Connecting PDFs from phenomenology and lattice QCD

13th European Research Conference
on Electromagnetic Interactions with Nucleons and Nuclei

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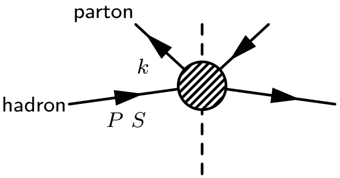
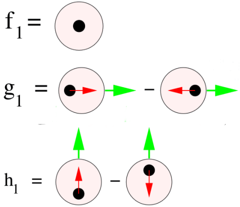
MARIE CURIE ACTIONS

Foreword: (collinear) leading twist PDF map





$$\phi_{ij}(k;P,S) = 2\pi \sum_X \int \frac{d^3\mathbf{P}_X}{2E_X}\delta^4(P-k-P_X)\langle P,S|\bar{\psi}_j(0)|X\rangle\langle X|\psi_i(0)|P,S\rangle$$

$$\boxed{\phi(x,S) = \frac{1}{2}\left[\mathbf{f_1}(x)\not{n}_+ + S_L g_1(x)\gamma^5\not{n}_+ + h_1 i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu\right]}$$

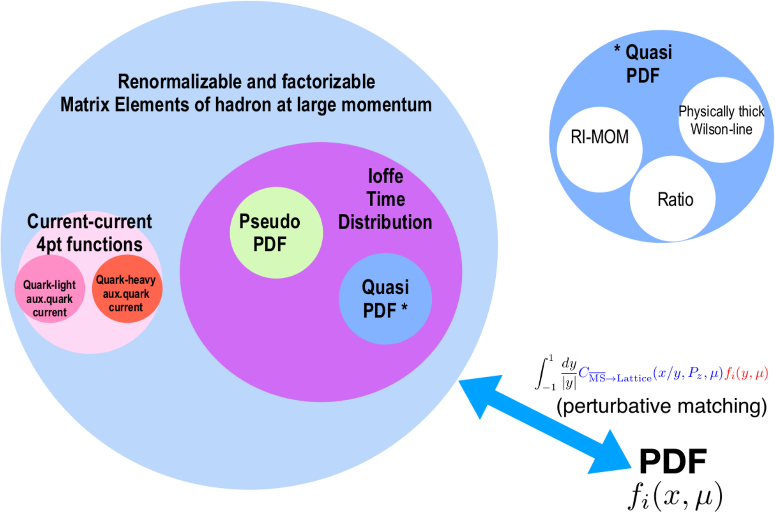
In this talk $\mathbf{f_1} \to f$, $g_1 \to \Delta f$ and $h_1 \to \delta f$

$$f(x) = \frac{1}{4\pi}\int dy^- e^{-ixP^+y^-}\langle P,S|\bar{\psi}_f(0,0,\mathbf{0}_\perp)\gamma^+\mathcal{P}\psi_f(0,y^-,\mathbf{0}_\perp)|P,S\rangle$$

$$\Delta f(x) = \frac{1}{4\pi}\int dy^- e^{-ixP^+y^-}\langle P,S|\bar{\psi}_f(0,0,\mathbf{0}_\perp)\gamma^+\gamma^5\mathcal{P}\psi_f(0,y^-,\mathbf{0}_\perp)|P,S\rangle$$

$$h_1(x) = \frac{1}{4\pi}\int dy^- e^{-ixP^+y^-}\langle P,S|\bar{\psi}_f(0,0,\mathbf{0}_\perp)i\sigma^{1+}\gamma^5\mathcal{P}\psi_f(0,y^-,\mathbf{0}_\perp)|P,S\rangle$$

Strategies to reconstruct PDFs from lattice QCD



Y.Q.Ma and J.W.Qiu, PRL120 (2018),022003

X. Ji, PRL110 (2013), 262002

A. Radyushkin, PRD98 (2017),034025

W. Detmold and C.J. D. Lin, PRD73 (2006) 014501
V. M. Braun and D. Muller, EPJ C55, 349 (2008)

R.S. Sufian et al, PRD (2019), 074507

[Courtesy of N. Karthik]

[See talks by J. Qiu and D. Richards]

Strategy to fit PDFs from data

1. Collinear, leading-twist factorisation of physical observables

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} C_{If}(y,\alpha_s(\mu^2)) \otimes f(y,\mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

2. Parametrisation: general, smooth, flexible at an initial scale Q_0^2

$$x f_i(x,Q_0^2) = A_{f_i}\, x^{a_{f_i}}\, (1-x)^{b_{f_i}}\, \mathscr{F}(x,\{c_{f_i}\})$$

small x

$$x f_i(x,Q^2) \xrightarrow{x\to 0} x^{a_{f_i}}$$

(Regge theory)

$$\mathscr{F}(x,\{c_{f_i}\}) \xrightarrow[x\to 1]{x\to 0} \text{finite}$$

$$\xrightarrow[\text{smooth interpolation in between}]{}$$

(polynomials, neural networks)

large x

$$x f_i(x,Q^2) \xrightarrow{x\to 1} (1-x)^{b_{f_i}}$$

(quark counting rules)

3. A prescription to determine/compute expectation values and uncertainties

$$\chi^2 = \sum_{i,j}^{N_{\text{dat}}} [T_i[\{\vec{a}\}] - D_i](\text{cov}^{-1})_{ij}[T_j[\{\vec{a}\}] - D_j]$$

$$E[\mathcal{O}] = \int \mathcal{D}\Delta f \mathcal{P}(\Delta f|data)\mathcal{O}(\Delta f) \qquad V[\mathcal{O}] = \int \mathcal{D}\Delta f \mathcal{P}(\Delta f|data)[\mathcal{O}(\Delta f) - E[\mathcal{O}]]^2$$

Monte Carlo: $\mathcal{P}(\Delta f|data) \longrightarrow \{\Delta f_k\}$

$$E[\mathcal{O}] \approx \tfrac{1}{N}\textstyle\sum_k \mathcal{O}(\Delta f_k)$$

$$V[\mathcal{O}] \approx \tfrac{1}{N}\textstyle\sum_k [\mathcal{O}(\Delta f_k) - E[\mathcal{O}]]^2$$

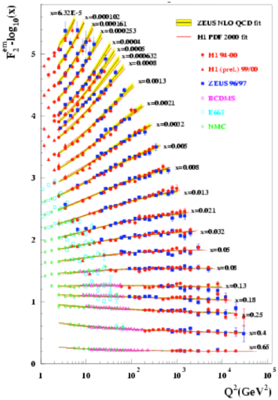
Maximum likelihood: $\mathcal{P}(\Delta f|data) \longrightarrow \Delta f_0$

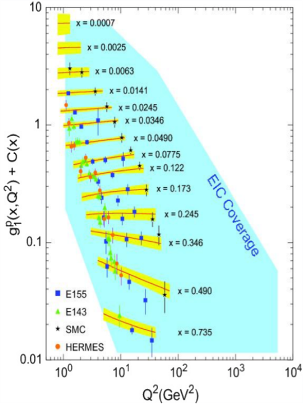
$$E[\mathcal{O}] \approx \mathcal{O}(\Delta f_0)$$

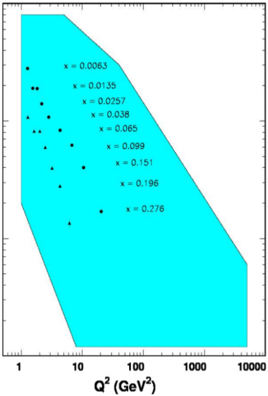
$$V[\mathcal{O}] \approx \text{Hessian}, \Delta\chi^2\text{envelope}, \ldots$$

Kinematic coverage

World data for F_2^p	World data for g_1^p	World data for h_1



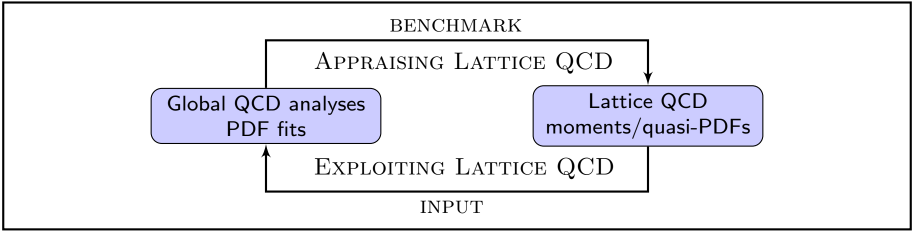




Fits of f from **thousands** of data CT, MMHT, NNPDF, . . . [See talk by L. Harland-Lang]	Fits of Δf from hundreds of data DSSV, JAM, NNPDF, . . . [See talk by N. Sato]	Fits of δf from tens of data Kang; Anselmino; Bacchetta

[See also F. Olness' talk in the general workshop track]

Connecting two faces of the same world



Define a mutually agreed conventional notation
for relevant PDF-related quantities, such as PDF moments.

Assess the sources of systematic uncertainties in lattice-QCD calculations.

Identify a best-set of quantities
to benchmark lattice-QCD calculations against global-fit determinations.

Set precision targets for lattice-QCD calculations
with respect to global-fit determinations.

Assess the impact of lattice-QCD calculations
on global-fit determinations within their current/projected precision.

PDFLattice2017, Balliol College, Oxford, 22-24 March 2017 [Prog.Part.Nucl.Phys. 100 (2018) 107]

PDFLattice2019, Kellogg Biological Station, Hickory Corners, 25-27 September 2019

1. Appraising lattice QCD

Define a quantitative benchmark for PDF moments

Benchmark quantities

f $\langle x\rangle_{u^+-d^+} = \int_0^1 dx\, x \left[u^+(x,Q^2) - d^+(x,Q^2)\right]$

$\langle x\rangle_{q^+} = \int_0^1 dx\, xq^+(x,Q^2),\ q=u,d,s \qquad \langle x\rangle_g = \int_0^1 dx\, xg(x,Q^2)$

Δf $g_A = \langle 1\rangle_{\Delta u^+-\Delta d^+} = \int_0^1 dx\, \left[\Delta u^+(x,Q^2) - \Delta d^+(x,Q^2)\right]$

$\langle 1\rangle_{\Delta q^+} = \int_0^1 dx\, \Delta q^+(x,Q^2),\ q=u,d,s \qquad \langle x\rangle_{\Delta u^- - \Delta d^-} = \int_0^1 x dx\, \left[\Delta u^-(x,Q^2) - \Delta d^-(x,Q^2)\right]$

δf $g_T = \langle 1\rangle_{\delta u^+-\delta d^+} = \int_0^1 dx\, \left[h_1^{u^+}(x,Q^2) - h_1^{d^+}(x,Q^2)\right]$

$g_T^q = \langle 1\rangle_{\delta q^+} = \int_0^1 dx\, h_1^{q^+}(x,Q^2),\ q=u,d,s$

Benchmark criteria

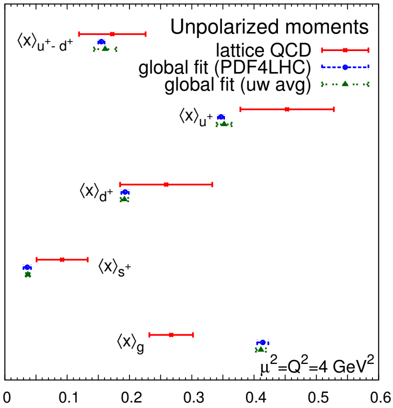
	★	○	■
discretisation	$\{a_1,\dots,a_i,\dots\}\quad i\geq 3$ $a_l, a_m < 0.1$ fm $\left(\frac{a_{\max}}{a_{\min}}\right)^2 \geq 2$	$\{a_1,\dots,a_i,\dots\}\quad i\geq 2$ $a_l < 0.1$ fm $\left(\frac{a_{\max}}{a_{\min}}\right)^2 \geq 1.4$	otherwise
chiral extrapolation	$m_{\pi,i},\ i\geq 3$ $m_{\pi,1,2} < 250$ MeV $m_{\pi,3} < 200$ MeV	$m_{\pi,i},\ i\geq 3$ $m_{\pi,1,2} < 300$ MeV	otherwise
finite volume	$m_{\pi,\min}L \geq 4$ $L_1 \neq L_2 \neq L_3 > 2.5$ fm	$m_{\pi,\min}L \geq 3.4$ $L_1 \neq L_2 > 2.5$ fm	otherwise
renormalisation	non-perturbative (RI-MOM)	perturbative (one-loop or ohigher)	otherwise
excited states	(source-sink)$_i$ $i\geq 3\ \forall\ m_\pi, L$	(source-sink)$_i$ $i\geq 2\ \forall\ m_\pi, L$	otherwise

Moments of f

Mom.	Collab.	Ref.	N_f	discretisation	chiral extrapolation	finite volume	renormalisation	excited states		Value
$\langle x \rangle_{u^+ - d^+}$	LHCP 14	[PLB 734 (2014) 290]	2+1	■	★	★	★	★		0.140(21)
	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	★	★	*	0.194(9)(11)
	RQCD 14	[PRD 90 (2014) 074510]	2	■	■	○	★	★	▶	0.217(9)
$\langle x \rangle_{u^+}$	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	★	★	*,⊢	0.453(57)(48)
$\langle x \rangle_{d^+}$	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	★	★	*,⊢	0.259(57)(47)
$\langle x \rangle_{s^+}$	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	★	★	*,⊢	0.092(41)
$\langle x \rangle_{g}$	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	○	★	*	0.267(22)(27)
	χQCD 13	[PRD 91 (2015) 014505]	0	■	■	■	○	★	⋗	0.334(55)
	QCDSF 12	[PLB 714 (2012) 312]	0	■	■	★	★	—	⋈	0.43(7)(5)
$\langle x^2 \rangle_{u^- - d^-}$	LHPC/SESAM 02	[PRD 66 (2002) 034506]	2	■	■	■	○	■		0.145(69)
	QCDSF 05	[PRD 71 (2005) 114511]	0	■	■	■	★	■		0.083(17)
$\langle x^2 \rangle_{u^-}$	χQCD 09	[PRD 79 (2009) 094502]	0	■	■	■	○	■	⊩	0.117(18)
$\langle x^2 \rangle_{d^-}$	χQCD 09	[PRD 79 (2009) 094502]	0	■	■	■	○	■	⊩	0.052(9)

* Study employing a single physical pion mass ensemble.
▶ Study employing a single ensemble with $m_\pi = 150$ MeV.
⊢ Nonsinglet renormalisation is applied.
⊩ Only the connected contribution is included.
⋗ The connected contribution is only evaluated at one $t_{\rm sep}$.
⋈ The lightest m_π has $Lm_\pi \geq 4.0$, however, $L \sim 1.6$ fm.

Moments of f



Moment	Lattice QCD	Global Fit	PDF4LHC
$\langle x\rangle_{u^+-d^+}$	0.119–0.226	0.161(18)	0.155(5)
$\langle x\rangle_{u^+}$	0.453(75)†	0.352(12)	0.347(5)
$\langle x\rangle_{d^+}$	0.259(74)†	0.192(6)	0.193(6)
$\langle x\rangle_{s^+}$	0.092(41)†	0.037(3)	0.036(6)
$\langle x\rangle_{g}$	0.267(35)†	0.411(8)	0.414(9)

† Single lattice result [PRL 119 (2017) 142002].

$q^\pm = q \pm \bar{q}$, $q = u, d, s$; $Q = 2$ GeV.
For details, see [Prog.Part.Nucl.Phys. 100 (2018) 107].

$$\langle x\rangle_{u^+-d^+} = \int_0^1 dx\, x\left[u^+(x,Q^2) - d^+(x,Q^2)\right]$$

$$\langle x\rangle_{q^+} = \int_0^1 dx\, xq^+(x,Q^2)$$

$$\langle x\rangle_{g} = \int_0^1 dx\, xg(x,Q^2)$$

Moments of Δf

Mom.	Collab.	Ref.	N_f	discretisation	chiral extrapolation	finite volume	renormalisation	excited states		Value
g_A	CalLat 17	[arXiv:1704.01114]	2+1+1	■	★	■	★	★	◇	1.278(21)(26)
	PNDME 16	[PRD 94 (2016) 054508]	2+1+1	○	★	○	★	★		1.195(33)(20)
	LHPC 14	[PLB 734 (2014) 290]	2+1	■	★	★	★	★		0.97(8)
	Mainz 17	[IJMP A34 (2019) 1950009]	2	★	○	★	★	★		$1.278(68)(^{+0}_{-0.087})$
	ETMC 17	[PRD 96 (2017) 054507]	2	■	★	■	★	★	*	1.212(33)(22)
	RQCD 15	[PRD 91 (2015) 054501]	2	○	○	○	★	○	‡	1.280(44)(46)
	QCDSF 14	[PLB 732 (2014) 41]	2	○	○	○	★	■	‡	1.29(5)(3)
$\langle 1\rangle_{\Delta u^+}$	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	★	★	*	$0.830(26)(4)$
$\langle 1\rangle_{\Delta d^+}$	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	★	★	*	$-0.386(16)(6)$
$\langle 1\rangle_{\Delta s^+}$	χQCD 17	[PRD 95 (2017) 114509]	2+1	■	○	○	★	★	†,◁	-0.0403(44)(78)
	Engelhardt 12	[PRD 86 (2012) 114510]	2+1	■	■	○	★	★	◁	-0.031(17)
	ETMC 17	[PRL 119 (2017) 142002]	2	■	★	■	★	★	*	-0.042(10)(2)
$\langle x\rangle_{\Delta u^- - \Delta d^-}$	RBC/ UKQCD 10	[PRD 82 (2010) 014501]	2+1	■	■	★	★	■		0.256(23)/ 0.205(59)
	LHPC 10	[PRD 82 (2010) 094502]	2+1	■	■	○	○	■		0.1972(55)
	ETMC 15	[PRD 92 (2015) 114513]	2	■	★	■	★	★	*	0.229(33)

* Study employing a single physical pion mass ensemble.

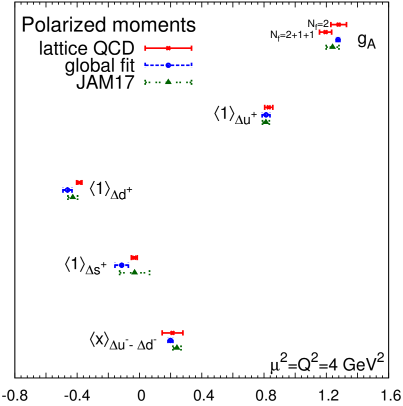
‡ g_A is determined via the ratio g_A/f_π employing the physical value for f_π.

◇ Approach inspired by the Feynman-Hellmann method is employed.

† Partially quenched simulation with $m_\pi = 330$ MeV.

◁ Some parts of the renormalisation are estimated.

Moments of Δf



Moment	Lattice QCD	Global Fit	JAM17
g_A	1.195(39)* 1.279(50)**	1.275(12)	1.240(41)
$\langle 1\rangle_{\Delta u+}$	0.830(26)†	0.813(25)	0.812(22)
$\langle 1\rangle_{\Delta d+}$	-0.386(17)†	-0.462(29)	-0.428(31)
$\langle 1\rangle_{\Delta s+}$	-0.052−-0.014	-0.114(43)	-0.038(96)
$\langle x\rangle_{\Delta u^- - \Delta d^-}$	0.146–0.279	0.199(16)	0.241(26)

* $N_f = 2$.
** $N_f = 2+1+1$.
† Single lattice result [PRL 119 (2017) 142002].

$\Delta q^\pm = \Delta q \pm \Delta\bar{q}$, $q = u, d, s$; $Q = 2$ GeV.
For details, see [Prog.Part.Nucl.Phys. 100 (2018) 107]

$$g_A = \langle 1\rangle_{\Delta u^+ - \Delta d^+} = \int_0^1 dx\, \left[\Delta u^+(x,Q^2) - \Delta d^+(x,Q^2)\right]$$

$$\langle 1\rangle_{\Delta q^+} = \int_0^1 dx\, \Delta q^+(x,Q^2)$$

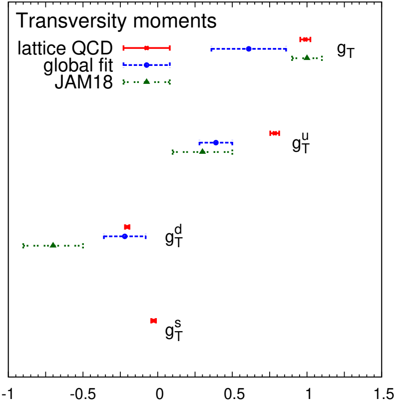
$$\langle x\rangle_{\Delta u^- - \Delta d^-} = \int_0^1 x dx\, \left[\Delta u^-(x,Q^2) - \Delta d^-(x,Q^2)\right]$$

Moments of δf

Mom.	Collab.	Ref.	N_f	discretisation	chiral extrapolation	finite volume	renormalisation	excited states		Value
g_T	ETMC 19	[arXiv:1909.00485]	2+1+1	○	○	★	★	★		0.926(32)
	PDNME 18	[PRD 98 (2018) 034503]	2+1+1	★	★	★	★	★	*	0.989(32)(10)
	LHPC 19	[PRD 99 (2019) 114505]	2+1	○	○	★	★	★	*	0.972(41)
	Mainz 19	[PRD 100 (2019) 034513]	2+1	○	★	★	★	★		$0.965(38)(^{+13}_{-41})$
	JLQCD 18	[PRD 98 (2018) 054516]	2+1	○	■	○	★	★		1.08(3)(3)(9)
	LHCP 12	[PRD 86 (2012) 114509]	2+1	★	■	★	★	★		1.038(11)(12)
	ETMC 17	[PRD 96 (2017) 099906]	2	○	■	○	★	★		1.004(21)902)(19)
	RQCD 14	[PRD 91 (2015) 054501]	2	★	○	★	★	■		1.005(17)(29)
g_T^u	PNDME 18	[PRD 98 (2018) 034503]	2+1+1	★	★	★	★	★	*	0.784(28)(10)
	JLQCD 18	[PRD 98 (2018) 054516]	2+1	■	○	○	★	★		0.85(3)(2)(7)
	ETMC 17	[PRD 96 (2017) 099906]	2	■	○	○	★	★		0.782(16)(2)(13)
g_T^d	PNDME 18	[PRD 98 (2018) 034503]	2+1+1	★	★	★	★	★	*	-0.204(11)(10)
	JLQCD 18	[PRD 98 (2018) 054516]	2+1	■	○	○	★	★		-0.24(2)(0)(2)
	ETMC 17	[PRD 96 (2017) 099906]	2	■	○	○	★	★		-0.219(10)(2)(13)
g_T^s	PNDME 18	[PRD 98 (2018) 034503]	2+1+1	★	★	★	★	★	*	-0.0027(16)
	JLQCD 18	[PRD 98 (2018) 054516]	2+1	■	○	○	★	★		-0.012(16)(8)
	ETMC 17	[PRD 96 (2017) 099906]	2	■	○	○	★	★		-0.00319(69)(2)(22)

* Not fully $\mathcal{O}(a)$ improved by requiring an additional lattice spacing.

Moments of δf



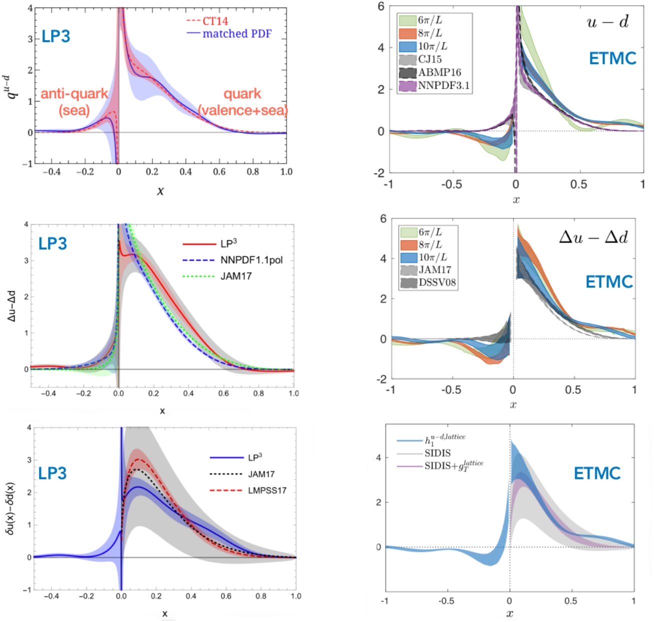
Moment	Lattice QCD	Global Fit	JAM18
g_T	0.989(32)(10)	0.61(25)	1.0(1)
g_T^u	0.784(28)(10)	0.39(11)	0.3(2)
g_T^d	-0.204(11)(1)	-0.22(14)	-0.7(2)
g_T^s	-0.027(16)	—	—

$q^+ = q + \bar{q}$, $q = u, d, s$; $Q = 2$ GeV.
Lattice results from the 2019 FLAG review.
Global fit [PRD 93 (2016) 014009]
JAM18 [PRL 120 (2018) 152502]

$$g_T = \langle 1 \rangle_{\delta u^+ - \delta d^+} = \int_0^1 dx \left[h_1^{u^+}(x, Q^2) - h_1^{d^+}(x, Q^2) \right]$$

$$g_T^q = \langle 1 \rangle_{\delta q^+} = \int_0^1 dx\, h_1^{q^+}(x, Q^2)$$

Qualitative comparison of lattice QCD and global PDF fits



2. Exploiting lattice QCD

Impact of lattice QCD moments on f

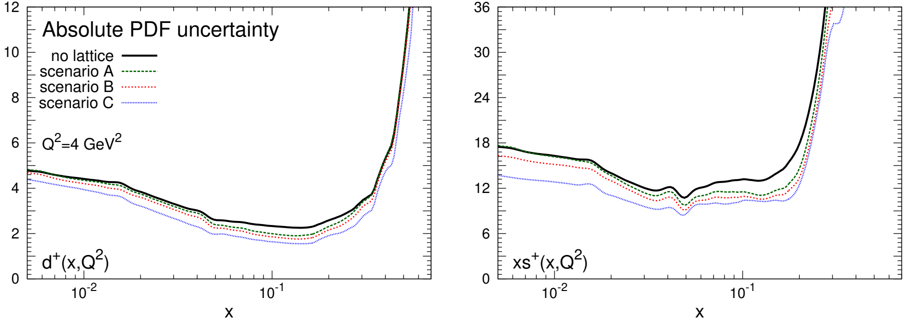
Generate lattice QCD pseudodata assuming NNPDF3.1 central values for

$$\langle x\rangle_{u^+},\ \langle x\rangle_{d^+},\ \langle x\rangle_{s^+},\ \langle x\rangle_{g},\ \langle x\rangle_{u^+-d^+}$$

Assume percentage uncertainties according to three scenarios

scenario	$\langle x\rangle_{u^+}$	$\langle x\rangle_{d^+}$	$\langle x\rangle_{s^+}$	$\langle x\rangle_{g}$	$\langle x\rangle_{u^+-d^+}$
A	3%	3%	5%	3%	5%
B	2%	2%	4%	2%	4%
C	1%	1%	3%	1%	3%
current	17%	30%	45%	13%	60%

Reweight NNPDF3.1 with lattice pseudodata and look at the impact



Impact of lattice QCD moments on Δf

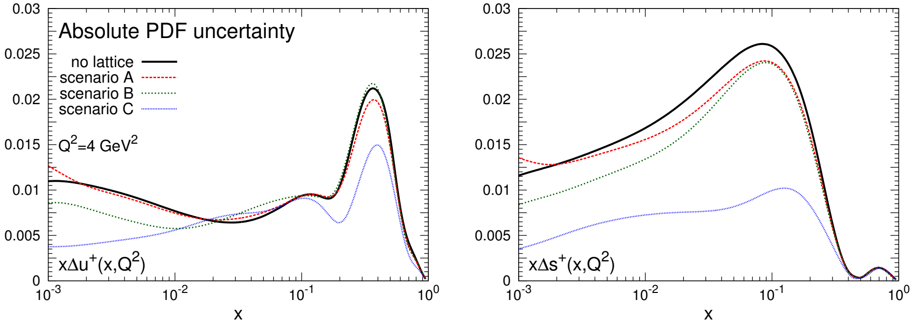
Generate lattice QCD pseudodata assuming NNPDFpol1.1 central values for

$$g_A \equiv \langle 1 \rangle_{\Delta u^+ - \Delta d^+},\ \langle 1 \rangle_{\Delta u^+},\ \langle 1 \rangle_{\Delta d^+},\ \langle 1 \rangle_{\Delta s^+},\ \langle x \rangle_{\Delta u^- - \Delta d^-}$$

Assume percentage uncertainties according to three scenarios

scenario	g_A	$\langle 1 \rangle_{\Delta u^+}$	$\langle 1 \rangle_{\Delta d^+}$	$\langle 1 \rangle_{\Delta s^+}$	$\langle x \rangle_{\Delta u^- - \Delta d^-}$
A	5%	5%	10%	100%	70%
B	3%	3%	5%	50%	30%
C	1%	1%	2%	20%	15%
current	3%	3%	5%	70%	65%

Reweight NNPDFpol1.1 with lattice pseudodata and look at the impact



Impact of lattice QCD moments on δf

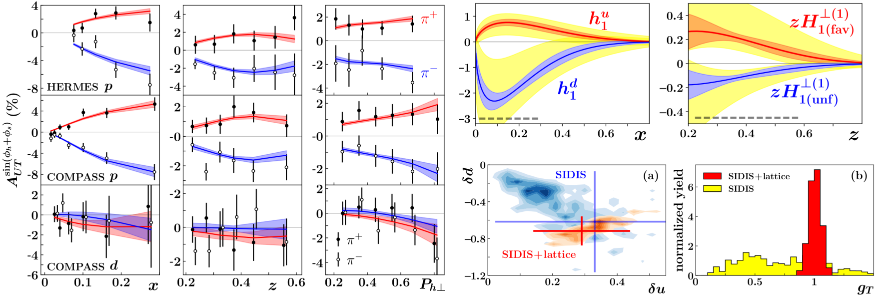
Simultaneous fit to the Collins asymmetry data from HERMES and COMPASS of

$$f_1^q(x, k_\perp^2) \qquad h_1^q(x, k_\perp^2) \qquad D_1^{h/q}(z, p_\perp^2) \qquad H_1^{\perp h/q}(z, p_\perp)$$

and to three lattice *data sets* with an estimate of systematic uncertainties

PDNME [Bhattacharya et al. (2016)] RQCD [Bali et al. (2015)] LHPC [Green et al. (2012)]

using Monte Carlo techniques for the representation of uncertainties



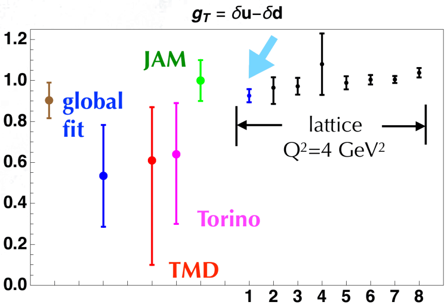
[PRL 120 (2018) 152502]

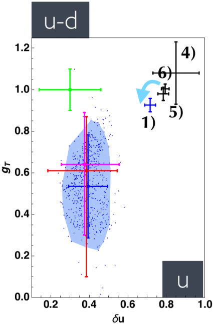
Excellent description of the data with and without lattice results ($\chi^2/N_{\rm dat} = 0.65$)

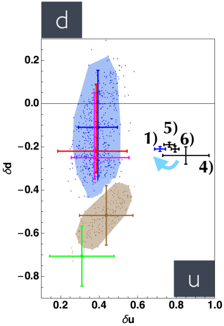
Lattice results seem compatible with measured asymmetries

Lattice results are able to reduce the uncertainty on h_1 and $H_1^\perp$ significantly

Impact of lattice QCD moments on δf [Courtesy of M. Radici]



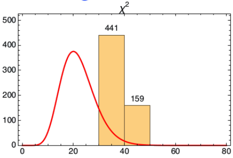




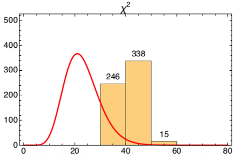
[1] ETMC 19; [2] Mainz 19; [3] LHPC 19; [4] JLQCD 18; [5] PNDME 18; [6] ETMC 17; [7] RQCD 14; [8] LHPC 12

global fit [Radici, in progress]; **JAM** [PRL 120 (2018) 152502]; **TMD** [PRD 93 (2016) 014009]; **Torino** [PRD 92 (2015) 114023]

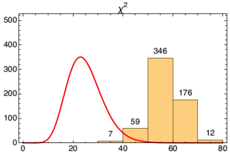
global fit → **global fit + g_T** → **global fit + g_T + δu + δd**



χ^2/dof = 1.76 ± 0.11
22 d.o.f.

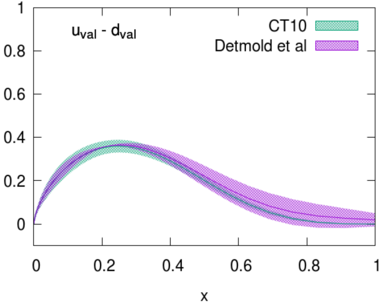


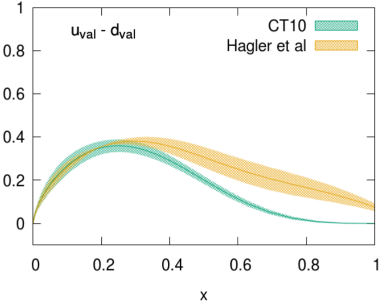
χ^2/dof = 1.82 ± 0.25
23 d.o.f.



χ^2/dof = 2.29 ± 0.25
25 d.o.f.

Reconstructing PDFs from lattice moments





Detmold *et al.* [EPJ direct 3 (2001) 13]

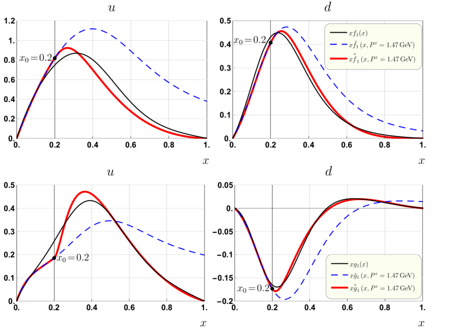
$u - d$ from the lowest few lattice moments, ensure the correct behavior in the chiral and heavy quark limits

Haegler *et al.* [PRD 77 (2008) 094502]

non-perturbative renormalization factor for the axial vector current, only connected diagrams are included

Bacchetta *et al.* [PRD 95 (2017) 014036]

supplement lattice moments with quasi-PDFs (using results of a diquark spectator model) matched at a fixed point x_0

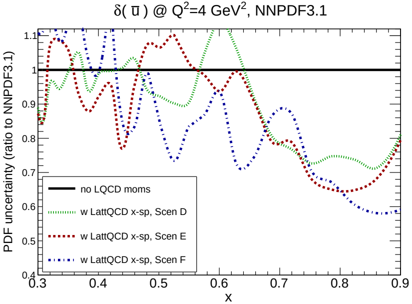


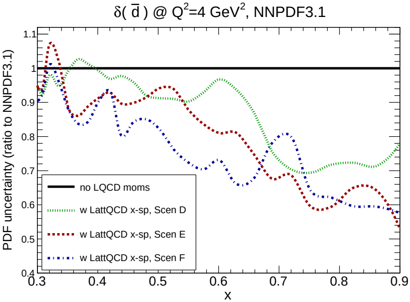
Impact of lattice calculations of x-space PDFs

Apply Bayesian reweighting to the isotriplet PDF combinations

$$\begin{array}{llll} f & \text{NNPDF3.1} & u(x_i,Q^2)-d(x_i,Q^2) & \bar{u}(x_i,Q^2)-\bar{d}(x_i,Q^2) \\ \Delta f & \text{NNPDFpol1.1} & \Delta u(x_i,Q^2)-\Delta d(x_i,Q^2) & \Delta\bar{u}(x_i,Q^2)-\Delta\bar{d}(x_i,Q^2) \end{array} \quad i=1,\dots,N_x$$

Consider uncorrelated lattice pseudodata $Q^2 = 4$ GeV^2 and $x_i = 0.70, 0.75, 0.80, 0.85, 0.90$
for three scenarios: (D) $\delta_L^{(i)} = 12\%$; (E) $\delta_L^{(i)} = 6\%$; (F) $\delta_L^{(i)} = 3\%$





No large differences among the three scenarios (PDF variations are correlated)

Moderate precision required for lattice QCD to make an impact on antiquarks at large x

Caveat: rough assumptions ($\{x_i\}$, $\delta_L^{(i)}$); can we do something better?

Impact of lattice calculations of x-space PDFs

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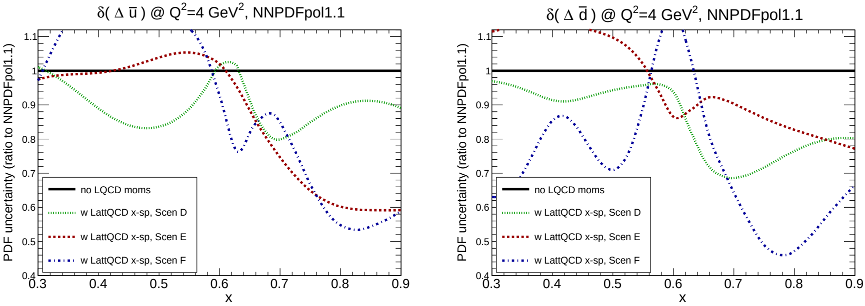

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Impact of lattice calculations of x-space PDFs [JHEP 1910, (2019) 137]

Quasi-PDFs defined as momentum-dependent nonlocal static matrix elements for nucleon states at finite momentum, with an ultraviolet cut-off scale $\Lambda \sim 1/a$

$$\widetilde{q}(x,\Lambda,p_z) = \int \frac{dz}{4\pi} e^{-ixzp_z} \frac{1}{2} \sum_{s=1}^{2} \langle p,s|\,\bar{\psi}(z)\gamma_\alpha e^{ig\int_0^z A_z(z')dz'}\psi(0)\,|p,s\rangle$$

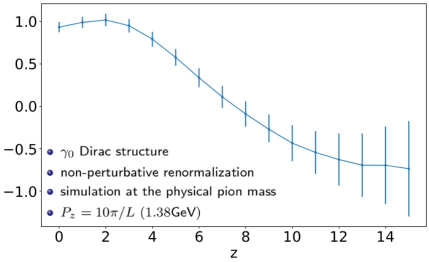
Must be related to the corresponding light-front PDF, usually within LaMET

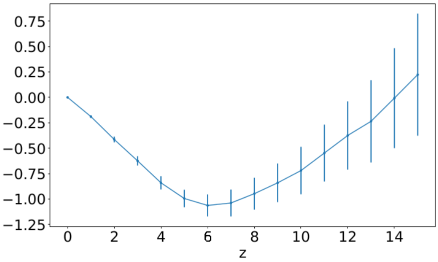
$$\widetilde{q}(x,\Lambda,p_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{p_z},\frac{\Lambda}{p_z}\right)_{\mu^2=Q^2} q(y,Q^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{p_z^2},\frac{m^2}{p_z^2}\right)$$

Restrict to the isotriplet distributions and consider ETMC lattice data

$$V_3 = u - \bar{u} - [d - \bar{d}] \qquad\qquad T_3 = u + \bar{u} - [d + \bar{d}]$$

$$\mathcal{O}^{\mathrm{Re}}_{\gamma^0}(z,\mu) \equiv \mathrm{Re}[h_{\gamma^0,3}(zp_z,z^2,\mu^2)] = \mathcal{C}_3^{\mathrm{Re}} \circledast V_3 \qquad \mathcal{O}^{\mathrm{Im}}_{\gamma^0}(z,\mu) \equiv \mathrm{Im}[h_{\gamma^0,3}(zp_z,z^2,\mu^2)] = \mathcal{C}_3^{\mathrm{Im}} \circledast T_3$$



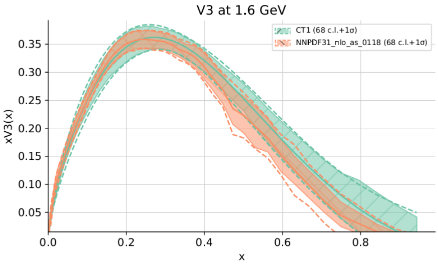


Impact of lattice calculations of x-space PDFs [JHEP 1910, (2019) 137]

Consider various scenarios for systematic uncertainties

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

Percentage values for scenarios S1-S3 should be understood as a given fraction of the central value of the matrix element
Absolute values for scenarios S4-S6 are shifts independent from the matrix element



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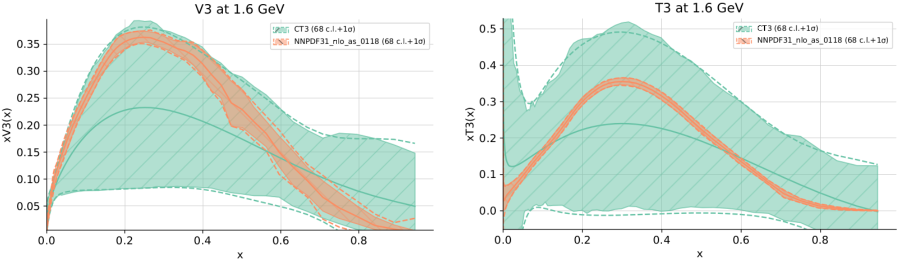

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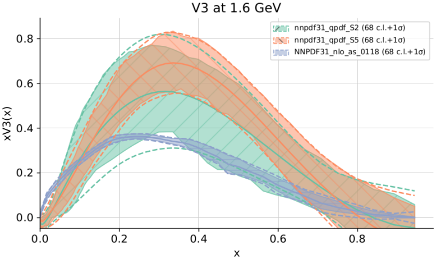
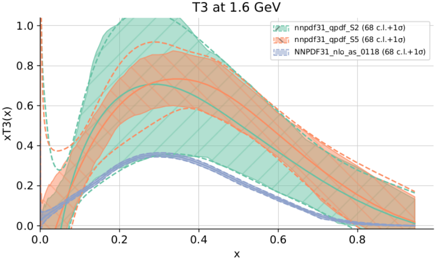




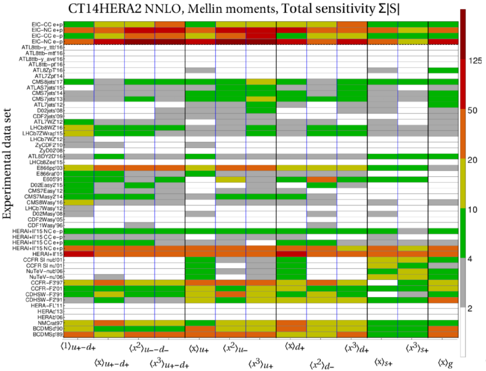
A synergy between lattice QCD and HEP phenomenology?

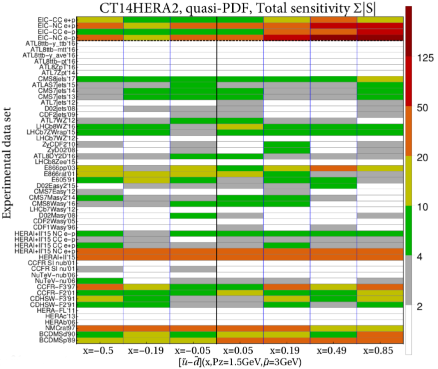
What can HEP phenomenology do for lattice QCD? [arXiv:1904.00022]

$$r_i(\vec{a}) = \frac{1}{s_i}\left[T_i(\vec{a}) - D_i^{sh}(\vec{a})\right] \qquad C_f(x_i,\mu_i) = \mathrm{Corr}[f, r_i(x_i,\mu_i)]$$

$$\mathrm{Corr}[\mathrm{X},\mathrm{Y}] = \frac{1}{4\Delta X \Delta Y}\sum_{l=1}^{N}(X_l^+ - X_l^-)(Y_l^+ - Y_l^-) \qquad \Delta X = \frac{1}{2}\sqrt{\sum_{l=1}^{N}(X_l^+ - X_l^-)^2}$$

$$\boxed{S_f = \frac{\Delta r_i}{\langle r_0\rangle_E} C_f}$$





3. Final remarks

Summary

There has been an undeniable progress in the determination of PDFs
from both the global fit to data and the lattice QCD sides

Such a progress cannot be ignored

Opportunity to gain further knowledge
by improving cross-talk between the two sides

Attempt to realise such an opportunity within the PDFLattice joint effort
benchmark + impact studies

Some substantial effort is ongoing
the definition and renormalisation of the non-local operators involved in the lattice simulation

the proof of the factorization theorem between PDFs and quasi-PDFs

the computation of the matching coefficients
relating lattice-computable quantities to PDFs in different renormalization schemes

the implementation of efficient methods
to incorporate lattice QCD information into global PDF fit determinations (and *viceversa*)

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Thank you