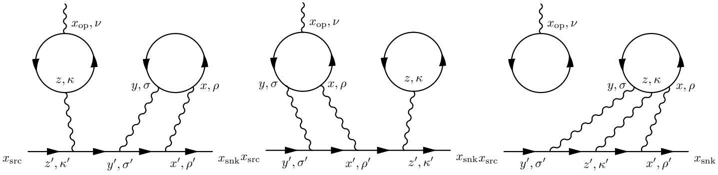
- Importance sample point source propagators at x and y



- Three diagrams together enforce Ward Identity on each configuration
- Moment method allows computation of $F_2(q^2)$ directly at $q = 0$

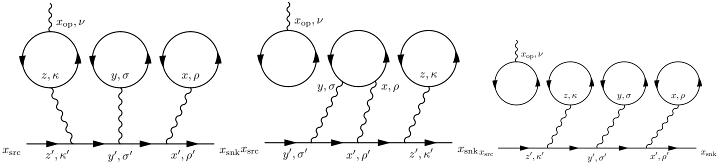
Techniques produce huge improvement in statistical error over original non-perturbative QED method [Blum et al., 2015]

Quark-line disconnected diagrams



Leading

$O(m_s - m_{u,d})$

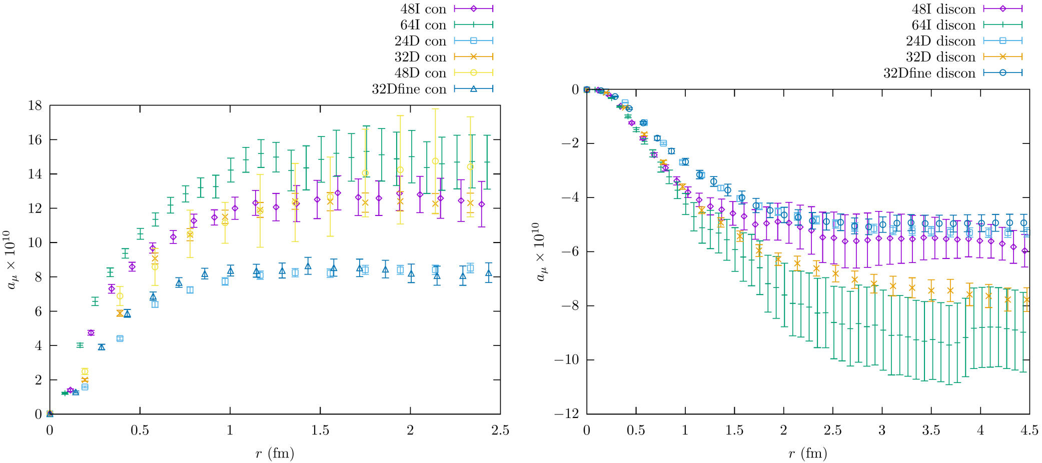


$O(m_s - m_{u,d})^2$

and higher

- only 1 diagram (upper-left) does not vanish in SU(3) flavor limit
- Perms. of internal photons, gluons within and connecting quark loops not shown

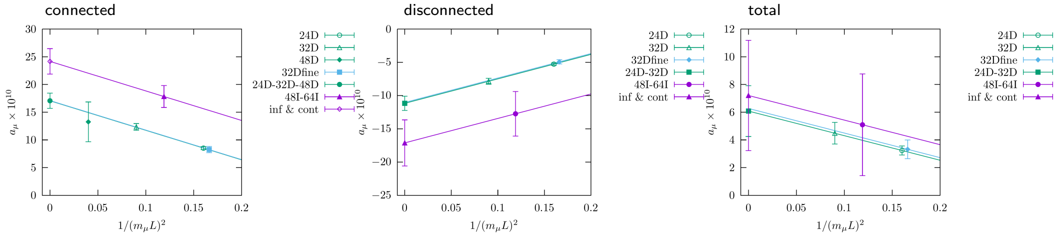
HLbL, QED$_L$, $m_\pi \approx 140$ MeV, $4.8 \lesssim L \lesssim 9.6$ fm, $1 \lesssim a^{-1} \lesssim 2.3$ GeV



- Cumulative sum up to distance r, max between sampled points

HLbL, QED$_L$, $m_\pi \approx 140$ MeV, ∞ Volume and $a \to 0$ limits

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\left(1 - \frac{b_1}{(m_\mu L)^2} - c_1(a^{\mathrm{I}})^2 - c_1(a^{\mathrm{D}})^2 + c_2(a^{\mathrm{D}})^4\right)$$

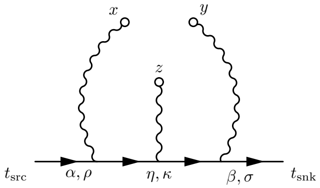


(RBC, preliminary)

$$a^{\mathrm{cHLbL}} = 24.16(2.30)(5.10) \times 10^{-10}$$
$$a^{\mathrm{dHLbL}} = -17.12(3.46)(4.37) \times 10^{-10}$$
$$a^{\mathrm{HLbL}} = 7.2(4.0)(1.7) \times 10^{-10}$$

Infinite volume QED$_\infty$ [Asmussen et al., 2016, Blum et al., 2017]

QCD in finite volume, QED in ∞ volume, continuum (*c.f.* HVP contribution)

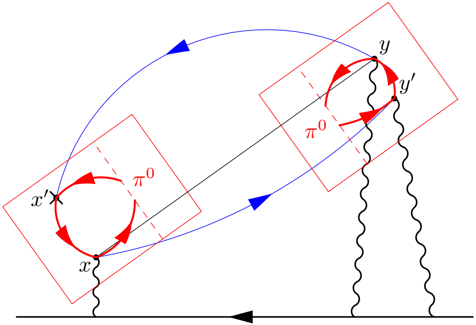


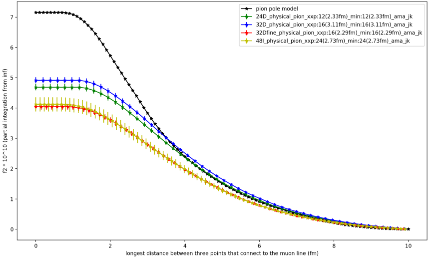
analytic integral, computed numerically for each triplet x, y, z

- Subtract terms that vanish as $a \to 0$, $L \to \infty$ to reduce $O(a^2)$ errors [Blum et al., 2017]
- Leading FV error is exponentially suppressed (*c.f.* HVP) instead of $O(1/L^2)$

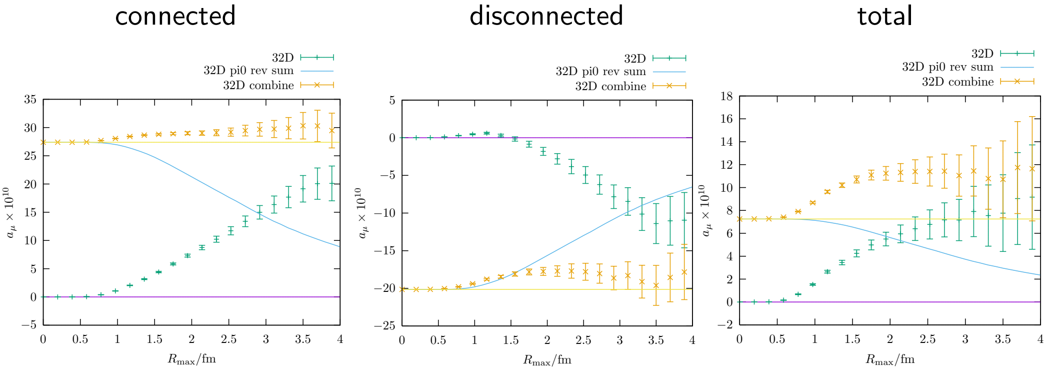
Pion pole from lattice (Cheng Tu) N. Christ, Muon Theory g-2 Initiative HLbL Workshop, UConn 2018

- Long distance part computed in position space on lattice QCD, $\langle J_\mu(x) J_\nu(x')|\pi\rangle$





HLbL, QED$_\infty$+π^0-pole (LMD), $m_\pi = 142$ MeV, $a = 0.2$ fm, $L = 6.4$ fm

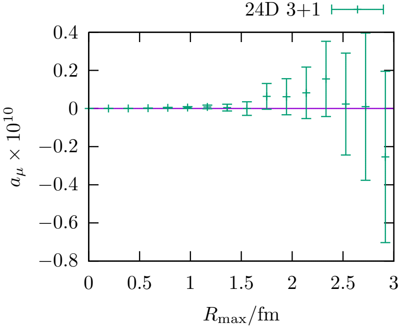


(RBC, preliminary)

At 2.5 fm, the combination gives $a_\mu = 11.47 \pm 1.27_{stat} \times 10^{-10}$

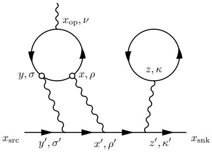
Non-leading disconnected contribution to HLbL, QED$_\infty$ (preliminary)

RBC



- Partial sum upto $R_{\max}$

  $R_{\max} = \max(|x-y|, |x-z|, |y-z|)$



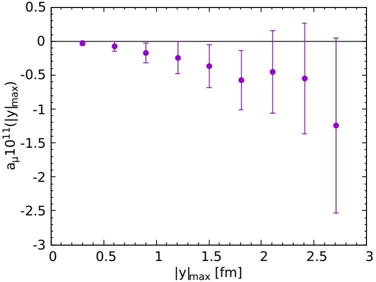
- 24D: $24^3 \times 64$

  $L = 4.8$ fm

- $a^{-1} = 1.015$ GeV

  $M_\pi = 142$ MeV

  $M_K = 512$ MeV



Mainz (220 MeV pion)

negligible contribution compared to error on leading contributions

Outline I

Lattice calculations crucial for Standard Model test with experiment (FNAL E989, J-PARC J34)

White paper with new consensus theory value by end of year

Acknowledgments

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Outline I

Aoyama, T., Hayakawa, M., Kinoshita, T., and Nio, M. (2012).
Complete Tenth-Order QED Contribution to the Muon g-2.
Phys.Rev.Lett., 109:111808.

Asmussen, N., Green, J., Meyer, H. B., and Nyffeler, A. (2016).
Position-space approach to hadronic light-by-light scattering in the muon $g-2$ on the lattice.
PoS, LATTICE2016:164.

Aubin, C., Blum, T., Tu, C., Golterman, M., Jung, C., and Peris, S. (2019).
Light quark vacuum polarization at the physical point and contribution to the muon $g-2$.

Bennett, G. et al. (2006).
Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL.
Phys.Rev., D73:072003.

Bernecker, D. and Meyer, H. B. (2011).
Vector Correlators in Lattice QCD: Methods and applications.
Eur.Phys.J., A47:148.

Blum, T. (2003).

Lattice calculation of the lowest order hadronic contribution to the muon anomalous magnetic moment.

Phys.Rev.Lett., 91:052001.

Blum, T., Boyle, P. A., Glpers, V., Izubuchi, T., Jin, L., Jung, C., Jttner, A., Lehner, C., Portelli, A., and Tsang, J. T. (2018).

Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment.

to be published, Phys. Rev. Lett.

Blum, T., Chowdhury, S., Hayakawa, M., and Izubuchi, T. (2015).

Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment from lattice QCD.

Phys.Rev.Lett., 114(1):012001.

Blum, T., Christ, N., Hayakawa, M., Izubuchi, T., Jin, L., Jung, C., and Lehner, C. (2017).

Using infinite volume, continuum QED and lattice QCD for the hadronic light-by-light contribution to the muon anomalous magnetic moment.

Blum, T., Christ, N., Hayakawa, M., Izubuchi, T., Jin, L., and Lehner, C. (2016).
Lattice Calculation of Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment.
Phys. Rev., D93(1):014503.

Bouchiat, C. and Michel, L. (1961).
La rsonance dans la diffusion mson ?? mson ? et le moment magntique anormal du mson ?
J. Phys. Radium, 22(2):121–121.

Colangelo, G., Hoferichter, M., Nyffeler, A., Passera, M., and Stoffer, P. (2014).
Remarks on higher-order hadronic corrections to the muon g?2.
Phys. Lett., B735:90–91.

Davier, M., Hoecker, A., Malaescu, B., and Zhang, Z. (2017).
Reevaluation of the hadronic vacuum polarisation contributions to the Standard Model predictions of the muon $g-2$ and $\alpha(m_Z^2)$ using newest hadronic cross-section data.

Eur. Phys. J., C77(12):827.

Gnendiger, C., Stckinger, D., and Stckinger-Kim, H. (2013).
The electroweak contributions to $(g-2)_\mu$ after the Higgs boson mass measurement.
Phys.Rev., D88:053005.

Keshavarzi, A., Nomura, D., and Teubner, T. (2018).
Muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis.
Phys. Rev., D97(11):114025.

Kurz, A., Liu, T., Marquard, P., and Steinhauser, M. (2014).
Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order.
Phys.Lett., B734:144–147.

Lautrup, B., Peterman, A., and De Rafael, E. (1971).
On sixth-order radiative corrections to a(mu)-a(e).
Nuovo Cim., A1:238–242.

Prades, J., de Rafael, E., and Vainshtein, A. (2009).
Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment.