13th European Research Conference on Electromagnetic Interactions with Nucleons and Nuclei

27 October – 02 November 2019

Paphos, Cyprus

Workshop 1 – Distribution functions: Lattice QCD meets phenomenology

Overview of Direct Evaluation of Parton Distribution Functions

Jianwei Qiu

Theory Center, Jefferson Lab

See: *"Parton distributions and lattice QCD calculations: a community white paper"*
H.W. Lin et al. Prog. Part. Nucl. Phys. 100, 107 (2018), arXiv: 1711.07916
"A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results"
K. Cichy and M. Constantinou, Adv. High Energy Phys. 2019, 3036904 (2019), arXiv:1811.07248
Plenary talk by K. Cichy on Thursday afternoon, ...

Jefferson Lab

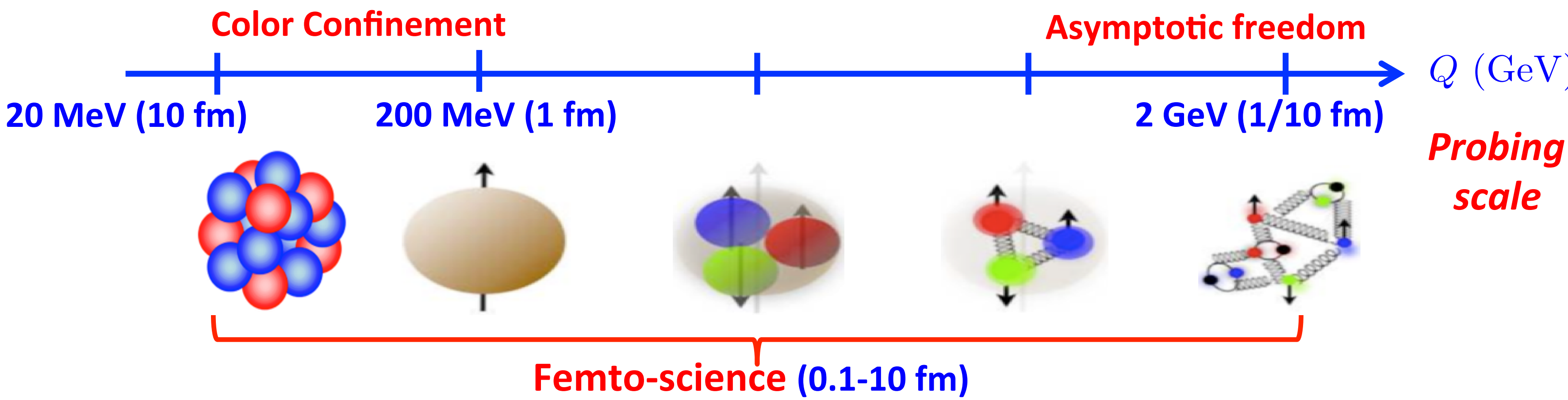
TMD Collaboration

U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

Hadron Structure: Emergent Phenomena of QCD

- QCD landscape of nucleon and nuclei?



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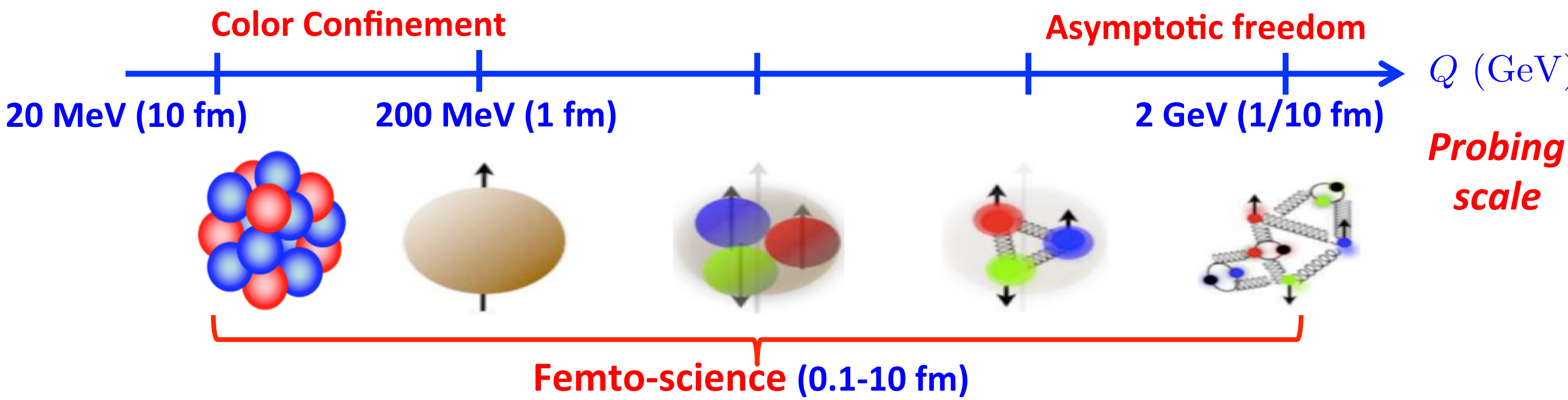
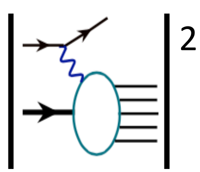


- Any cross section with identified hadron(s) is NOT perturbatively calculable!

$$\sigma_{\text{DIS}}(x, Q^2) =$$



Hadron Structure: Emergent Phenomena of QCD

- **QCD landscape of nucleon and nuclei?**

Color Confinement | Asymptotic freedom

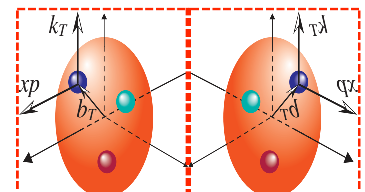
Q (GeV)

20 MeV (10 fm) | 200 MeV (1 fm) | 2 GeV (1/10 fm)

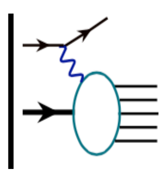
Probing scale

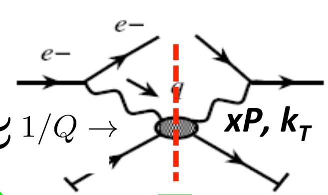
Femto-science (0.1-10 fm)

- **Any cross section with identified hadron(s) is NOT perturbatively calculable!**

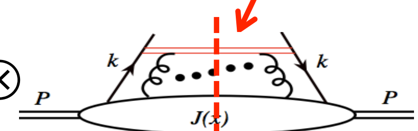


$$\sigma_{\rm DIS}(x,Q^2) = \left| \right|^2 \approx$$





$\otimes$



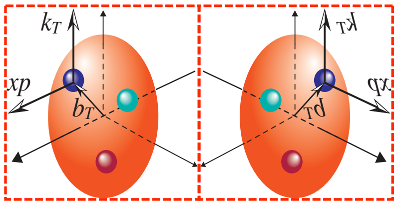
$$+\ O\left(\frac{1}{QR}\right)$$

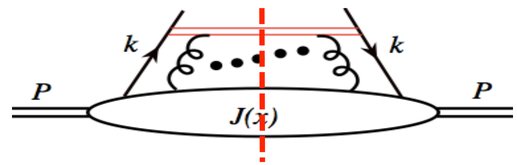
Factorization | Hard-part *Probe* | Parton-distribution *Structure* | Power corrections *Approximation!*

Hadron structure at short-distance: $\langle P|\mathcal{O}(\overline{\psi},\psi,A)|P\rangle$ ***with operators on light-cone***

Quantify Hadron's Partonic Structure

❑ Parton distribution functions (PDFs):





k_T - integrated

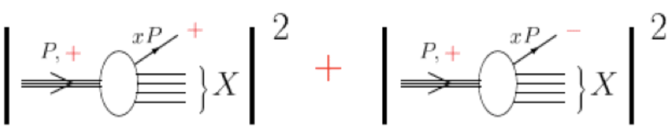
$$f_{q/p}(x,\mu^2)$$

$$f_q(x,\mu^2) \equiv \int \frac{dP^+\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \langle P|\overline{\psi}(\xi^-)\frac{\gamma^+}{2P^+}\exp\left\{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right\}\psi(0)|P\rangle$$

Boost invariant – *Dominated by the region*:

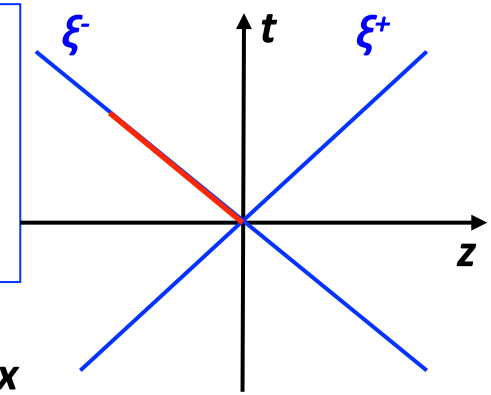
$$\xi^- \lesssim 1/(xP^+) \sim 1/Q$$

Interpreted as:

$q(x) =$ 

Probability density to find a quark with a momentum fraction x

Quantum correlation of quark fields along ξ^- direction! (Conjugated to the large *P⁺*)



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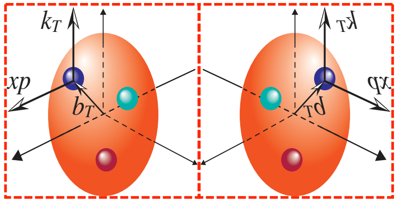

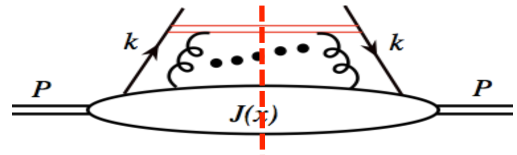

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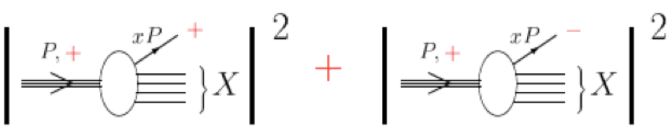
$q(x) =$ 

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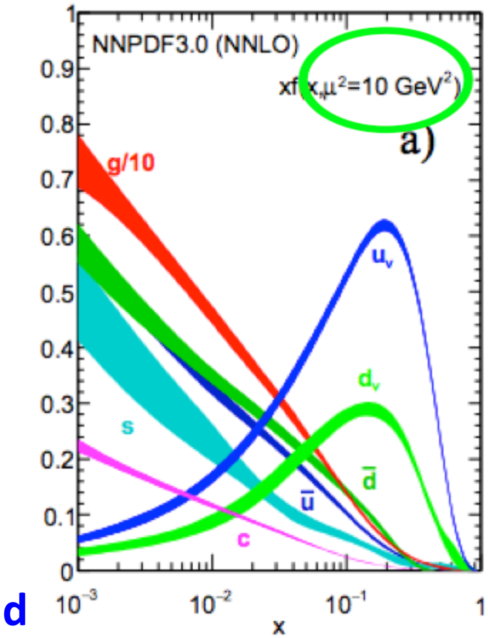


Good hadron cross sections:

1) can be measured experimentally with good precision,
2) can be factorized into universal matrix elements of quarks and
   – parton distribution functions *with controllable approximation*

Provide indirect access to hadron's partonic structure!

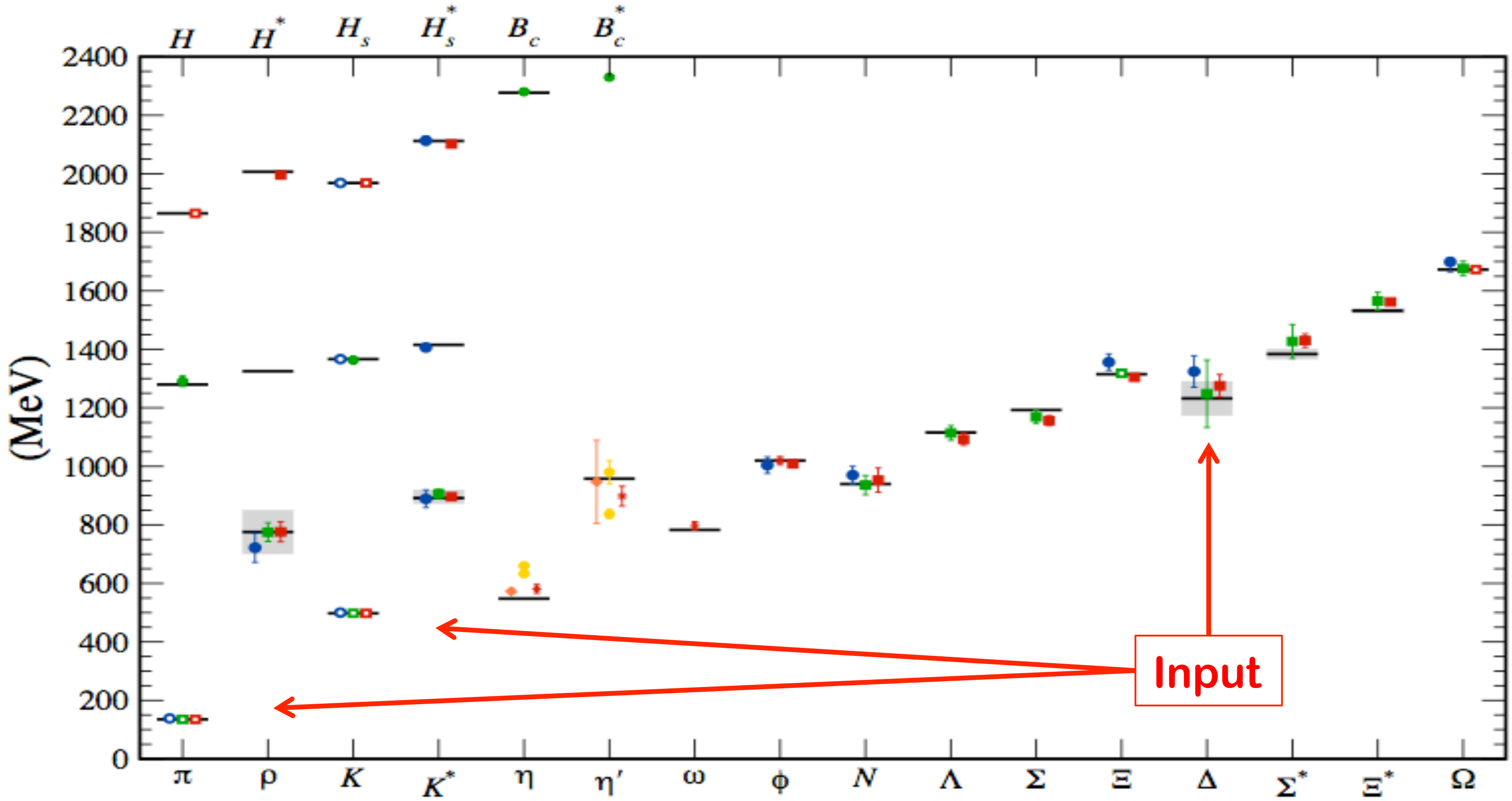
QCD global fits

[CT, NNPDF, JAM, …]

Lattice QCD

- Hadron masses: **Predictions with limited inputs**

See talk by P. Shanahan on Lattice QCD



- Lattice "time" is Euclidean: $\tau = i\,t$

Cannot calculate PDFs, TMDs, GPDs, …, directly, whose operators are time-dependent!

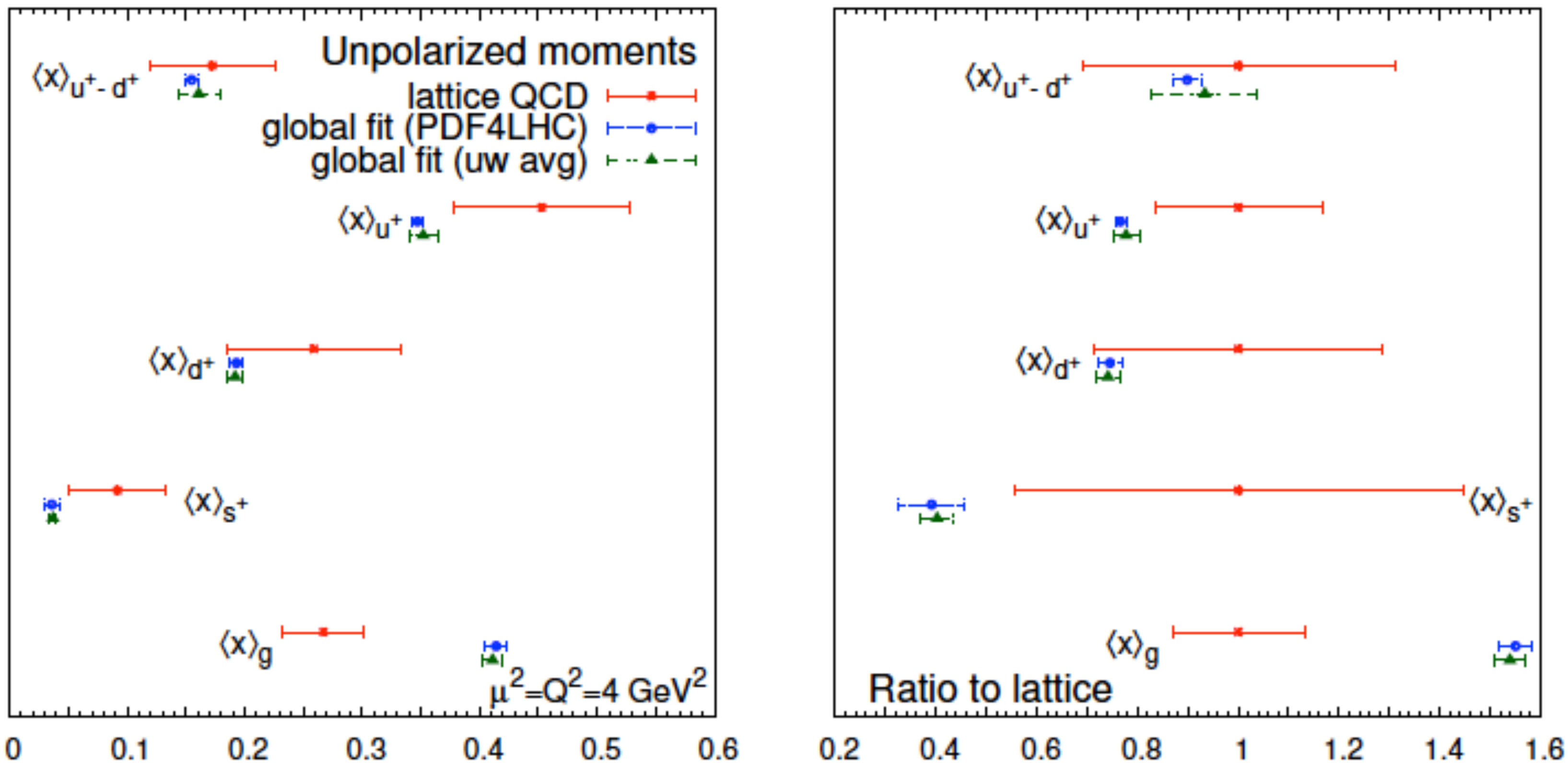
PDFs from Lattice QCD

talk by G. Koutsou tomorrow

- **Moments of PDFs – matrix elements of local operators**

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2) \qquad q^\pm \equiv q \pm \bar{q} \quad \text{and} \quad \Delta q^\pm \equiv \Delta q \pm \Delta\bar{q}$$

- **Works, but, hard and limited moments:**



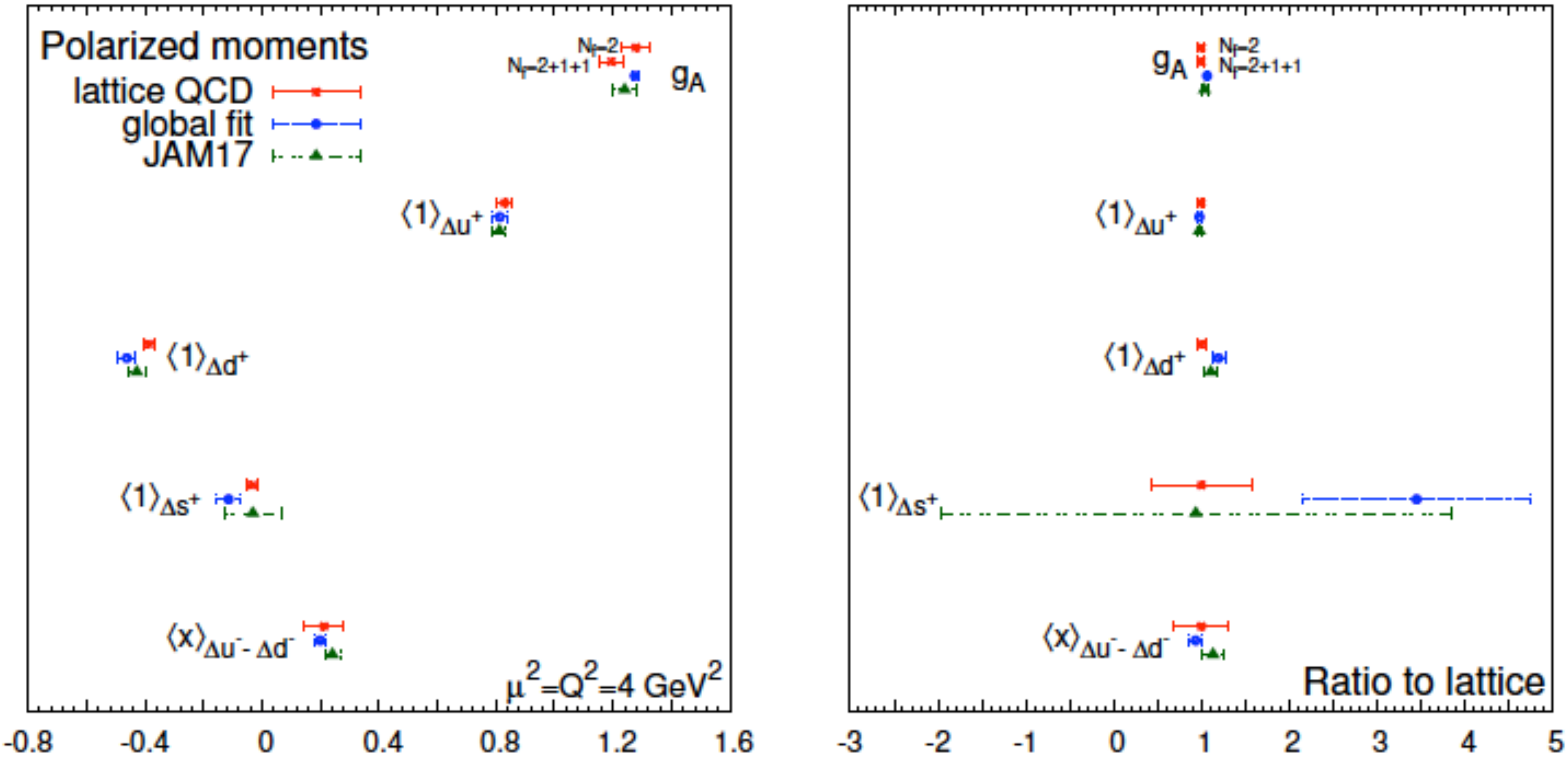
$\mu^2 = 4\ \mathrm{GeV}^2$

arXiv:1711.07916

PDFs from Lattice QCD

❑ Moments of polarized PDFs:

$\mu^2 = 4\ \mathrm{GeV}^2$



❑ Axial charge:

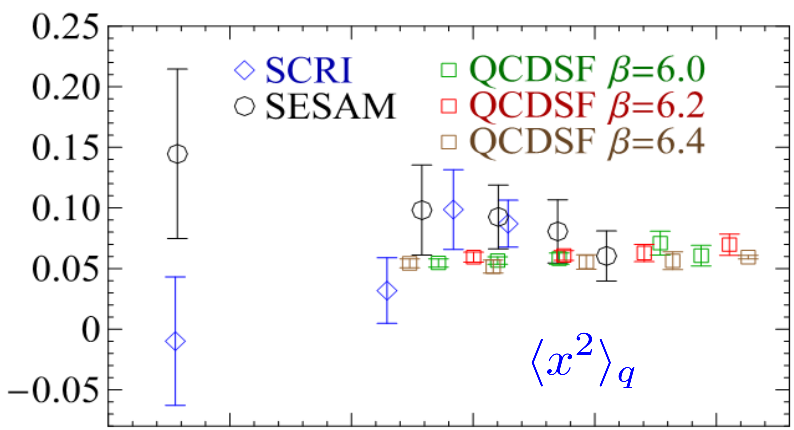
	Lattice QCD	Global Fit
$g_A \equiv \langle 1 \rangle_{\Delta u^+ - \Delta d^+}$	$1.195(39)\ (N_f = 2+1+1)$ $1.279(50)\ (N_f = 2)$	$1.275(12)$

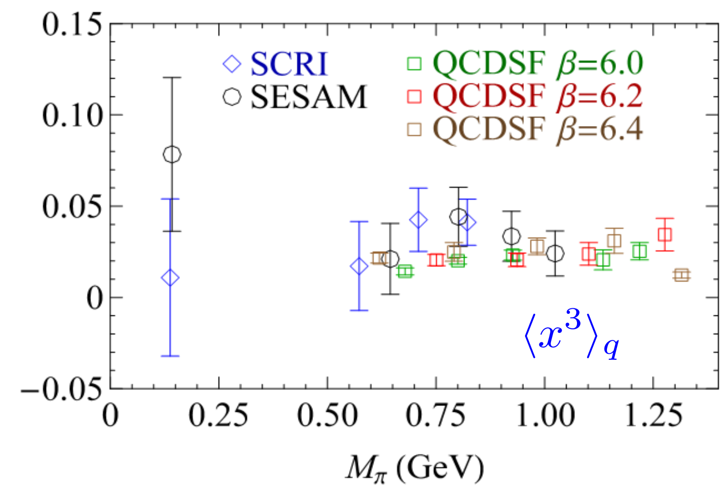
arXiv:1711.07916

X-dependent PDFs from Lattice QCD

❑ Limited moments:

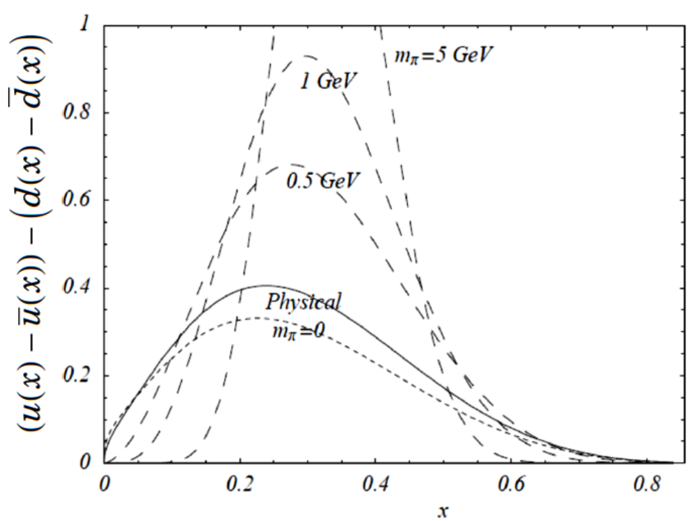
Dolgov et al., hep-lat/0201021 Gockeler et al., hep-ph/0410187





Limited moments – hard to get the full x-dependent distributions!

❑ Early efforts:



✧ **Assume a smooth functional form:**

$$xq(x) = a\, x^b (1-x)^c (1+\epsilon\sqrt{x}+\gamma\, x)$$

✧ **Fix parameters with LQCD moments:**

W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

Cannot distinguish valence quark contribution from sea quarks

X-dependent PDFs from Lattice QCD

M. Constantinou @ DNP19

- Lots of ideas, e.g.

  - ✧ **Hadronic tensor**
    [K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
  - ✧ **Auxiliary scalar quark**
    [U. Aglietti et al., arXiv:hep-ph/9806277, Phys. Lett. B441, 371 (1998)]
  - ✧ **Fictitious heavy quark**
    [W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
  - ✧ **Auxiliary scalar quark**
    [V. Braun & D. Mueller, arXiv:0709.1348, Eur. Phys. J. C55, 349 (2008)]
  - ✧ **Higher moments**
    [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
  - ✧ **Quasi-Parton Distributions (LaMET)**
    [X. Ji, arXiv:1305.1539, PRL 110 (2013) 262002; Sci. China PPMA. 57, 1407 (2014)]
  - ✧ **Good Lattice Cross Sections**
    [Y-Q Ma & J. Qiu, arXiv:1404.6860, arXiv:1709.03018, PRL 120, 022003 (2018)]
  - ✧ **Compton amplitude and OPE**
    [A. Chambers et al. (QCDSF), arXiv:1703.01153, PRL 118, 242001 (2017)]
  - ✧ **Pseudo-Parton Distributions**
    [A. Radyushkin, arXiv:1705.01488, Phys. Rev. D 96, 034025 (2017)]

All approaches are under investigation in lattice QCD!

arXiv:1811.07248

Quasi-Parton Distribution Functions

Ji, arXiv:1305.1539

Talk by X. Ji Thursday

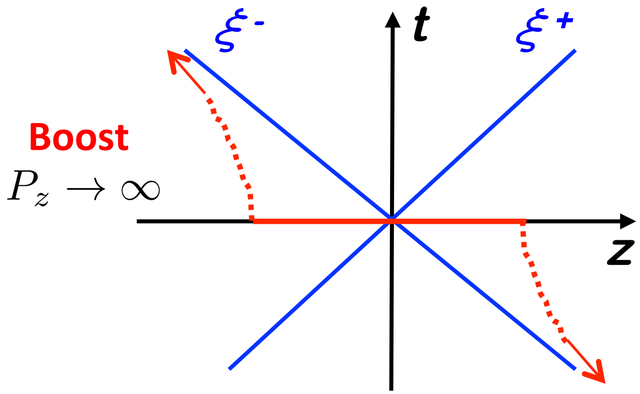
❑ Quasi-PDFs:

$$\tilde{q}(\tilde{x}, \mu_R^2, P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-i\tilde{x}P_z\xi_z} \langle P|\overline{\psi}(\frac{\xi_z}{2})\gamma_z \text{exp}\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(\frac{-\xi_z}{2})|P\rangle$$

Idea – Large Momentum Effective Theory (LaMET):

Quasi-PDFs are not boost invariant.

$$\tilde{q}(\tilde{x}, \mu_R^2, P_z) \longrightarrow q(x, \mu^2) \quad \textbf{when} \quad P_z \to \infty$$



Also see talks by K. Jansen and J. Zhang, and panel discussion on Thursday

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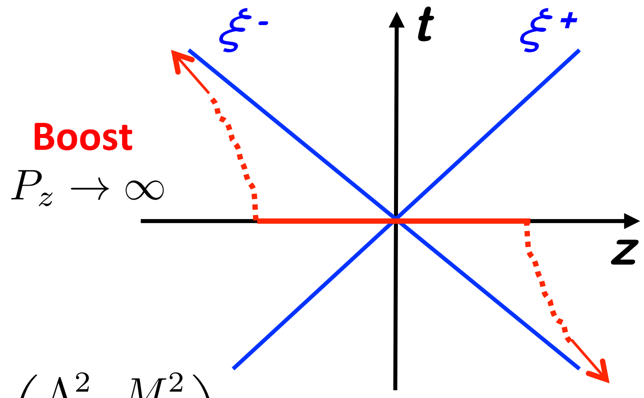

Note:

In Lattice QCD calculation, difficult to take $P_z \to \infty$ limit

Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Ji, arXiv:1305.1539

Caution:

Power corrections could be of the form: $\dfrac{\Lambda_{QCD}^2 R}{x^2(1-x)P_z^2}$

V. Braun et al, arXiv:1810.00048

Mixing with gluon and other flavor contribution beyond LO

Power UV divergence - μ_R does not obey DGLAP

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Ji, arXiv:1305.1539

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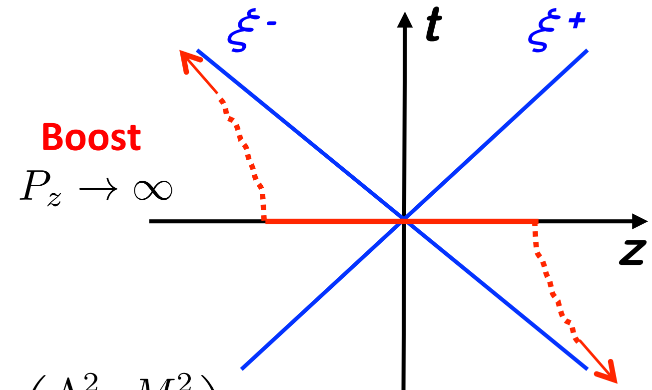

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Progress:

Power UV divergences are multiplicatively renormalizable

Quark: 1706.08962, 1707.03107, 1707.07152
Gluon: 1808.10824, 1809.01836

Also see talks by K. Jansen and J. Zhang, and panel discussion on Thursday

Pseudo-Parton Distribution Functions

A. Radyushkin, arXiv:1705.01488

Talk by D. Richards

- **Pseudo-PDFs:**

  ✧ **Lattice calculation with $\alpha = 0$:** **Ioffe time:** $\nu = p\cdot\xi \to -p_z\xi_z$

$$\mathcal{M}^\alpha(\nu = p\cdot\xi, \xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^\alpha\Phi_v(0,\xi,v\cdot A)\psi(\xi)|p\rangle$$ **with** $\xi^2 < 0$

Fourier transform of ν

$$\equiv 2p^\alpha\mathcal{M}_p(\nu,\xi^2) + \xi^\alpha(p^2/\nu)\mathcal{M}_\xi(\nu,\xi^2) \approx 2p^\alpha\mathcal{M}_p(\nu,\xi^2)$$

$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\nu}{2\pi}\, e^{ix\,\nu}\mathcal{M}_{p=p^0}(\nu,\xi^2)/\mathcal{M}_{p=p^0}(0,\xi^2)$$

Remove UV!

  ✧ **Off-light-cone extension of PDFs:** $f(x) = \mathcal{P}(x,\xi^2=0)$ **with** $\xi^\mu = (0^+,\xi^-,0_\perp)$

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❑ Quasi-PDFs vs Pseudo-PDFs:

See also Constantinou, Nikhil @ DNP2019

$$\frac{\langle H(t_{\rm sink})\, \overline{\psi}(0)\, W(0,z)\, \gamma_t\psi(z)\, H^\dagger(0)\rangle}{\langle H(t_{\rm sink}) H^\dagger(0)\rangle} \xrightarrow{t_{\rm sink}\to\infty} h(z,P_z) = \langle P_z|\overline{\psi}(0)\, W(0,z)\, \gamma_t\psi(z)|P_z\rangle$$

Renormalization

$$h_R(z, P_z, \mu_R) \xrightarrow[\text{Fixed } P_z]{\text{Fourier } z} \widetilde{q}(\tilde{x}, P_z, \mu_R)$$

LaMET $P_z \to \infty$

$$f(x, \mu^2)$$

$$\mathcal{M}(zP_z, z^2) = \frac{h(z,P_z)}{h(z,0)} \xrightarrow[\text{Fixed } z^2]{\text{Fourier } zP_z} \mathcal{P}(x, z^2)$$

$z^2 \to 0$ **Light-cone OPE**

Each step has systematic uncertainties and challenges!

Good "Lattice Cross Sections"

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- **Single hadron matrix elements:**

$$\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$$ **with** $\omega \equiv P \cdot \xi,\ \xi^2 \neq 0,\ \text{and}\ \xi_0 = 0;$ **and**

1) can be calculated in lattice QCD with precision,
has a well-defined continuum limit (UV+IR safe perturbatively), and

2) can be factorized into universal matrix elements of quarks and gluons
with controllable approximation

$P \to \sqrt{s}$ ***and*** $\xi \to 1/Q$ ***define collision kinematics***

Collaboration between lattice QCD and perturbative QCD!

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Collaboration between lattice QCD and perturbative QCD!

❑ Current-current correlators:

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2} Z_{j_1}^{-1} Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

with

d_j : Dimension of the current

Z_j : Renormalization constant of the current

Sample currents:

$$j_S(\xi) = \xi^2 Z_S^{-1} [\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1} [\overline{\psi}_q \gamma \cdot \xi \psi_q](\xi),$$

$$j_{V'}(\xi) = \xi Z_{V'}^{-1} [\overline{\psi}_q \gamma \cdot \xi \psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1} [-\frac{1}{4} F^c_{\mu\nu} F^c_{\mu\nu}](\xi), \ldots$$

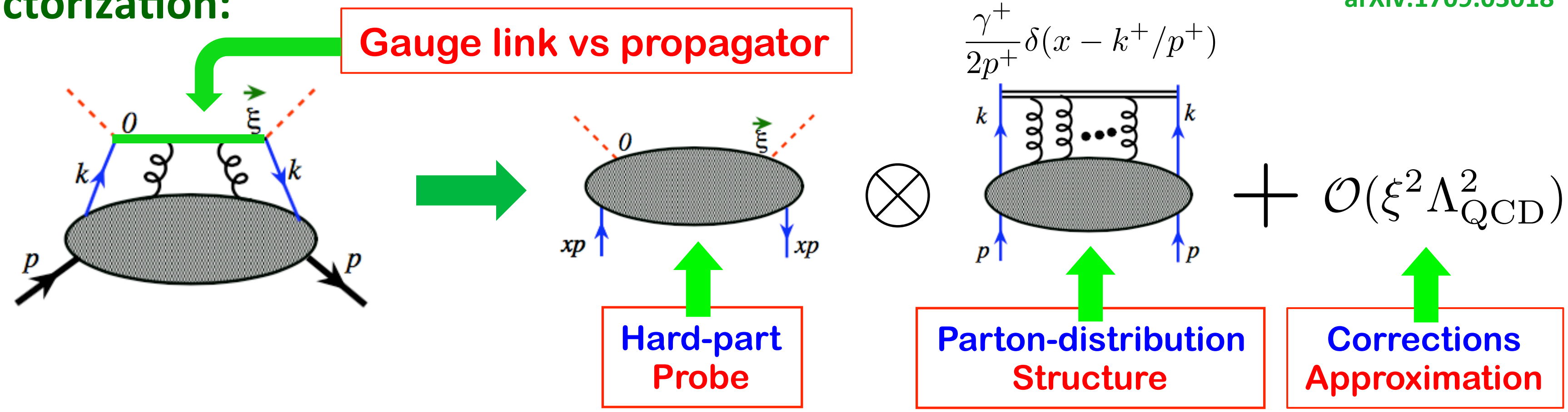
❑ Quasi- and pseudo-PDFs – special cases:

$$\mathcal{O}_q(\xi) = Z_q^{-1}(\xi^2) \overline{\psi}_q(\xi)\, \gamma \cdot \xi \Phi(\xi, 0)\, \psi_q(0) \qquad \Phi(\xi, 0) = \mathcal{P} e^{-ig \int_0^1 \xi \cdot A(\lambda \xi)\, d\lambda}$$

Lattice Data + QCD Global Analyses

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- **Factorization:**



$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x}\, f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda^2_{\rm QCD})$$

with $f_a(x,\mu^2) = -f_a(-x,\mu^2)$

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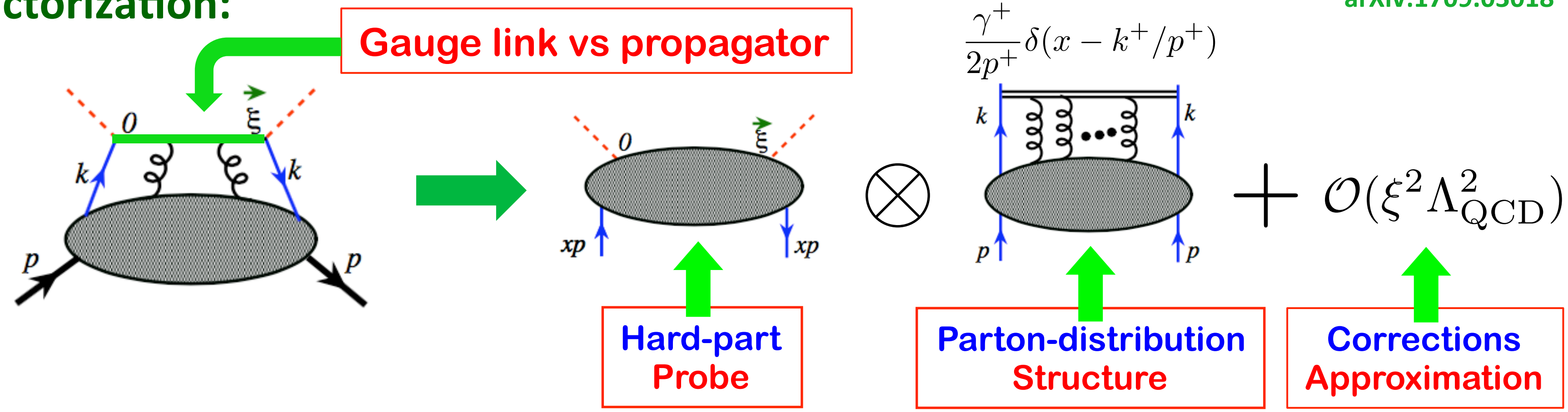

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with $\quad f_a(x,\mu^2) = -f_a(-x,\mu^2)$

- ❑ **QCD Global analysis:**

  Need data of "many" good lattice cross sections to be able to extract the x, Q, flavor dependence of the hadron structure, …

  Complementary to existing Global analysis of experimental data

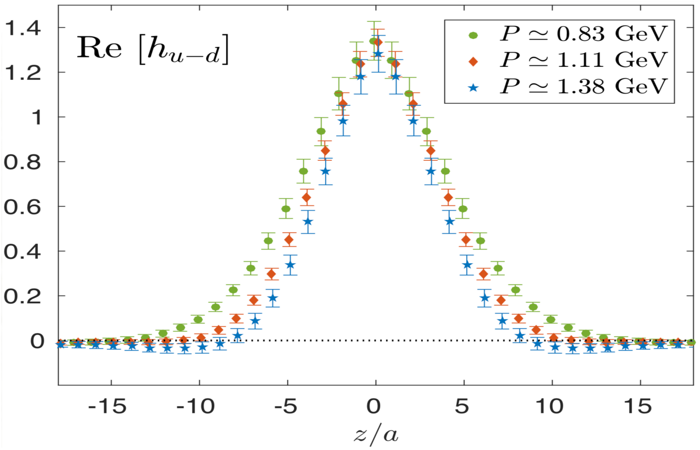
- ❑ **Tremendous potentials:**

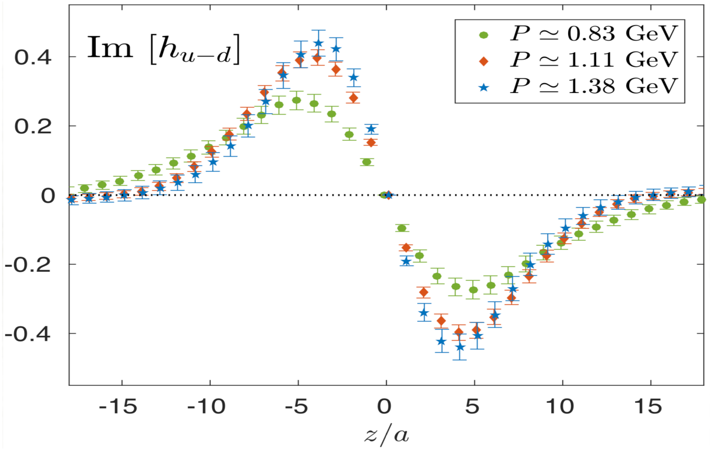
  - ✧ ***Neutron PDFs, … (no free neutron target!)***
  - ✧ ***Meson PDFs, such as pion, …***
  - ✧ ***More direct access to gluons – gluonic current, quark flavor, …***

Nucleon unpolarized PDFs – Quasi-PDF approach

C. Alexandrou et al., arXiv:1803.02685,
C. Alexandrou et al., arXiv:1807.00232

- Bare matrix elements with γ^0 – no mixing:





Nucleon unpolarized PDFs – Quasi-PDF approach

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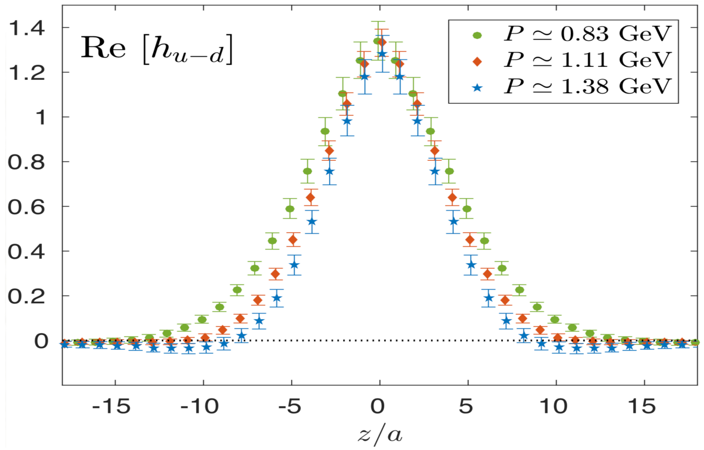
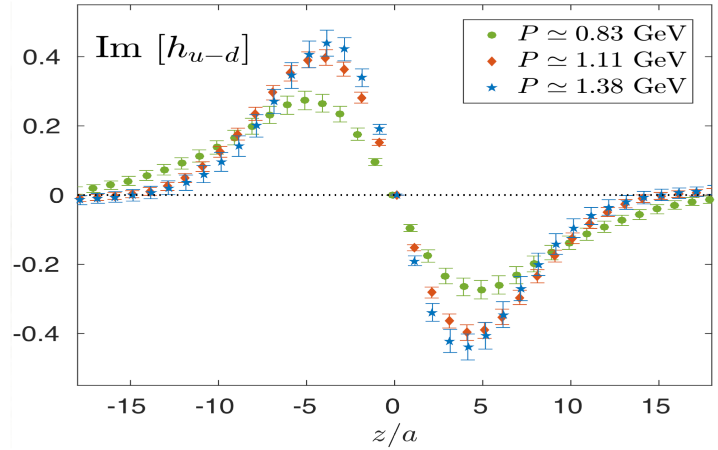
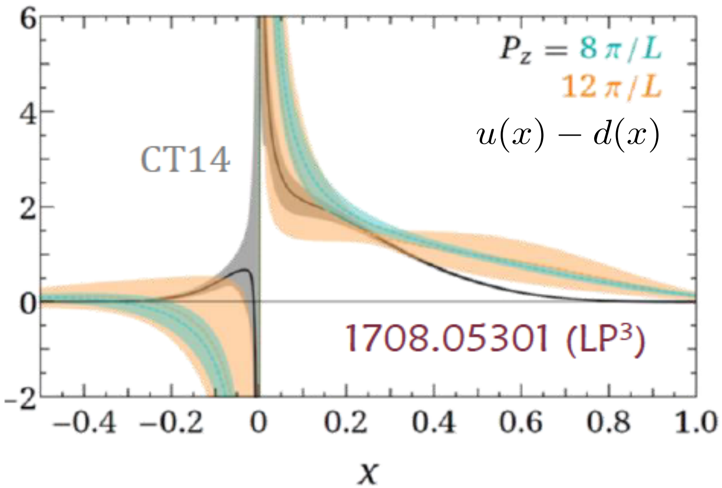
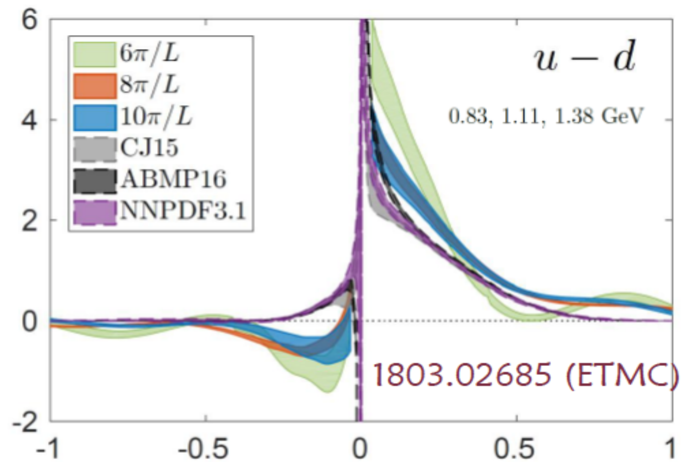




- Both LP3 and ETMC obtained their results at physical pion mass!



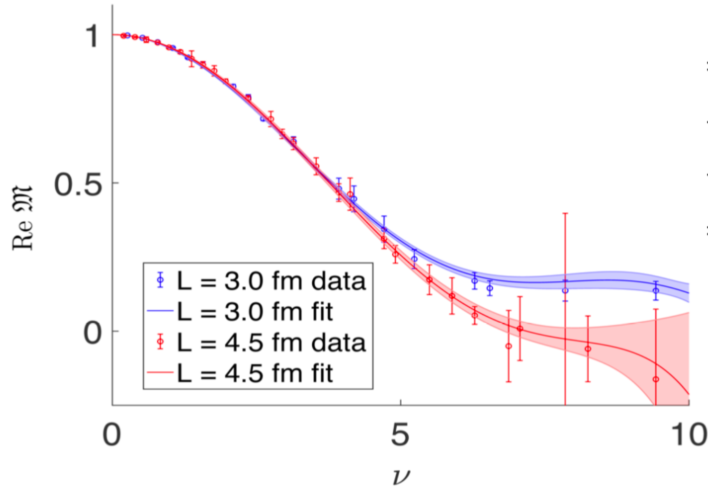


✧ **With one-loop perturbative matching and target mass correction**
✧ **Checked excited states contamination – single/two states fits**

Nucleon unpolarized PDFs – Pseudo-PDF approach

[B. Joo et al. (JLab-W&M), arXiv:1908.09771]
S. Zafeiropoulos
Thursday

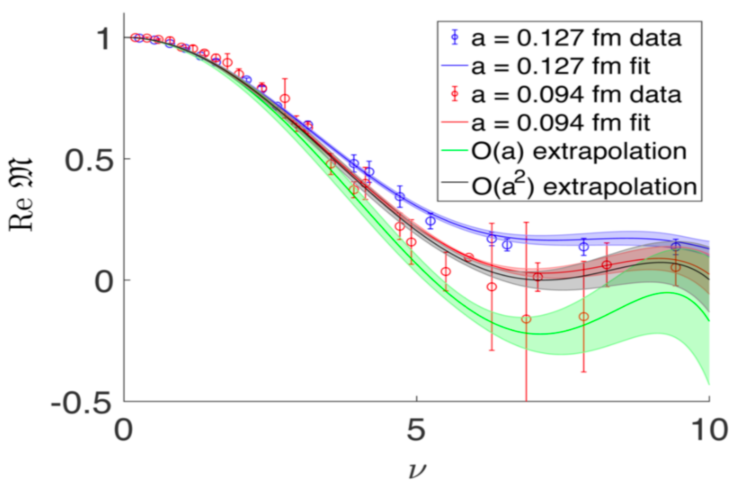
❑ Volume effect:



a(fm)	M_π(MeV)	$L^3 \times T$
0.127(2)	415(23)	$24^3 \times 64$
0.127(2)	415(23)	$32^3 \times 96$
0.094(1)	390(71)	$32^3 \times 64$

N_f=2+1 clover fermions (3 ensembles):

❑ Discretization effect:



Too strong "L" & "a" dependence – limited the range of ν!

Nucleon unpolarized PDFs – Pseudo-PDF approach

[B. Joo et al. (JLab-W&M), arXiv:1908.09771]
S. Zafeiropoulos
Thursday

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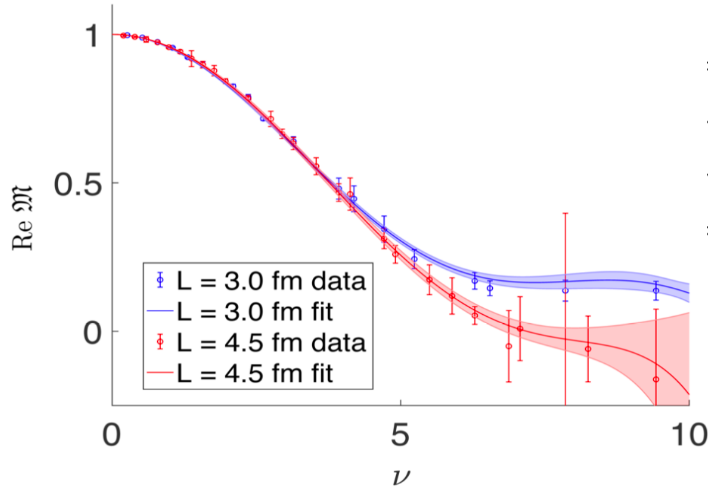

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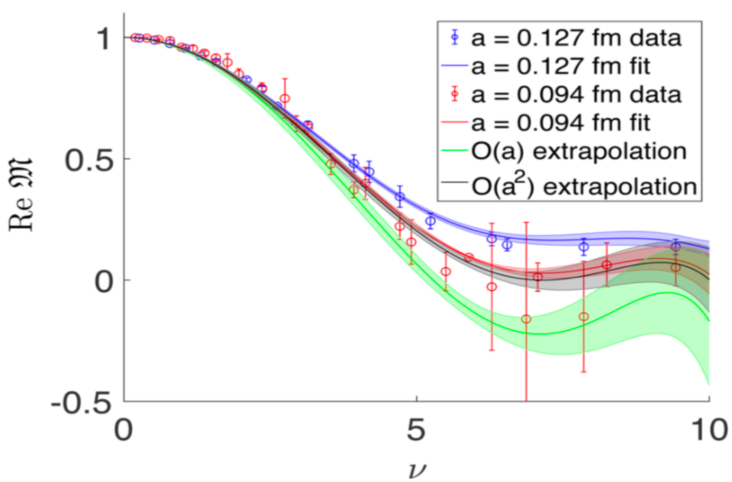
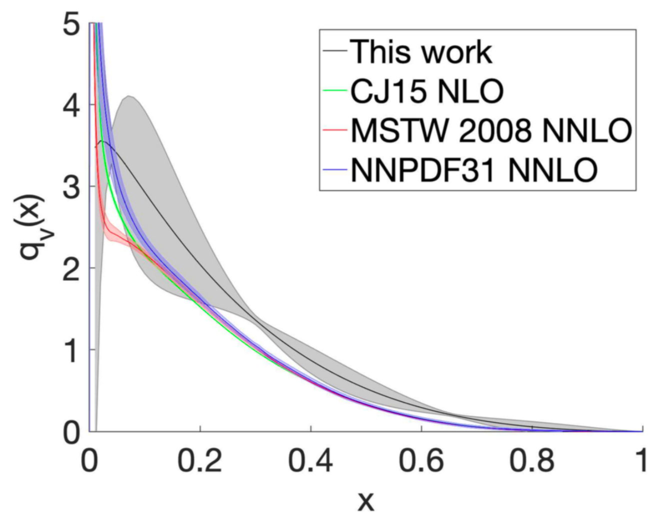


Too strong "L" & "a" dependence – limited the range of ν!

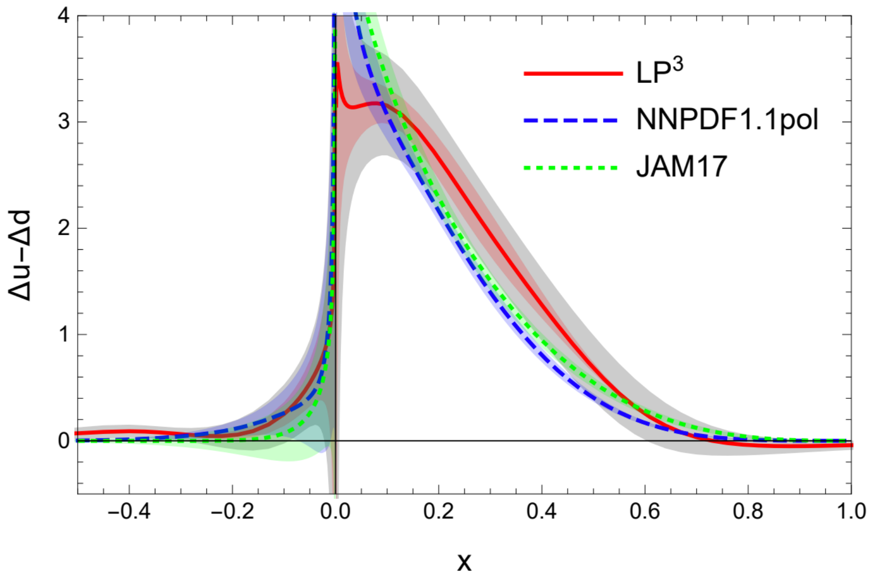
- Extract/fit PDF from lattice data with a functional form similar to CJ and MSTW



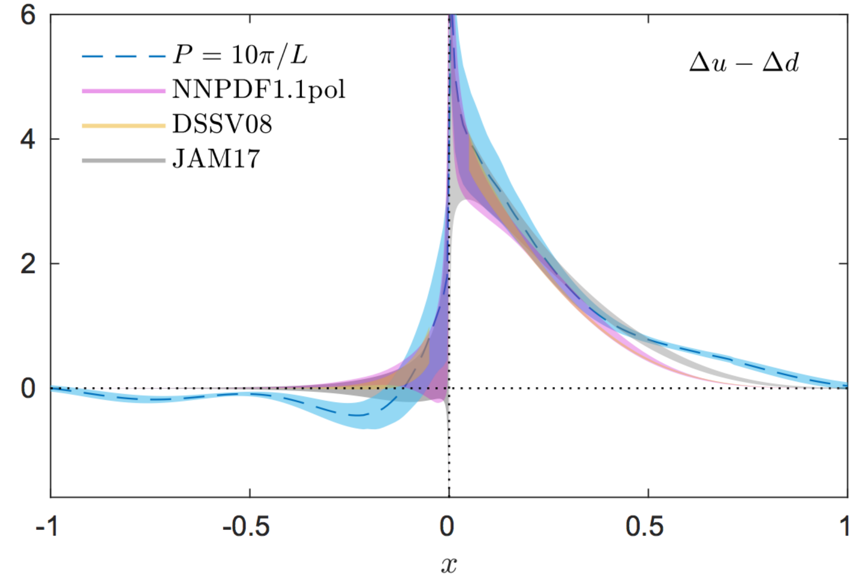
Large error – few data

Challenges due to lattice limitation
Results are encouraging!

Nucleon Helicity PDFs – Quasi-PDF approach



[H.-W. Lin et al. (LP3), PRL 121 (2018) 242003]



[C. Alexandrou et al. (ETMC), PRL 121 (2018) 112001]

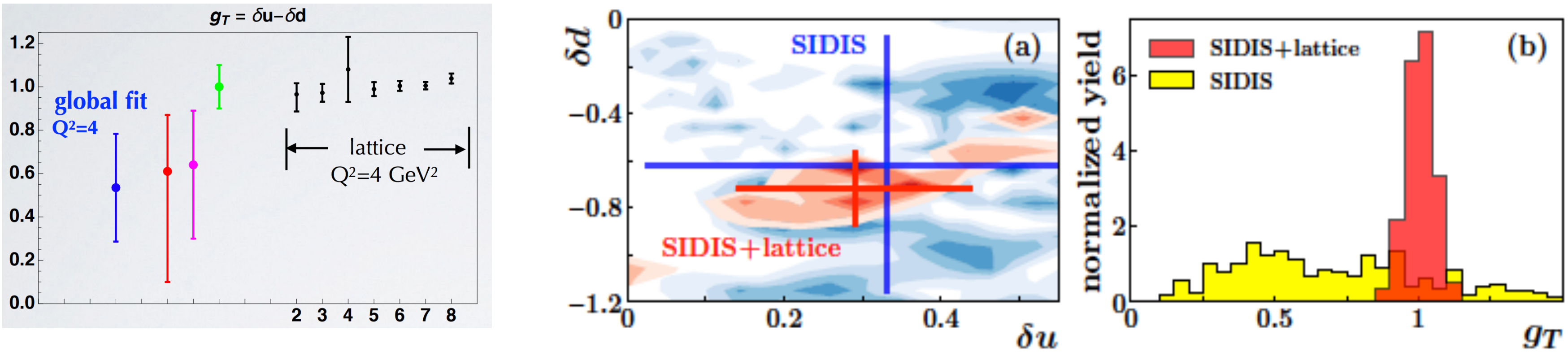
- ✧ Both calculations are at the physical pion mass
- ✧ Lattice data approach global fits
- ✧ Nucleon momentum is limited by lattice spacing (for both ETMC and LP3)
- ✧ Slightly different matching procedure led to different behavior for given x

  agreement (within statistical uncertainties) between ETMC & global fits for x<0.5, while for LP3 is x>0.6
- ✧ Negative-x region: anti-quark contribution suffers from uncertainties

Nucleon Transversity – Quasi-PDF approach

H.W. Lin et al, PRL 120, 152502 (2018)

❑ Tensor charge – lattice QCD helps global fit:



Nucleon Transversity – Quasi-PDF approach

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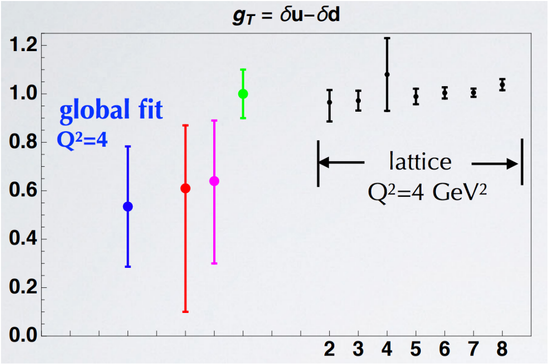
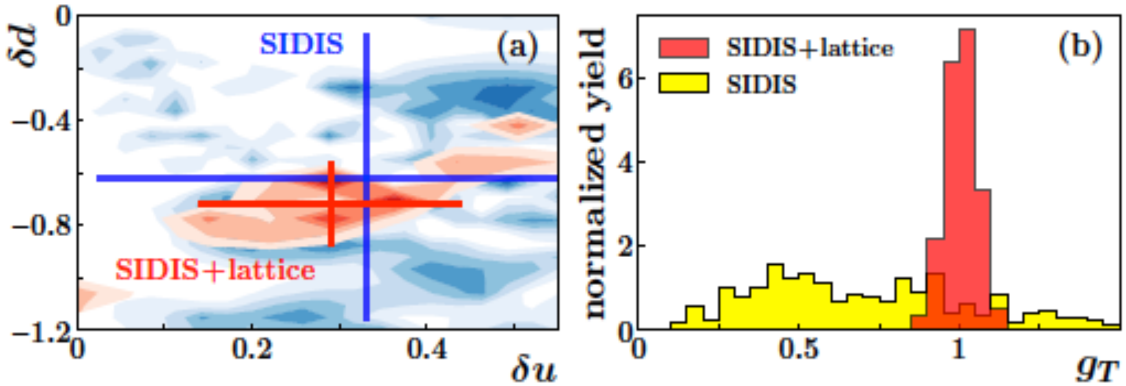
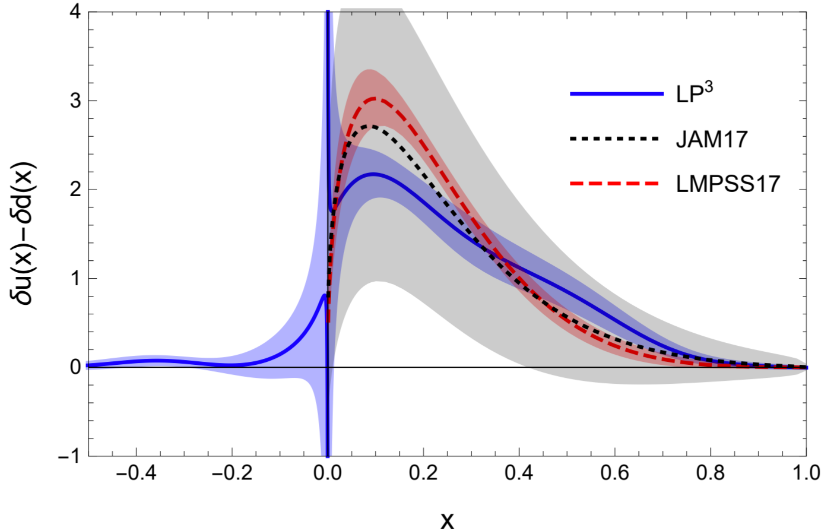
H.W. Lin et al, PRL 120, 152502 (2018)

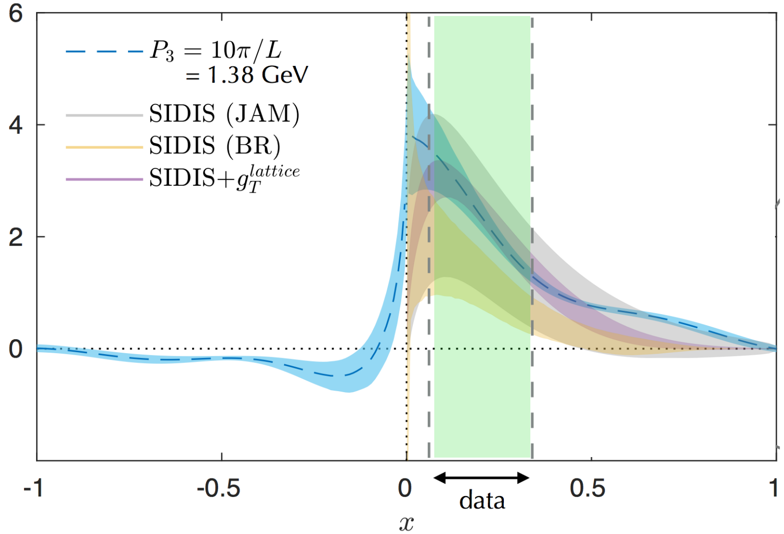




❑ X-dependent transversity:



[Y.-S. Liu et al. (LP3), arXiv:1807.00232]



[C. Alexandrou et al. (ETMC), arXiv:1807.00232]

Lattice data from quasi-PDF approach has less error than that from global fit !

Current-Current Correlators

Ma, Qiu, PRL (2018)

Sufian et al. JLab
PRD99 (2019) 074507

❑ Pion valence PDFs:

– using a vector-axial-vector correlation as an example

✧ Parity-Time-reversal invariance:

$$\frac{1}{2}\left[T^{\mu\nu}_{v5}(\xi,p)+T^{\mu\nu}_{5v}(\xi,p)\right]=\frac{\xi^4}{2}\langle h(p)|\left(\mathcal{J}^\mu_v(\xi/2)\,\mathcal{J}^\nu_5(-\xi/2)+\mathcal{J}^\mu_5(\xi/2)\,\mathcal{J}^\nu_v(-\xi/2)\right)|h(p)\rangle$$
$$\equiv \epsilon^{\mu\nu\alpha\beta}\,p_\alpha\xi_\beta\,\widetilde{T}_1(\omega,\xi^2)+(p^\mu\xi^\nu-\xi^\mu p^\nu)\,\widetilde{T}_2(\omega,\xi^2)$$

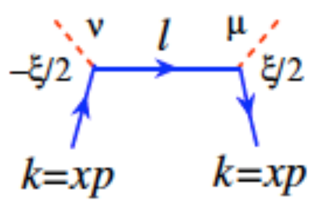
Vanishes under T

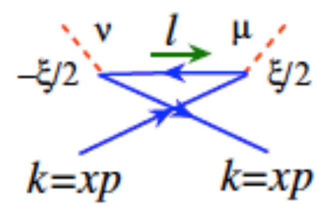
✧ Collinear factorization:

$$\widetilde{T}_i(\omega,\xi^2)=\sum_{f=q,\bar{q},g}\int_0^1\frac{dx}{x}\,f(x,\mu^2)\,C_i^f(\omega,\xi^2;x,\mu^2)+\mathcal{O}\left[|\xi|/\mathrm{fm}\right]$$

✧ Lowest order coefficient functions:

$$C_1^{q(0)}(\omega,\xi^2;x)=\frac{1}{\pi^2}\,x\left(e^{ix\omega}+e^{-ix\omega}\right)$$
$$C_2^{q(0)}(\omega,\xi^2)=0$$





$$T_1(\widetilde{x},\xi^2)\equiv\int\frac{d\omega}{2\pi}e^{-i\widetilde{x}\omega}\widetilde{T}_1(\omega,\xi^2)$$
$$=\frac{1}{\pi^2}\left(q(\widetilde{x},\mu^2)-\overline{q}(\widetilde{x},\mu^2)\right)\equiv\frac{1}{\pi^2}q_v(\widetilde{x},\mu^2)$$

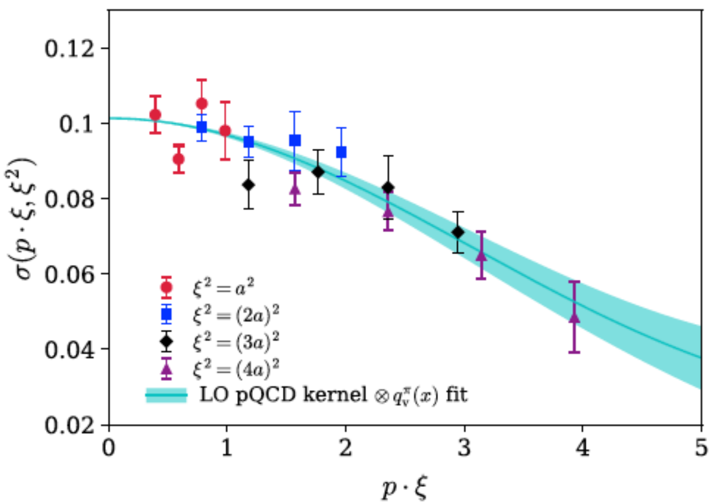
Sufian et al.
PRD99 (2019) 074507

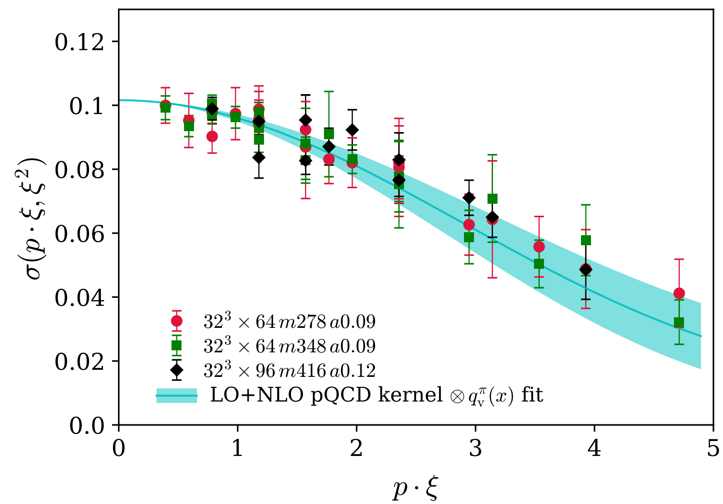
✧ Lattice QCD calculation results with 1-loop matching coefficient

Current-Current Correlators

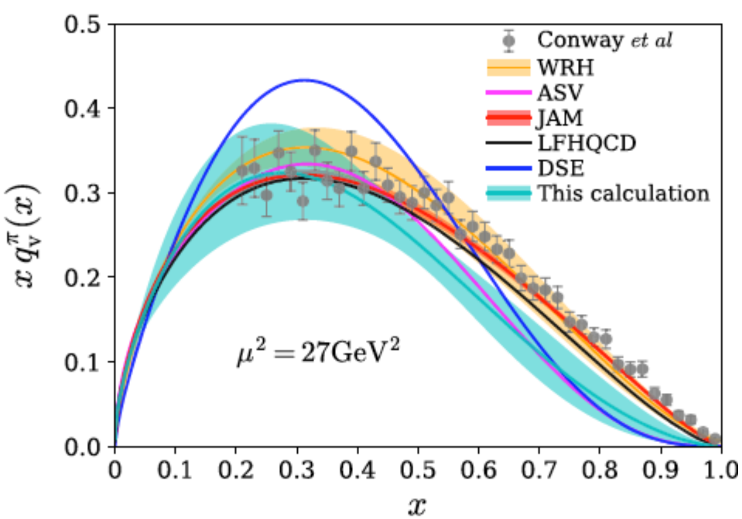
Sufian et al. JLab
PRD99 (2019) 074507

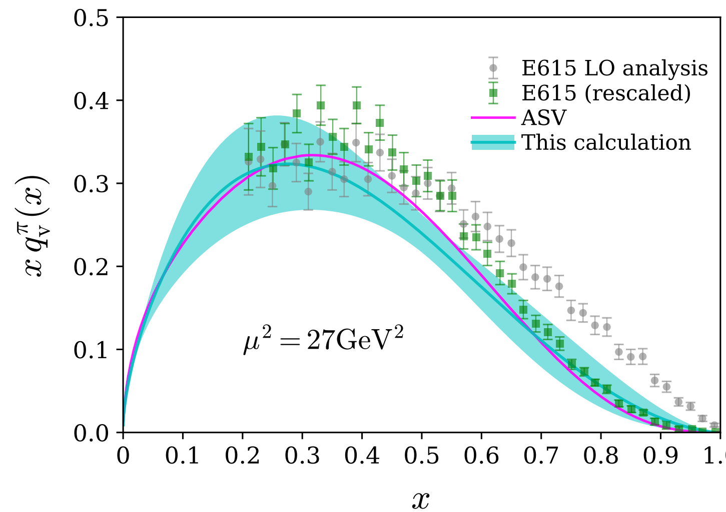
❑ “Lattice cross section” of V-A current correlator:





❑ Extracted pion valence quark distribution:





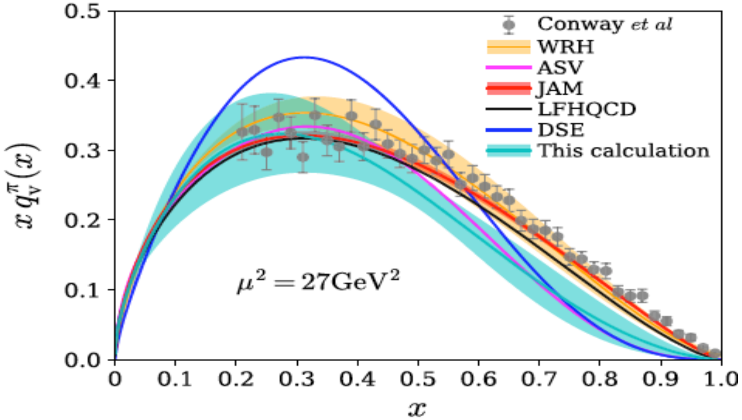
Sufian et al. @ DNP19

Comparison of pion PDF from different approaches

M. Constantinou @ DNP19

❑ "Lattice cross section":

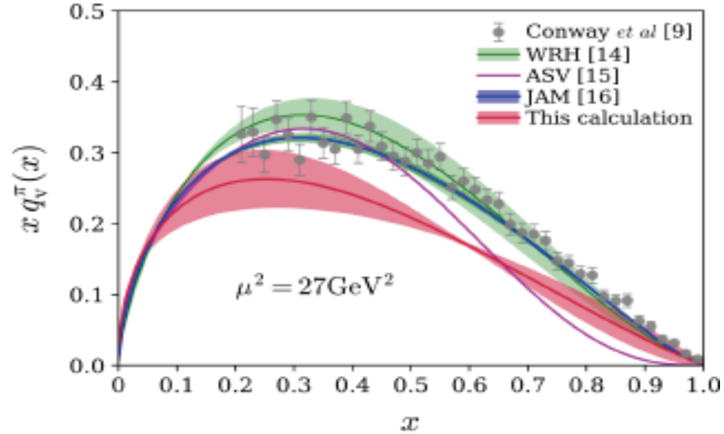
[R. Sufian et al. (JLab - W&M), Phys. Rev. D 99 (2019) 074507]



$m_\pi = 416\,\mathrm{MeV}\,,\ P = 0.6 - 1.5\,\mathrm{GeV}$

❑ Pseudo-PDF:

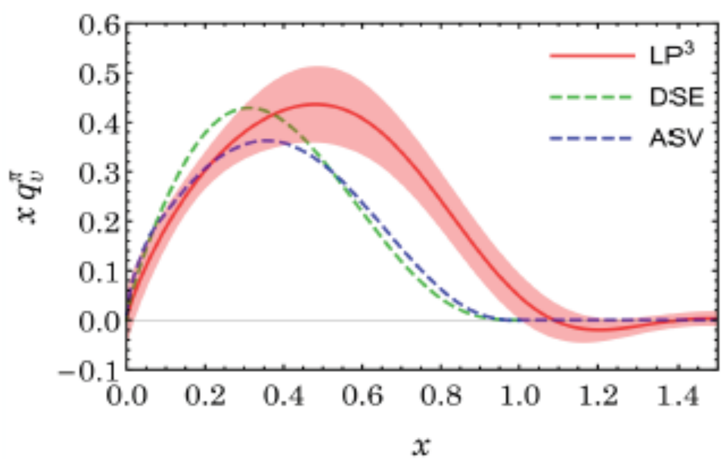
[B. Joo et al. (JLab-W&M), arXiv:1909.08517]



$m_\pi = 416\,\mathrm{MeV}\,,\ \nu \leq 4.7$

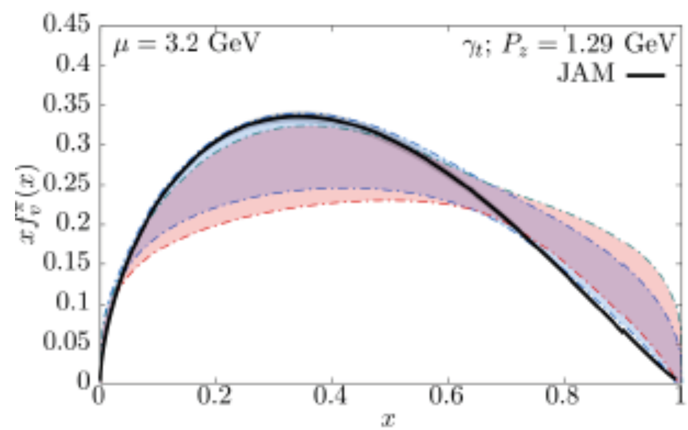
❑ Quasi-PDF:

[J.-H. Zhang et al. (LP3),Phys. Rev. D 100, 034505 (2019)]



$m_\pi = 310\,\mathrm{MeV}\,,\ P = 1.74\,\mathrm{GeV}$

[T. Izubuchi et al. (BNL-SBU-UConn), Phys. Rev. D 100, 034516 (2019)]



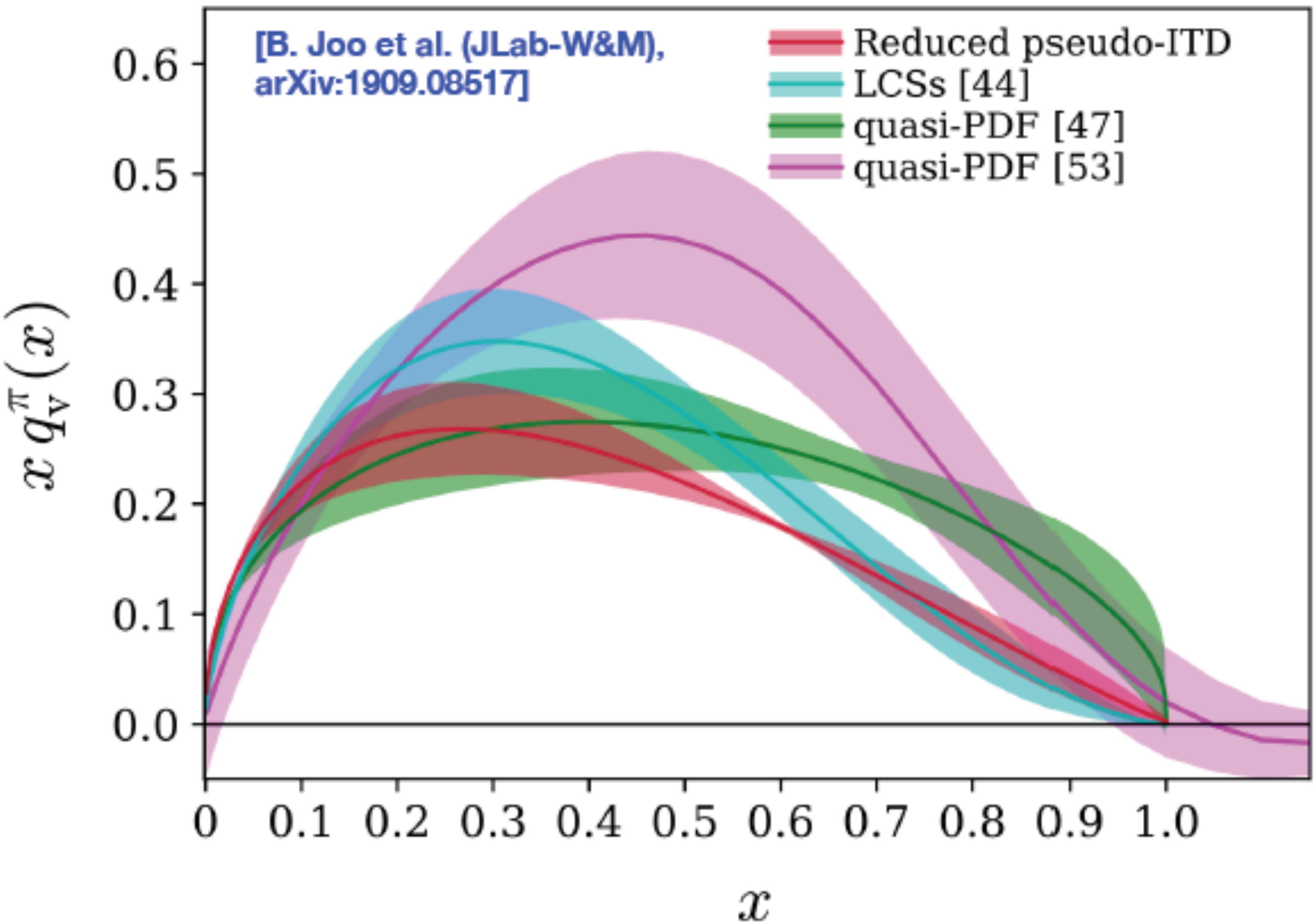
$m_\pi = 300\,\mathrm{MeV}\,,\ P = 1.29\,\mathrm{GeV}$

Comparison of pion PDF from different approaches

M. Constantinou @ DNP19

- ❑ "Lattice cross section":
- ❑ Pseudo-PDF:

[I



❑ C

4516 (2019)]

$m_\pi = 310\,\mathrm{MeV}\,,\ P = 1.74\,\mathrm{GeV}$

$m_\pi = 300\,\mathrm{MeV}\,,\ P = 1.29\,\mathrm{GeV}$

Lattice and Global fits

M. Constantinou @ DNP19

Fit ETMC lattice results directly within:

[C. Alexandrou et al. (ETMC), PRL 121 (2018) 112001, arXiv:1803.02685]

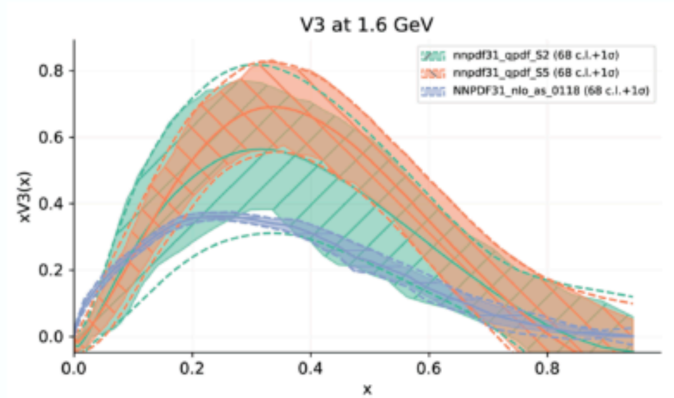
★ NNPDF framework

[K. Cichy, L. Del Debbio, T. Giani, accepted in JHEP, arXiv:1907.06037]

$$\mathscr{O}_{\gamma_0}^{\mathrm{Re/Im}}(z,\mu) = \mathscr{C}_3^{\mathrm{Re/Im}}(z,\mu/P_z) \circledast V_3/T_3(\mu)$$

$$T_3 = u_V - d_V + 2(u_S - d_S)$$
$$V_3 = u_V - d_V$$

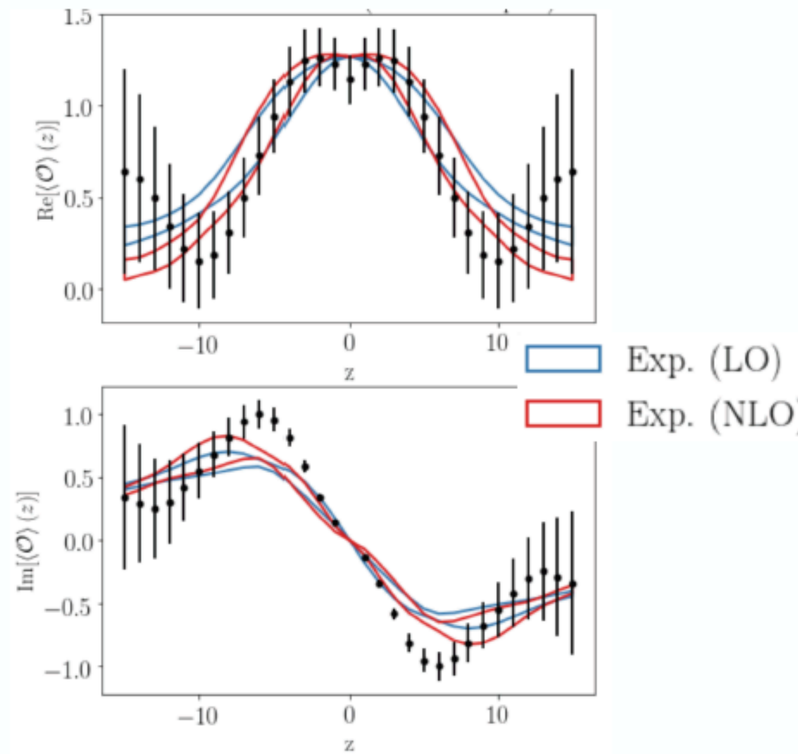
Scenario	Cut-off	FVE	Excited states	Truncation
S2	20%	5%	10%	20%
S5	0.2	0.05	0.1	0.2



Reasonable agreement, but a lot of work for the lattice to reduce uncertainties!

★ JAM framework

Talk by J. Bringewatt, Thu @ 10:06am



Good agreement between lattice and experiment for polarized PDFs (within larger uncertainties)

GPDs – Quasi-PDF and Pseudo-PDF approach

A. Scapellato
this afternoon

❑ Quark-GPD:

$$\widetilde{q}_{\Gamma}^{\mathrm{GPD}}(x,\xi,t,P_z,\mu_R) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle h(P_z+\Delta/2)|\overline{\psi}(z)\Gamma\Phi_z(z,0)\phi(0)|h(P_z-\Delta/2)\rangle_{\mu_R}$$

More variables:

- ✧ **Length of the Wilson line z (x in momentum space)**
- ✧ **Hadron momentum:** P_z
- ✧ **Momentum transfer:** $t = \Delta^2 = -Q^2$
- ✧ **Skewness:** $\xi = -\dfrac{Q_z}{P_z}$ **Quasi-skewness = light-cone skewness** $+\mathcal{O}(1/P_Z^2)$

Much higher computational cost compared to PDFs

❑ Matching:

- ✧ **Perturbative matching depends on skewness, but not on momentum transfer**
- ✧ **For ξ=0, the matching is the same as that of PDFs**
- ✧ **Matching for general ξ**

[X. Ji et al., PRD 92 (2015) 014039, arXiv:1506.00248]

[Y.-S. Liu et al., PRD 100, 034006 (2019), arXiv:1902.00307]

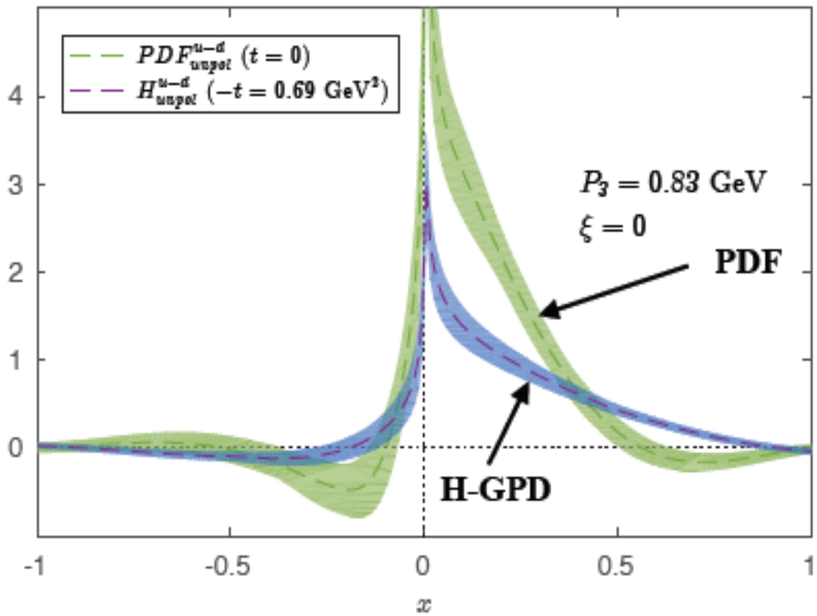
Nucleon unpolarized GPDs

- N_f=2+1+1 twisted mass fermions:

  [C. Alexandrou et al., (ETMC), 2019]

  $m_\pi = 260\,\mathrm{MeV},\ P = 0.83 - 1.66\,\mathrm{GeV}$

  $|\xi| \geq 0,\ a = 0.09\,\mathrm{fm},\ 32^3 \times 64$



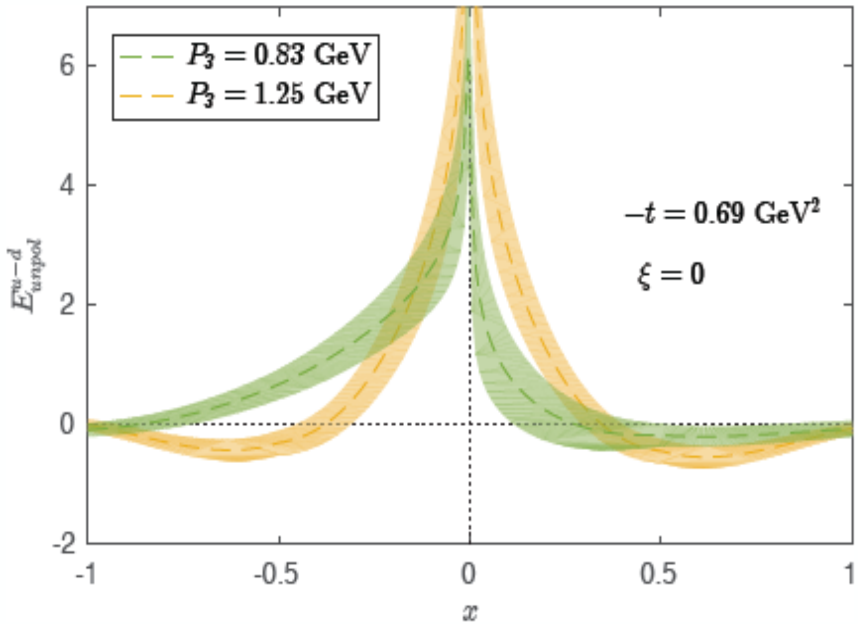
- GPD vs PDF at same P_z:

  H suppressed compared to PDF

- GPDs decomposition(unpolarized):

$$\mathcal{M}_{\gamma_0}(\Gamma_0) = \tilde{H}_Q C_H(\Gamma_0, p_i, p_f) + \tilde{E}_Q C_E(\Gamma_0, p_i, p_f)$$

$$\mathcal{M}_{\gamma_0}(\Gamma_\kappa) = i\,\tilde{H}_Q\, C_H(\Gamma_\kappa, p_i, p_f) + i\,\tilde{E}_Q\, C_E(\Gamma_\kappa, p_i, p_f)$$



- P_z dependence:

  Becomes symmetric in x as P_z increases

Nucleon helicity GPDs

- **N_f=2+1+1 twisted mass fermions:**

  [C. Alexandrou et al., (ETMC), 2019]

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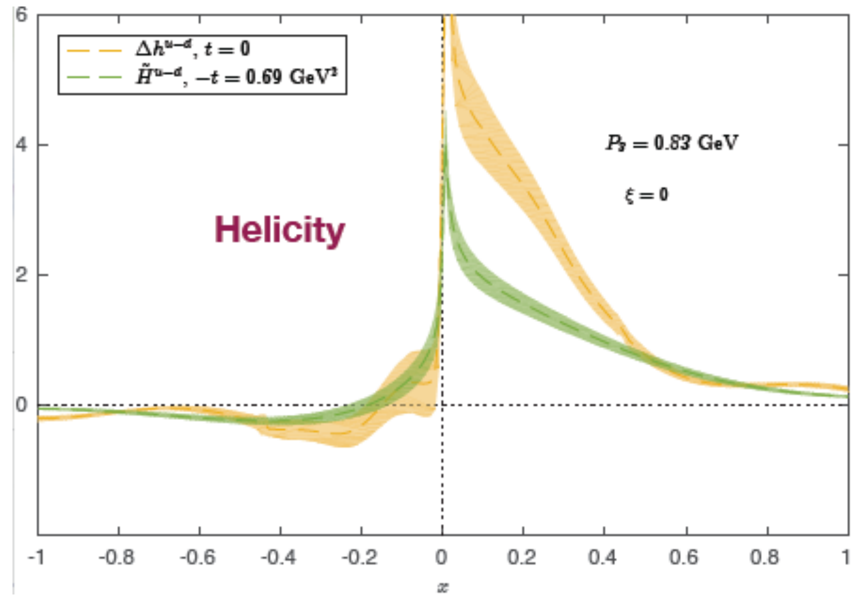

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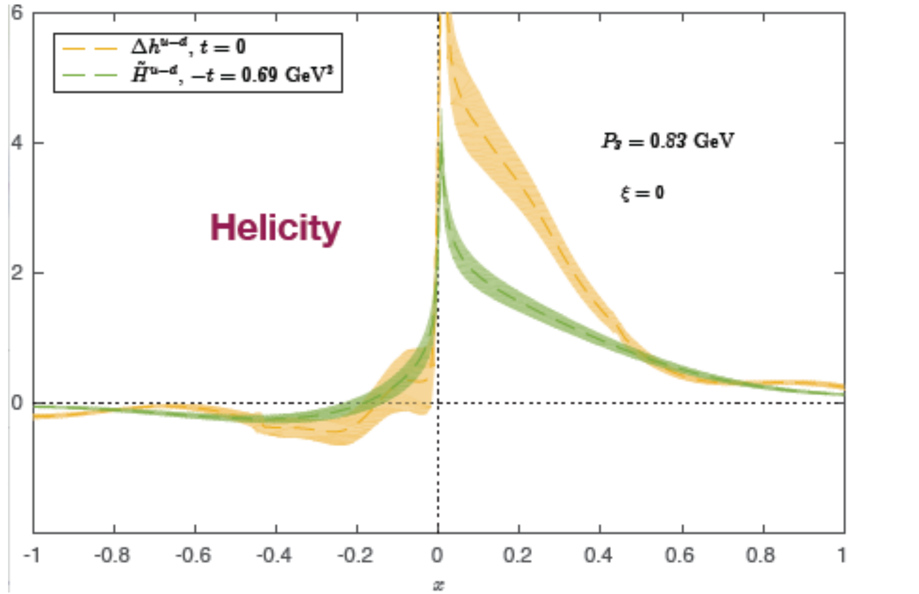
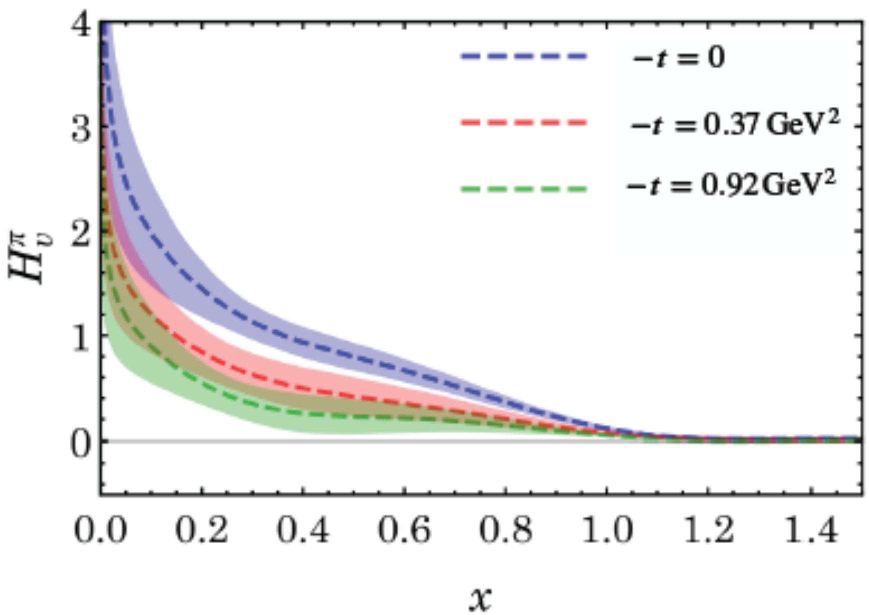


- Also see pion quasi-GPDs:

  [J.W. Chen et al., arXiv:1904.12376]



Exploratory study with Nf=2+1+1 HISQ fermions

$$m_\pi = 310\,\mathrm{MeV}\,, \quad P = 0.86, 1.32, 1.74\,\mathrm{GeV}$$
$$a = 0.12\,\mathrm{fm}\,, \quad 24^3 \times 48\,, (L \sim 3\mathrm{fm})$$

✧ t-dependent converges within uncertainties

Summary and outlook

- **Although lattice QCD cannot calculate parton distributions directly, many new ideas and approaches make it possible to extract PDFs, GPDs, TMDs, … from lattice QCD calculations**

- **Like extracting PDFs and partonic structure from hadronic cross sections, PDFs and non-perturbative partonic structure can be extracted from:**

  1) **Lattice QCD calculable hadronic matrix elements (quasi-, pseudo-, current-current correlators, … , which**
  2) **can be factorized/matched into PDFs or any universal partonic distributions**

- **Tremendous progresses have been made for extracting PDFs from lattice QCD calculations, with various and complementary approaches**

- **Lattice QCD can be used to study hadron structure, including PDFs, GPDs and TMDs, but, more works are still needed for understanding the systematic uncertainties, lattice artifacts, …**

Thank you!

PDFs from Lattice QCD

- **Isovector charges of nucleon:** $g_A^{u-d}, g_S^{u-d}, g_T^{u-d}$

arXiv:1806.09006