Quasi-GPDs using twisted mass fermions

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Outline

Introduction

Quasi-GPD approach

• Exploratory study of nucleon GPDs

Summary

Exploratory study of GPDs 0000000

Generalized parton distributions (GPDs)

- D. Muller, A. Radyushkin, X. Ji (1994-1997)
- Wealth of information contained in GPDs:
 - ► form factors and PDFs
 - longitudinal and transverse distribution of partons in a hadron
 - orbital angular momentum of quarks and gluons



• Experimentally accessed from e.g. DVCS and DVMP, but with uncertainties

[See e.g. [K. Kumericki et al., Eur. Phys. J. A52 6 (2016) 157, arXiv:1602.02763]]

- limited coverage of kinematic region
- independent measurements to disentangle different GPDs





DVCS: $ep \rightarrow ep\gamma$

Introduction		
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Definitions of GPDs

Matrix elements of quarks and gluons operators at a light-like separation

$$F^{q} = \frac{1}{2} \int \frac{dz^{-}}{2\pi} e^{ixP^{+}z^{-}} \langle P_{2} | \bar{\psi}(0) \gamma^{+} e^{ig \int_{0}^{z} dx' A^{+}} \psi(z) | P_{1} \rangle |_{z^{+}=0, \vec{z}=0}$$

$$= \frac{1}{2P^{+}} \left[H^{q}(x,\xi,t) \bar{u}(P_{2}) \gamma^{+} u(P_{1}) + E^{q}(x,\xi,t) \bar{u}(P_{2}) \frac{i\sigma^{+\alpha} \Delta_{\alpha}}{2m} u(P_{1}) \right]$$

Notation:

- $P^+ = \frac{P_1^+ + P_2^+}{2}$: momentum boost
- $t = \Delta^2 = (P_2 P_1)^2$
- $\xi = -\frac{\Delta^+}{2P^+}$: skewness



- In the forward limit $(P_1 = P_2)$, GPDs reduce to parton densities : $H^q(x, 0, 0) = q(x)$
- Form factors are moments of GPDs, e.g. $\int_{-1}^{+1} dx H^q(x,\xi,t) = F_1(t)$

How can we study GPDs?

- GPDs are non-perturbative quantities \Rightarrow Lattice QCD ?
- Light-cone dominance prevents a direct implementation of GPDs definition
- GPDs mostly studied through their moments (GFFs) and form factors (F_1 and F_2) [see Talk by G. Koutsou on Wed.]
- A tower of moments is needed to extract x-dependence, else information about longitudinal structure is lost
- New fields are growing up:

 quasi-GPDs
 [X.-D. Ji et al., Phys.Rev. D92 (2015) 014039]

 pseudo-GPDs
 [A. Radyushkin, arXiv:1909.08474 [hep-ph]]

 lattice cross-sections with momentum transfer

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lattice cross-sections with momentum transfer

Extensions of approaches used to study PDFs

[see Talk by K. Cichy on Thu.]

Quasi-GPDs in a nutshell

• Quasi-GPDs extracted from purely spatial matrix elements of fast moving hadrons

 $\tilde{q}_{\Gamma}^{GPD}(x,\xi,t,P_3,\mu^2) = \int \frac{dz}{4\pi} e^{-ixP_3 z} \langle H(P_3 \hat{z} + \vec{Q}/2) | \, \bar{\psi}(z) \Gamma W(z,0) \psi(0) | \, H(P_3 \hat{z} - \vec{Q}/2) \rangle_{\mu^2}$

* W(z): Wilson line of lenght z* P_3 : hadron momentum boost (finite) * $t = \Delta^2 = -Q^2$ $\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_3^2}, \frac{t}{P_3^2}, \frac{m_X^2}{P_3^2}\right)$

*
$$\tilde{\xi} = -\frac{Q_3}{2P_3}$$
: quasi-skewness $\tilde{\xi} = \xi + O(1/P_3^2)$



• Quasi-GPDs matched onto GPDs within LaMET framework

$$\tilde{q}_{\Gamma}^{GPD} = \int_{-1}^{1} \frac{dy}{|y|} \, \frac{\mathcal{C}_{\Gamma}}{\mathcal{C}_{\Gamma}} \, q_{\Gamma}^{GPD}(x,\xi,t,P_{3},\mu^{2}) + \mathcal{O}\left(\frac{\Lambda_{QCD}^{2}}{P_{3}^{2}},\frac{t}{P_{3}^{2}},\frac{m_{N}^{2}}{P_{3}^{2}}\right)$$

- $\star \ \mathcal{C}_{\Gamma}$ independent on Q^2
- $\star \ \mathcal{C}_{\Gamma}$ reduces to the one of PDFs for $\xi=0$

[X.Ji et al., Phys.Rev. D92 (2015) 014039]
 [X.Xiong, J-H. Zhang., Phys.Rev. D92 (2015) no.5, 054037]
 [Y-S. Liu et al., Phys.Rev. D100 (2019) no.3, 034006]

	Exploratory study of GPDs	
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Lattice setup

Gauge ensemble

• Configurations of $N_f = 2 + 1 + 1$ flavors & clover term [ETMC collaboration]

Ensemble	N_f	$V(L^3 \times T)$	lattice spacing a	m_{π}	$m_{\pi}L$
cA211.32	4	$32^3 \times 64$	0.093 fm	270 MeV	4



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Computations of GPDs in a Euclidean lattice	Exploratory study of GPDs	
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Nucleon momenta

Breit frame	aP_3	P_3 [GeV]	aP_i	aP_f	$Q^2 = -t \; [\mathrm{GeV}^2]$	ξ	N_{meas}
	2	0.83	(0,-1,2)	(0,1,2)	0.69	0	1600
$\vec{P_i} = P_3 \hat{z} - \vec{Q}/2$	3	1.25	(0,-1,3)	(0,1,3)	0.69	0	7520
n n n n n n n n n n	1.5	0.62	(0,-1,1)	(0,1,2)		1/3 -	1600
$P_f = P_3 z + Q/2$	2.5	1.04	(0,-1,3)	(0,1,2)	0.87	+1/5	1600

more will be added

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• Classes chosen such that:

- **I** different GPDs can be disentangled $(H(x,t,\xi), E(x,t,\xi), \tilde{H}(x,\xi,t), \ldots)$
- 2 P_3 -dependence and magnitude of the skewness can be studied

	Exploratory study of GPDs	
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• Matrix elements $\langle H(P_3\hat{z}+\vec{Q}/2)|\,\bar{\psi}(z)\Gamma W(z,0)\psi(0)|\,H(P_3\hat{z}-\vec{Q}/2)\rangle$ extracted from

$$\gamma^{2pt}(\Gamma_0, \vec{p}, t_f, t_i) = \sum_{\vec{x}_f} e^{-i(\vec{x}_f - \vec{x}_i) \cdot \vec{p}} \mathsf{Tr}[\Gamma_0 \langle \mathcal{I}_N(\vec{x}_f, t_f) \bar{\mathcal{I}}_N(\vec{x}_i, t_i) \rangle]$$

$$\begin{split} \Gamma_0 &= \frac{(1+\gamma_0)}{4} \\ \Gamma_k &= \frac{(1+\gamma_0)}{4} \gamma_5 \gamma_k, \ k = 1, 2, 3 \end{split}$$

$$C^{3pt}(\Gamma_k;\vec{p}_f,t_f;\vec{p}_i,t_i) = \sum_{\vec{x}_f,\vec{x}} e^{-i(\vec{x}_f-\vec{x})\cdot\vec{p}_f} e^{-i(\vec{x}-\vec{x}_i)\cdot\vec{p}_i} \operatorname{Tr}[\Gamma_k \langle \mathcal{I}_N(\vec{x}_f,t_f)\mathcal{O}(\vec{x},t,z) \bar{\mathcal{I}}_N(\vec{x}_i,t_i) \rangle]$$

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Lattice methods:

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Lattice methods:

• Sequential inversions through the sink $(T_{sink} = 12a \simeq 1.13 \text{ fm})$

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Lattice methods:

- Sequential inversions through the sink $(T_{sink} = 12a \simeq 1.13 \text{ fm})$
- Momentum smearing

$$S_{mom} = \frac{1}{1+6\alpha} \left(\psi(x) + \alpha \sum_{j=\pm 1}^{\pm 3} U_j(x) e^{i\vec{\xi} \cdot \vec{p}} \psi(x+\hat{j}) \right)$$

phase kept parallel to the nucleon momenta (at the source & sink)

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Bare unpolarized matrix elements of quasi-GPDs

• To disentangle $E(x, \xi, t)$ and $H(x, \xi, t)$, two matrix elements are needed:

$$\begin{split} M(\gamma_0, \Gamma_0) &= \mathcal{K}_H(P_i, P_f, \Gamma_0) H(z, t, \xi, t) + \mathcal{K}_E(P_i, P_f, \Gamma_0) E(z, t, \xi, t) \\ \hline M(\gamma_0, \Gamma_2) &= \mathcal{K}'_H(P_i, P_f, \Gamma_2) H(z, t, \xi, t) + \mathcal{K}'_E(P_i, P_f, \Gamma_2) E(z, t, \xi, t) \\ \end{split}$$



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- Faster approach to zero as P₃ increases
- Increased noise at P₃ = 1.25 GeV (≃ 4 times statistics than P₃ = 0.83 GeV)





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- Faster approach to zero as P_3 increases
- Increased noise at P₃ = 1.25 GeV (≃ 4 times statistics than P₃ = 0.83 GeV)



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 $-t = 0.87 \text{ GeV}^2$

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- Faster approach to zero as P₃ increases
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- $\operatorname{Im}[H]$ sensitive to the magnitude of P_3 and ξ
- Real and imaginary parts play a crucial role upon Fourier transform

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Unpolarized quasi-GPDs

Upon Fourier transform



• Quasi-H and -E affected differently on the momentum boost

- quasi-H is compatible within errors
- quasi-E becomes symmetric in x (larger momenta will shed light on the behavior of the quasi-E)

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Unpolarized quasi-GPDs

Upon Fourier transform

 $\tilde{q}^{GPD} = \frac{2P_3}{4\pi} \sum_{z=-z_{\text{max}}}^{z=z_{\text{max}}} e^{-ixP_3 z} M E^{GPD}(z,t,\xi)$ $\xi = 0 \qquad \qquad \xi = 0$



• Quasi-H and -E affected differently on the momentum boost

quasi-H is compatible within errors

 quasi-E becomes symmetric in x (larger momenta will shed light on the behavior of the quasi-E) Still non-physical results, matching is needed

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 $\bullet~$ Matching affects both H and E largely

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Momentum dependence on $H(x,\xi,t)$ and $E(x,\xi,t)$, at $-t=0.69~{\rm GeV}^2$



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- H-GPD suppressed with respect to the PDF, as expected

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- H-GPD suppressed with respect to the PDF, as expected
- Remarkable P3-dependence in E-GPD
- $E\mbox{-}{\rm GPD}$ becomes symmetric in x at the larger P_3

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Momentum dependence on $H(x, \xi, t)$ and $E(x, \xi, t)$, at $-t = 0.69 \text{ GeV}^2$



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- H-GPD suppressed with respect to the PDF, as expected
- Remarkable P₃-dependence in E-GPD
- $\bullet \ E\mbox{-}{\rm GPD}$ becomes symmetric in x at the larger P_3
- ★ Lattice results will be compared to global fits to DVCS data

	Conclusions
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	Conclusions
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Exploratory study of nucleon GPDs

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	Conclusions
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Exploratory study of nucleon GPDs

- $\bullet~$ Computation performed using $N_f=2+1+1$ twisted mass fermions, at $m_\pi=270~{\rm MeV}$
- Quasi-distribution approach extended to quasi-GPDs (pion GPDs studied in [J-W. Chen et al., arXiv:1904.12376])
- More laborious calculation than for PDFs
 - more variables to control
 - signal-to-noise ratio worsens for non-zero momentum transfer

	Conclusions
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Exploratory study of nucleon GPDs

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Future perspectives

- increasing statistics for the kinematic setups already considered
- extending the kinematic coverage in (P_3, ξ, t) space
- testing theoretical properties of GPDs, e.g. $H(x, -\xi, t) = H(x, \xi, t), \ldots$
- studying excited states contamination (other T_{sink} must be added)
- applying advanced reconstruction methods for the Fourier transform [J. Karpie et al., JHEP 1904 (2019) 057]

Thank you very much

for your attention.

Speaker: A. Scapellato

Paphos, 29 October 201

Helicity GPDs ($\xi = 0$)

At $\xi=0$ only \tilde{H} survives



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