

Nuclear Physics from Lattice QCD

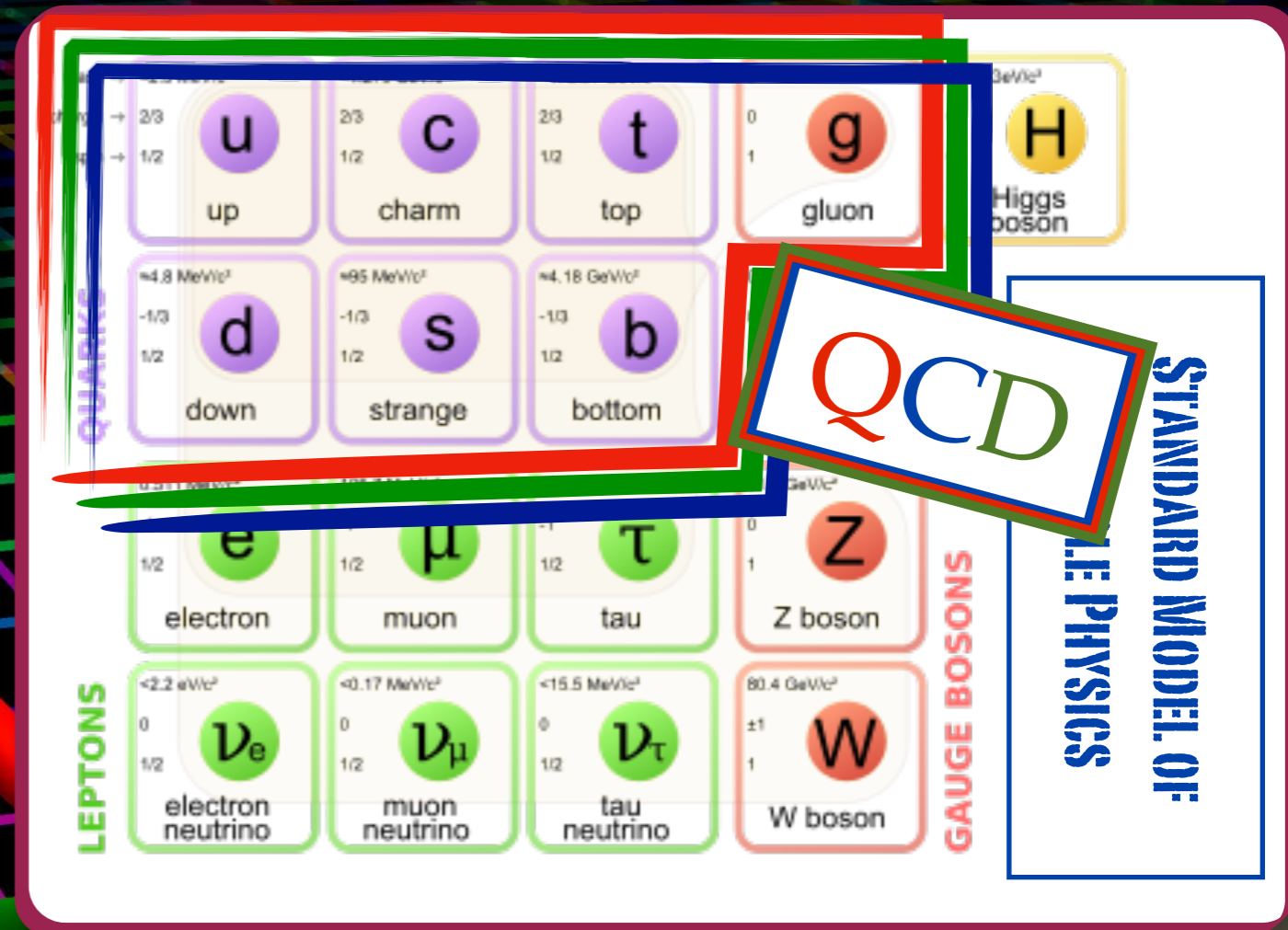
Amy Nicholson
UNC, Chapel Hill

EINN 2019 Paphos, Cyprus
October 30, 2019



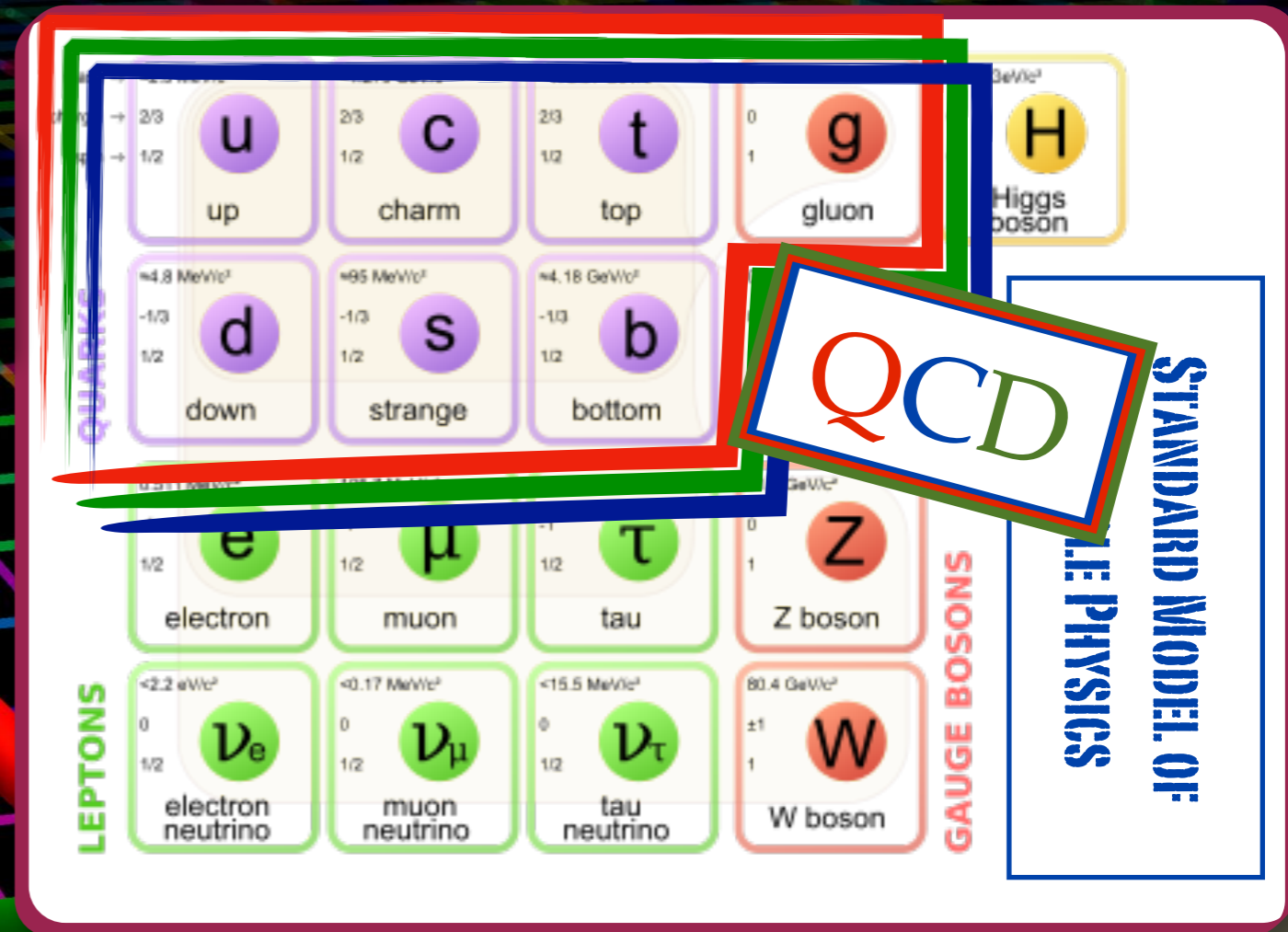
Lattice QCD

- Numerical solution to QCD:
 - Non-perturbative formulation of QCD in discretized, finite spacetime
 - Currently our only reliable technique for solving QCD at low energies
- All uncertainties are quantifiable and may be systematically removed
 - Extrapolations to continuum, infinite volume, physical pion mass

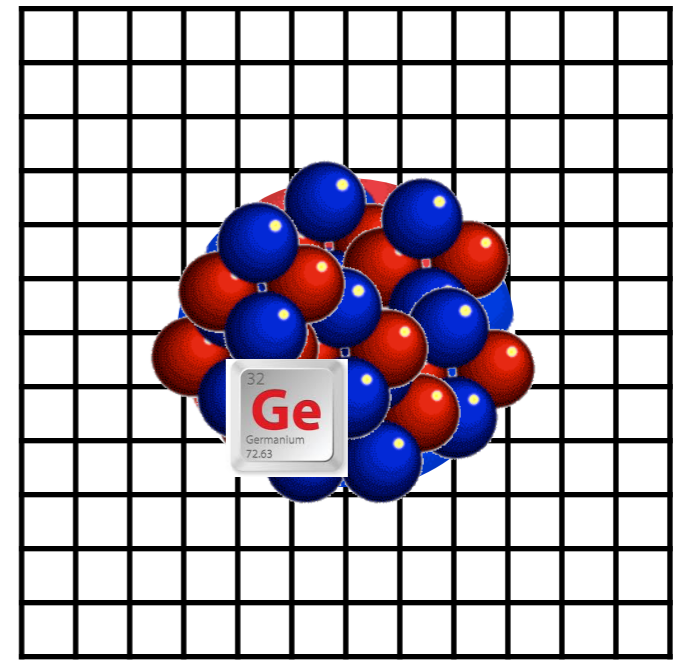


Lattice QCD

- Why LQCD for nuclear physics?
 - Test the SM
 - Match experimental signals to new physics models
 - Extract experimentally difficult quantities
 - Hadron interactions with non-zero strangeness
 - Three-neutron interactions
 - Understand quark mass dependence (fine-tuning?)
 -



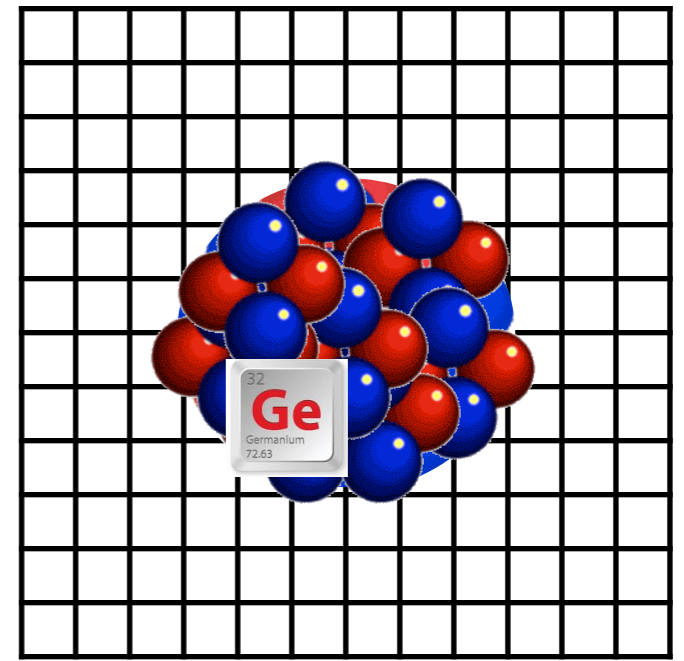
LQCD won't be used to
directly calculate heavy nuclei



Why?

LQCD won't be used to directly calculate heavy nuclei

- Need extremely large lattices
- Large range of scales
- Tiny energy splittings

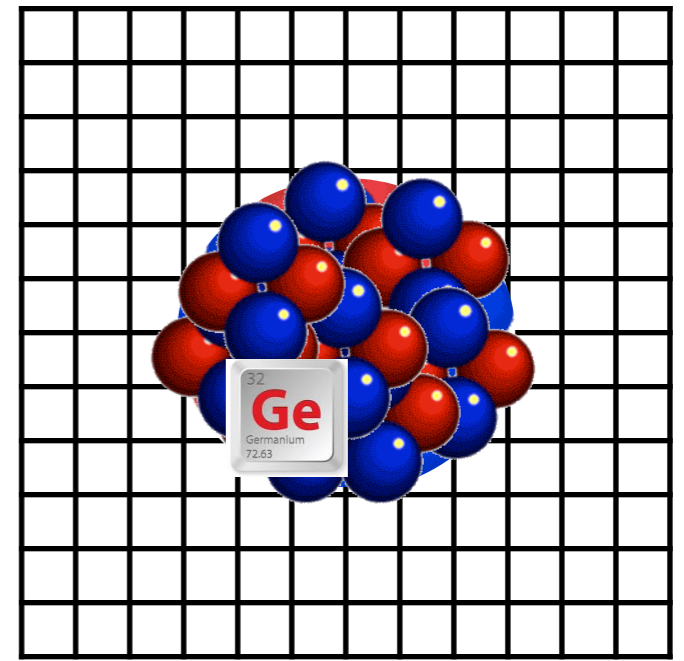


Why?

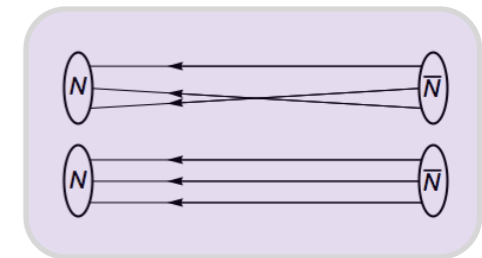
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- Wick contractions:
 $(A+Z)! \times (2A-Z)!$

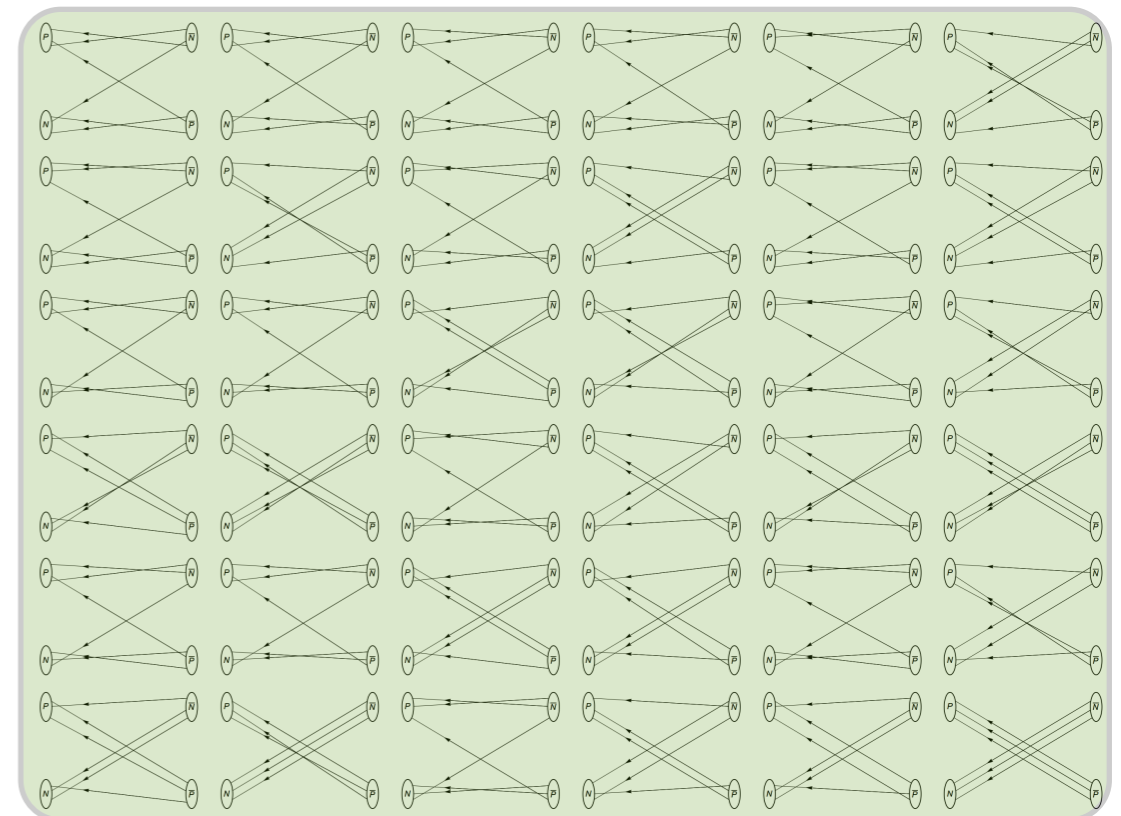
$\text{He}^4 : 518400$



Nucleon:

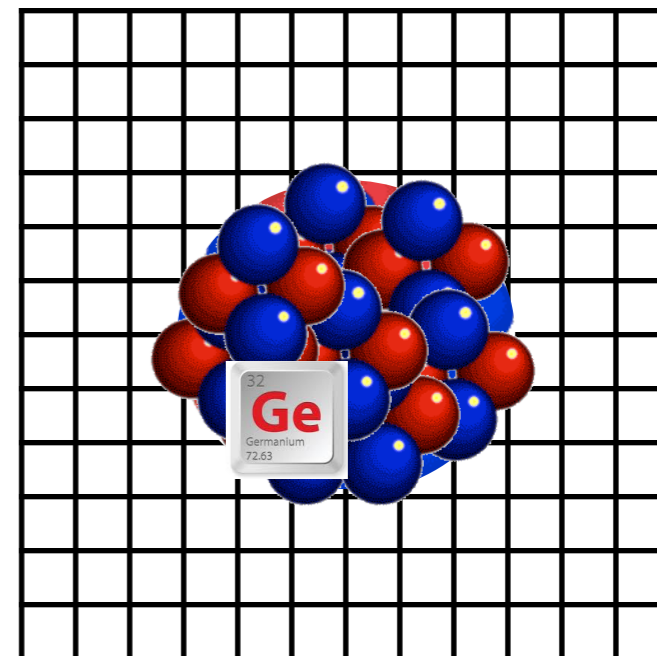


Deuteron:

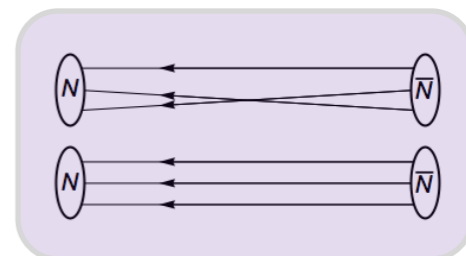


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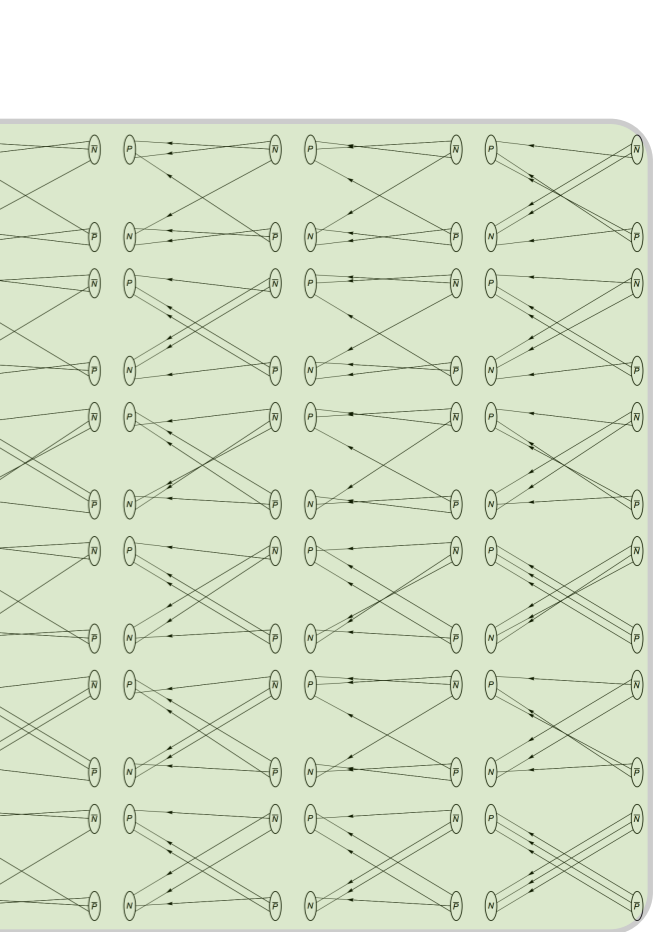
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Nucleon:



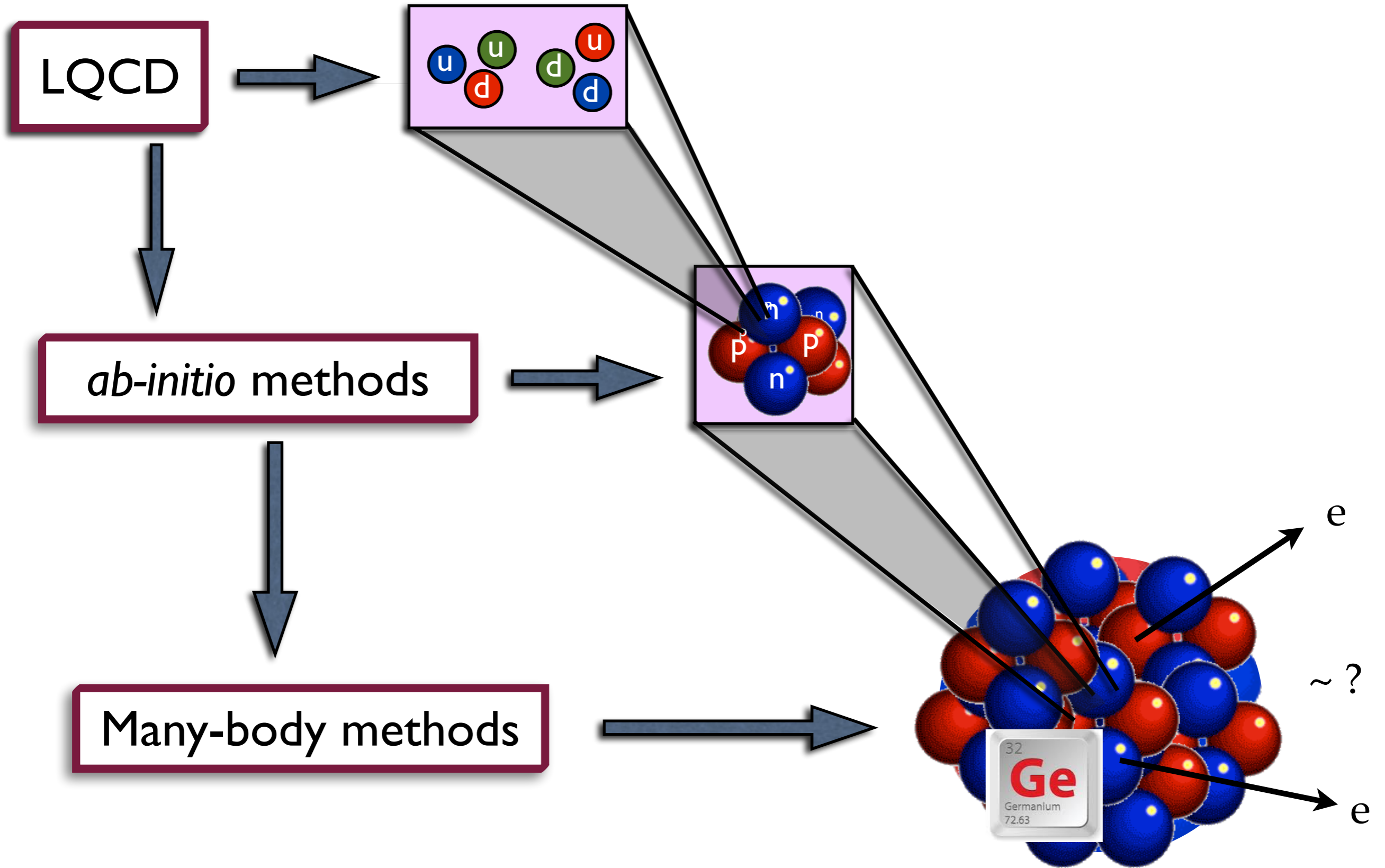
Deuteron:

- Wick contractions:
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He⁴ : 518400

- **Nucleon noise/sign problem**
signal/noise

$$\sim e^{-A(M_n - 3/2m_\pi)t}$$



Precision era for (single nucleon) LQCD

Neutron-proton mass difference:
accurate to 300 KeV (BMW 2015)

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REPORT



Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi¹, S. Durr^{1,2}, Z. Fodor^{1,2,3,*}, C. Hoelbling¹, S. D. Katz^{3,4}, S. Krieg^{1,2}, L. Lellouch⁵, T. Lippert^{1,2}, A. Portelli^{5,6}, K. K. Szabo^{1,2}, B. C. Toth¹

¹Department of Physics, University of Wuppertal, D-42119 Wuppertal, Germany.

²Jülich Supercomputing Centre, Forschungszentrum Jülich, D-52428 Jülich, Germany.

³Institute for Theoretical Physics, Eötvös University, H-1117 Budapest, Hungary.

⁴Lendület Lattice Gauge Theory Research Group, Magyar Tudományos Akadémia–Eötvös Loránd University, H-1117 Budapest, Hungary.

⁵CNRS, Aix-Marseille Université, Université de Toulon, CPT UMR 7332, F-13288, Marseille, France.

⁶School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK.

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Science 27 Mar 2015:

Vol. 347, Issue 6229, pp. 1452-1455

Axial charge of the nucleon:
 $g_A = 1.271(13)$ (CalLat 2018)

nature

International journal of science

Altmetric: 114

More detail >>

Letter | Published: 30 May 2018

A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

C. C. Chang, A. N. Nicholson, E. Rinaldi, E. Berkowitz, N. Garron, D. A. Brantley, H. Monge-Camacho, C. J. Monahan, C. Bouchard, M. A. Clark, B. Joó, T. Kurth, K. Orginos, P. Vranas & A. Walker-Loud ✉

Nature 558, 91–94 (2018) | Download Citation ↓

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Citation ↓

FLAG Review 2019

March 5, 2019

Flavour Lattice Averaging Group (FLAG)

S. Aoki,¹ Y. Aoki,^{2,3,*} D. Bećirević,⁴ T. Blum,^{5,3} G. Colangelo,⁶ S. Collins,⁷ M. Della Morte,⁸ P. Dimopoulos,⁹ S. Dür, ¹⁰ H. Fukaya,¹¹ M. Golterman,¹² Steven Gottlieb,¹³ R. Gupta,¹⁴ S. Hashimoto,^{2,15} U. M. Heller,¹⁶ G. Herdoiza,¹⁷ R. Horsley,¹⁸ A. Jüttner,¹⁹ T. Kaneko,^{2,15} C.-J. D. Lin,^{20,21} E. Lunghi,¹³ R. Mawhinney,²² A. Nicholson,²³ T. Onogi,¹¹ C. Pena,¹⁷ A. Portelli,¹⁸ A. Ramos,²⁴ S. R. Sharpe,²⁵ J. N. Simone,²⁶ S. Simula,²⁷ R. Sommer,^{28,29} R. Van De Water,²⁶ A. Vladikas,³⁰ U. Wenger,⁶ H. Wittig³¹

10 Nucleon matrix elements

Authors: S. Collins, R. Gupta, A. Nicholson, H. Wittig

A large number of experiments testing the Standard Model (SM) and searching for physics Beyond the Standard Model (BSM) involve either free nucleons (proton and neutron beams) or the scattering of electrons, protons, neutrinos and dark matter off nuclear targets. Necessary

Nucleon axial charge

PHYSICAL REVIEW LETTERS

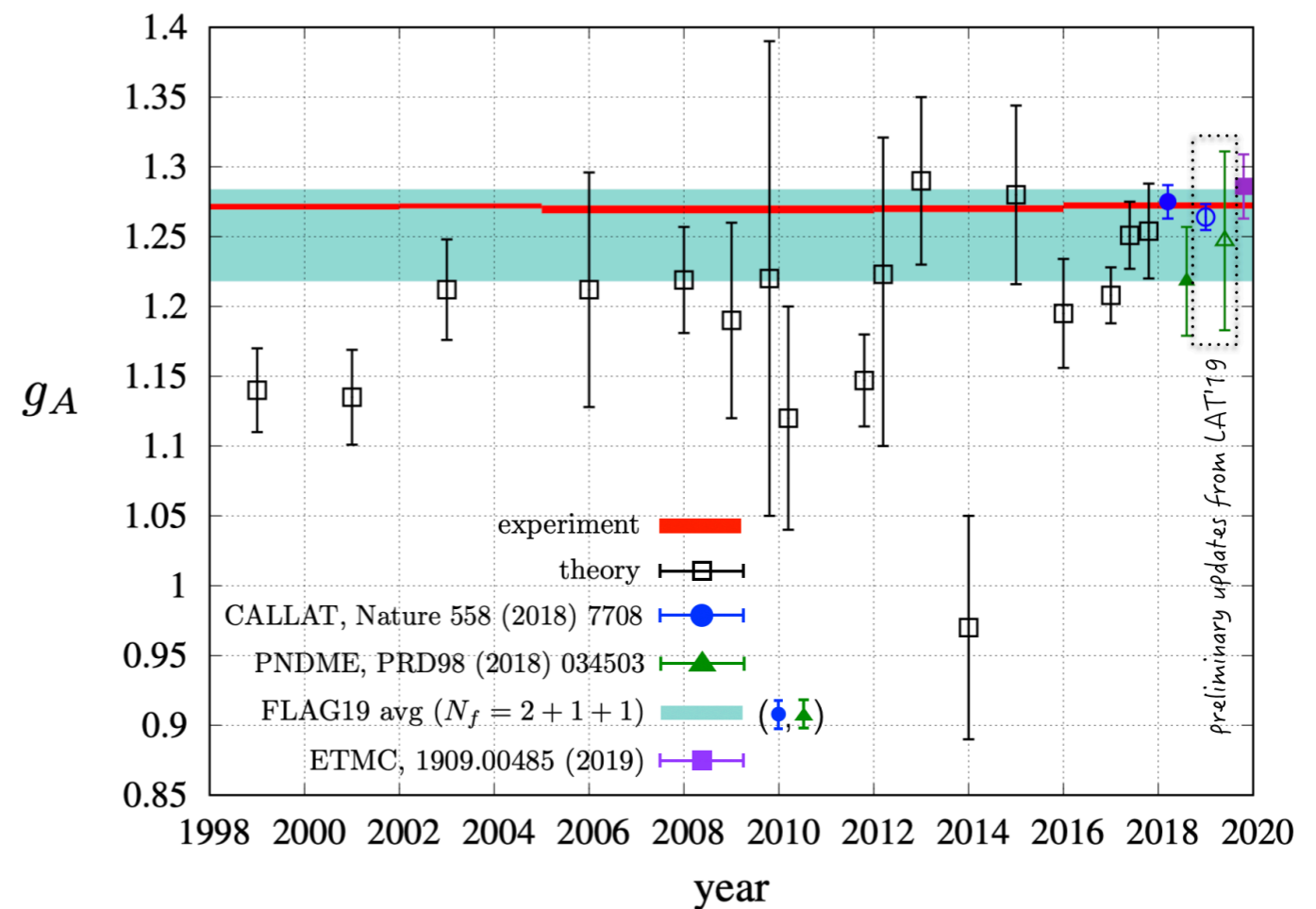
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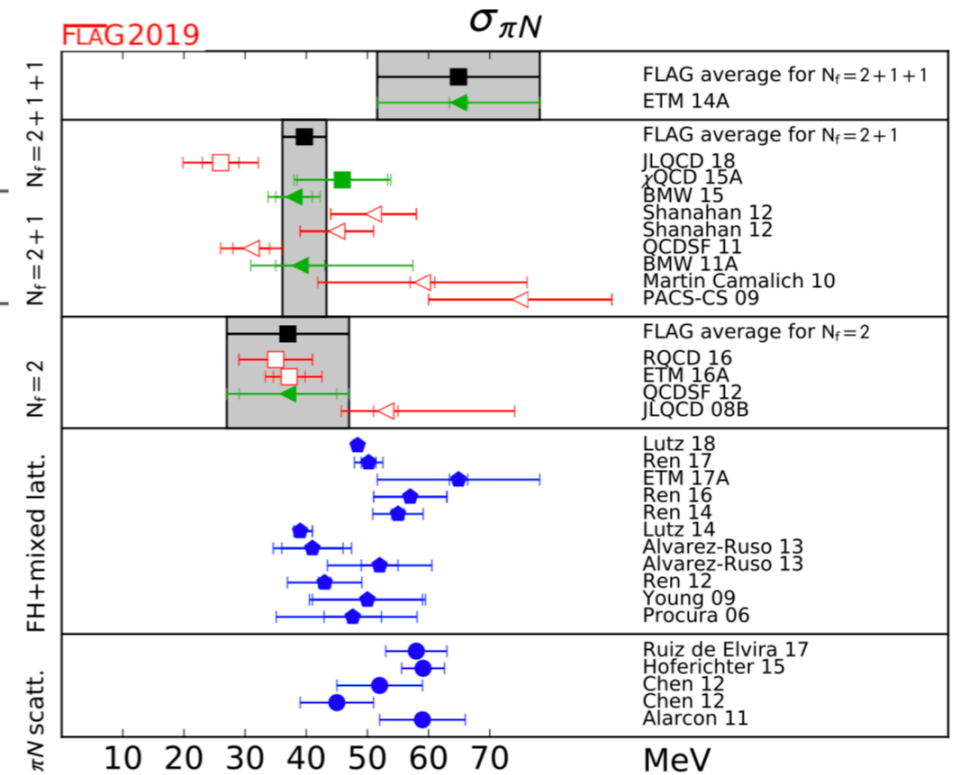
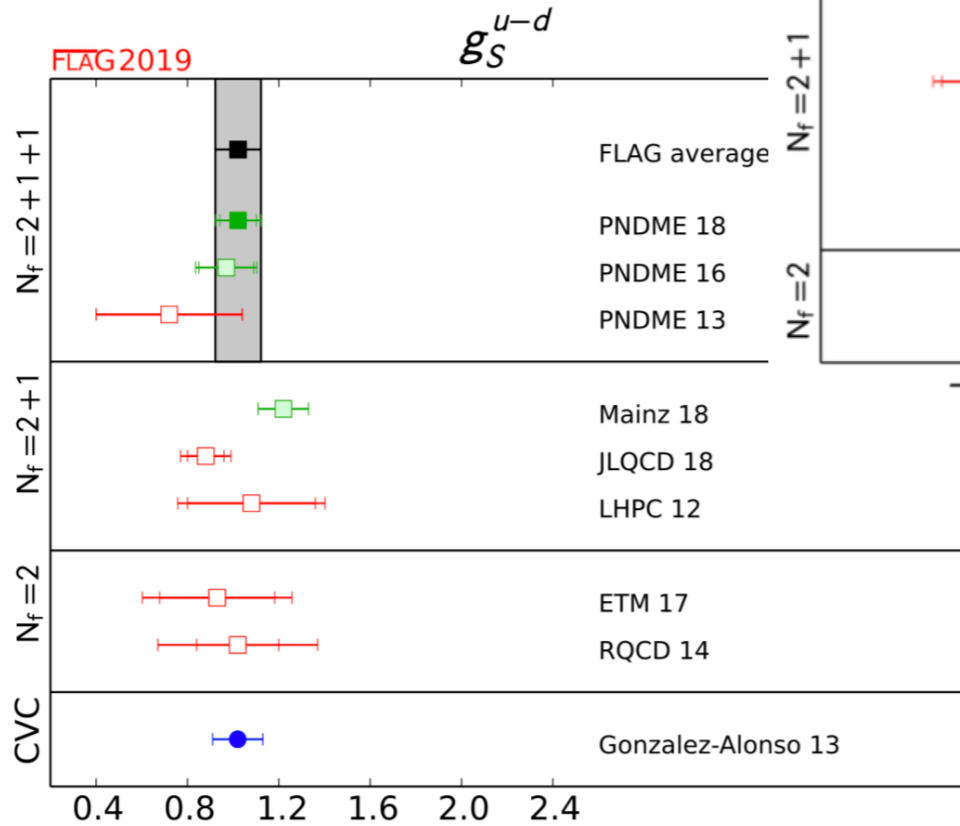
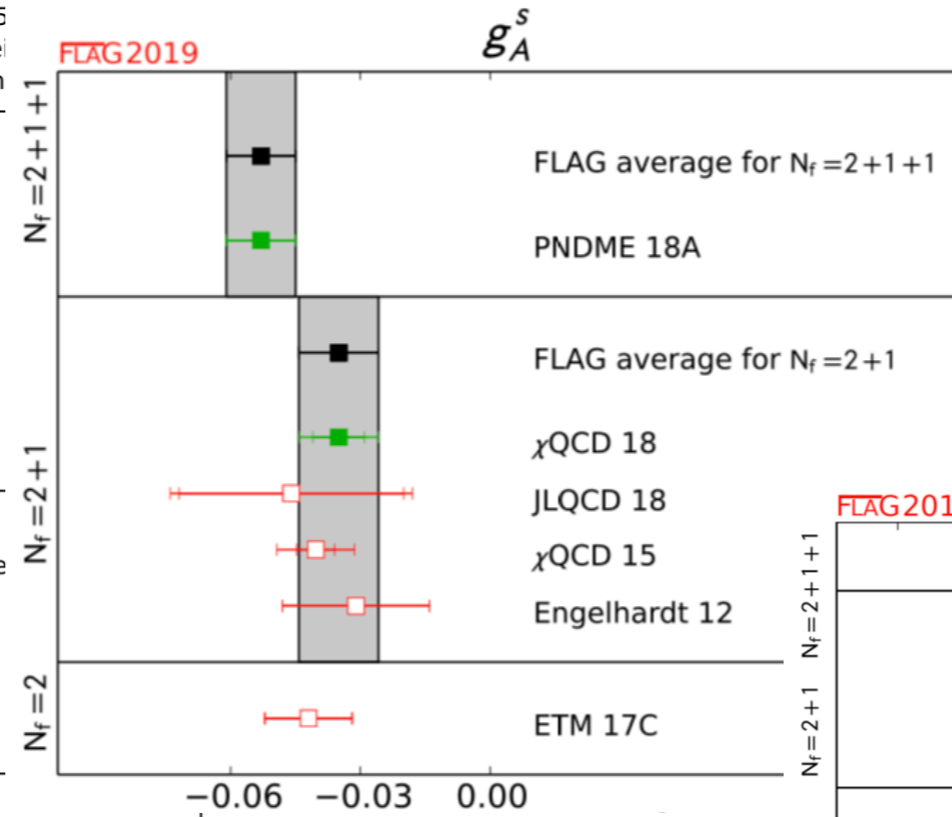
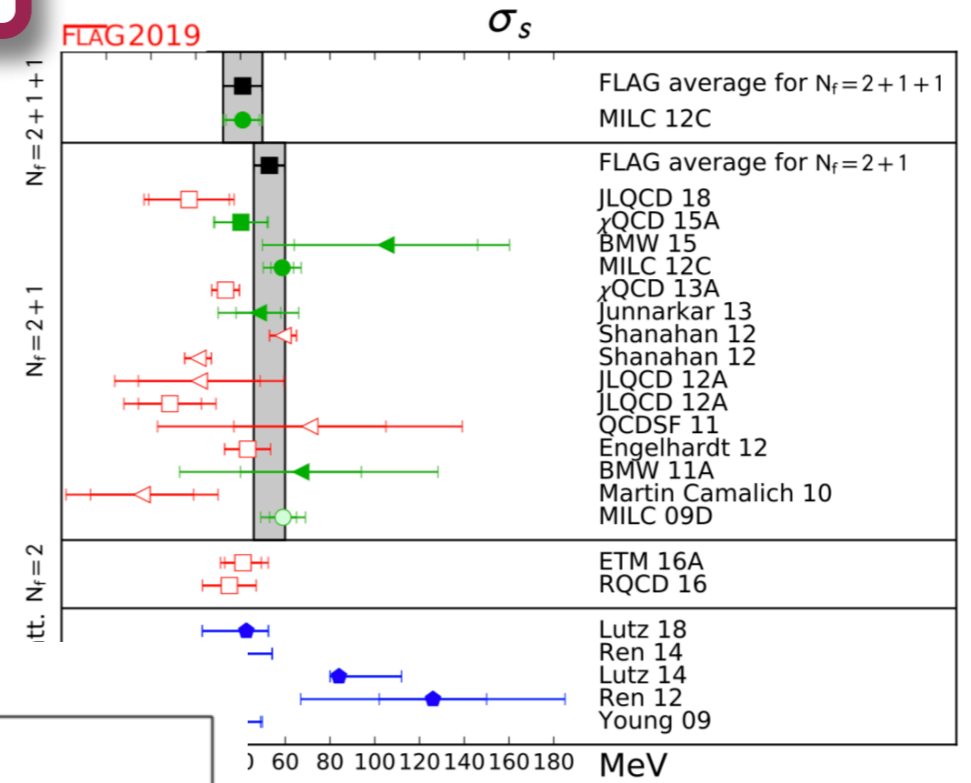
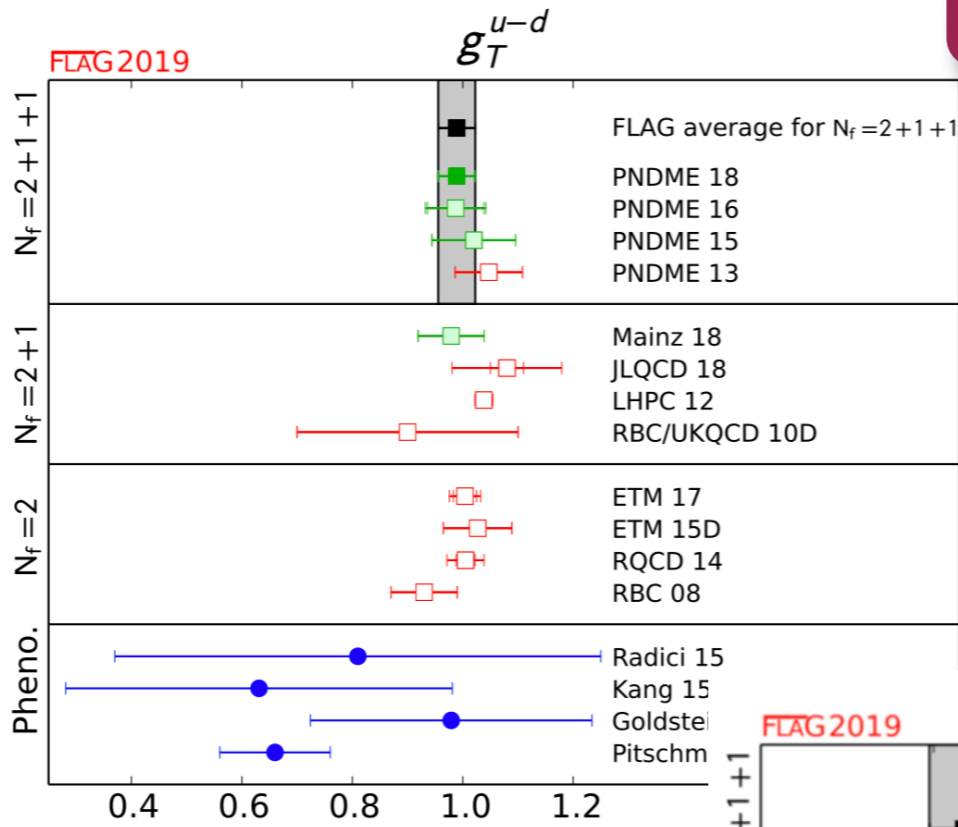
Access b

Nucleon Axial Charge in Full Lattice QCD

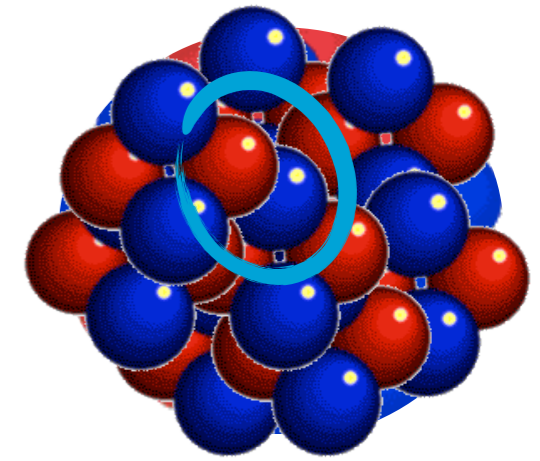
R. G. Edwards, G. T. Fleming, Ph. Hägler, J. W. Negele, K. Orginos, A. V. Pochinsky, D. B. Renner, D. G. Richards, and W. Schroers (LHPC Collaboration)
Phys. Rev. Lett. **96**, 052001 – Published 7 February 2006

The axial charge is the ideal starting point in the quest for precision lattice calculation of hadron structure for several reasons. It is accurately measured experimentally and the isovector combination $\langle 1 \rangle_{\Delta u} - \langle 1 \rangle_{\Delta d}$ has no contributions from disconnected diagrams, which are much more computationally demanding than the connected diagrams considered in this work. The functional dependence on both m_π^2 and volume is known at small masses from chiral perturbation theory (χ PT) [5,6] and renormalization of the lattice axial vector current can be performed accurately nonperturbatively using the five-dimensional conserved current for domain wall fermions. Thus, conceptually, it is a “gold plated” test of our ability to calculate hadron observables from first principles on the lattice. In addition, since it is known to be particularly sensitive to finite lattice volume effects that reduce the contributions of the pion cloud [7,8], it is also a stringent test of our control of finite volume artifacts.

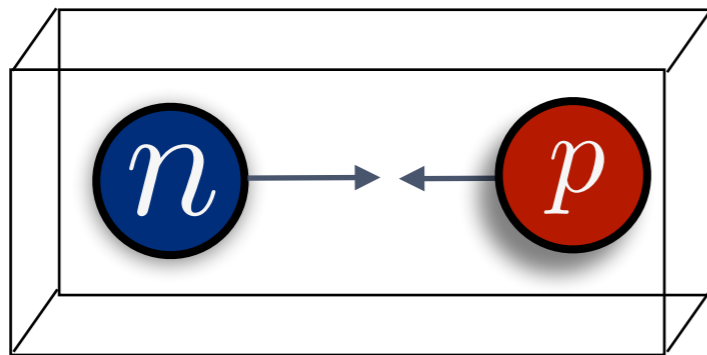




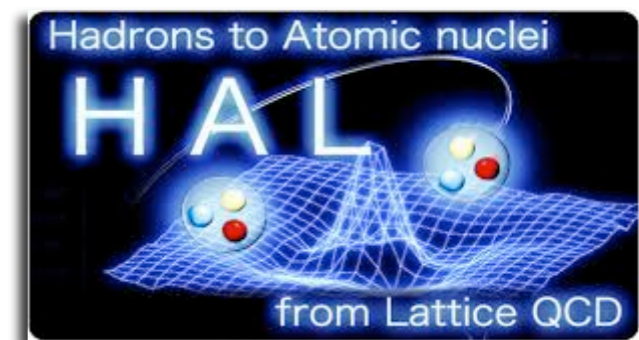
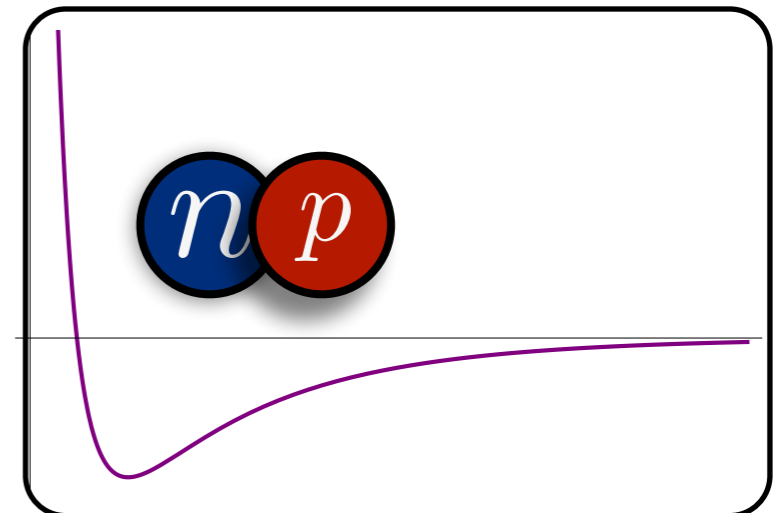
Two methods for calculating few-nucleon interactions from LQCD:



Spectroscopy + Lüscher Method

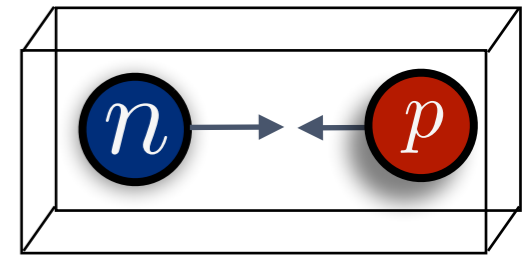


Potential Method



Yamazaki, et. al.

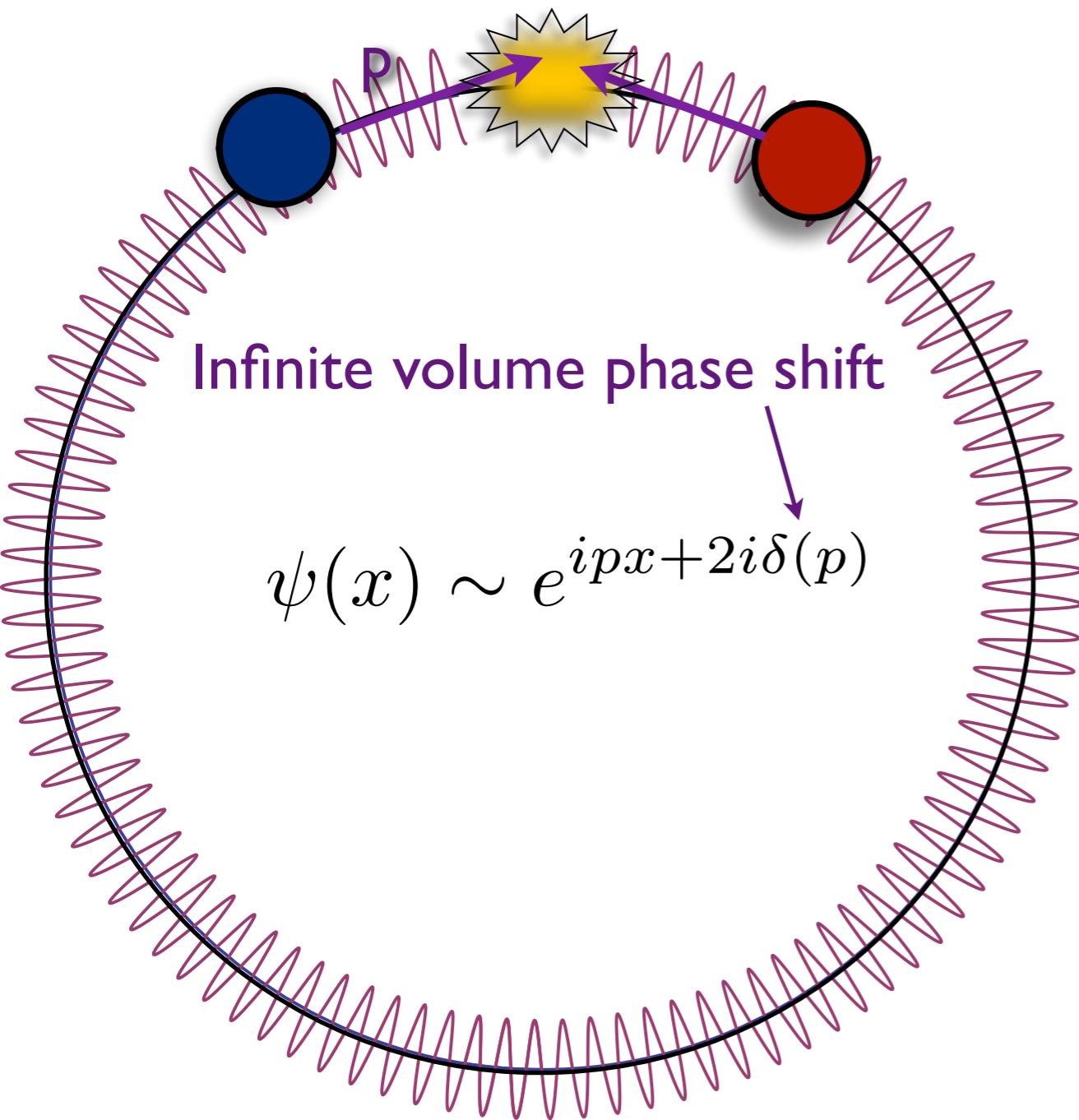
Lüscher



- Direct scattering “experiments” not possible in finite volume/Euclidean time
- Lüscher: measure discrete spectra of interacting particles in a box, and infer the interaction (scattering phase shift)

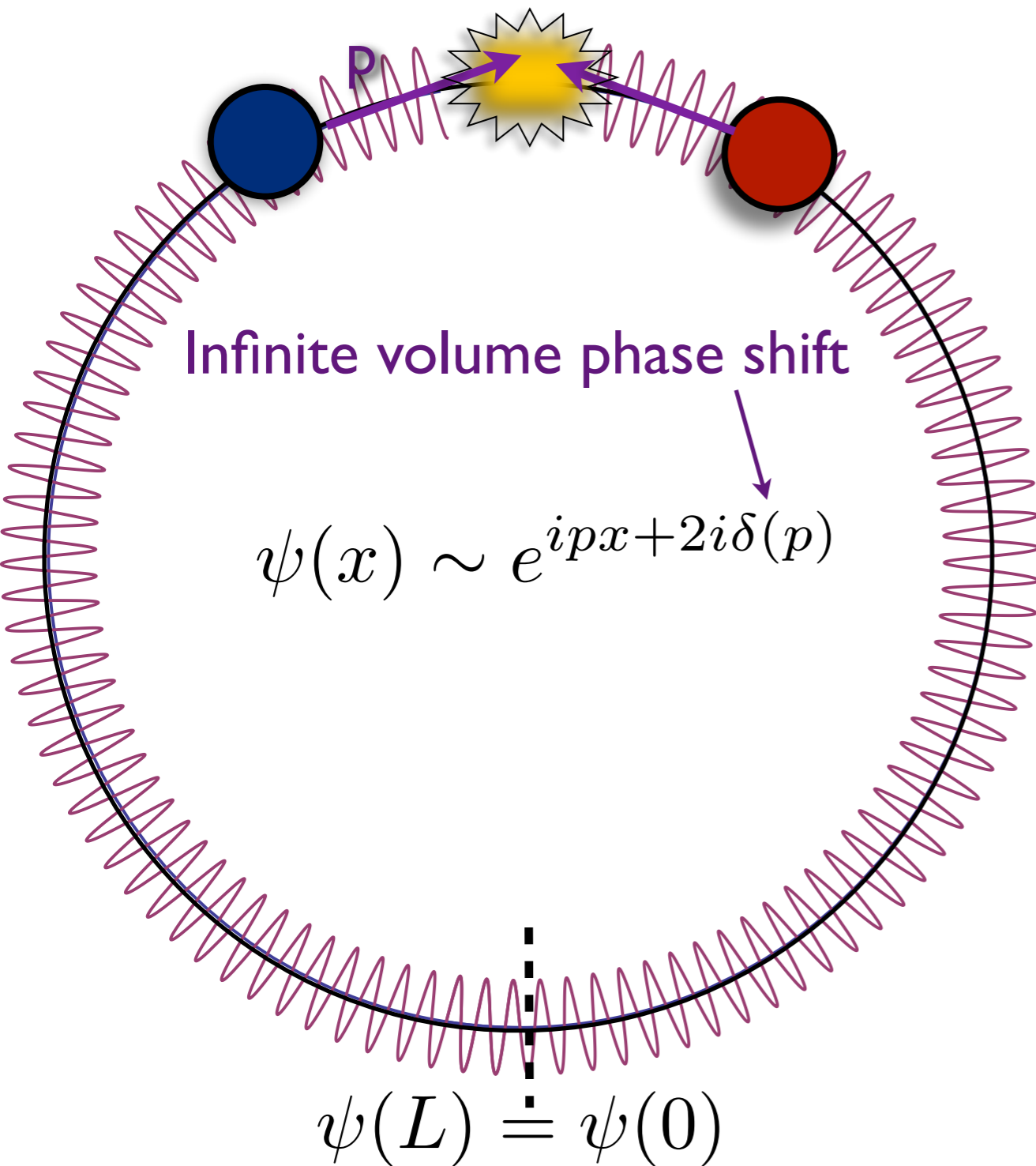
“Lüscher” in 1-d

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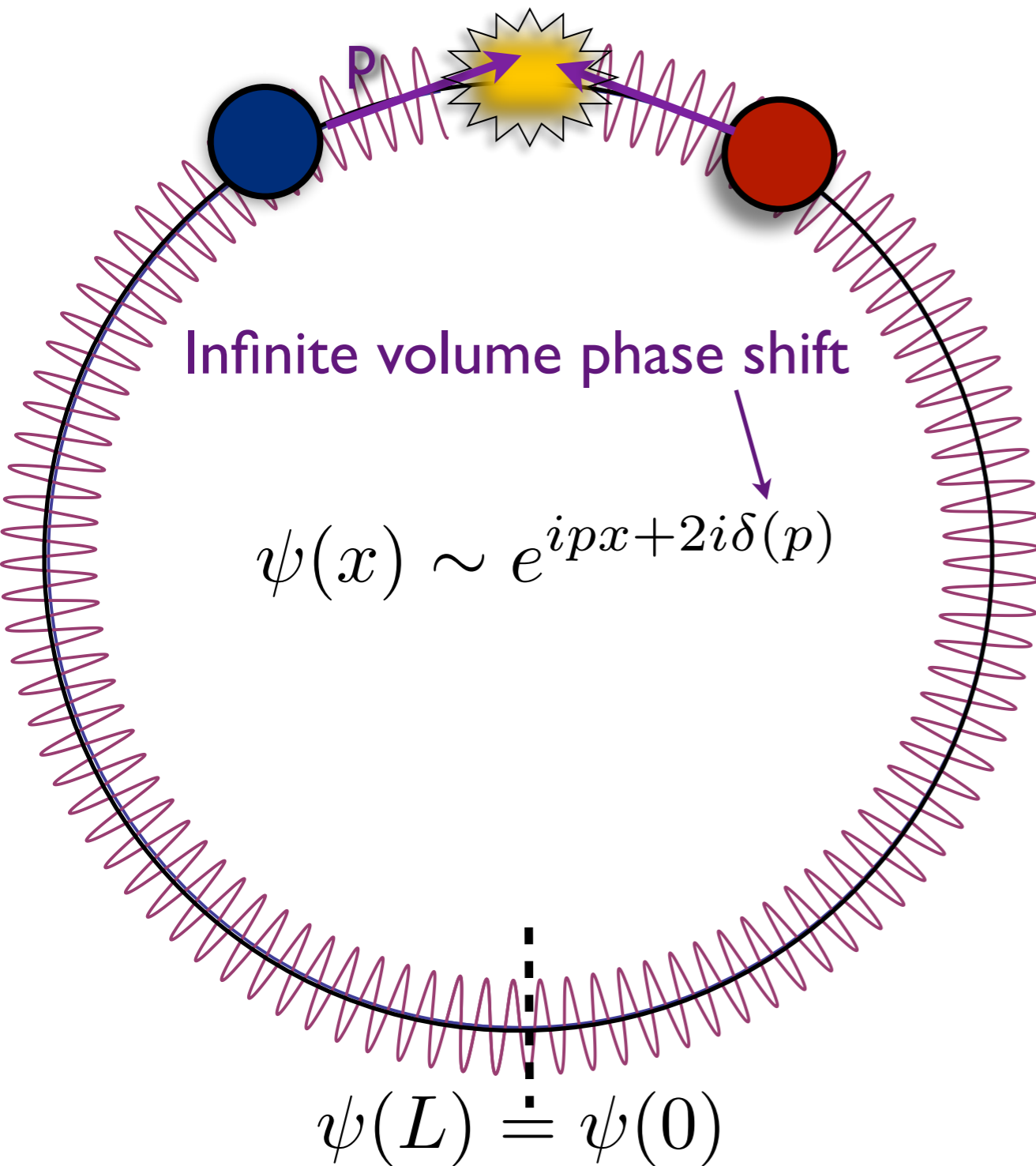


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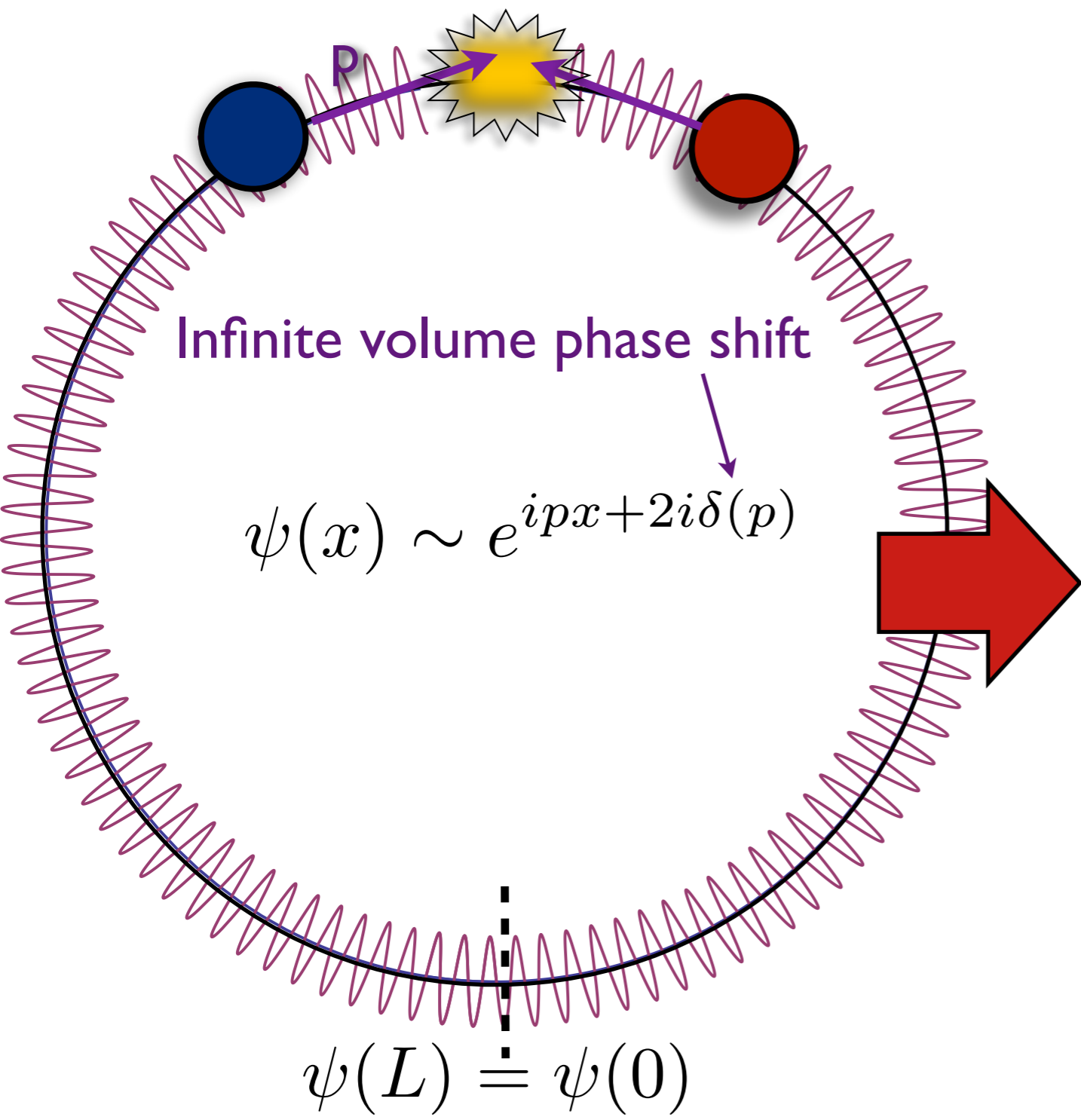
“Lüscher” in 1-d



Quantization condition:

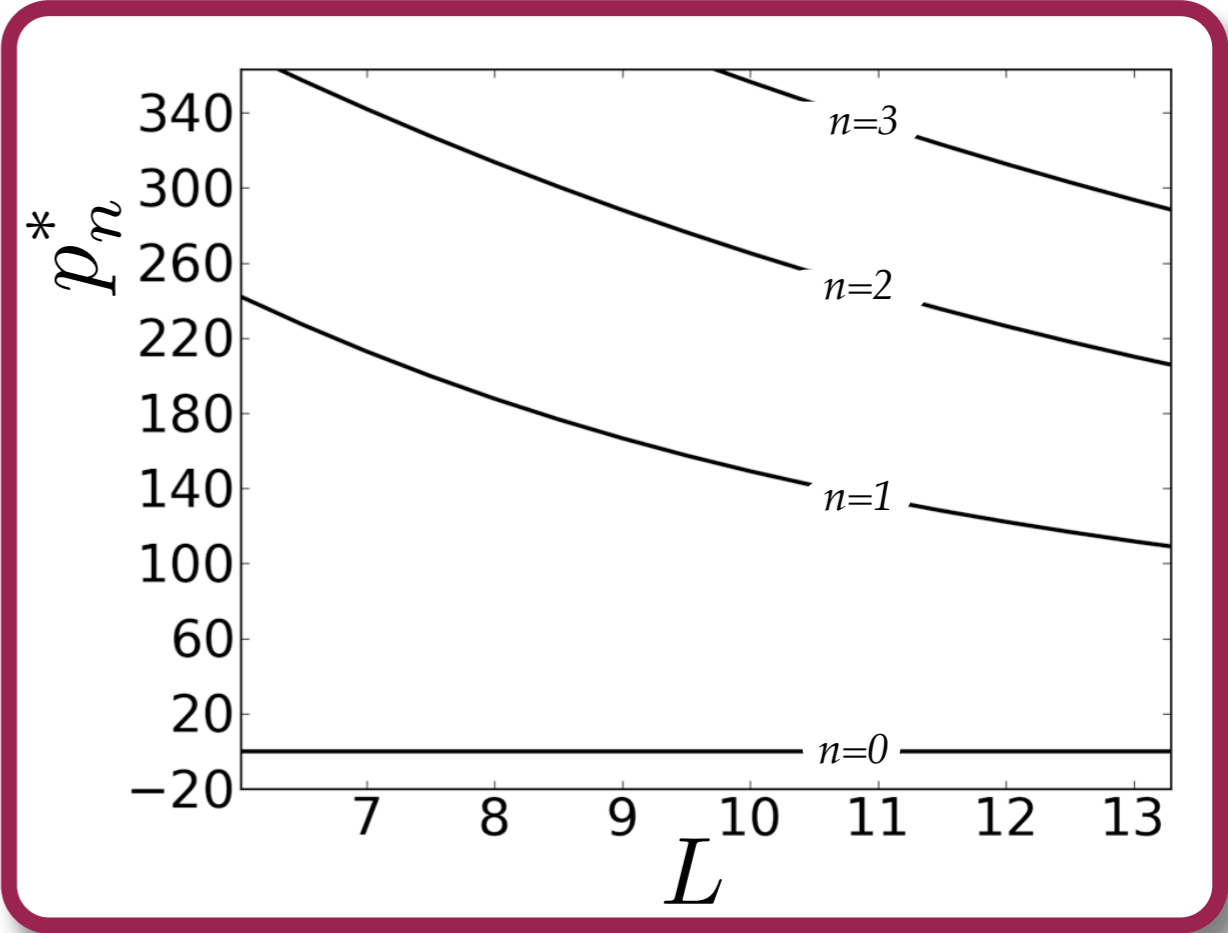
$$Lp_n^* + 2\delta(p_n^*) = 2\pi n$$

“Lüscher” in 1-d



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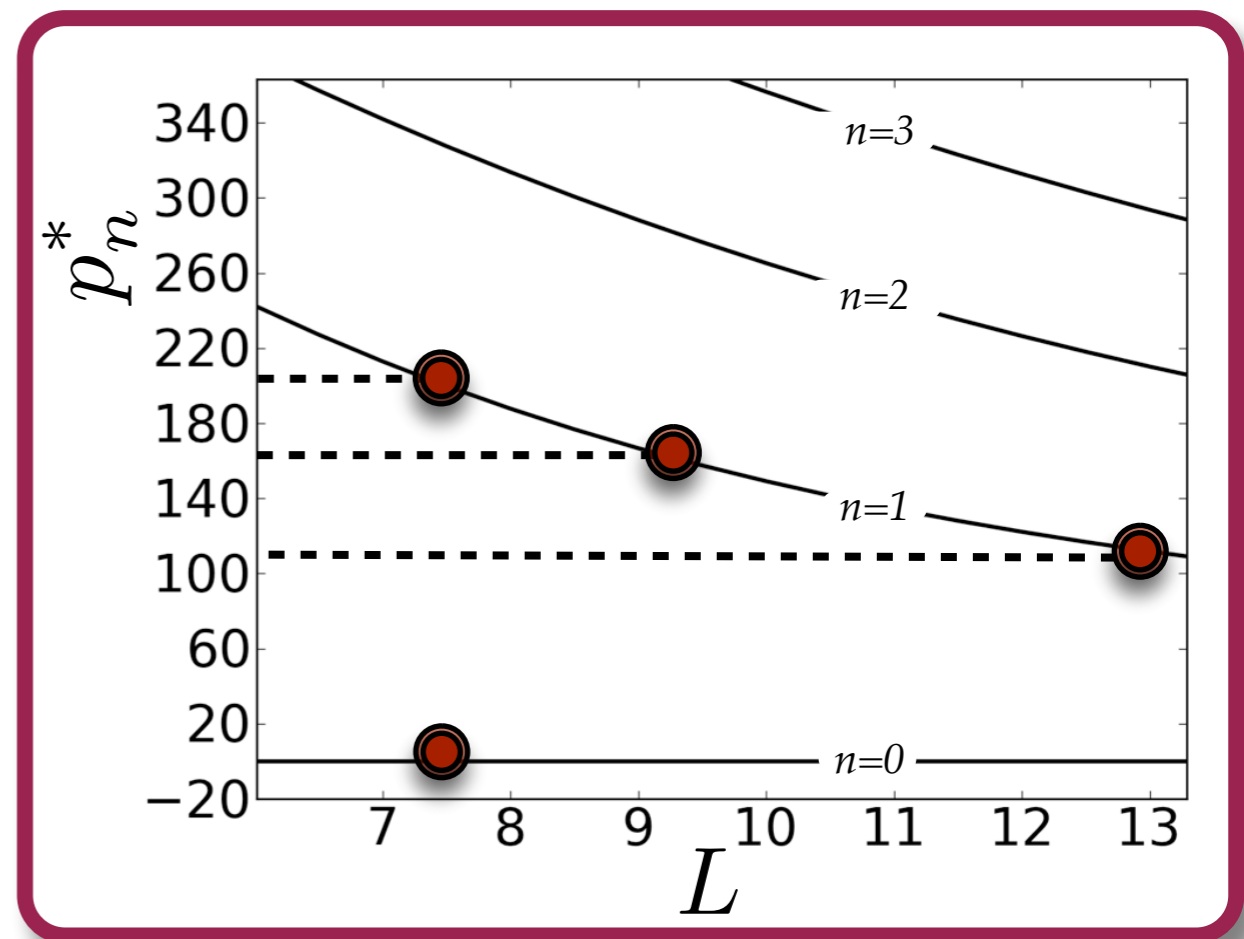
“Lüscher” in 1-d

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Quantization condition:

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Lattice: measure energies at a given L

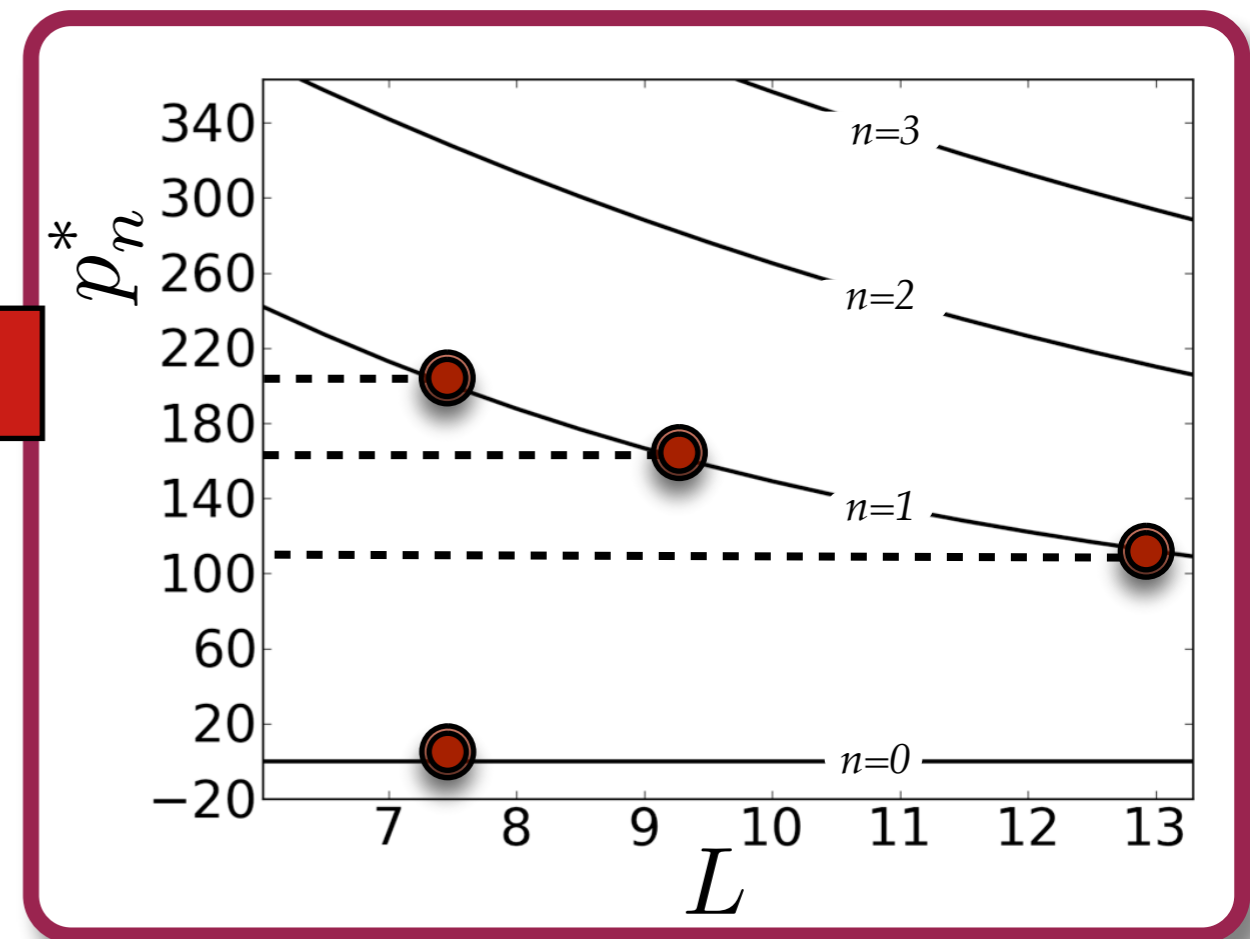
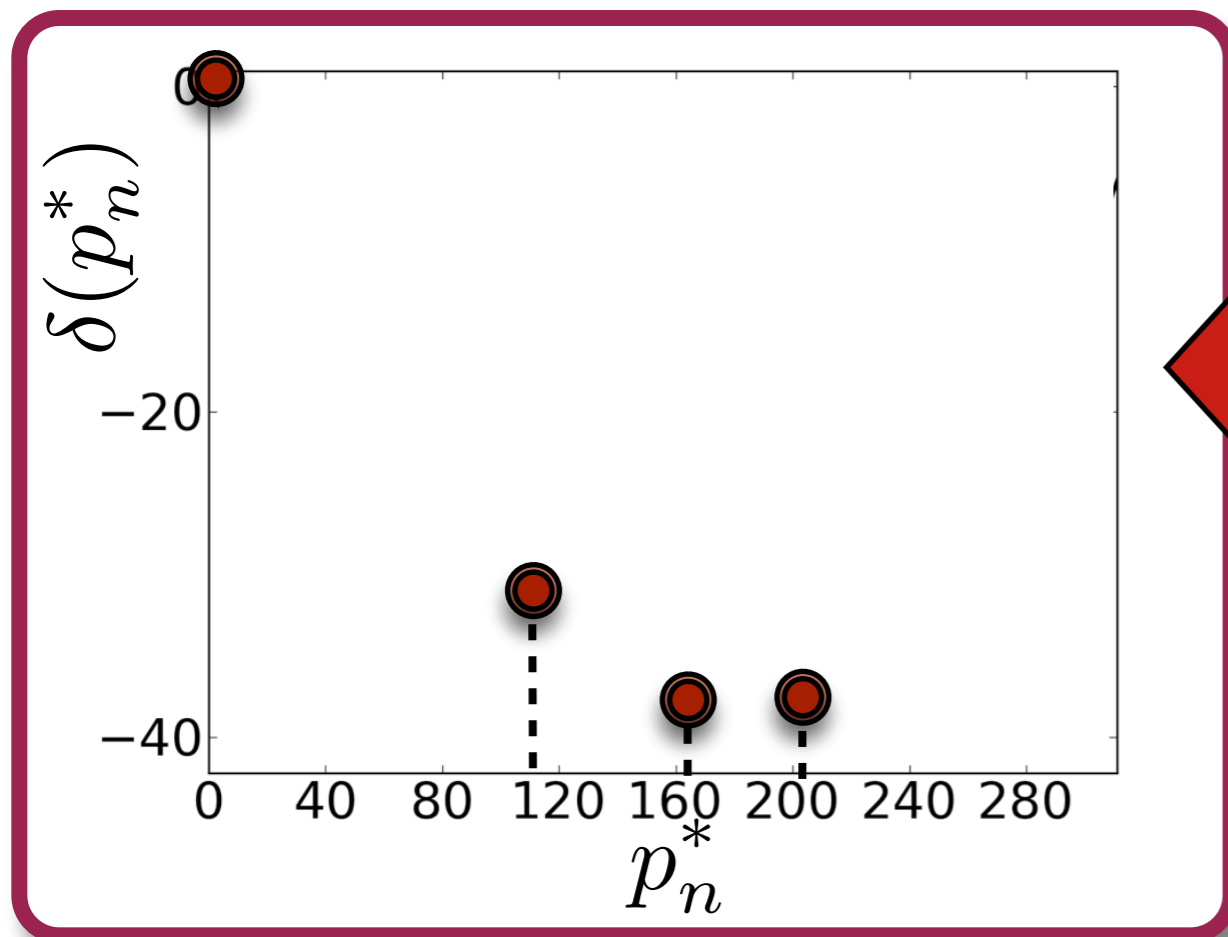


“Lüscher” in 1-d

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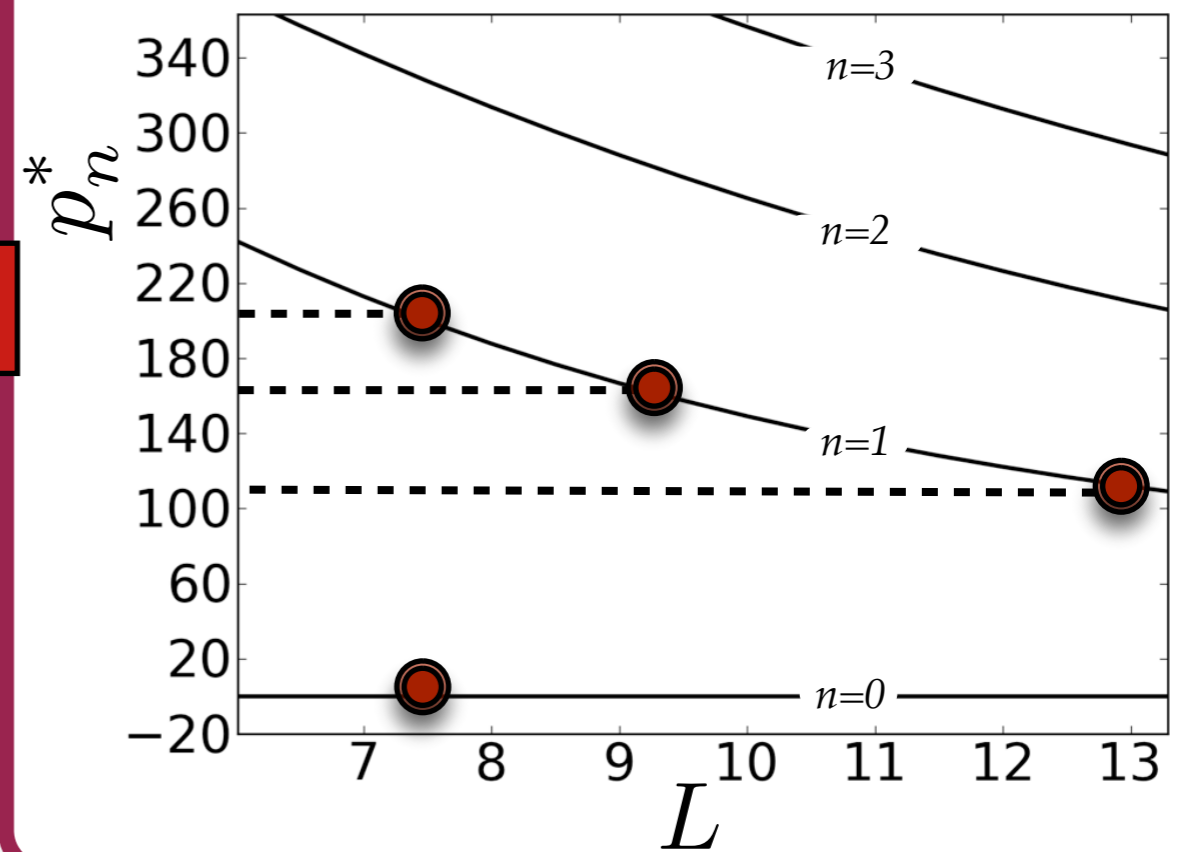
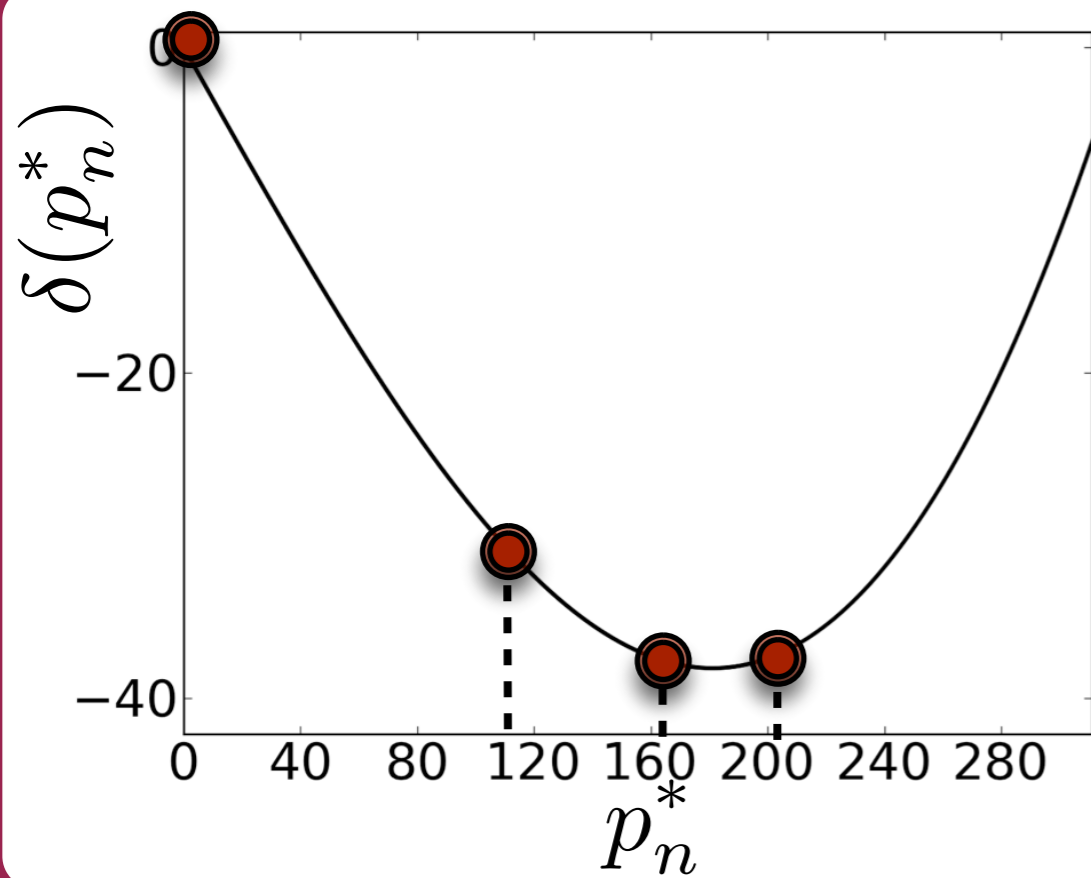


“Lüscher” in 1-d

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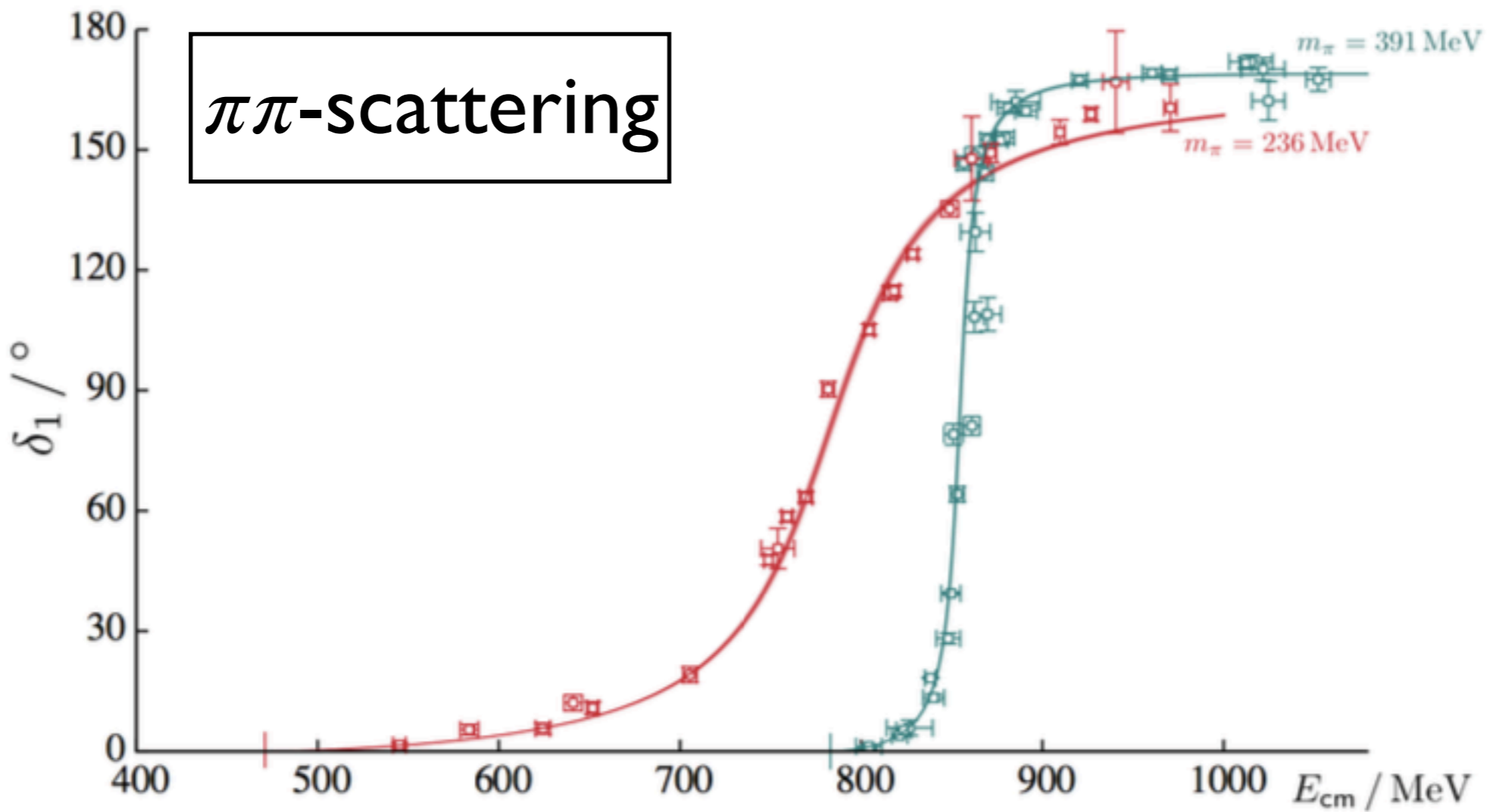
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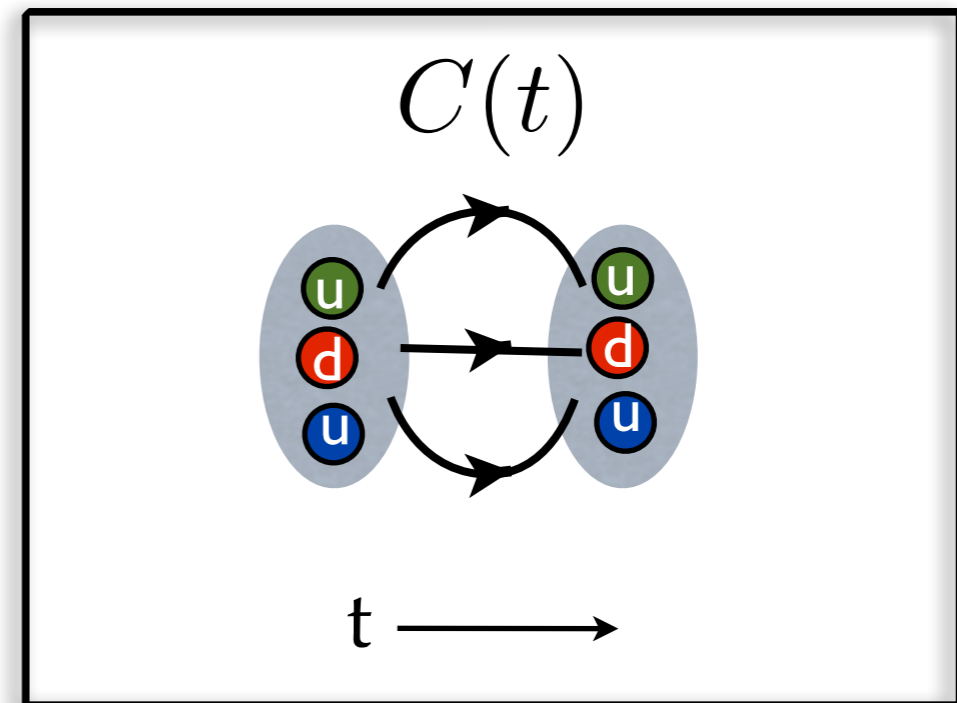
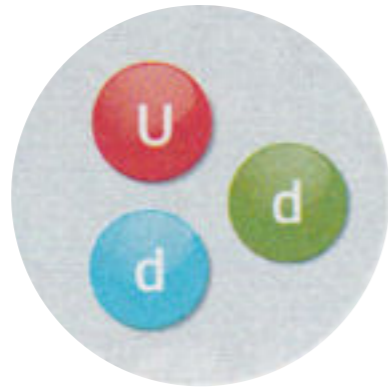
D. J. Wilson, R. A. Briceno, J. J. Dudek, R. G. Edwards and C. E. Thomas, Phys. Rev. D 92, 094502 (2015)

Quantization condition:



Calculating Observables

\mathcal{O} :

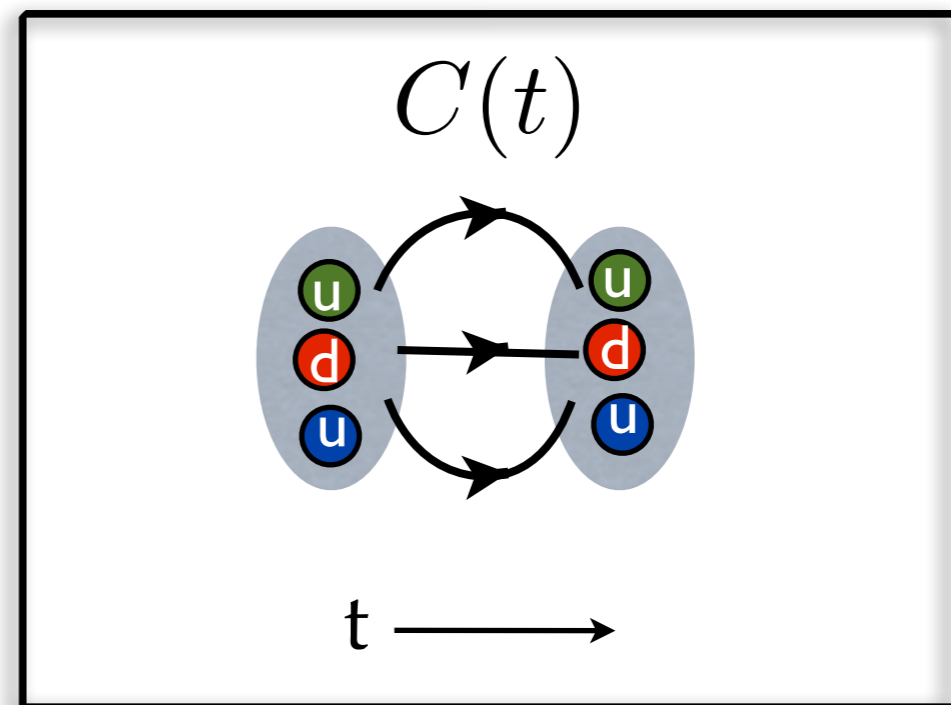


Calculating Observables

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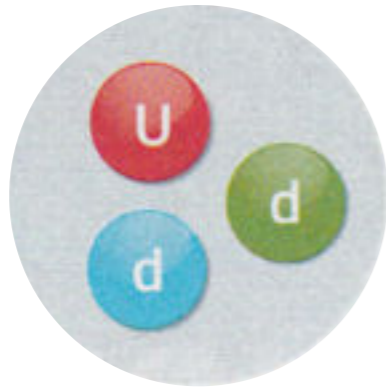


$$\langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle$$



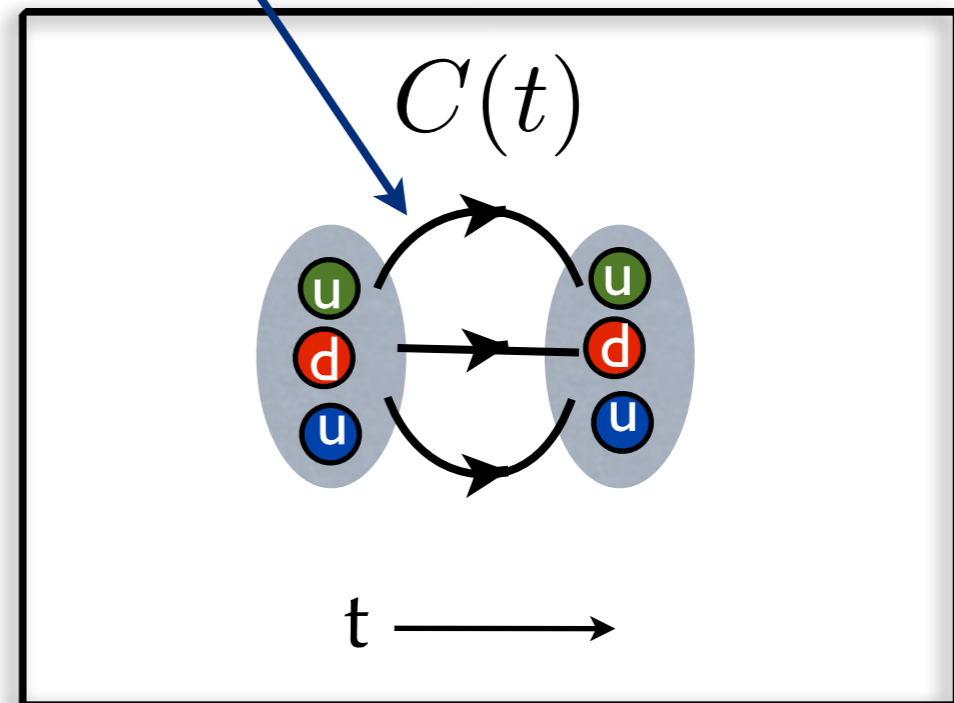
Calculating Observables

\mathcal{O} :



$$\langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \langle \mathcal{O}(0) e^{-Ht} \mathcal{O}(0) \rangle$$

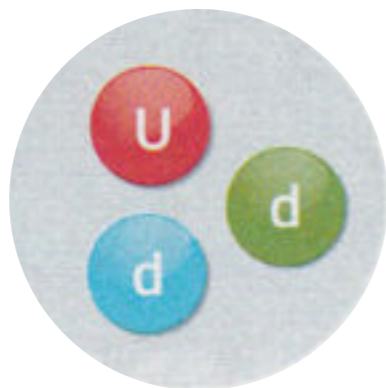
(Euclidean) Time evolution



Calculating Observables

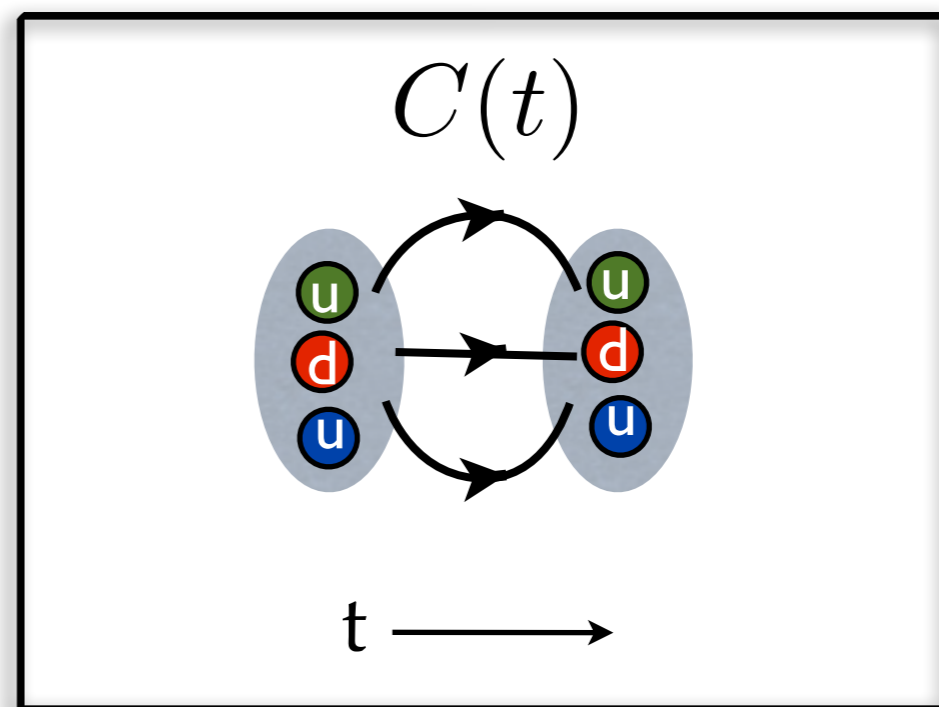
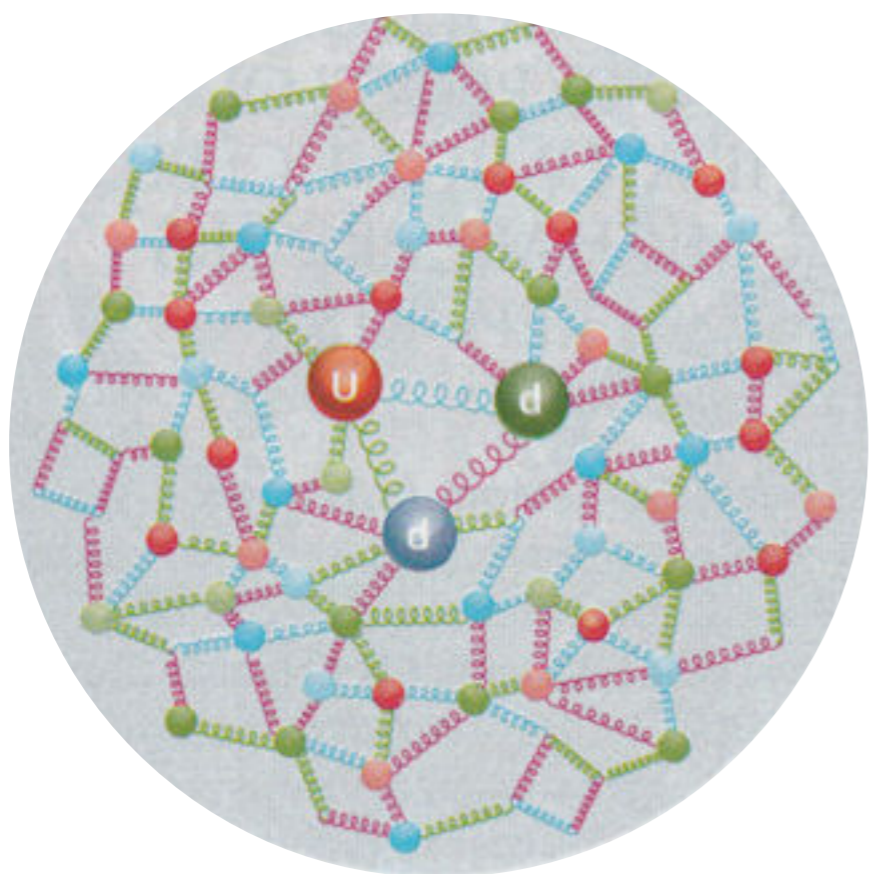
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\mathcal{O} :



(Euclidean) Time evolution
Complete set of states

ψ_n :



Calculating Observables

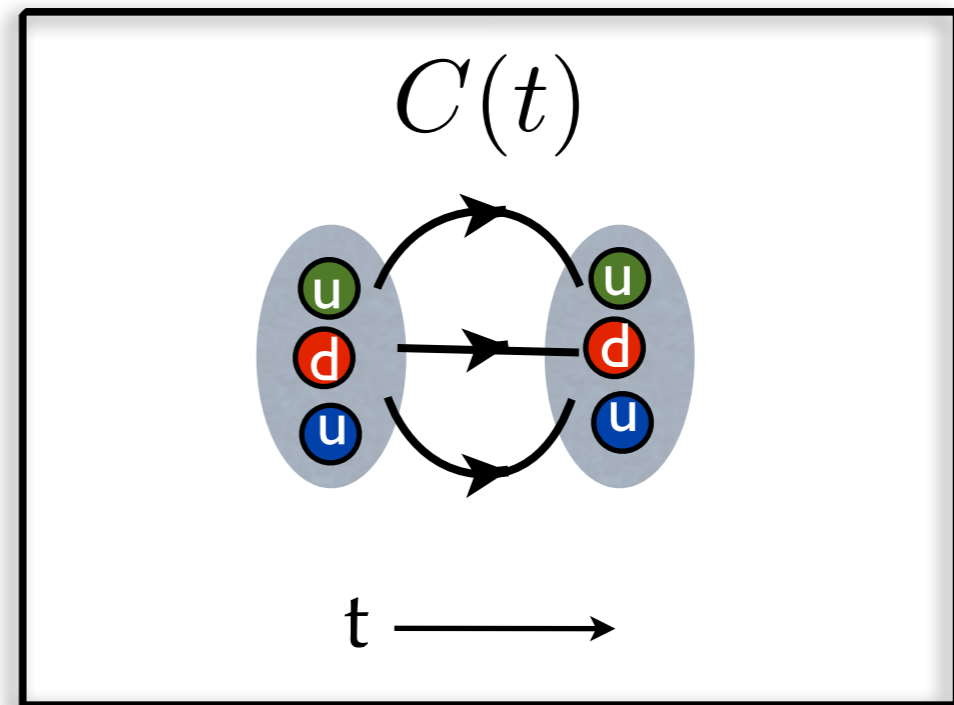
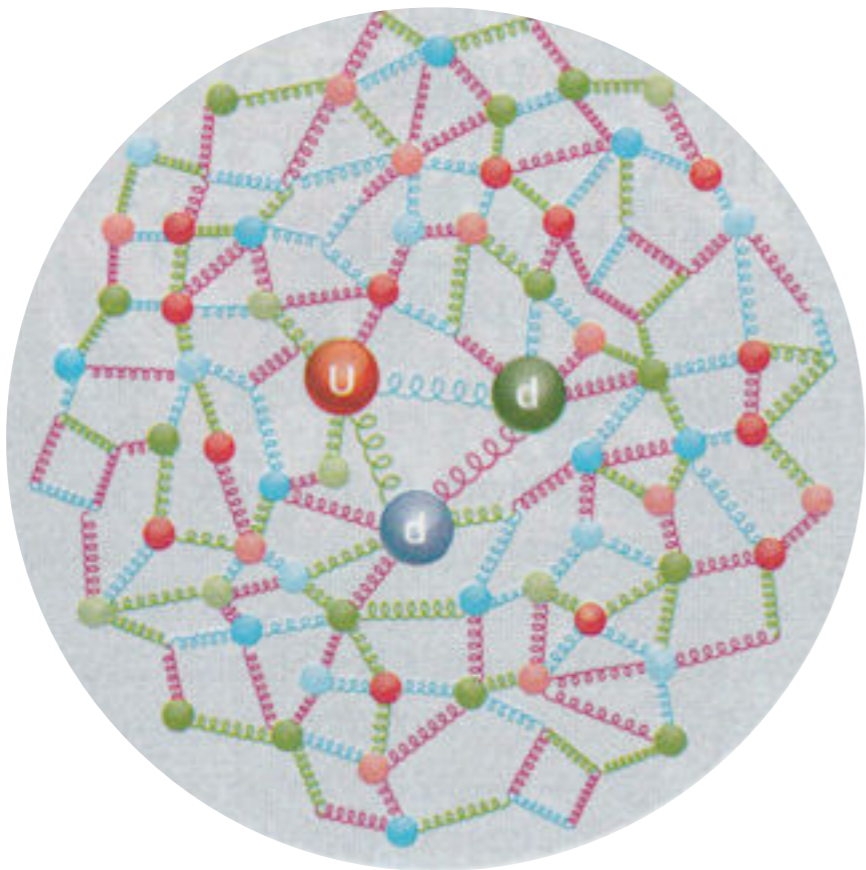
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Like a Boltzmann factor

ψ_n :



Calculating Observables

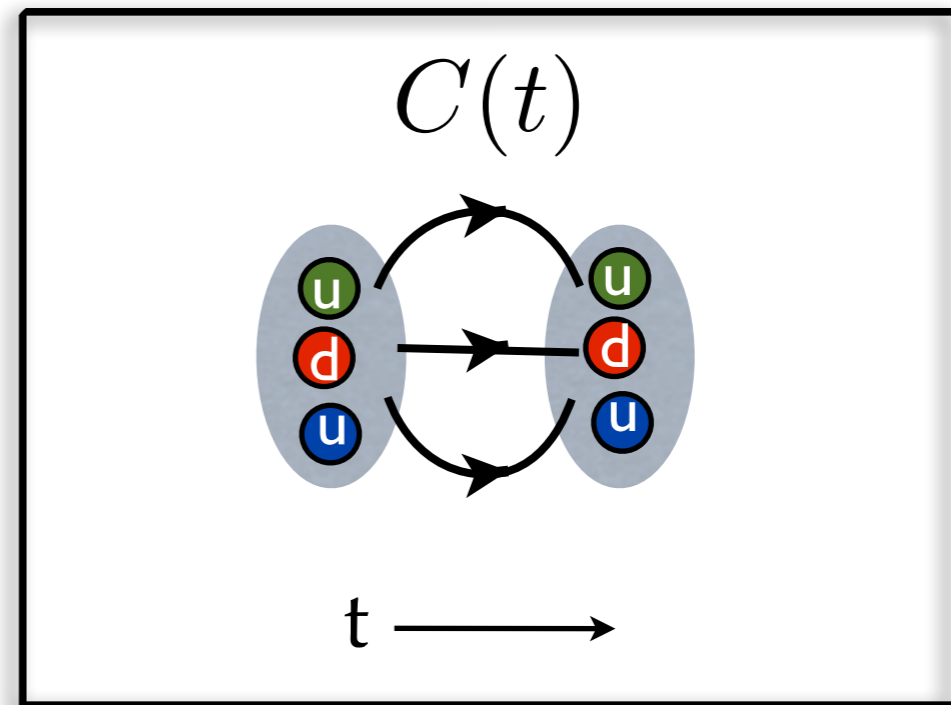
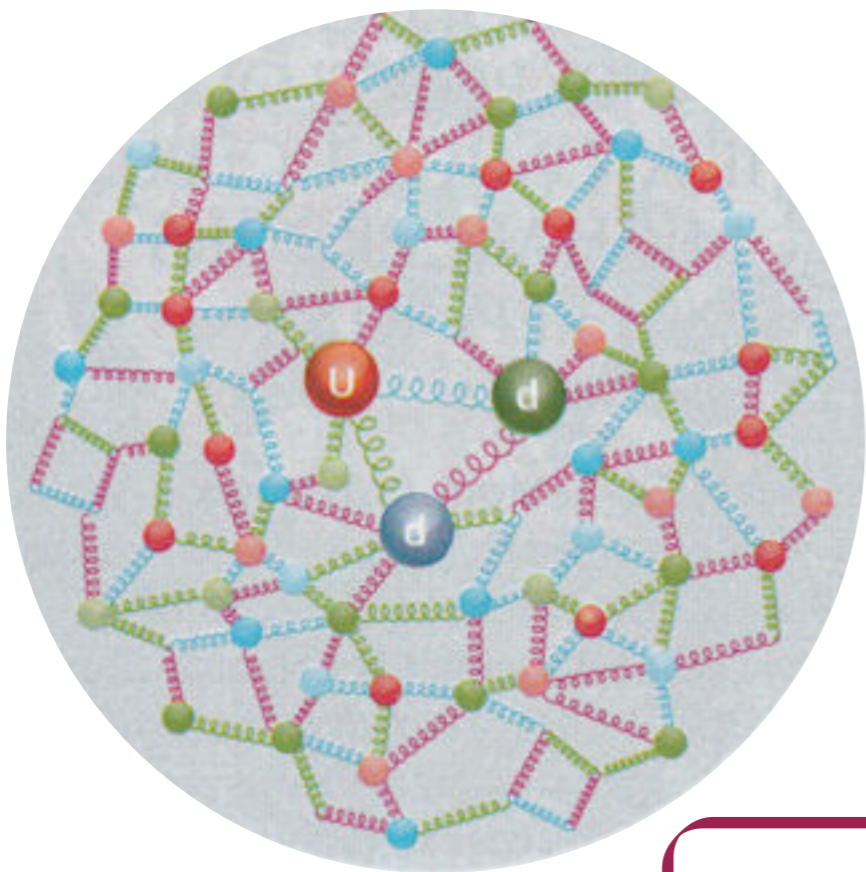
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Like a Boltzmann factor

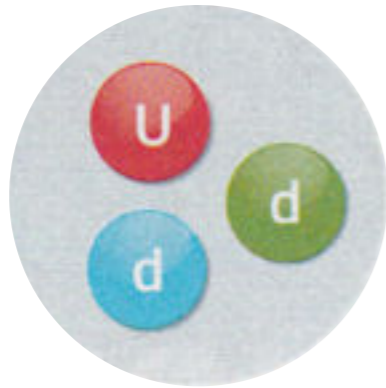
ψ_n :



$$C(t) = A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + A_2 e^{-E_2 t} + A_3 e^{-E_3 t} + \dots$$

Calculating Observables

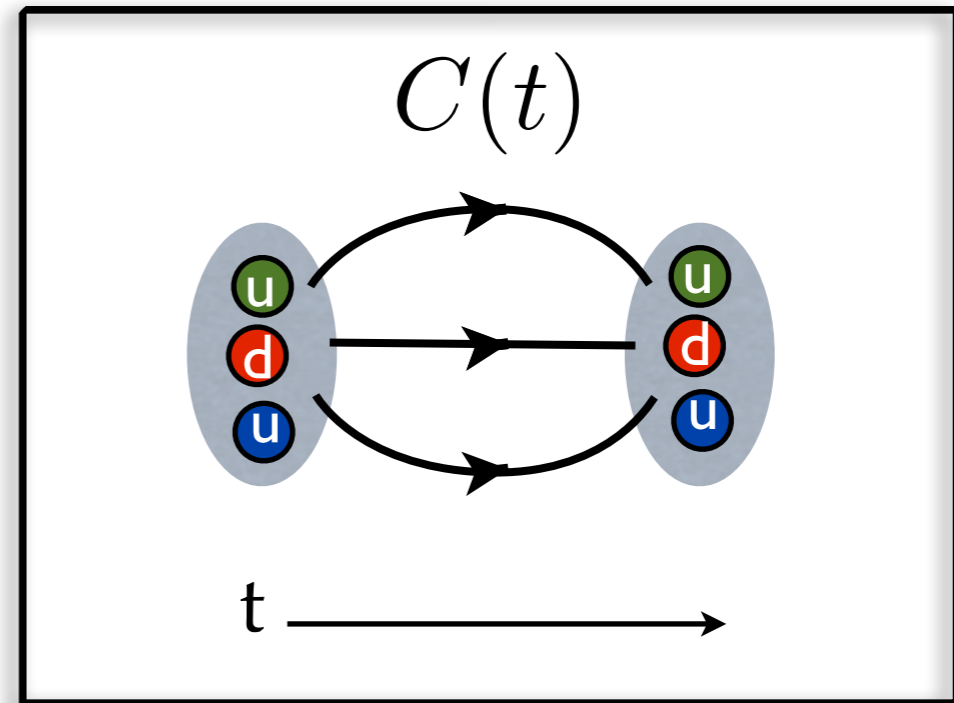
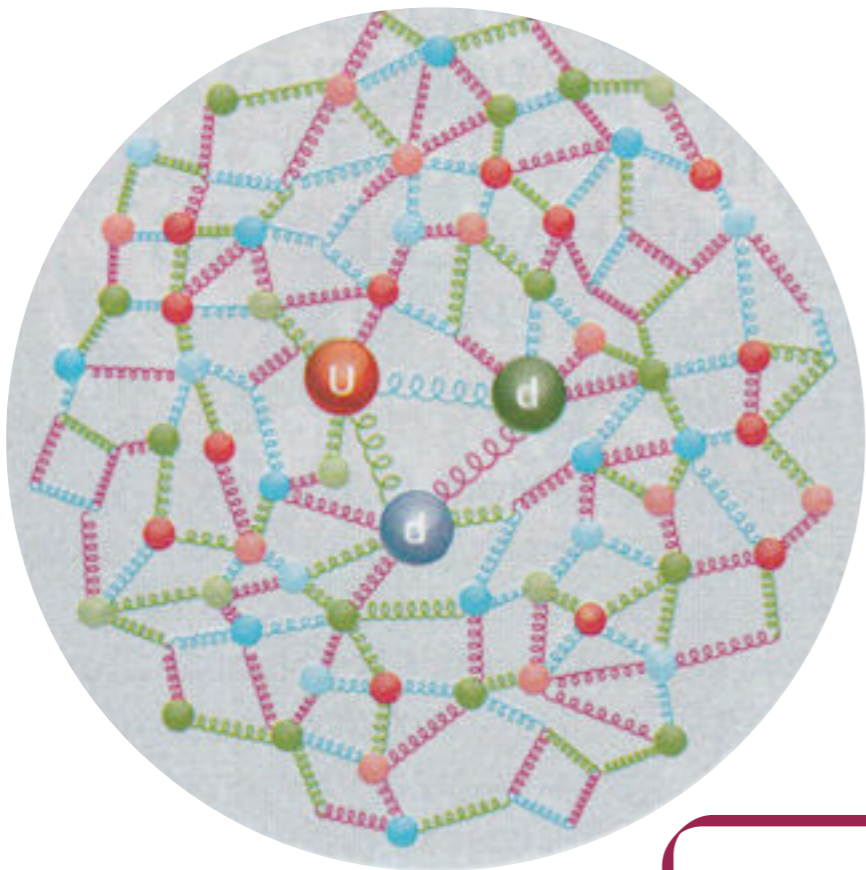
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Like a Boltzmann factor

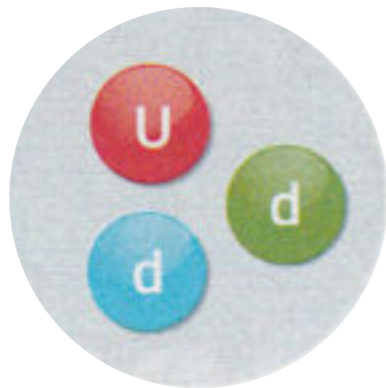
ψ_n :



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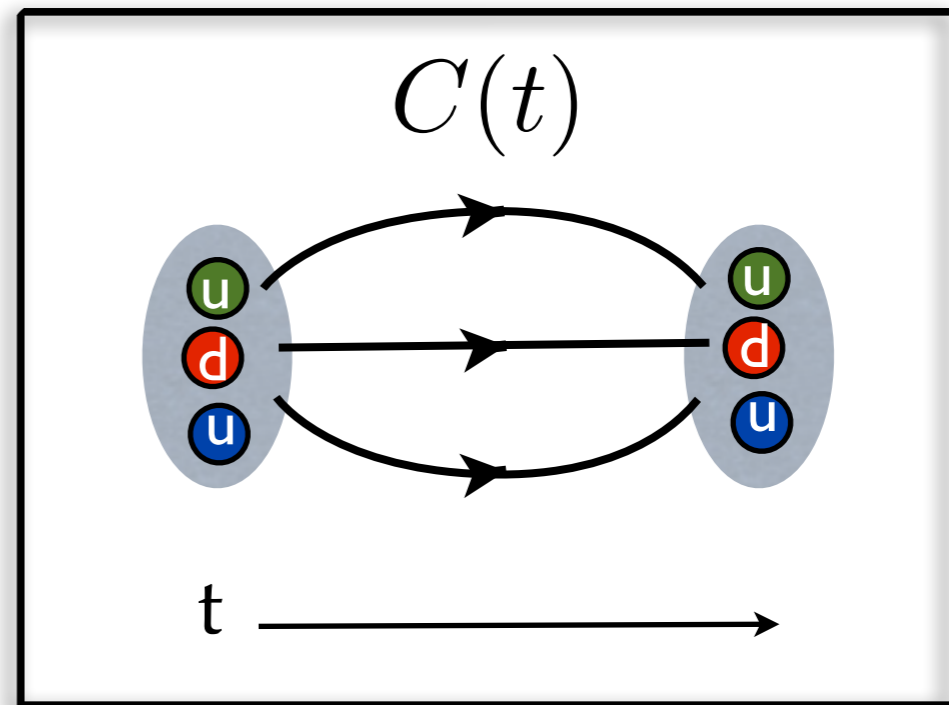
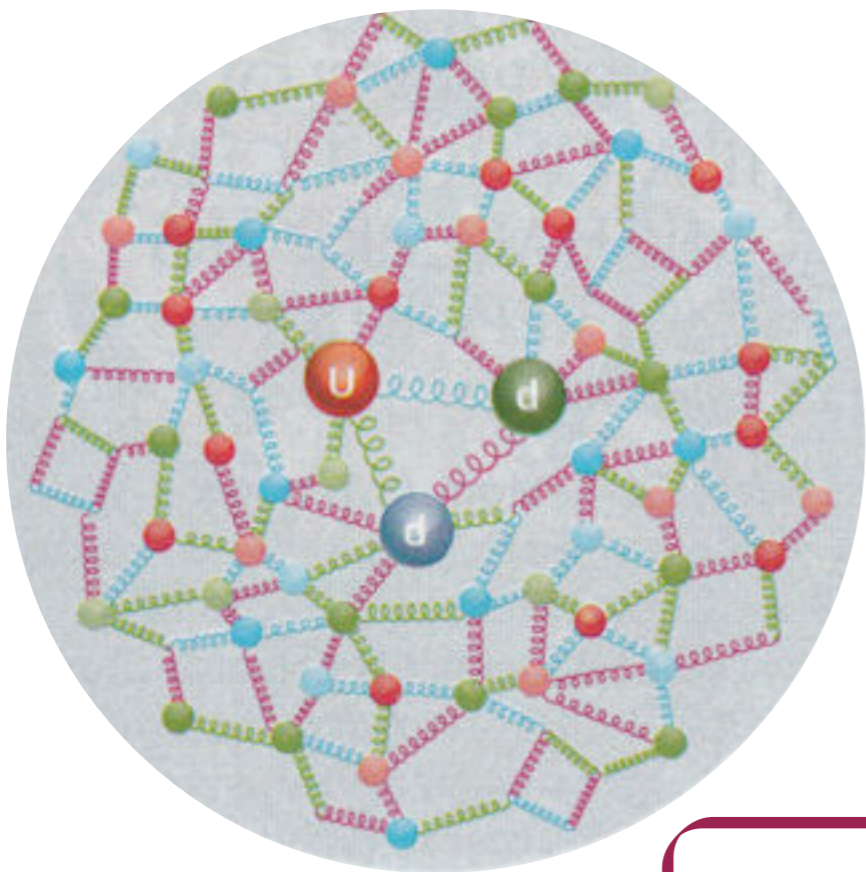
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Like a Boltzmann factor

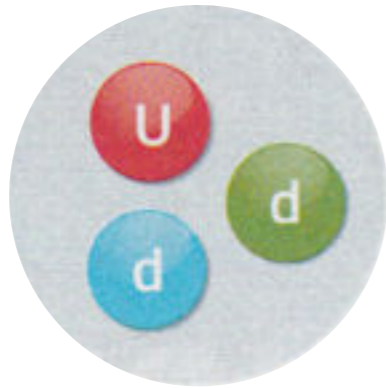
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Calculating Observables

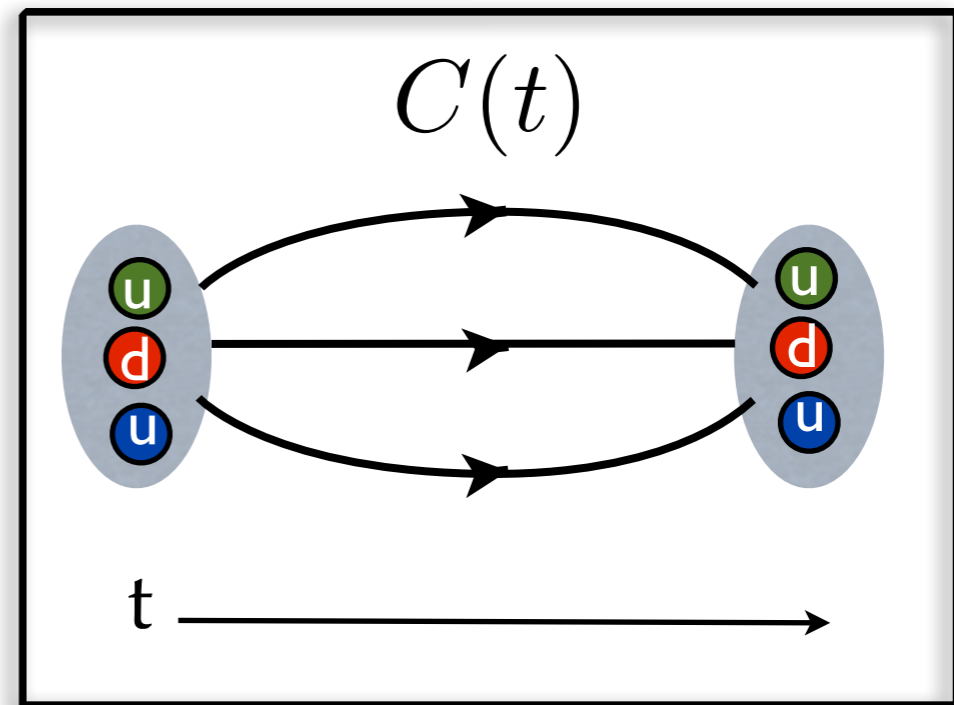
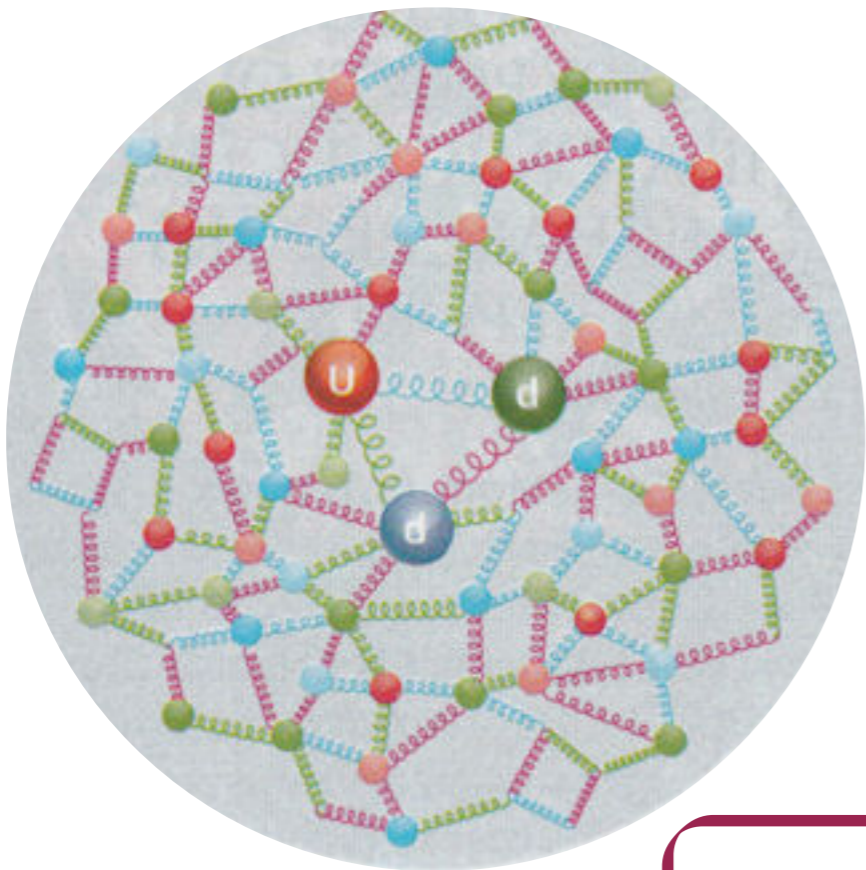
\mathcal{O} :



$$\langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \langle \mathcal{O}(0) e^{-Ht} \mathcal{O}(0) \rangle = \sum_n |\langle 0 | \mathcal{O} | n \rangle|^2 e^{-E_n t}$$

Like a Boltzmann factor

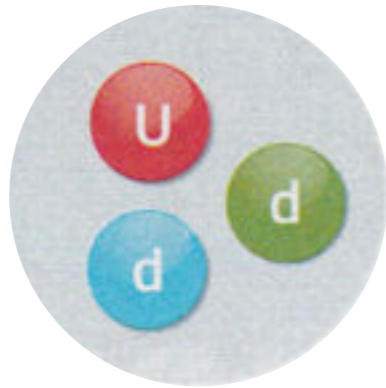
ψ_n :



$$C(t) = A_0 e^{-E_0 t} + \cancel{A_1 e^{-E_1 t}} + \cancel{A_2 e^{-E_2 t}} + \cancel{A_3 e^{-E_3 t}} + \dots$$

Calculating Observables

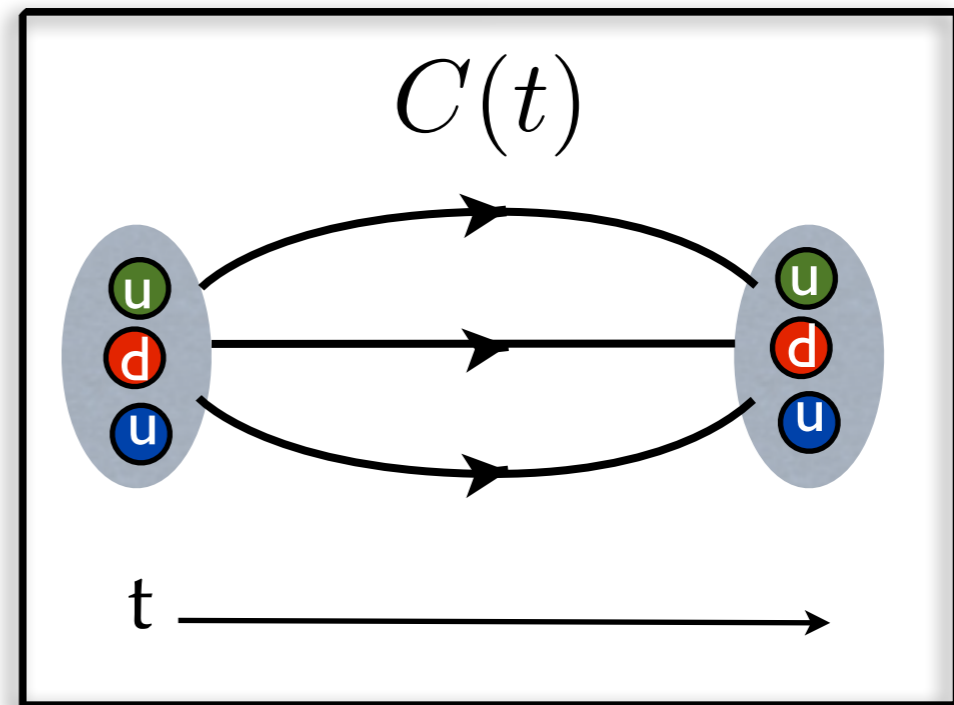
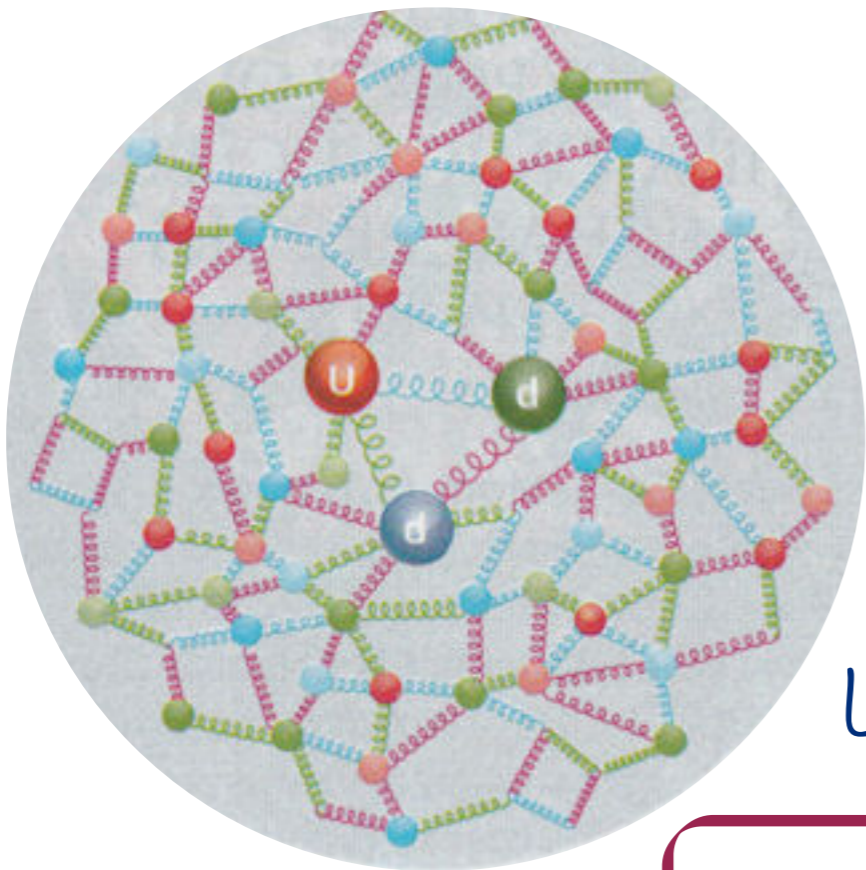
\mathcal{O} :



$$\langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \langle \mathcal{O}(0) e^{-Ht} \mathcal{O}(0) \rangle = \sum_n |\langle 0 | \mathcal{O} | n \rangle|^2 e^{-E_n t}$$

Like a Boltzmann factor

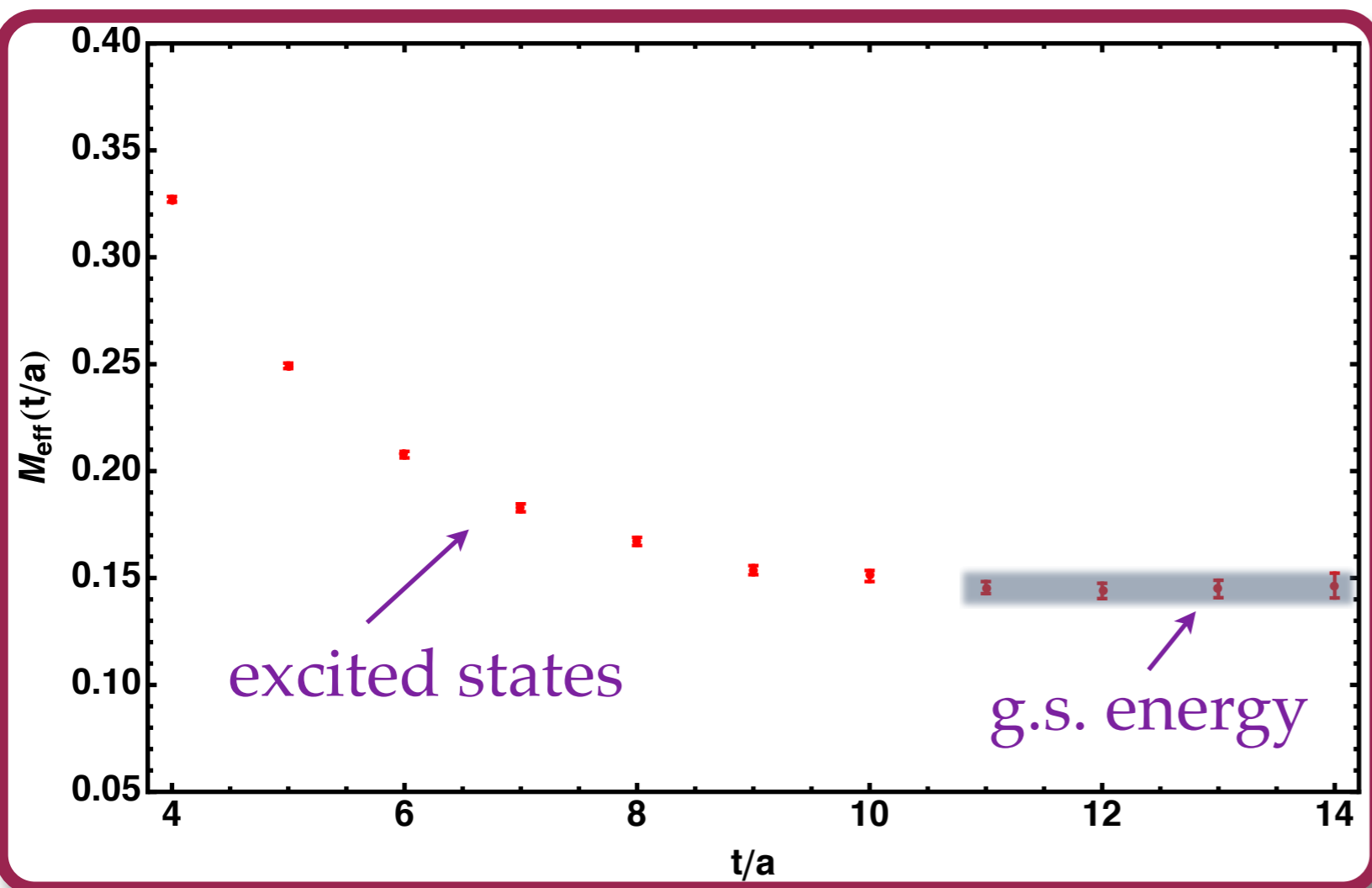
ψ_n :



Long time limit = zero temperature

$$C(t) = A_0 e^{-E_0 t} + \cancel{A_1 e^{-E_1 t}} + \cancel{A_2 e^{-E_2 t}} + \cancel{A_3 e^{-E_3 t}} + \dots$$

Calculating Observables



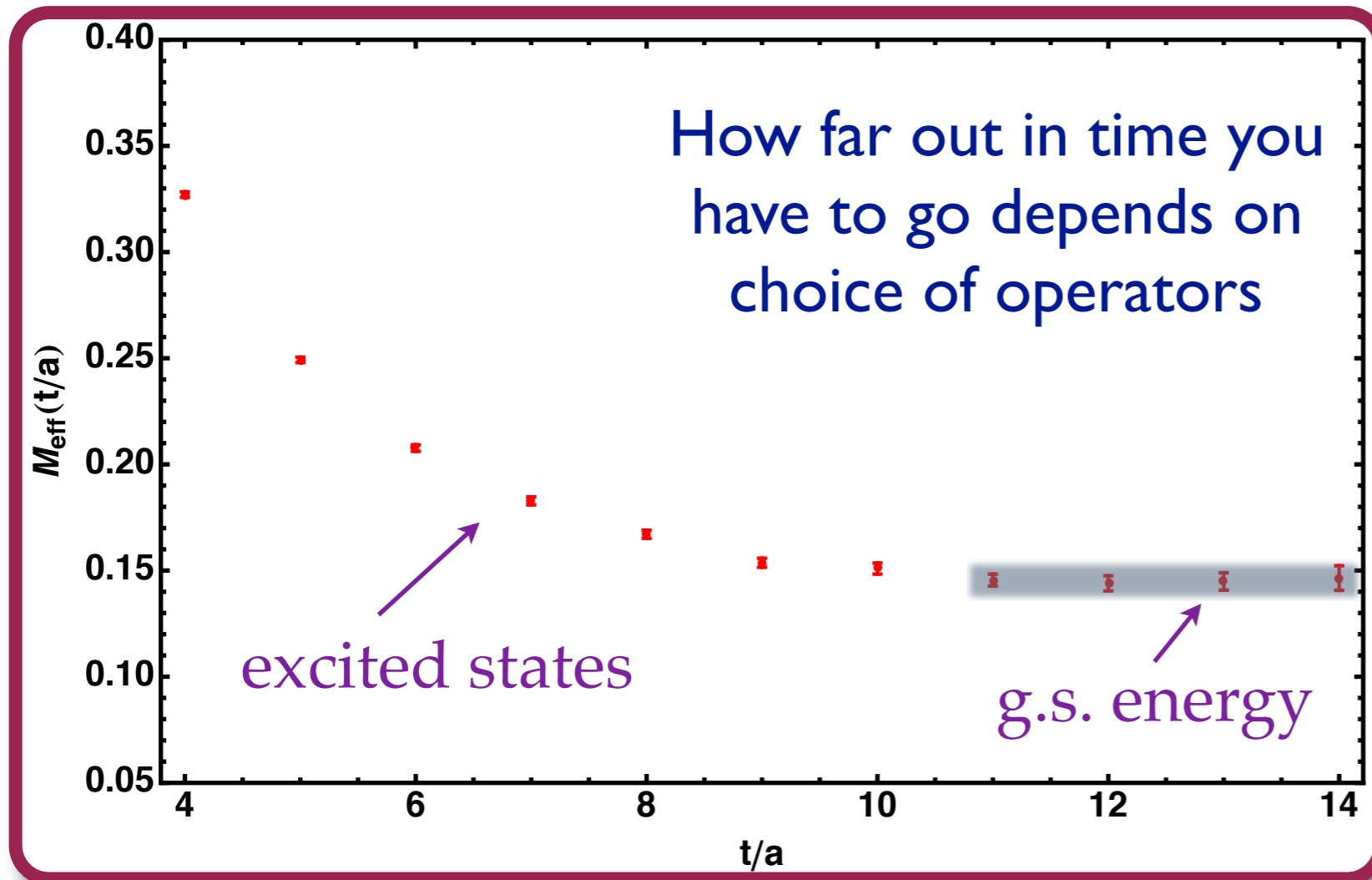
Effective mass plot:

$$M_{\text{eff}} \equiv \ln \frac{C(t)}{C(t+1)}$$
$$\xrightarrow[t \rightarrow \infty]{} E_0$$

Long time limit = zero temperature

$$C(t) = A_0 e^{-E_0 t} + \cancel{A_1 e^{-E_1 t}} + \cancel{A_2 e^{-E_2 t}} + \cancel{A_3 e^{-E_3 t}} + \dots$$

Calculating Observables



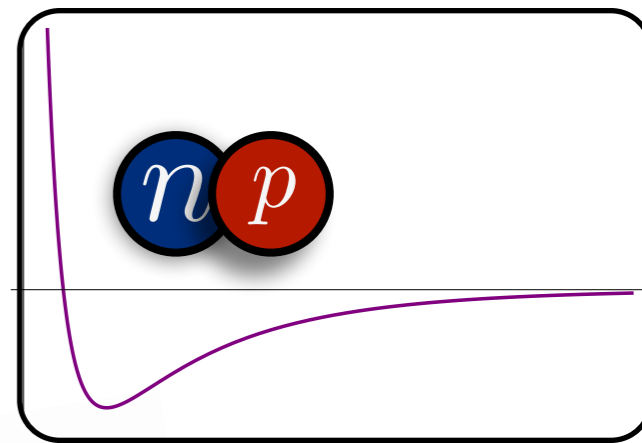
Effective mass plot:

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Long time limit = zero temperature

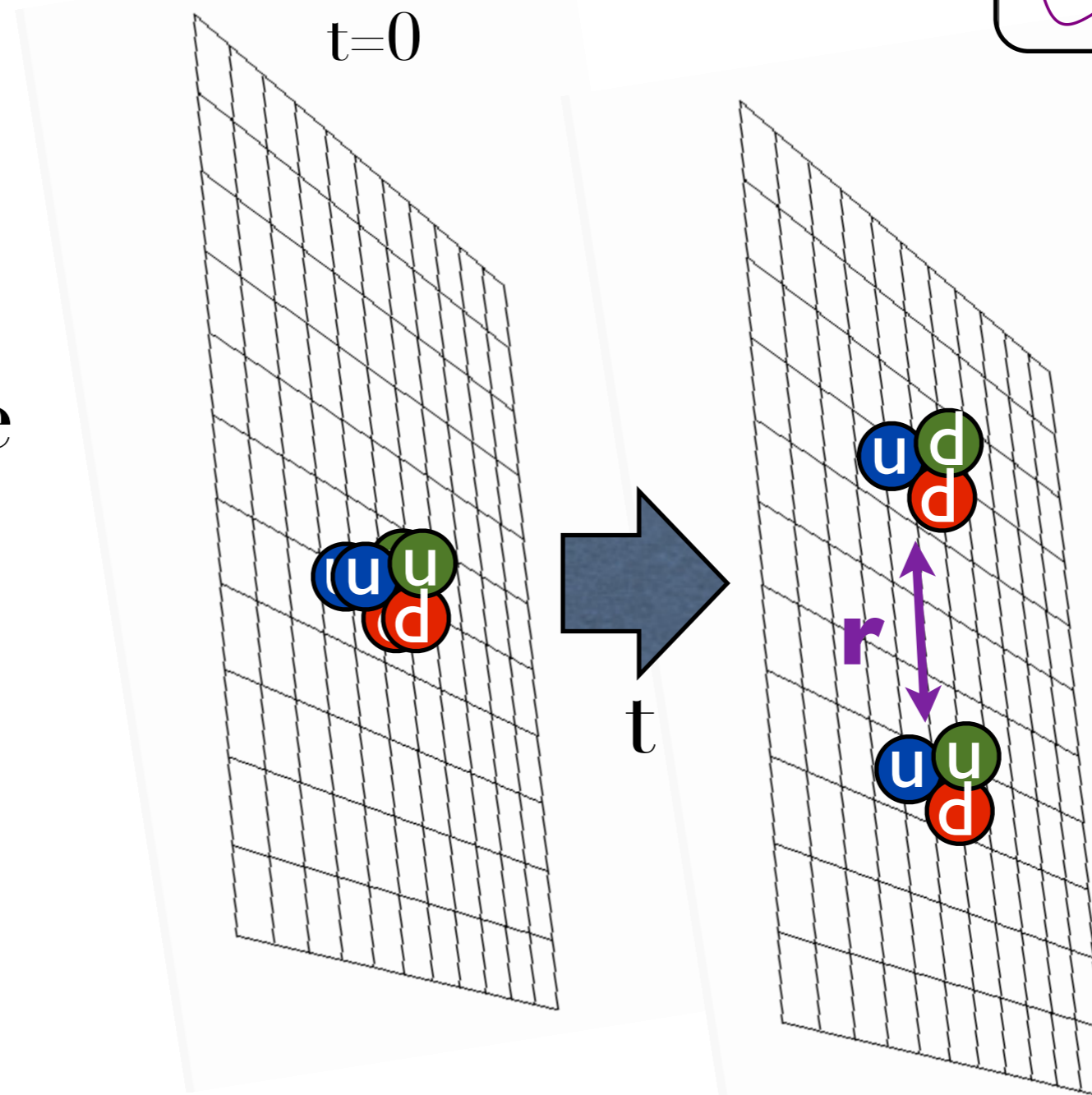
$$C(t) = A_0 e^{-E_0 t} + \cancel{A_1 e^{-E_1 t}} + \cancel{A_2 e^{-E_2 t}} + \cancel{A_3 e^{-E_3 t}} + \dots$$

Potential method

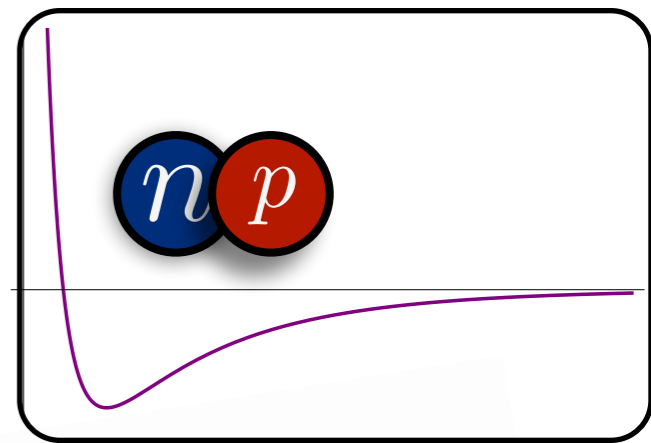


1. Create the following correlation function:

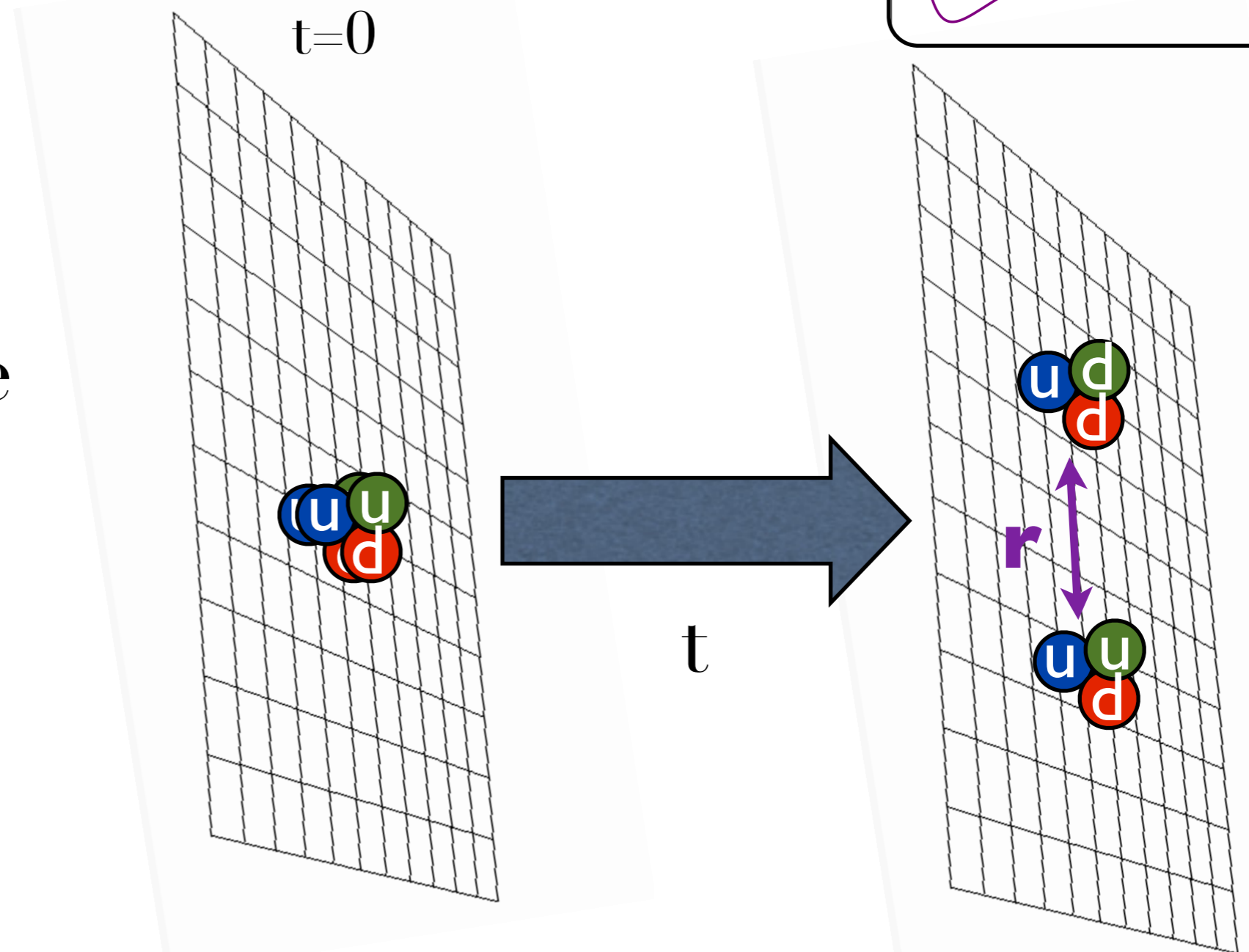
$$C_{NN}(\mathbf{r}, t)$$



Potential method

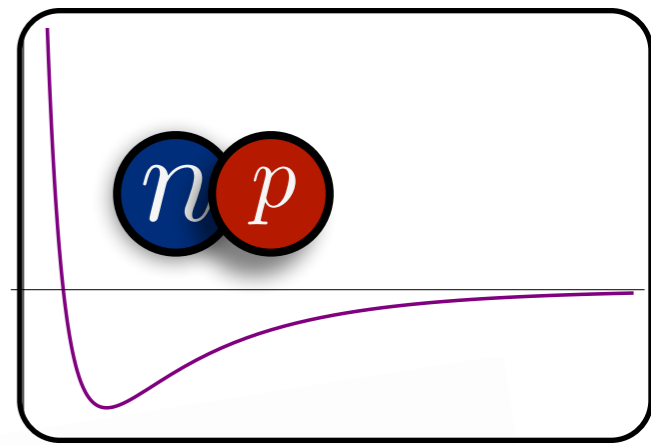


1. Create the following correlation function:

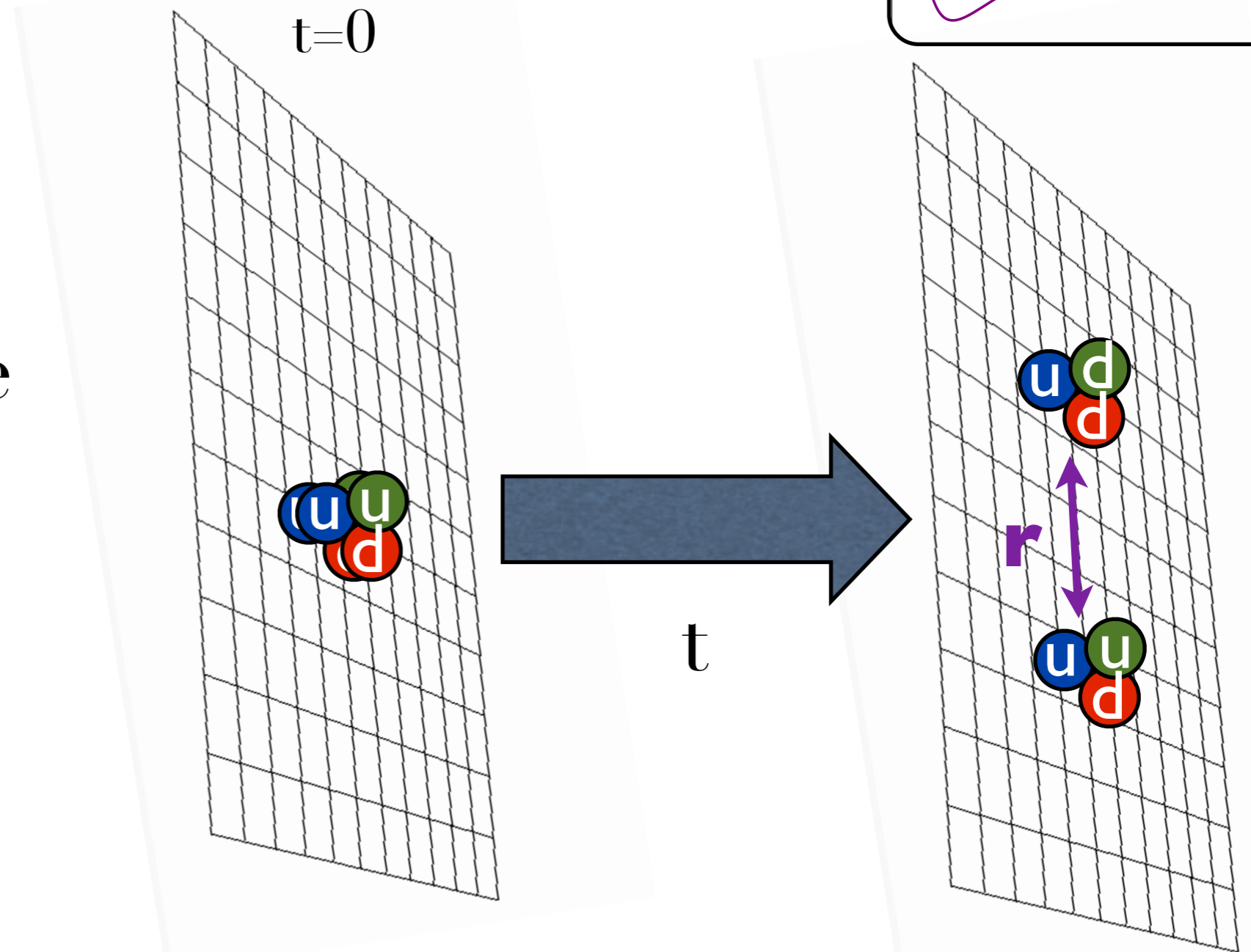


$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) =$$

Potential method

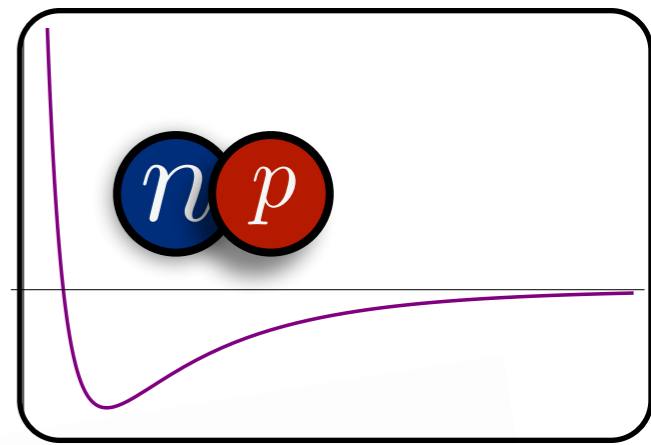


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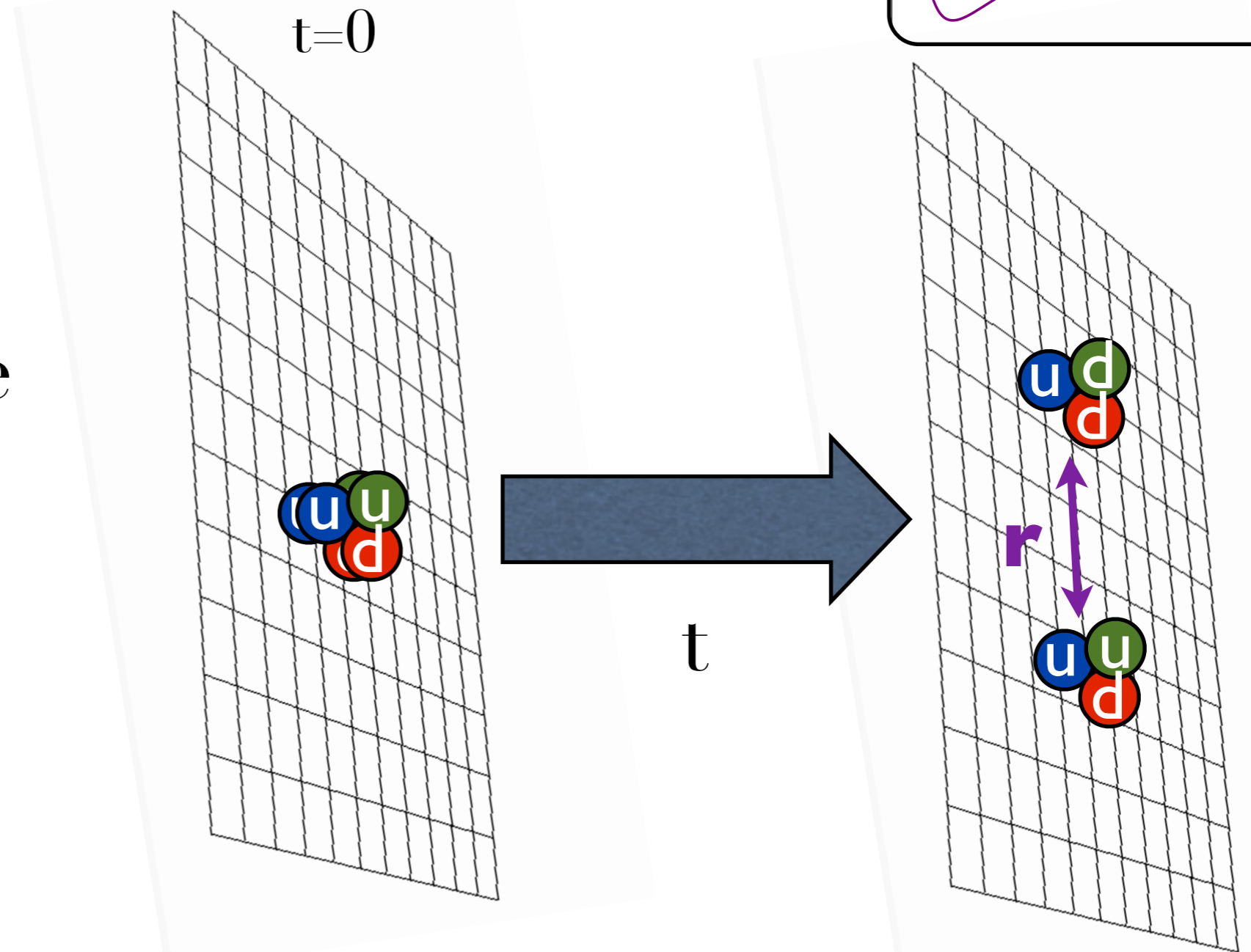


$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger$$

Potential method

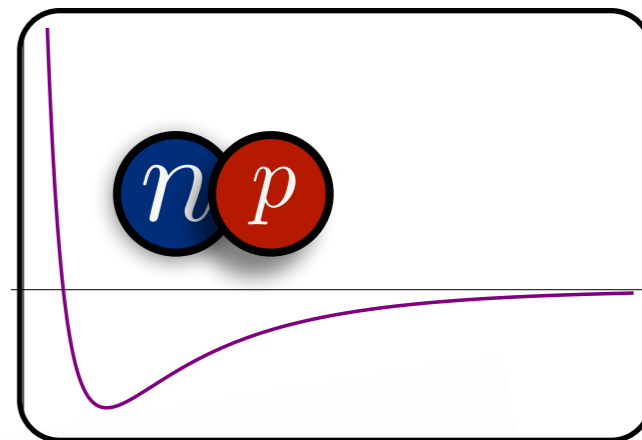


1. Create the following correlation function:

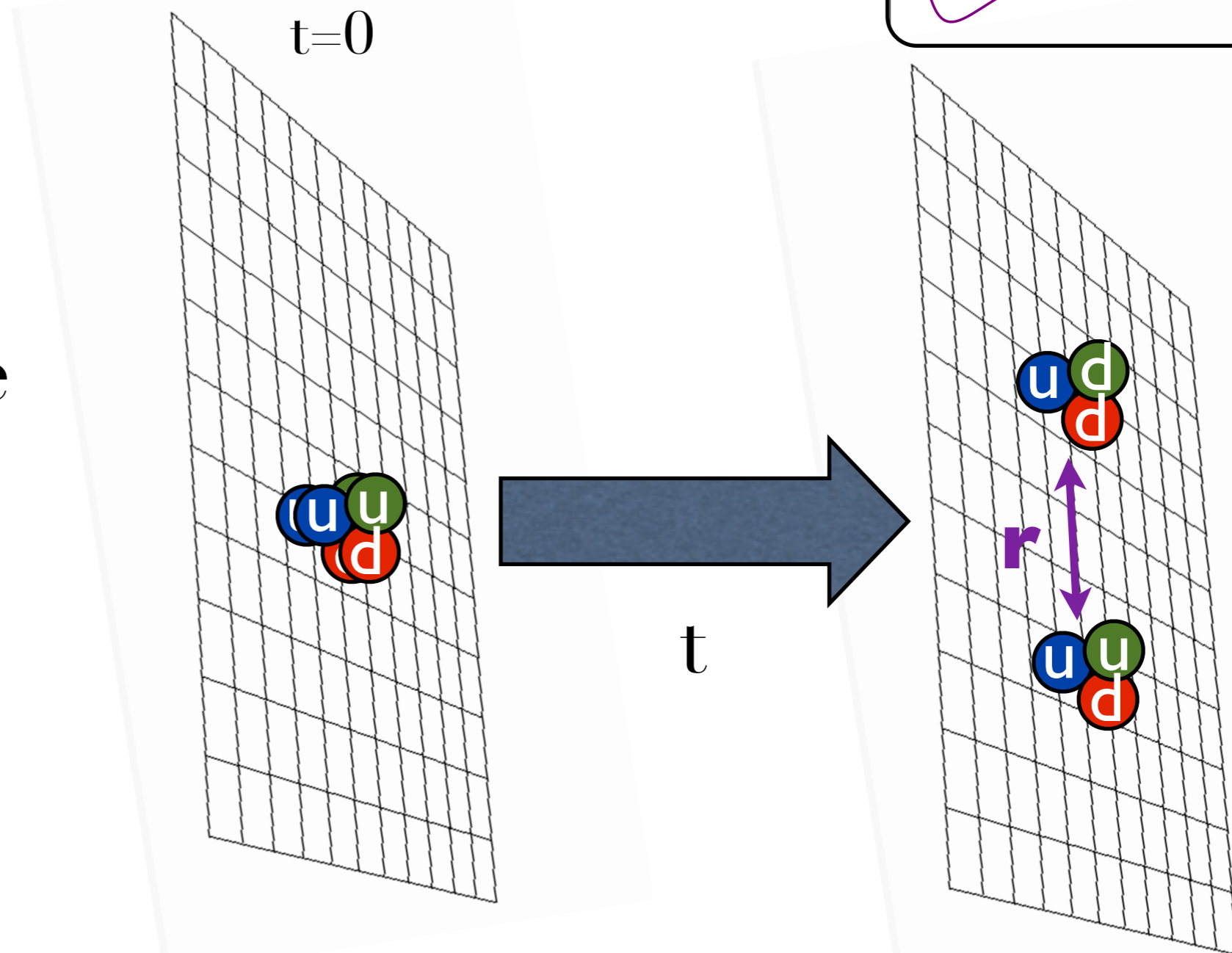


$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger \chi e^{-E_0 t}$$

Potential method

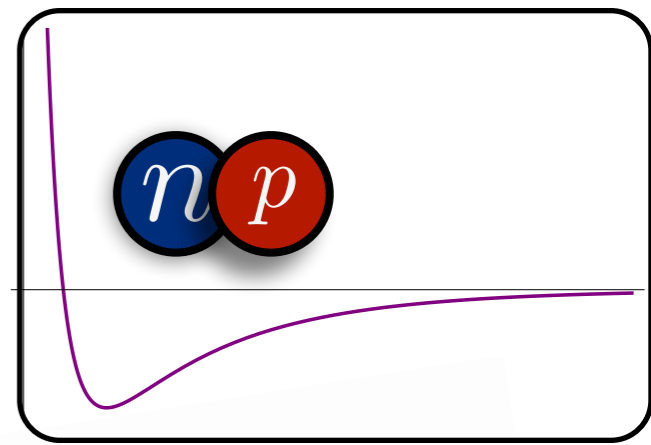


1. Create the following correlation function:

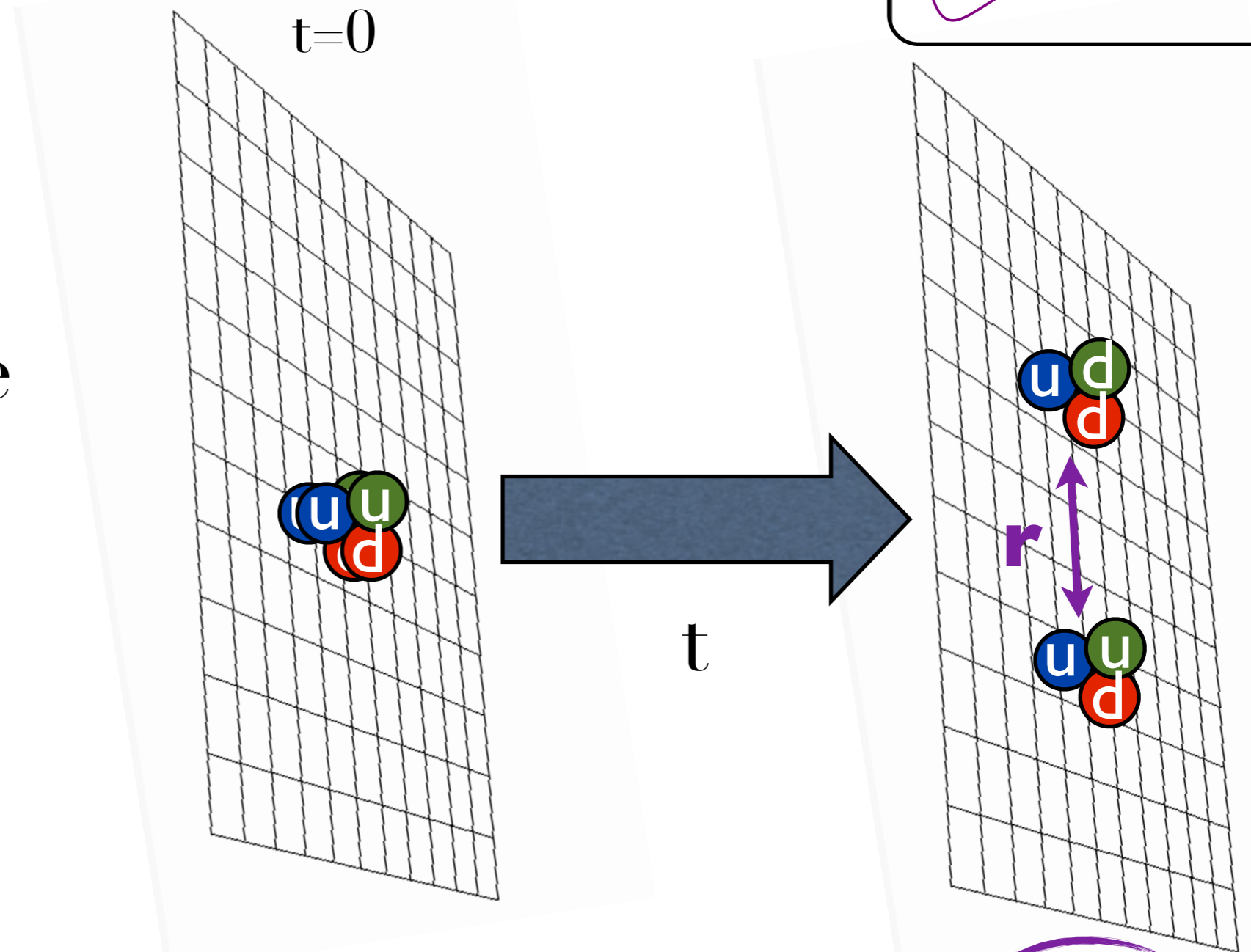


$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger \chi \cdot e^{-E_0 t} \chi \psi_0(\mathbf{r})$$

Potential method



1. Create the following correlation function:

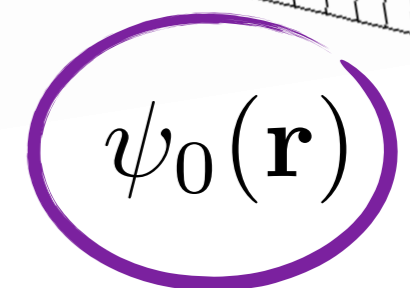
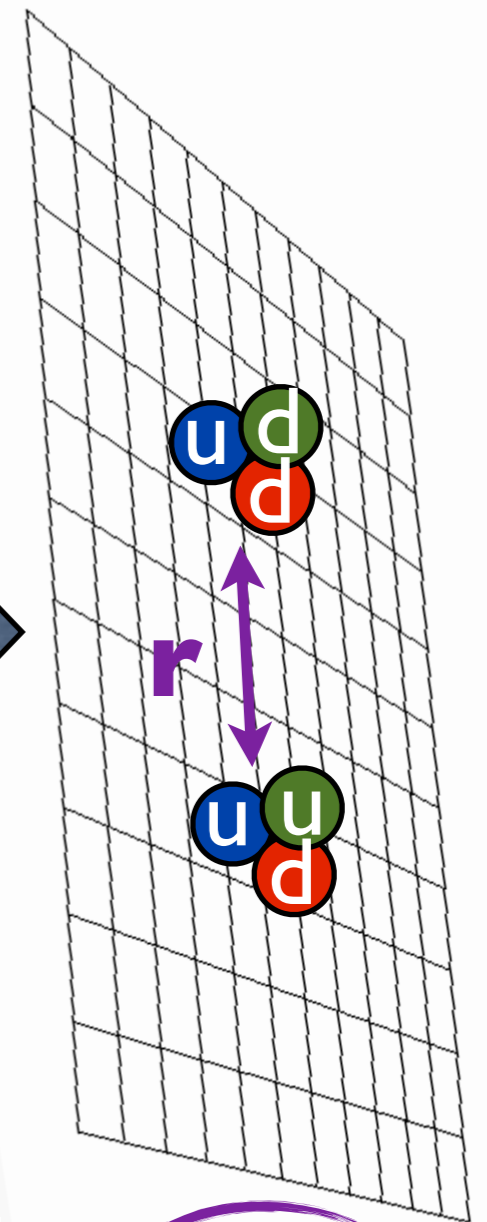
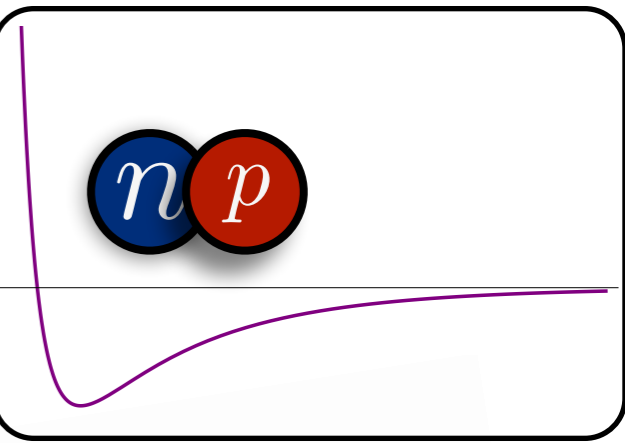
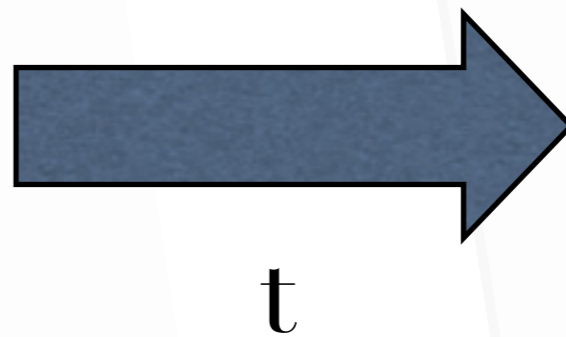
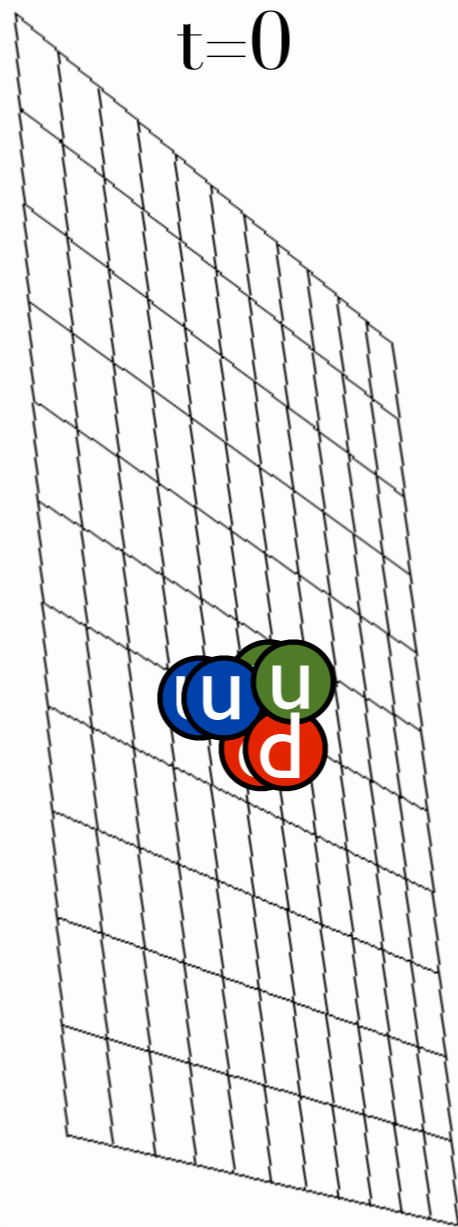


$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger \chi \cdot e^{-E_0 t} \chi \psi_0(\mathbf{r})$$

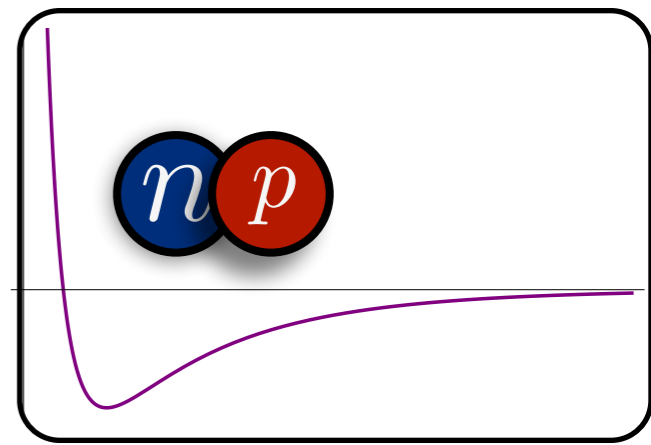
Potential method

2. Plug NBS wave-function into Schrödinger Eq. to determine the potential:

$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0 \right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{p}}(\mathbf{r}') \leftarrow \psi_0(\mathbf{r})$$



Potential method



2. Plug NBS
wave-function
into Schrödinger
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$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0 \right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{p}}(\mathbf{r}') \longleftarrow \psi_0(\mathbf{r})$$

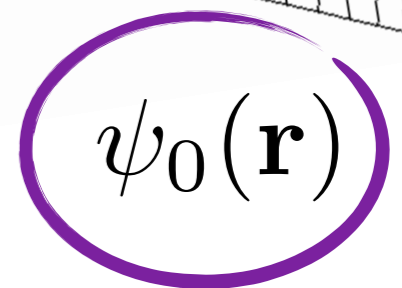
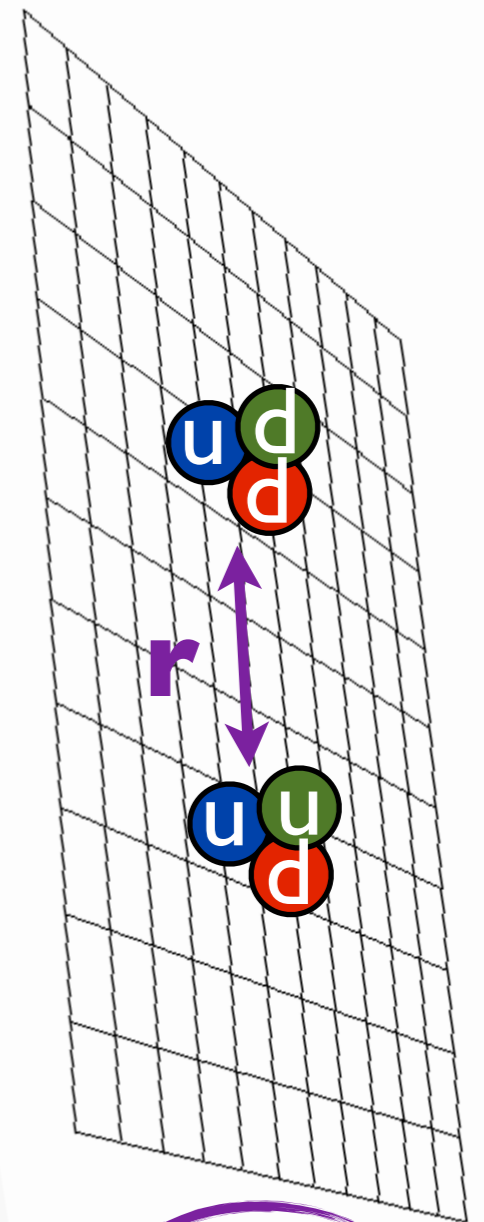
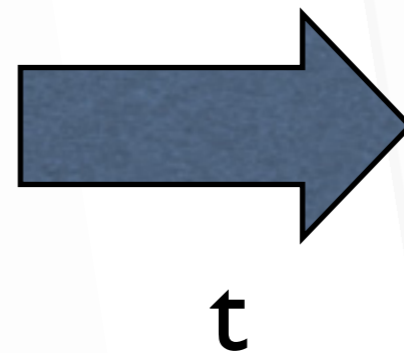
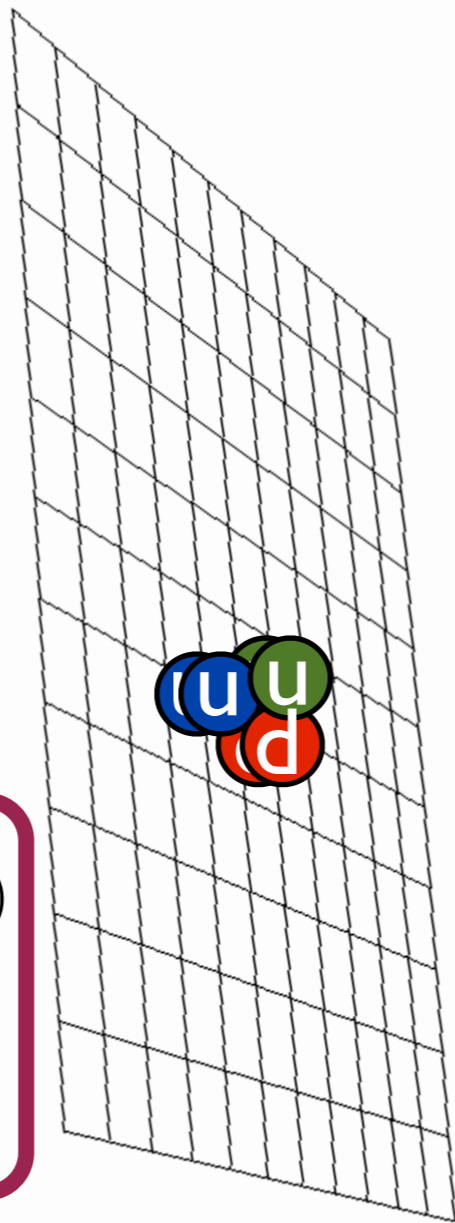
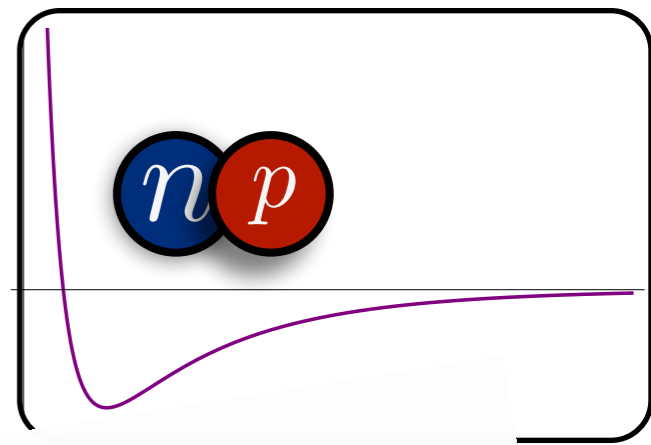
Potential method

3. Use derivative expansion to determine the leading order potential:

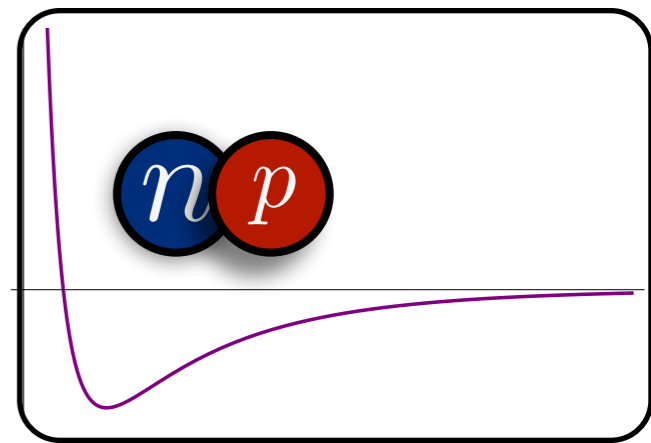
$$U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$

$$V_C(\mathbf{r}) \simeq \frac{\mathbf{p}^2}{2\mu} + \lim_{t \rightarrow \infty} \frac{1}{2\mu} \frac{\nabla_{\mathbf{r}}^2 C_{NN}(\mathbf{r}, t)}{C_{NN}(\mathbf{r}, t)}$$

$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0 \right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{p}}(\mathbf{r}') \leftarrow \psi_0(\mathbf{r})$$

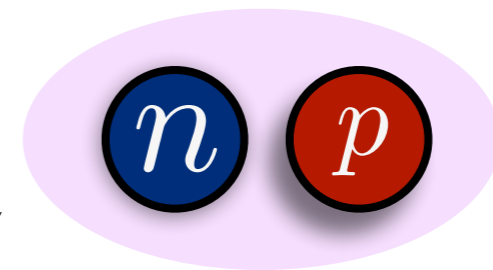


Potential method

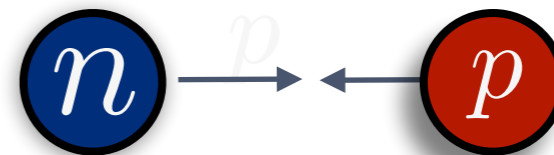


3. Use derivative expansion to determine the leading order potential:

Binding energies



Phase shifts

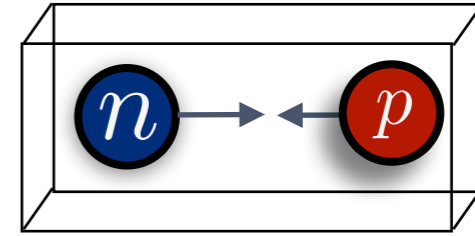


$$U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$

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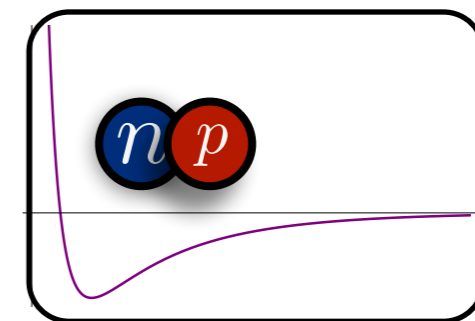
$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0 \right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{p}}(\mathbf{r}') \leftarrow \psi_0(\mathbf{r})$$

Some comparisons between methods



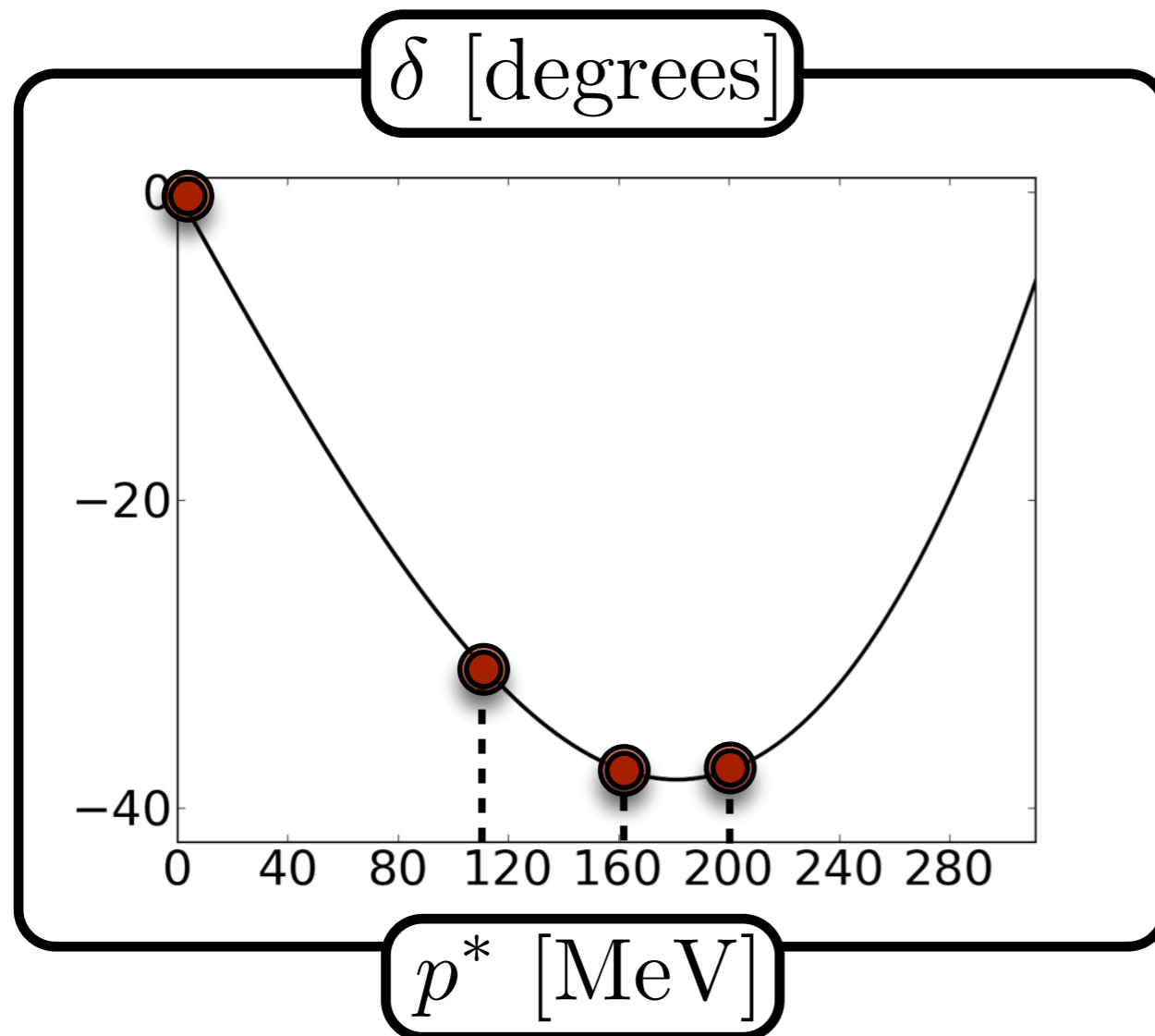
Lüscher

- discrete phase shifts
- need single state saturation
- no volume extrapolation
- no uncontrolled approximations



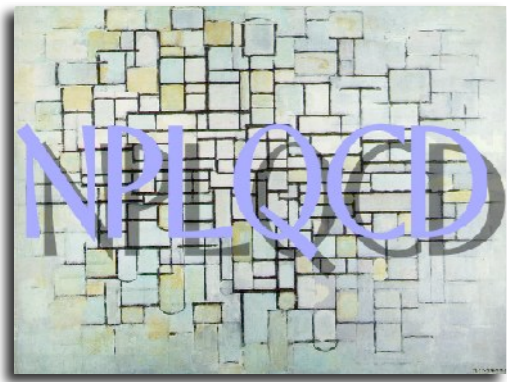
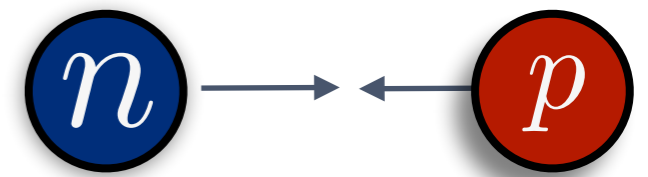
Potential

- nearly continuous phase shifts
- only need elastic state saturation
- need volume extrapolation
- cutoff in gradient expansion

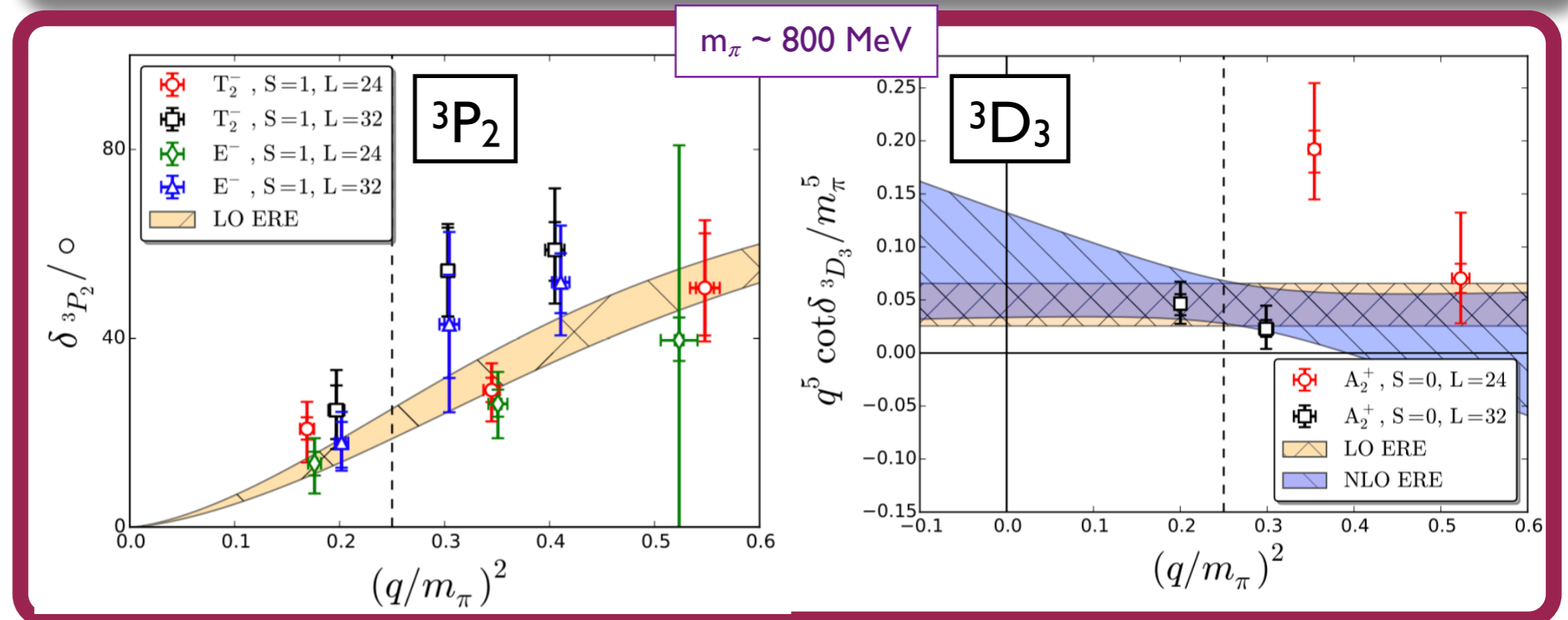
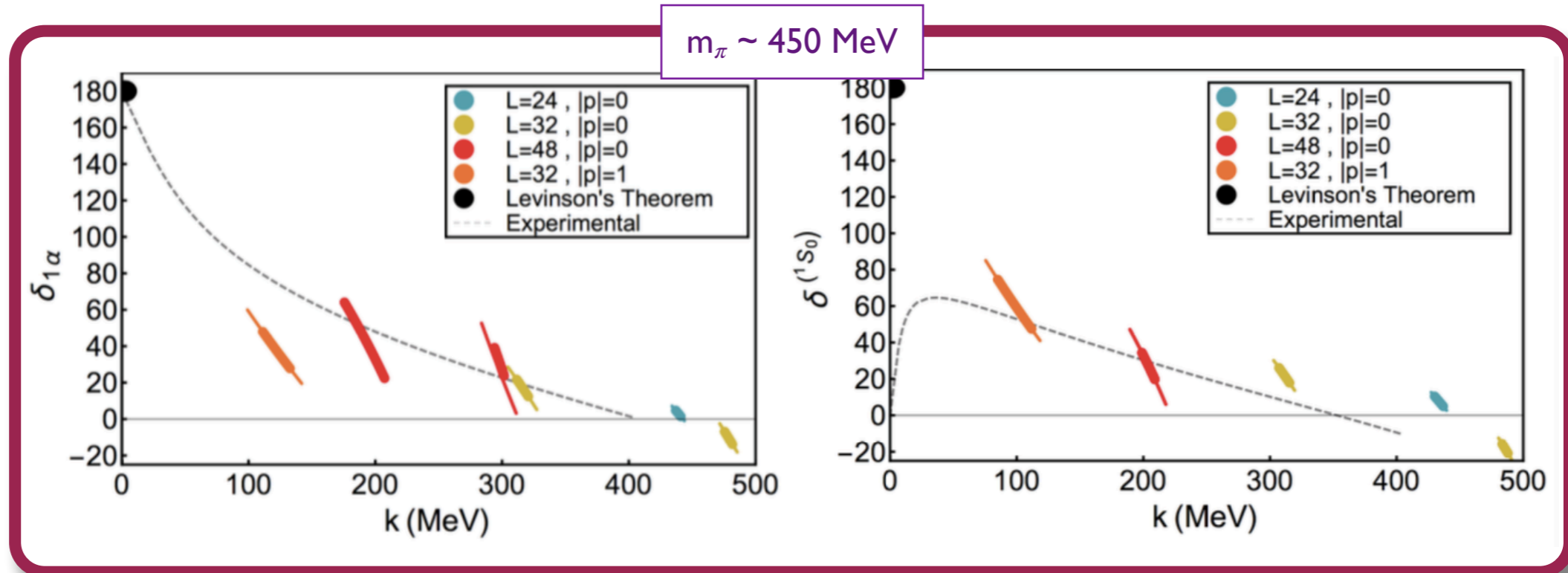


$$U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$

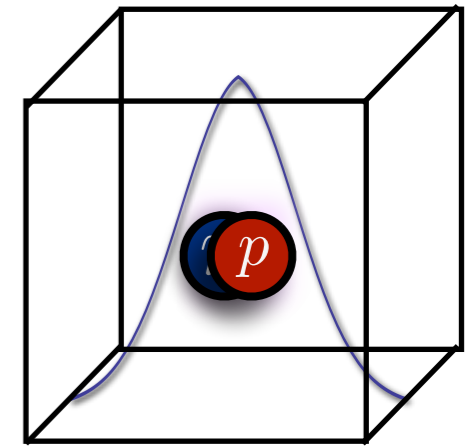
NN scattering: Lüscher



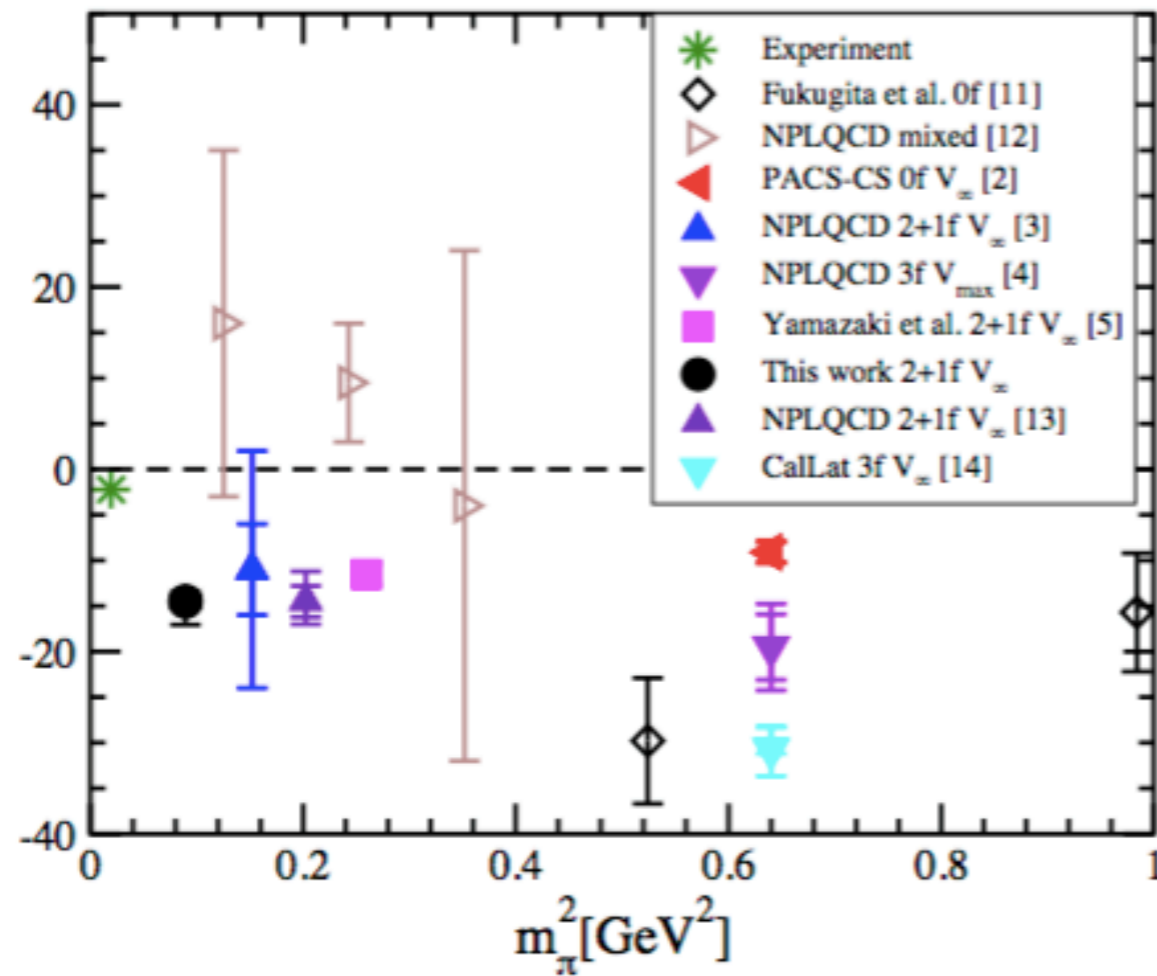
Phys.Rev. D92 (2015)
no.11, 114512



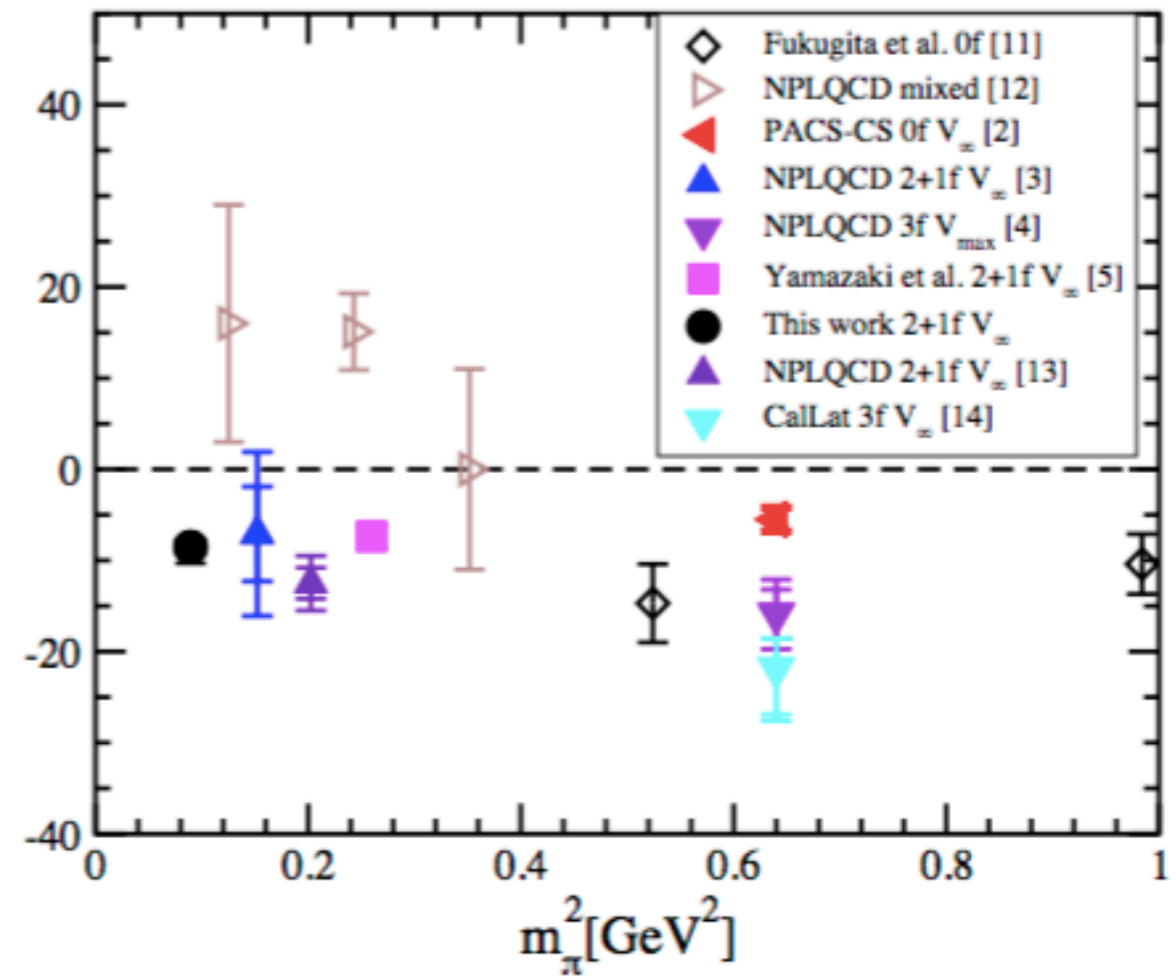
NN Binding energies



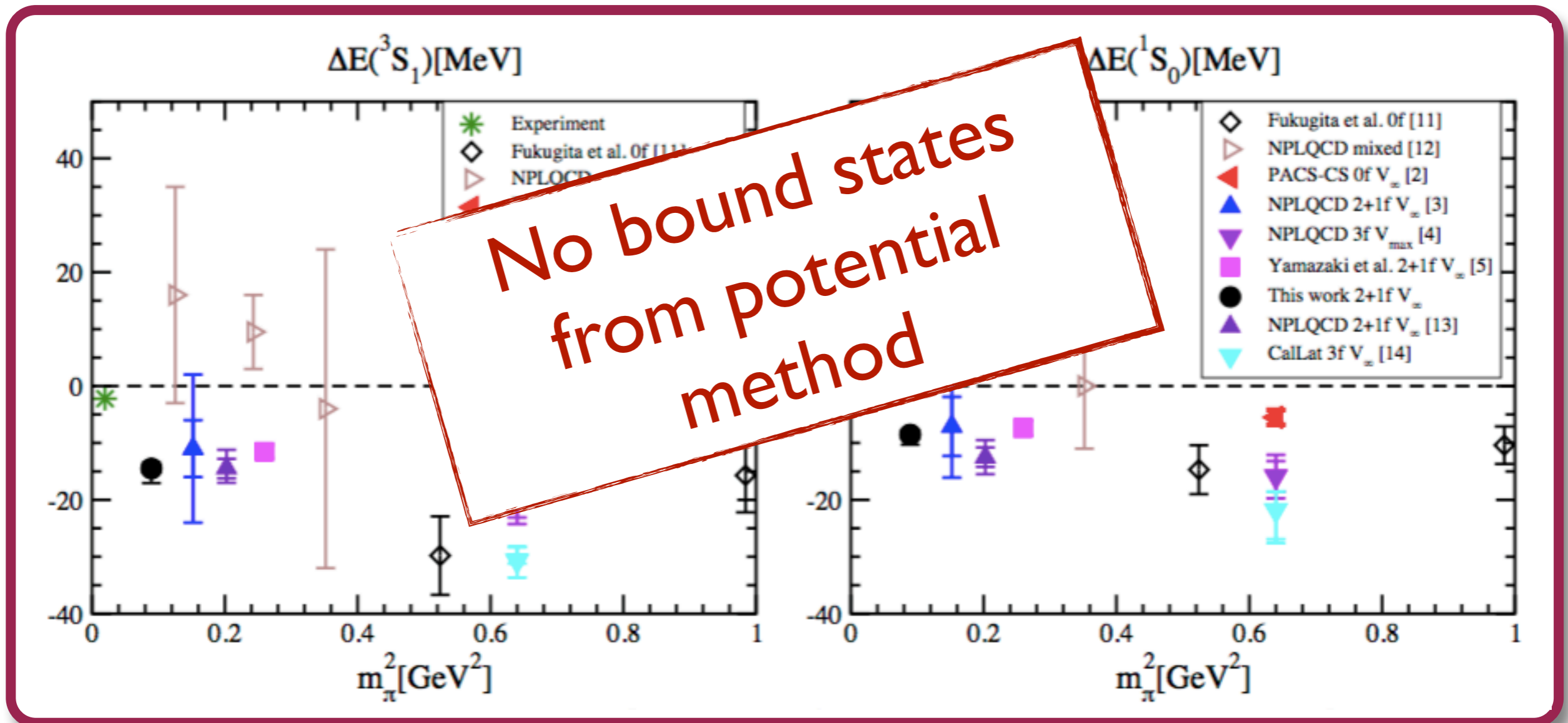
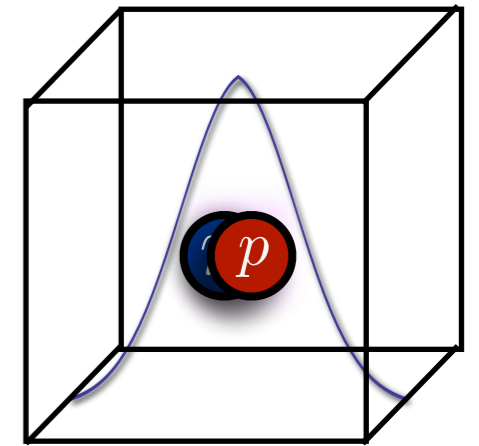
$\Delta E(^3S_1)$ [MeV]



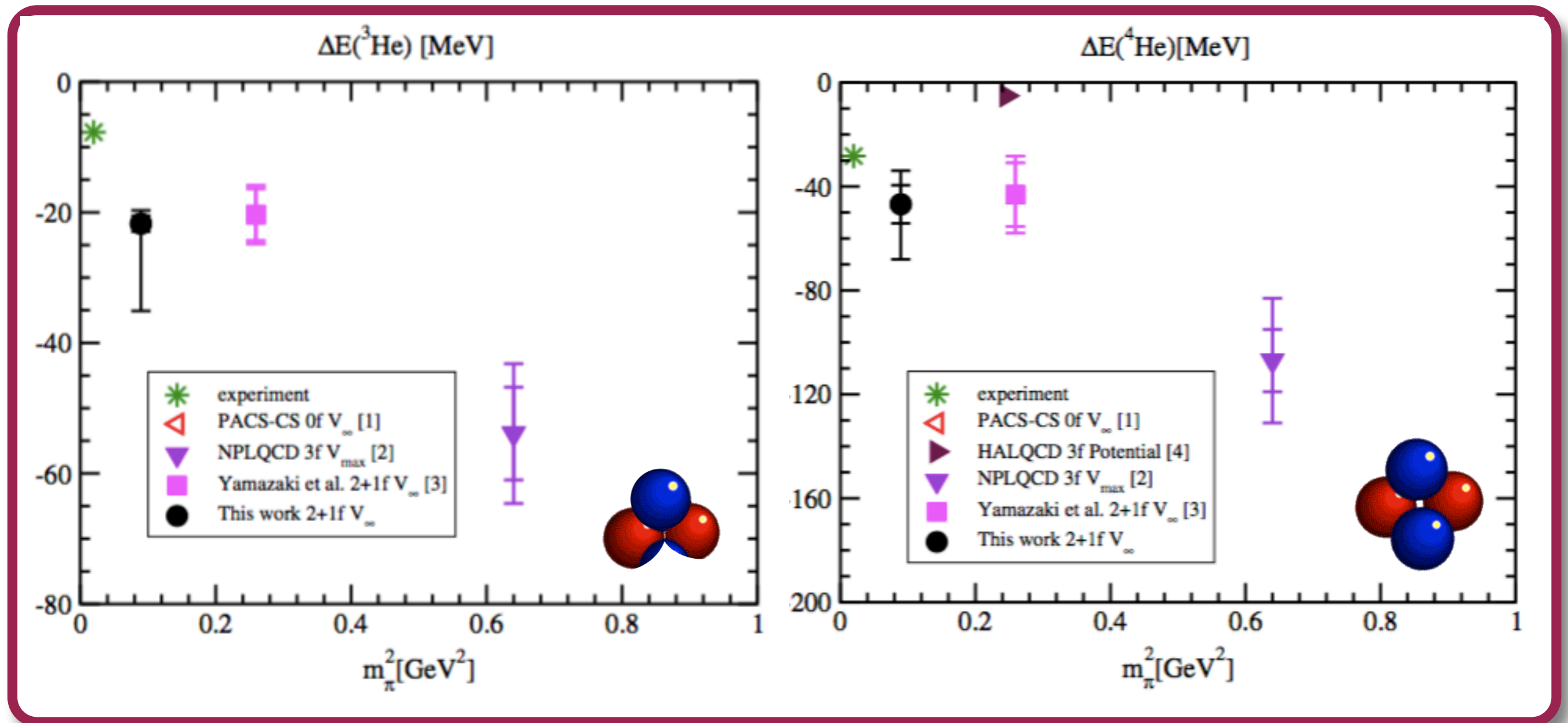
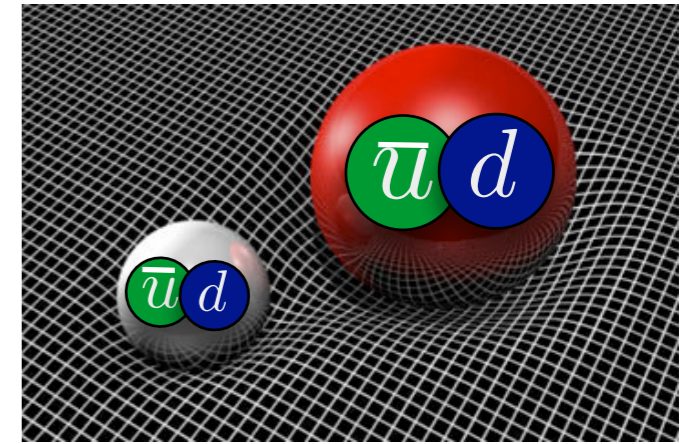
$\Delta E(^1S_0)$ [MeV]



NN Binding energies



Few-body systems

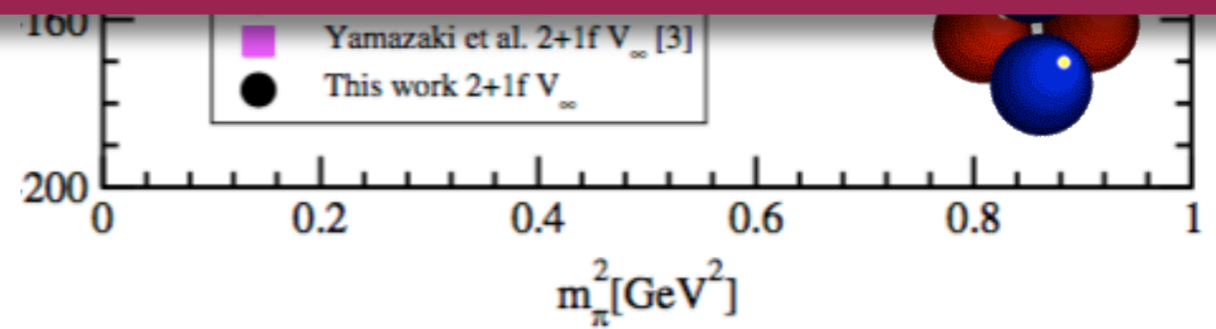
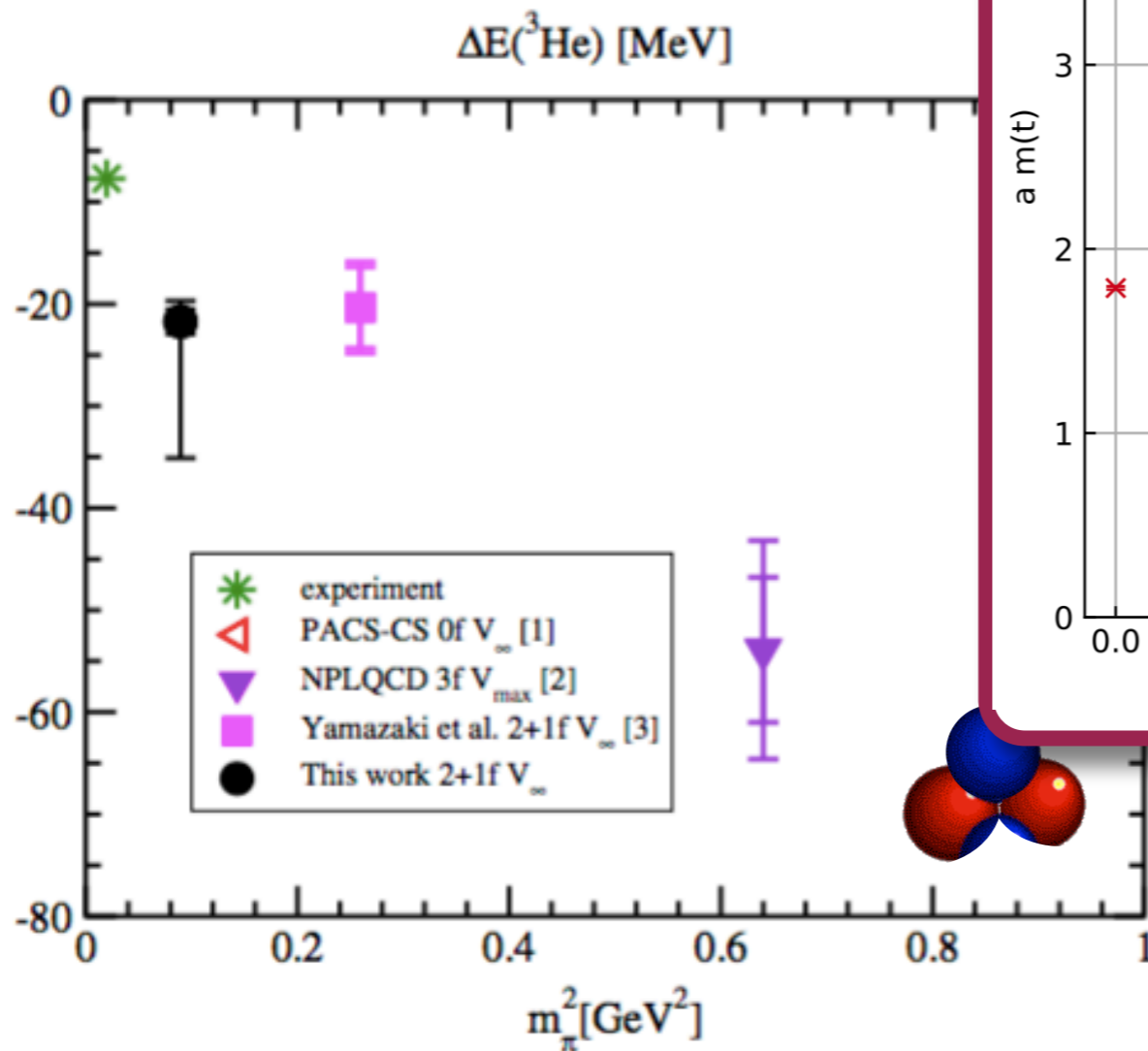
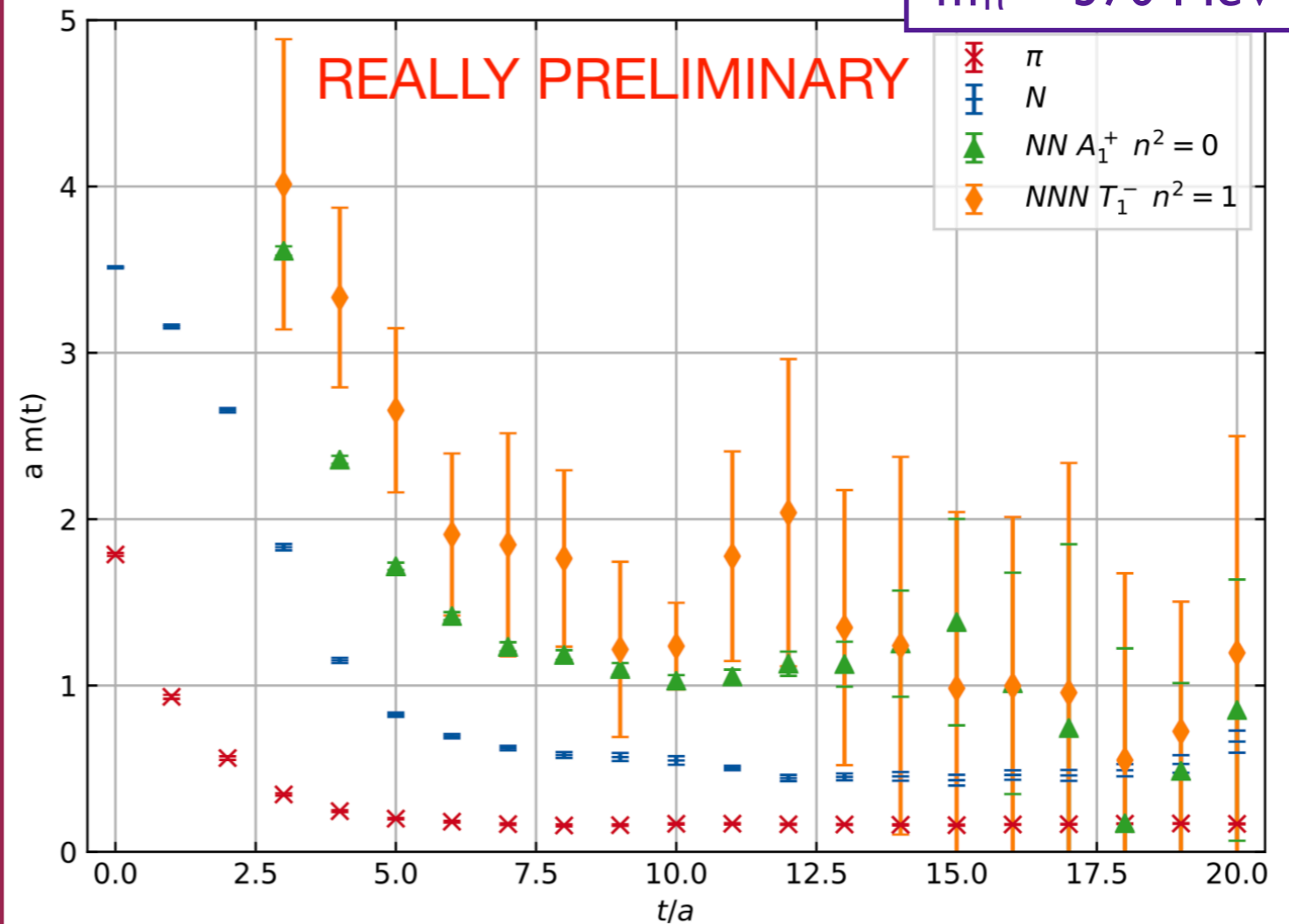


Few-body syst

Three Neutrons In A Box

Jan-Lukas Wynen, EB, Tom Luu, Andrea Schindler, John Bulava

$m_\pi \sim 370$ MeV

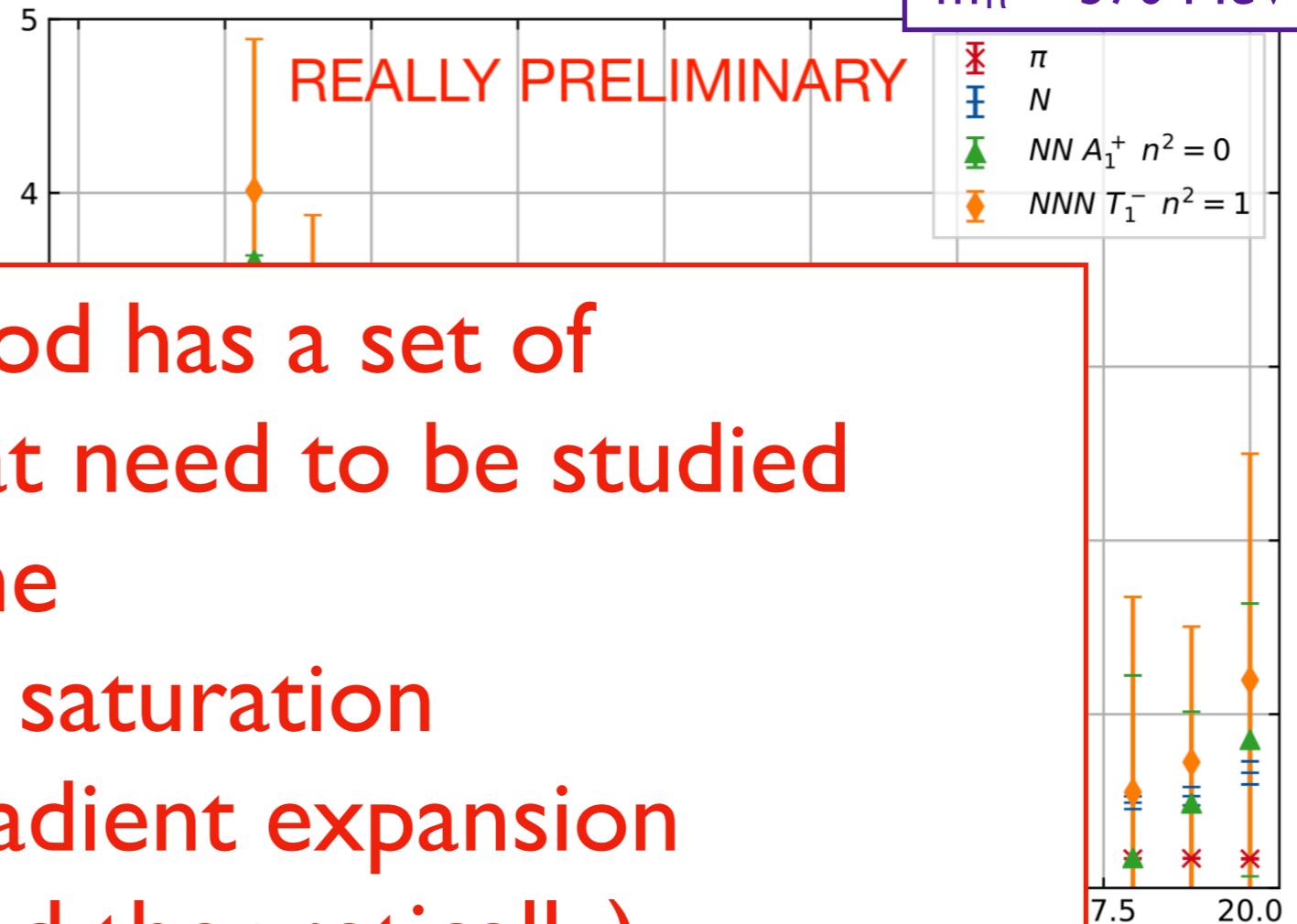


Few-body syst

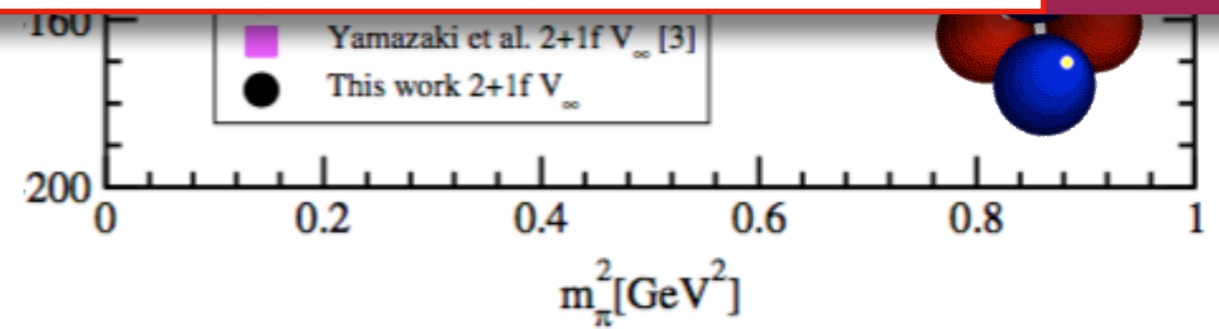
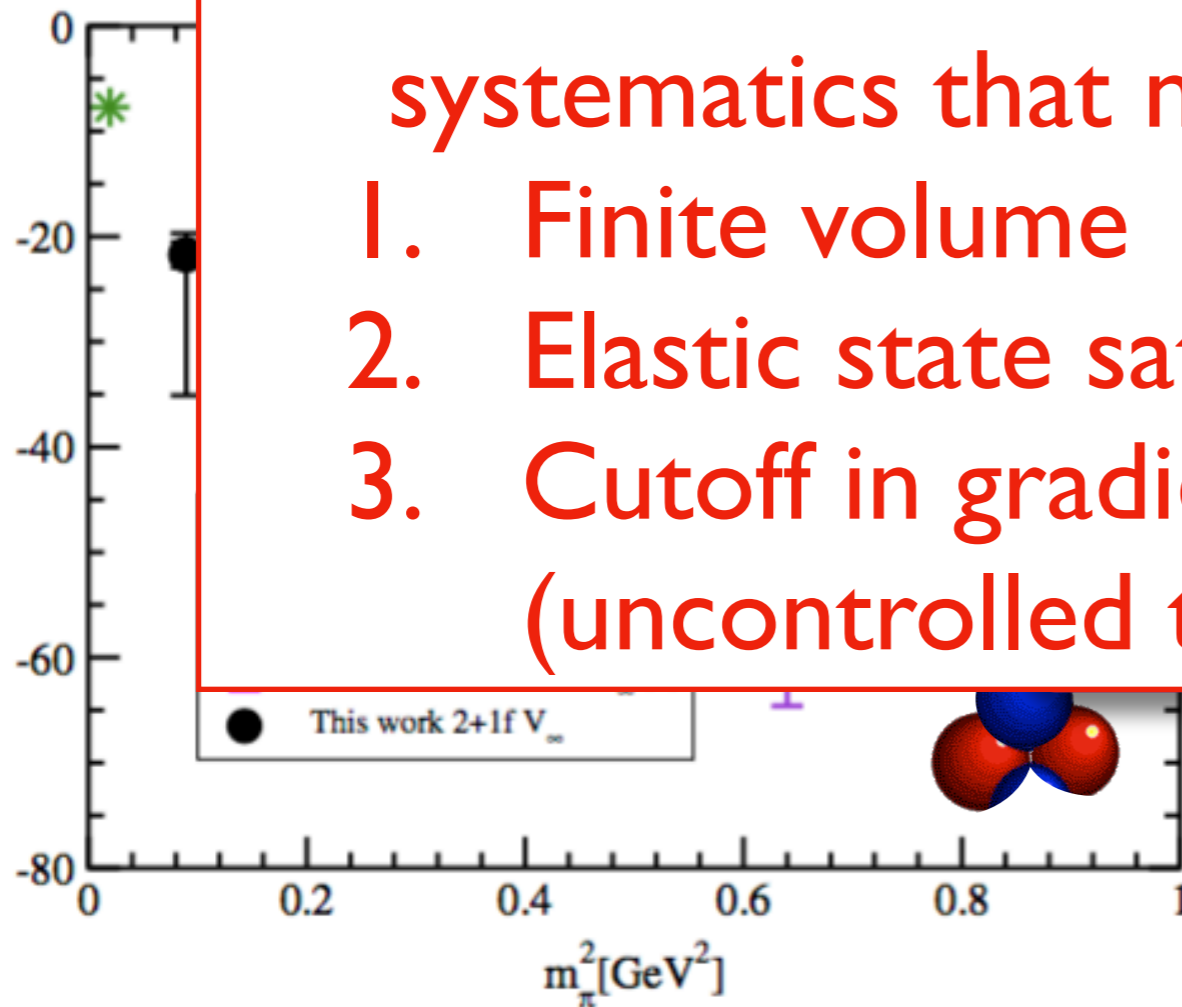
Three Neutrons In A Box

Jan-Lukas Wynen, EB, Tom Luu, Andrea Schindler, John Bulava

$m_\pi \sim 370 \text{ MeV}$



- I. Potential method has a set of systematics that need to be studied
 1. Finite volume
 2. Elastic state saturation
 3. Cutoff in gradient expansion (uncontrolled theoretically)

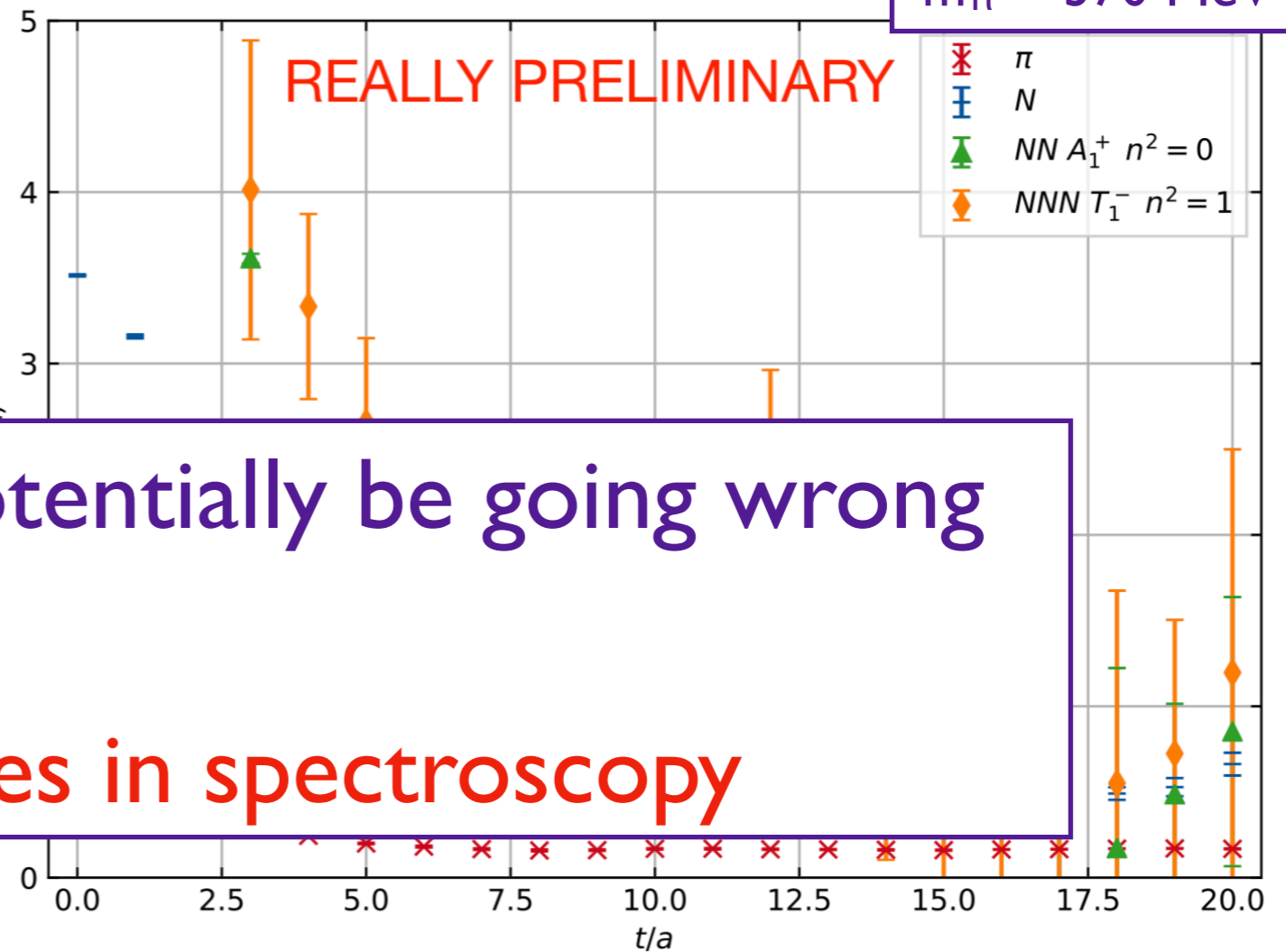


Few-body syst

Three Neutrons In A Box

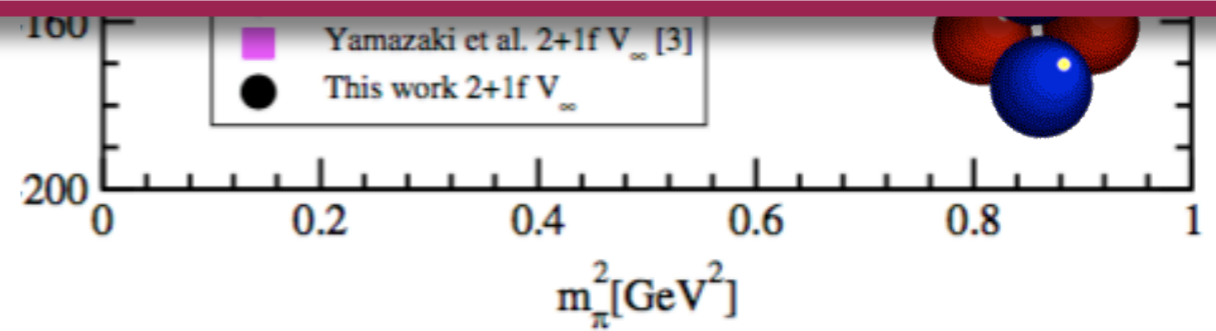
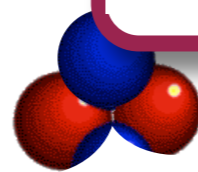
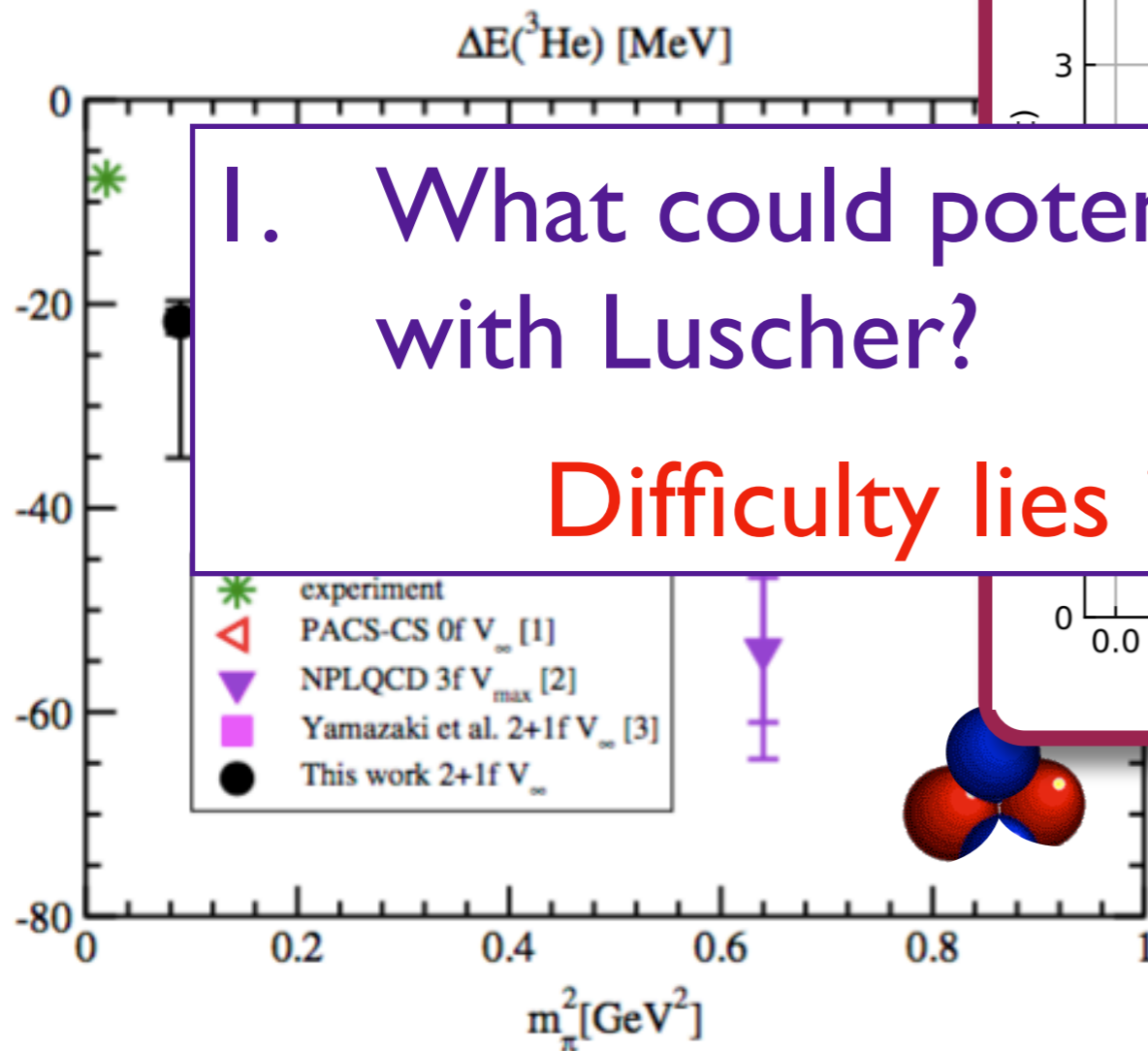
Jan-Lukas Wynen, EB, Tom Luu, Andrea Schindler, John Bulava

$m_\pi \sim 370 \text{ MeV}$



I. What could potentially be going wrong with Luscher?

Difficulty lies in spectroscopy

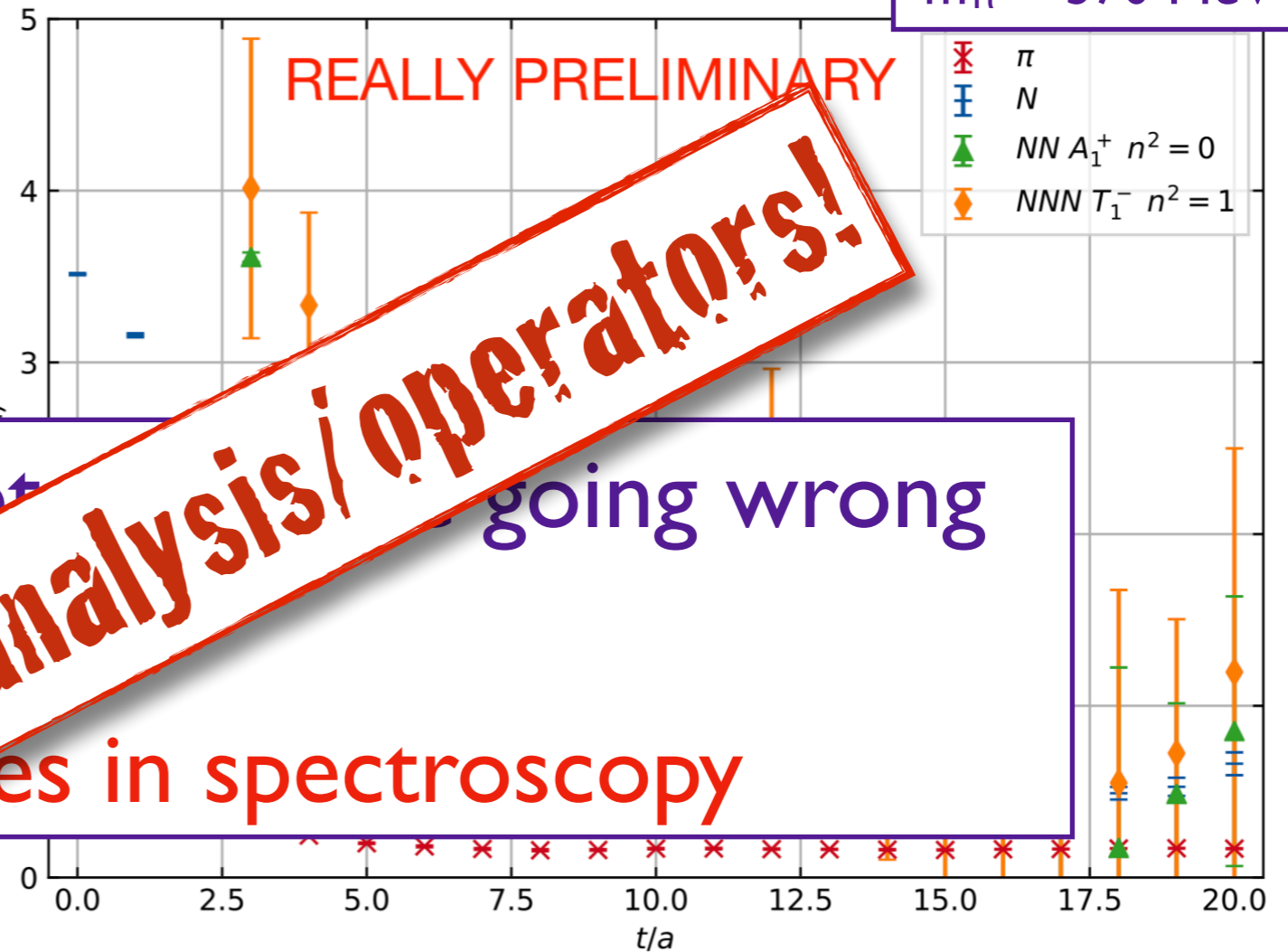


Few-body syst

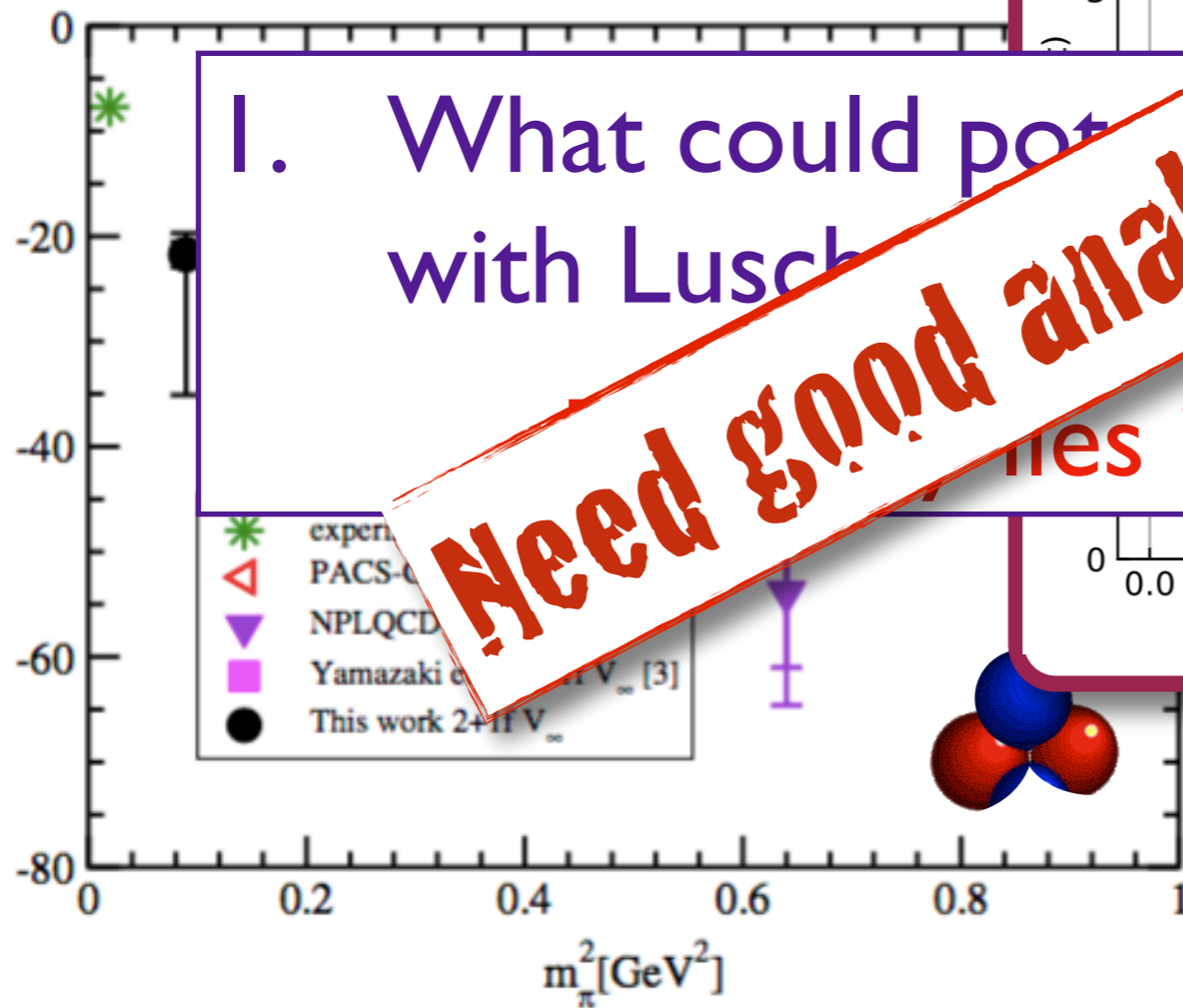
Three Neutrons In A Box

Jan-Lukas Wynen, EB, Tom Luu, Andrea Schindler, John Bulava

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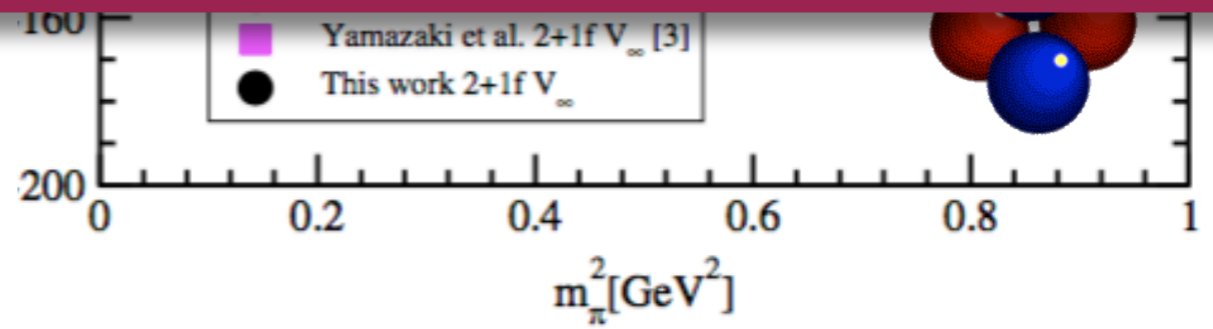
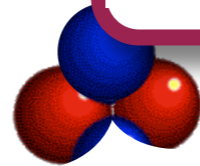


$\Delta E(^3\text{He})$ [MeV]

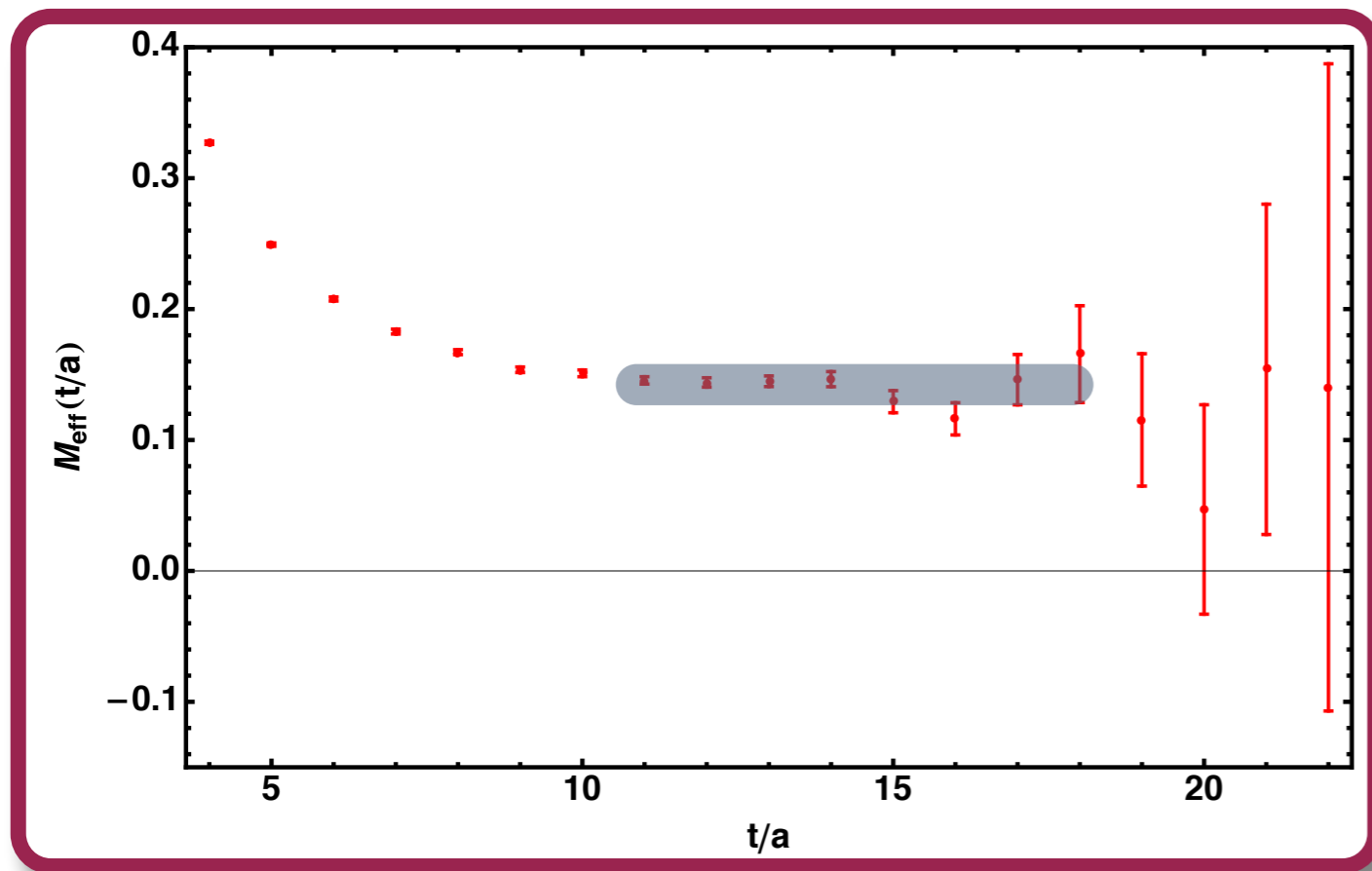


I. What could potentially be going wrong with Luscher's method in spectroscopy

Need good analysis/operators!

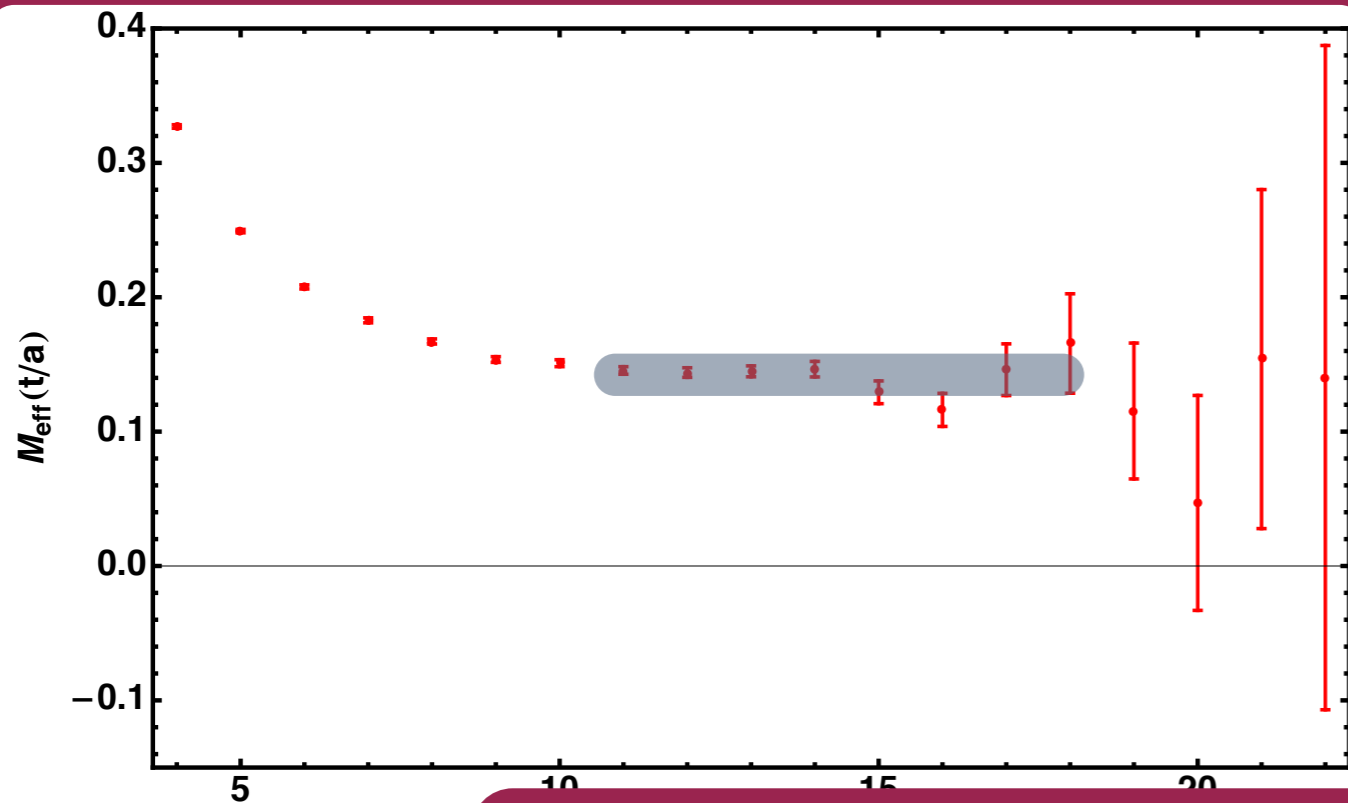


Nucleons: Spectroscopy

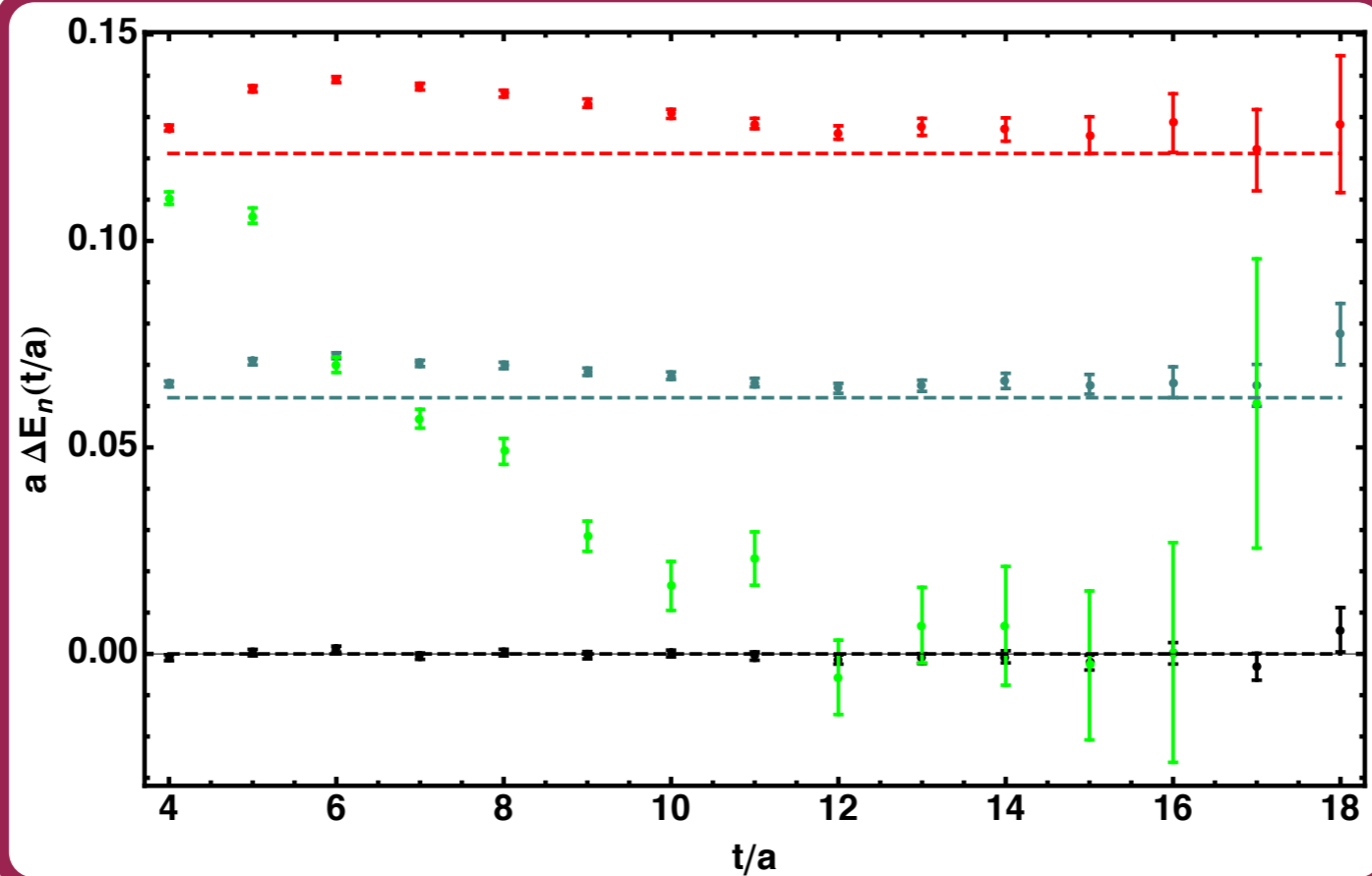


$$\sim e^{A(M_n - 3/2m_\pi)t}$$

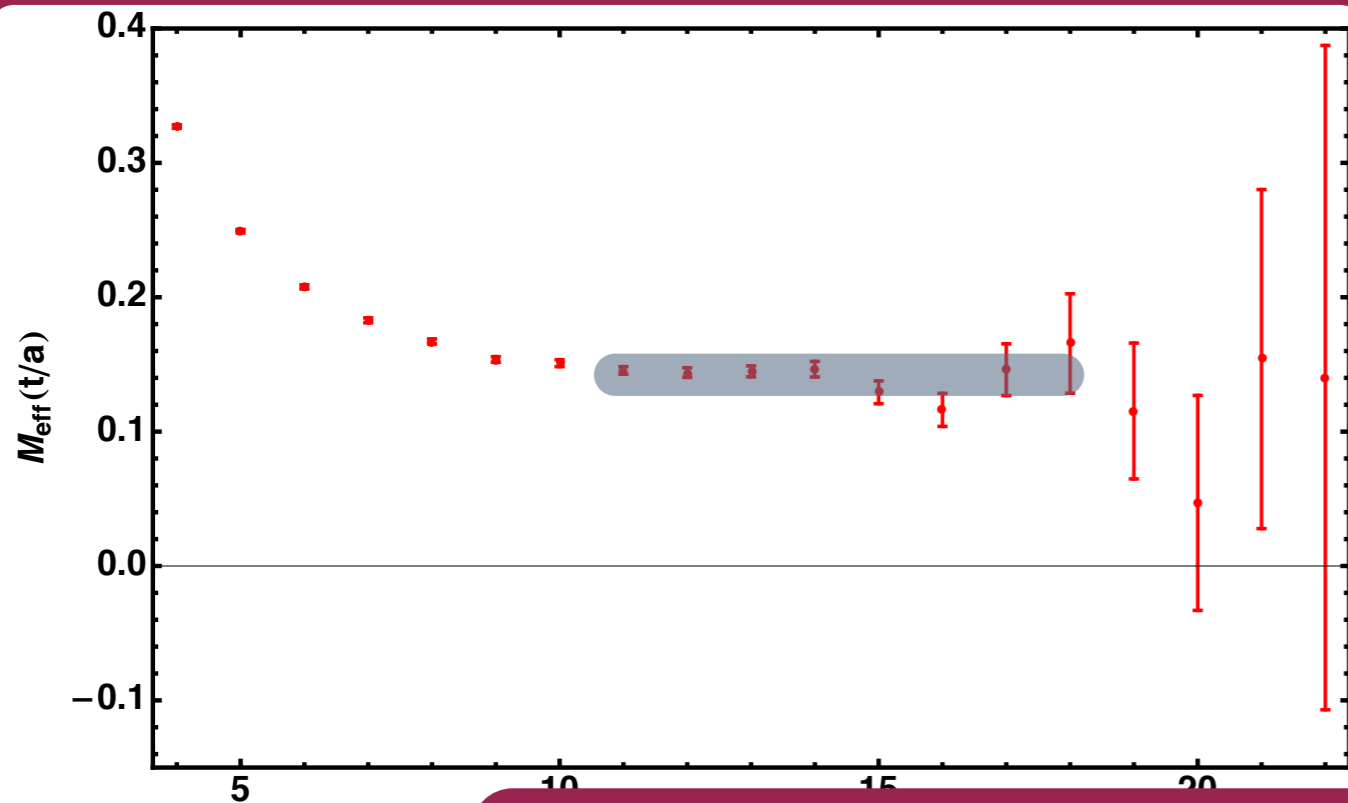
Nucleons: Spectroscopy



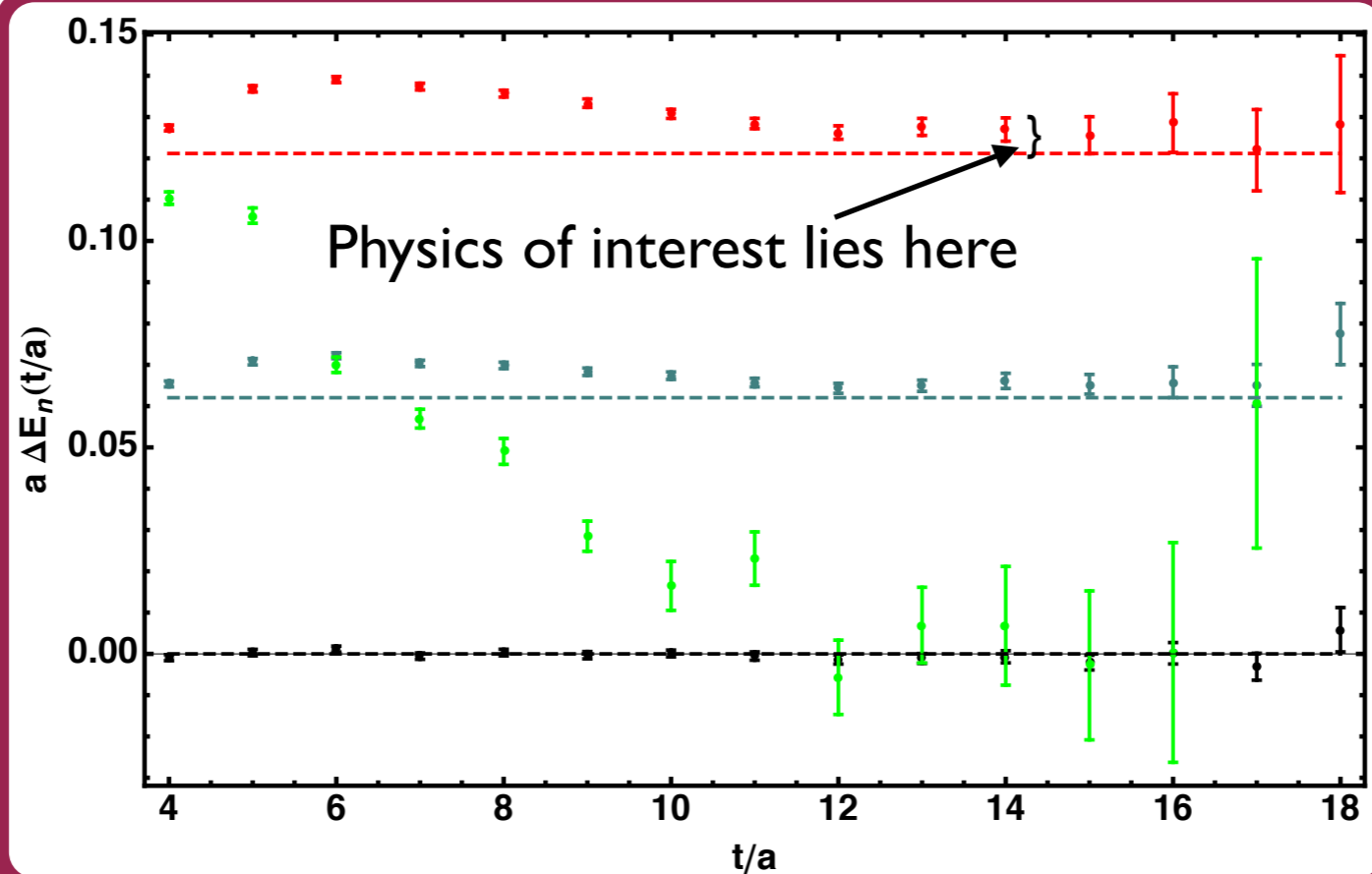
} $\sim e^{A(M_n - 3/2m_\pi)t}$



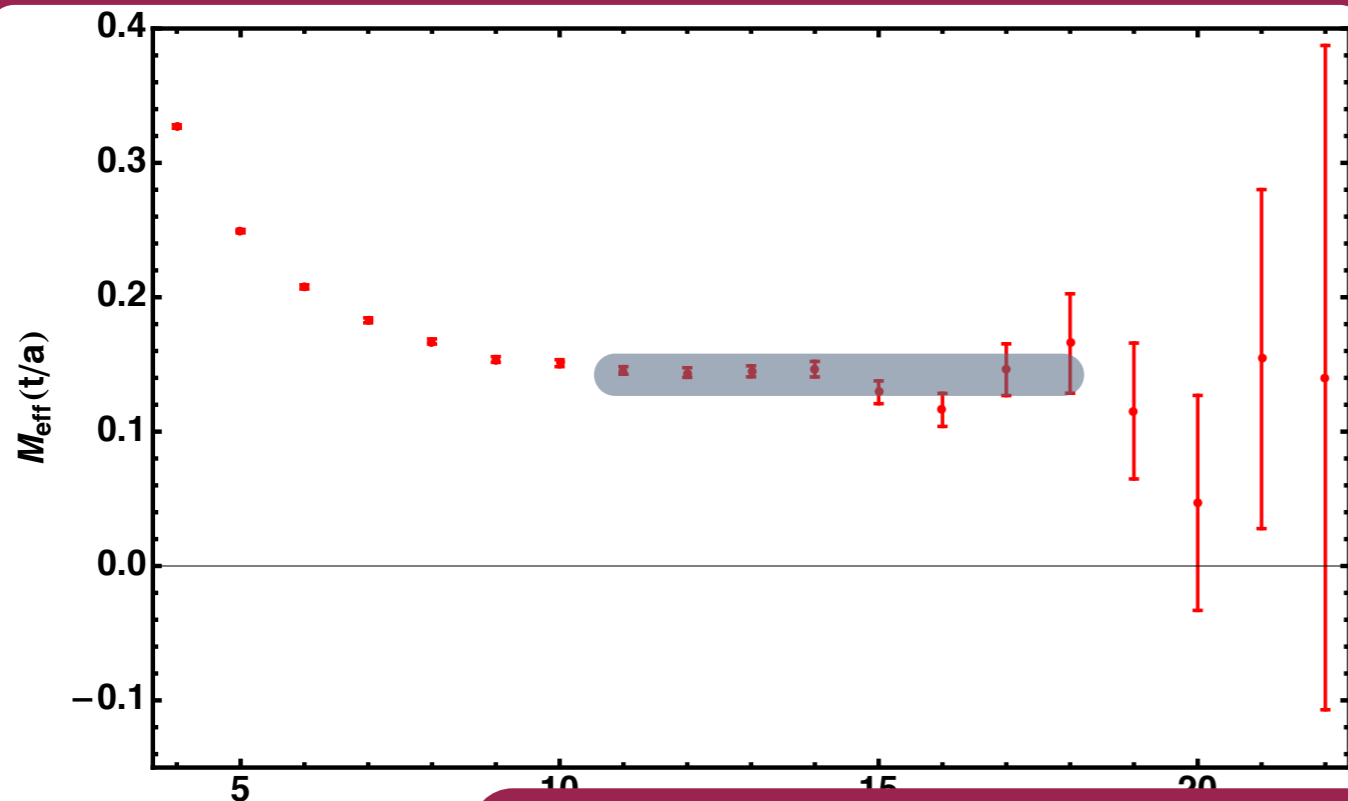
Nucleons: Spectroscopy



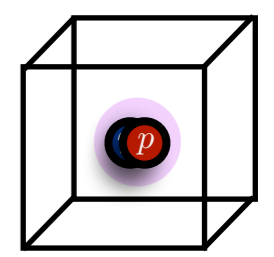
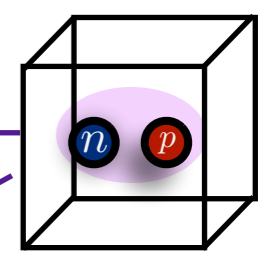
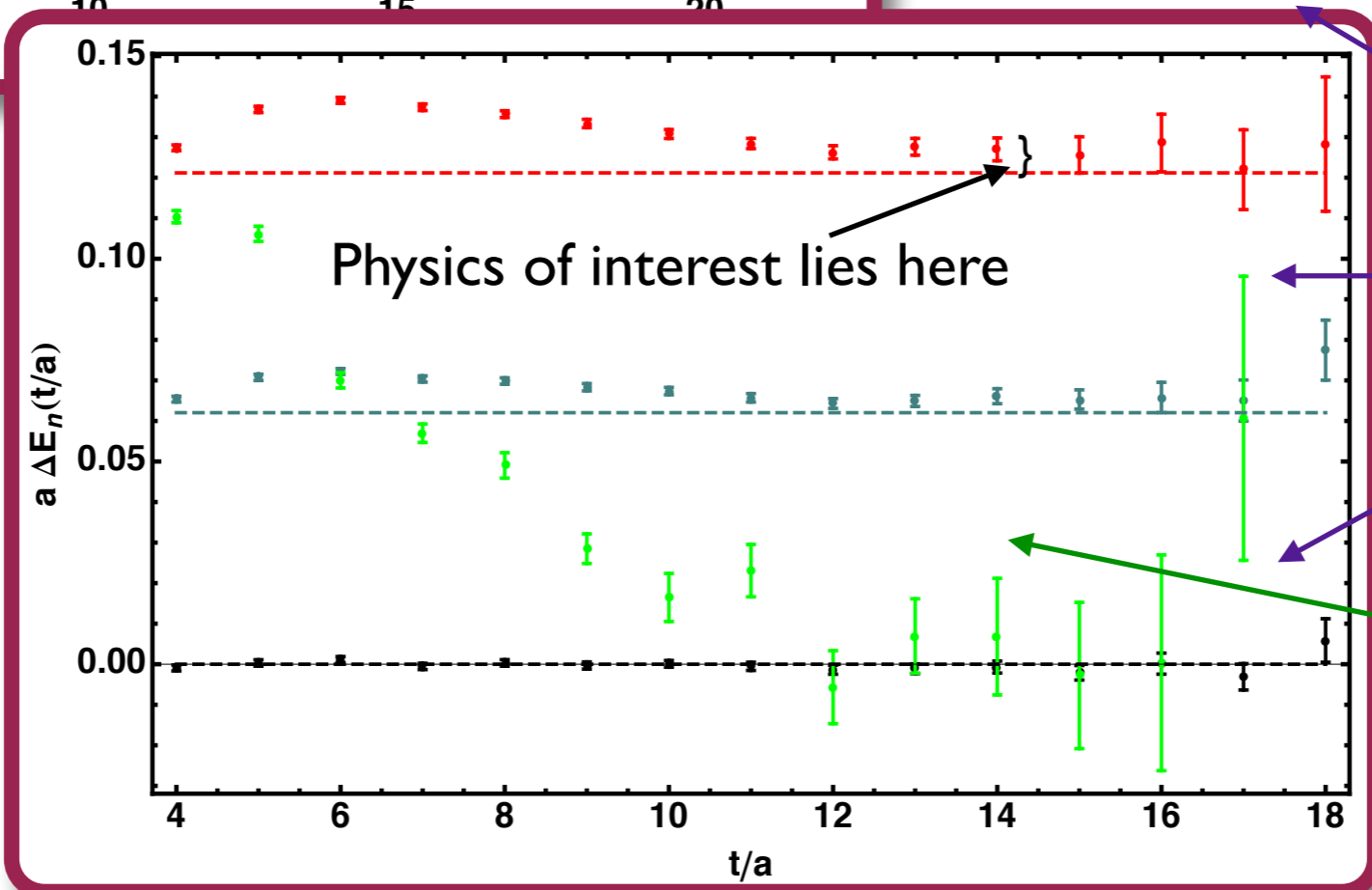
} $\sim e^{A(M_n - 3/2m_\pi)t}$



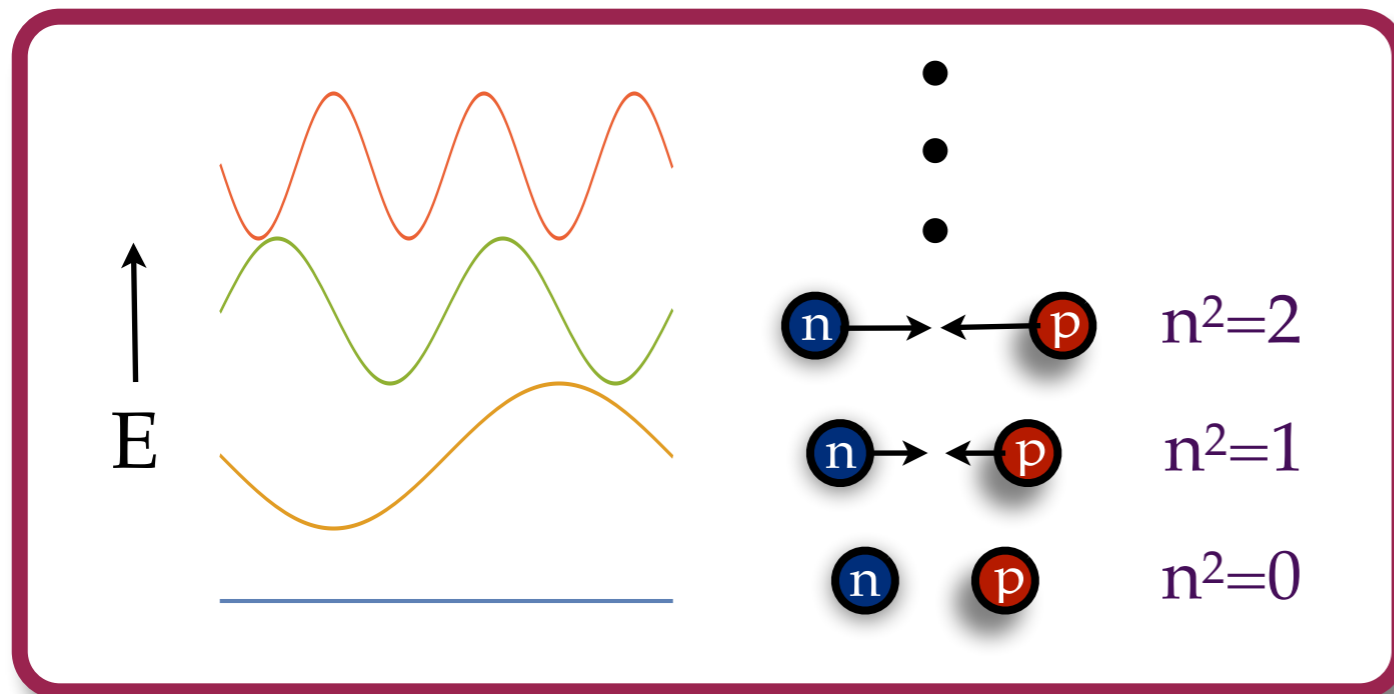
Nucleons: Spectroscopy



$$\sim e^{A(M_n - 3/2m_\pi)t}$$

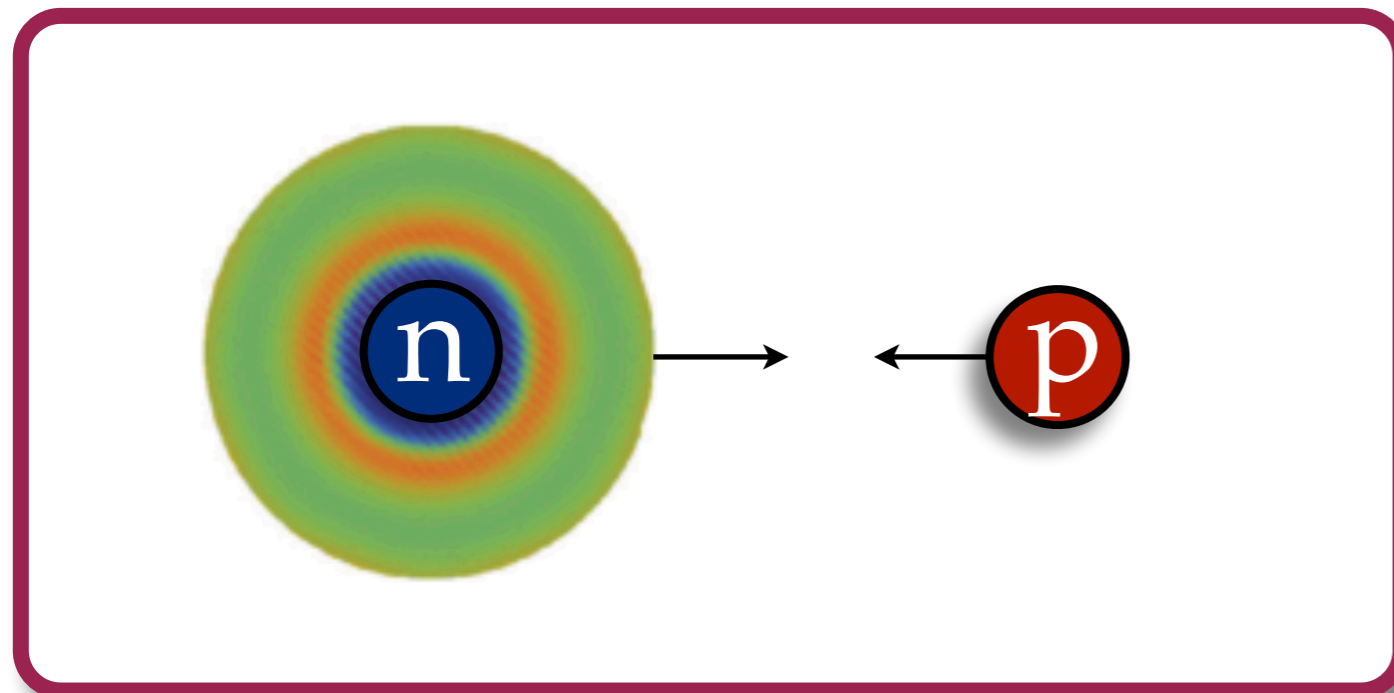


Excited state contamination



Elastic scattering
(2-body)

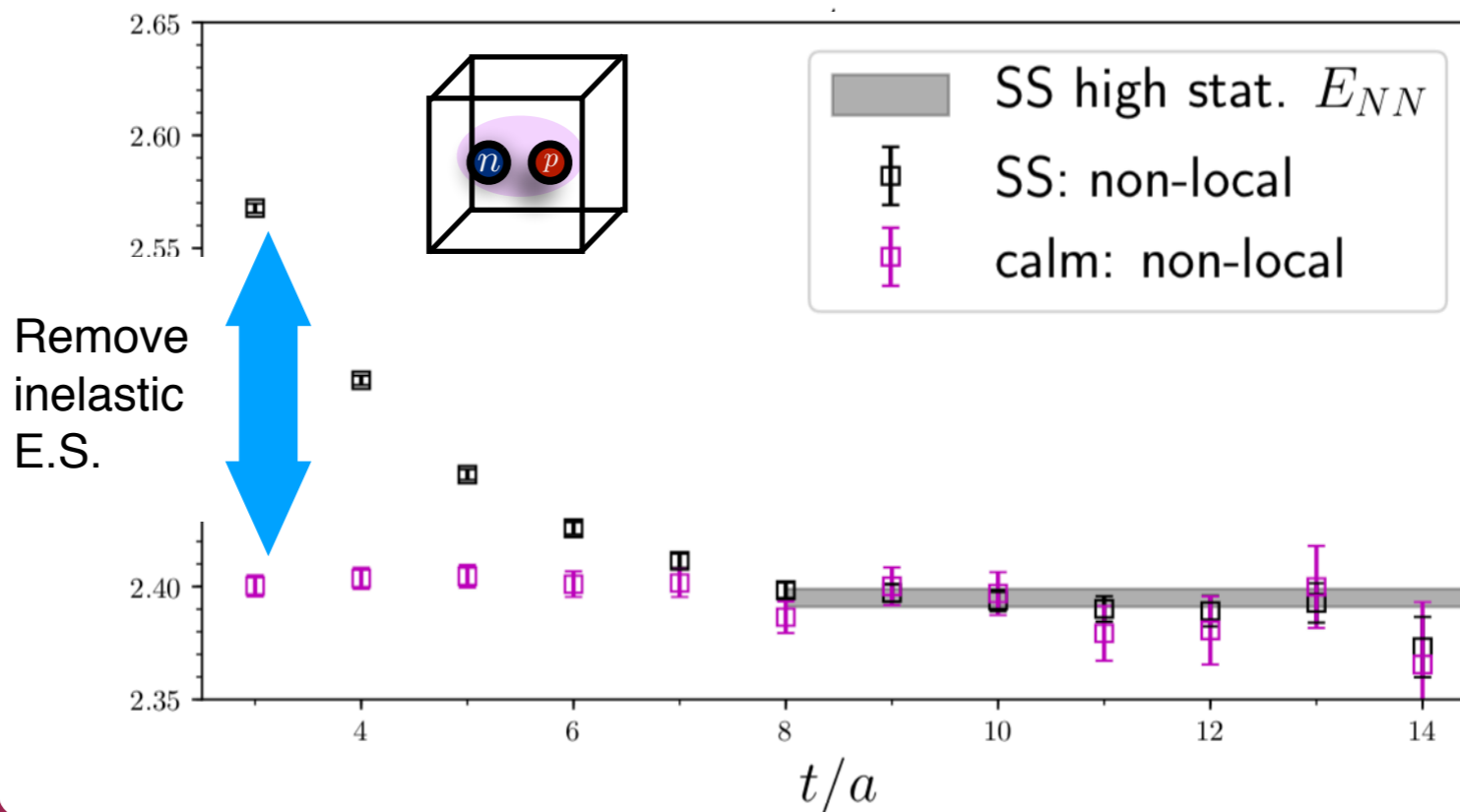
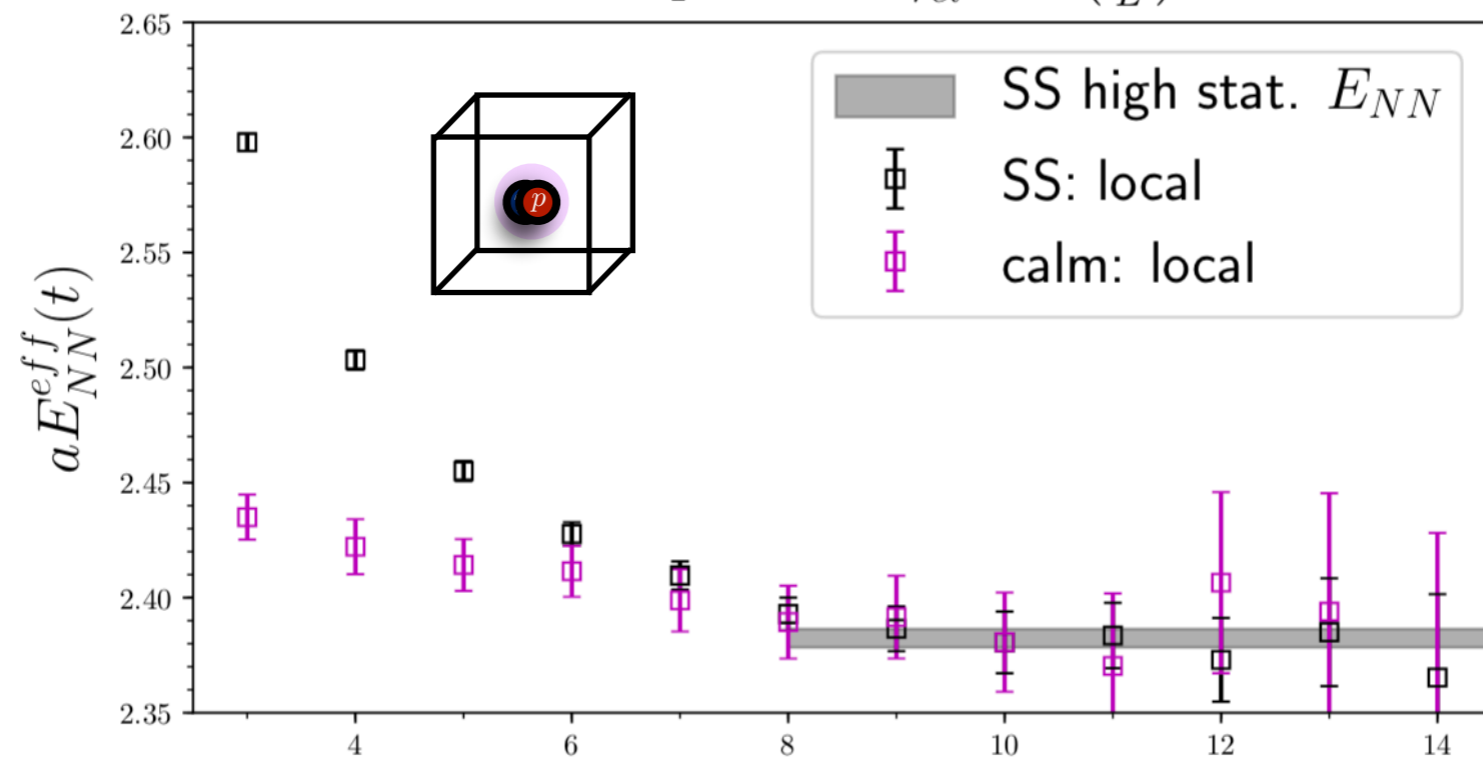
$$\Delta E \sim 50 \text{ MeV}$$



Inelastic single body

$$\Delta E \sim m_{\pi}$$

$$NN : T_1^+ : {}^3S_1 : p_{rel}^2 = 0 \left(\frac{2\pi}{L}\right)^2$$



Remove inelastic E.S.

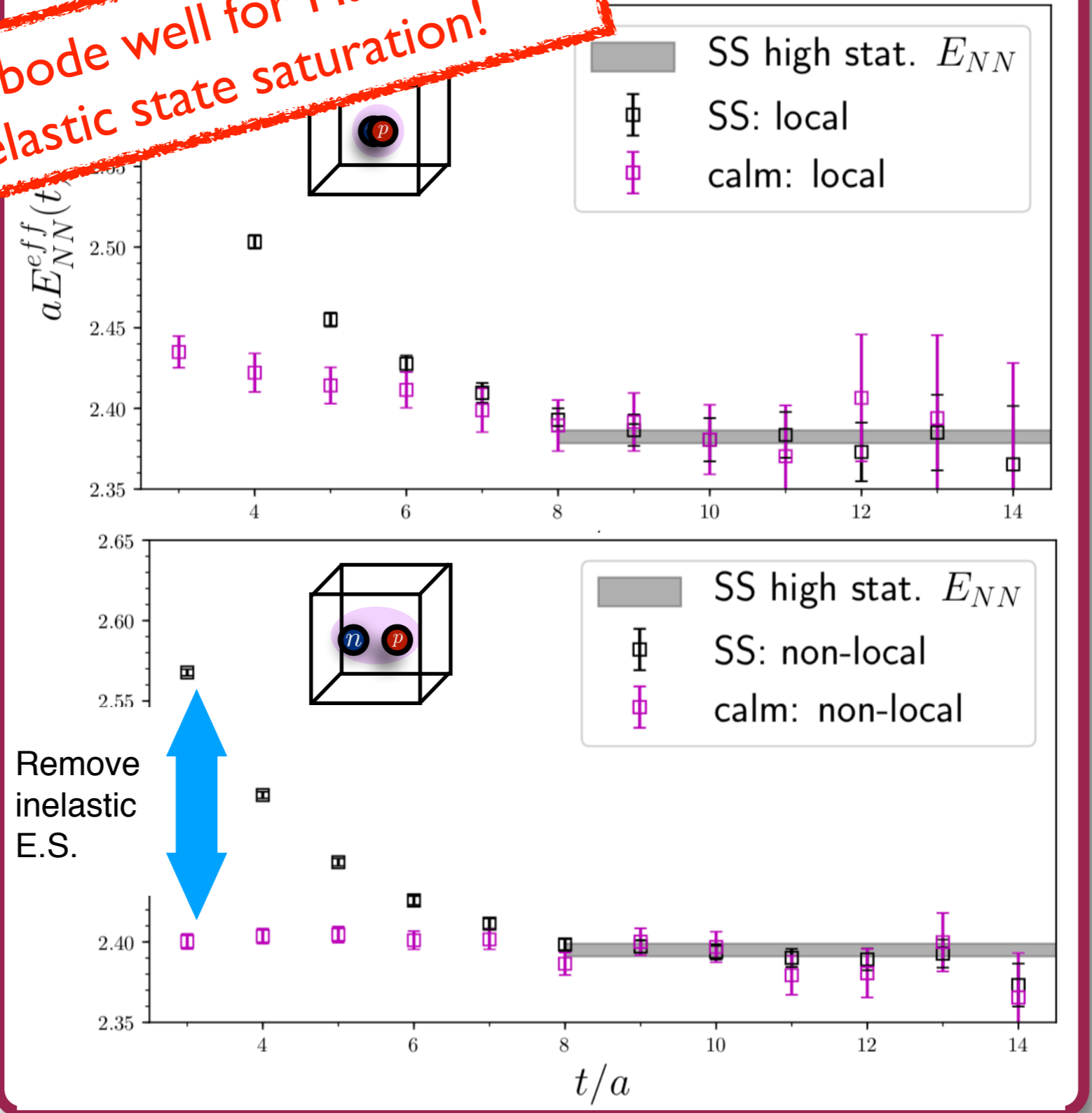
Remove elastic E.S.



CalLat (2017)
Matrix Prony:
NPLQCD (2009)

This doesn't bode well for HaIQCD's claim of elastic state saturation!

$$p_{rel}^2 = 0 \left(\frac{2\pi}{L}\right)^2$$



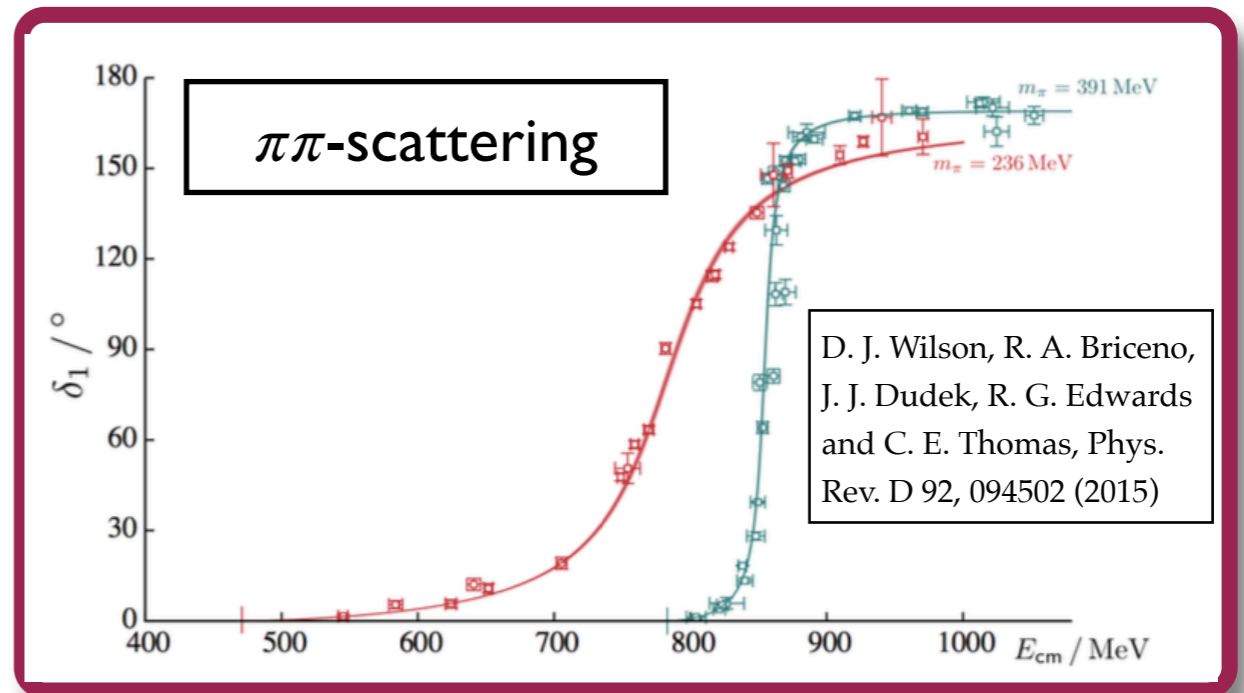
Remove inelastic E.S.

Remove elastic E.S.



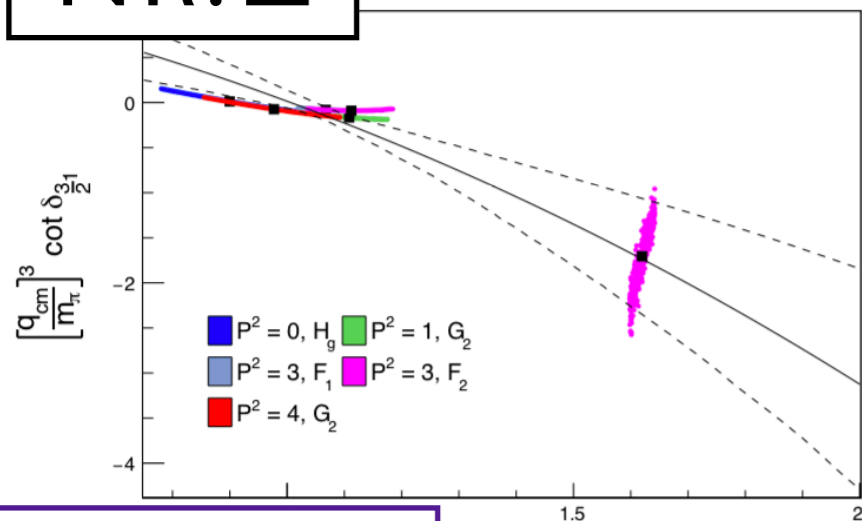
CalLat (2017)
 Matrix Prony:
 NPLQCD (2009)

The future: GEVP approaches

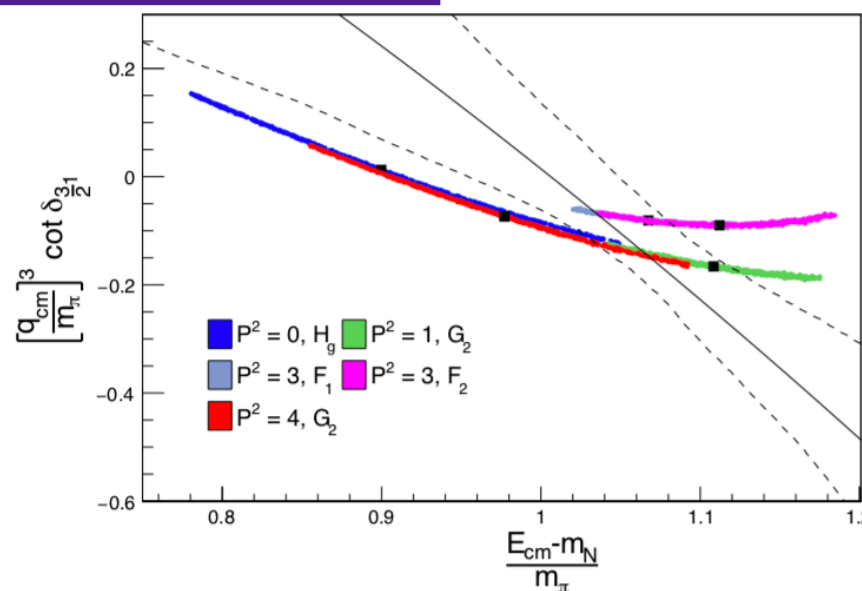


The future: GEVP approaches

$N\pi: \Delta$

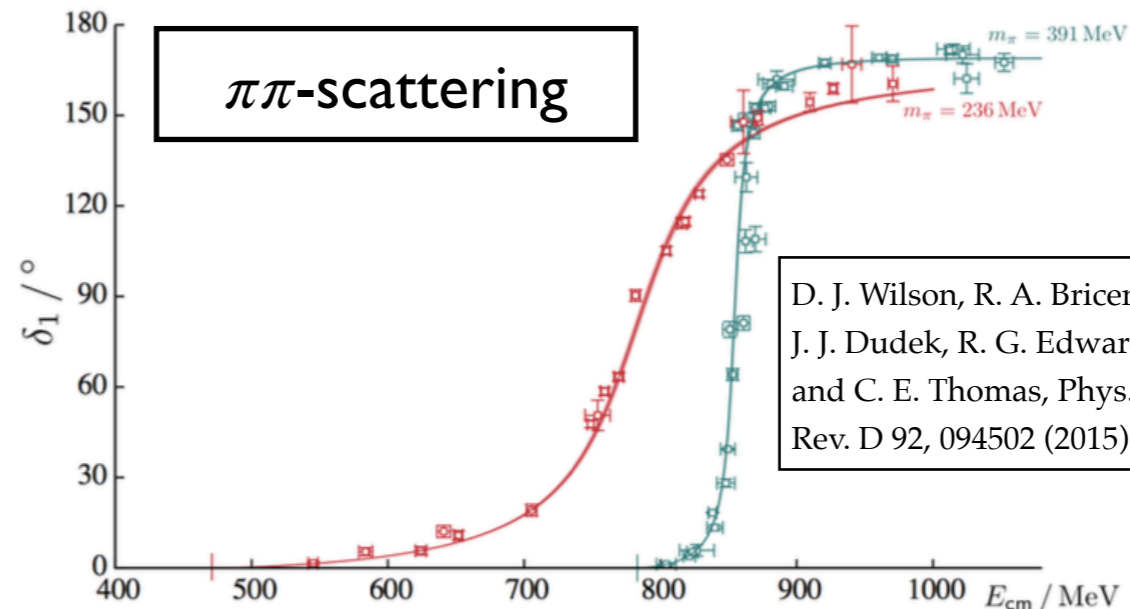


$m_\pi \sim 280, 460 \text{ MeV}$



Andersen, Bulava, Horz, Morningstar (2018)

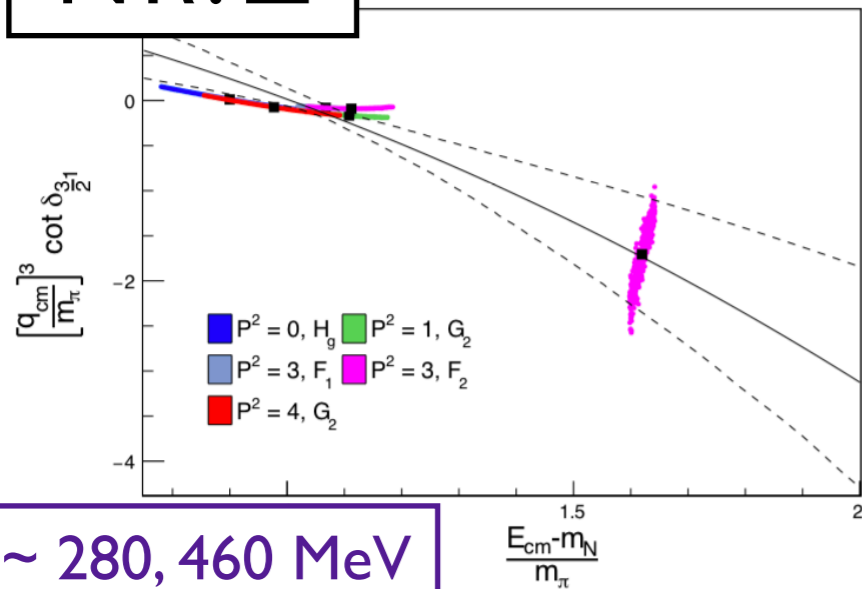
$\pi\pi$ -scattering



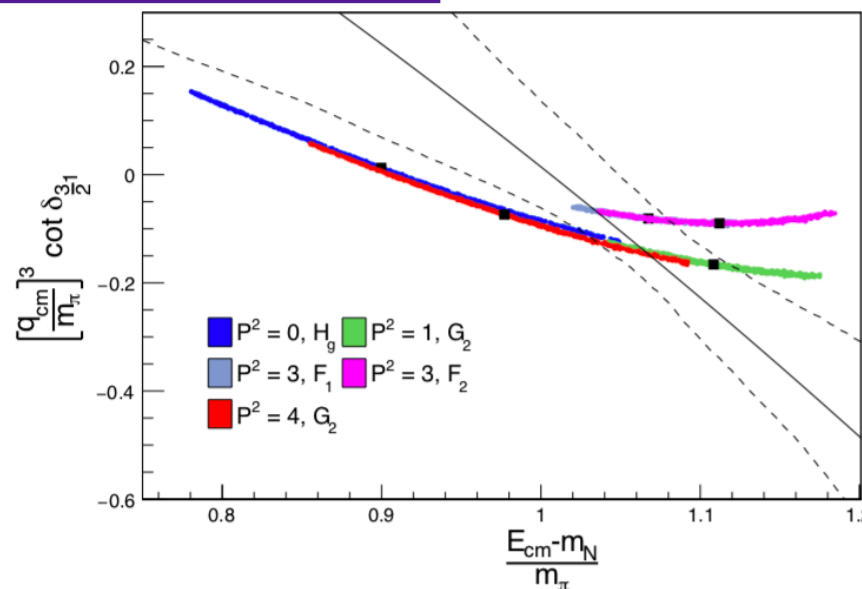
D. J. Wilson, R. A. Briceno,
J. J. Dudek, R. G. Edwards
and C. E. Thomas, Phys.
Rev. D 92, 094502 (2015)

The future: GEVP approaches

$N\pi: \Delta$

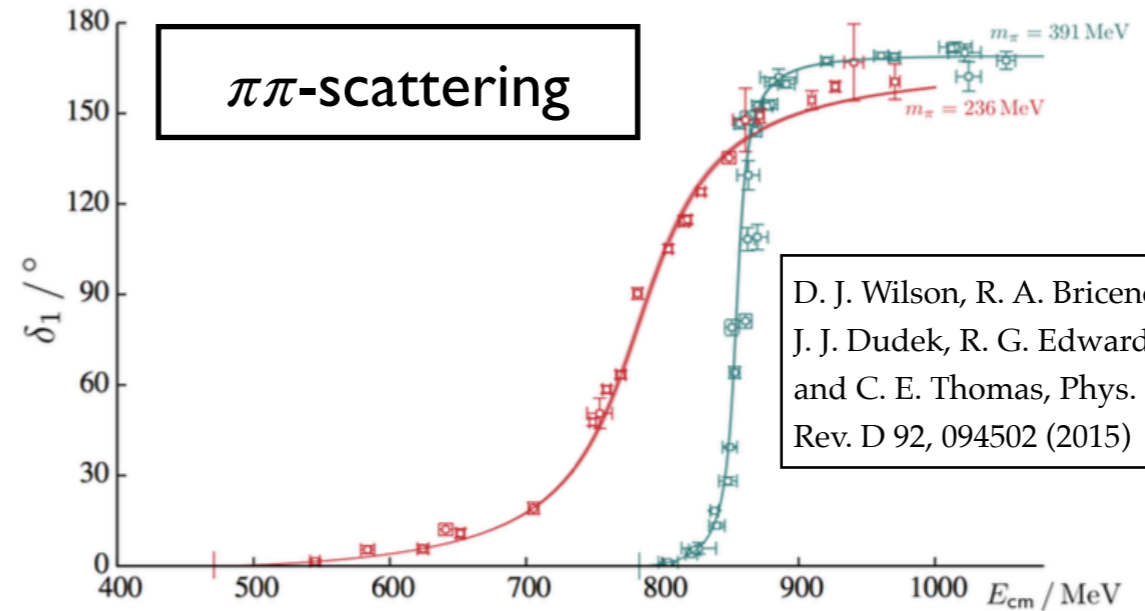


$m_\pi \sim 280, 460 \text{ MeV}$



Andersen, Bulava, Horz, Morningstar (2018)

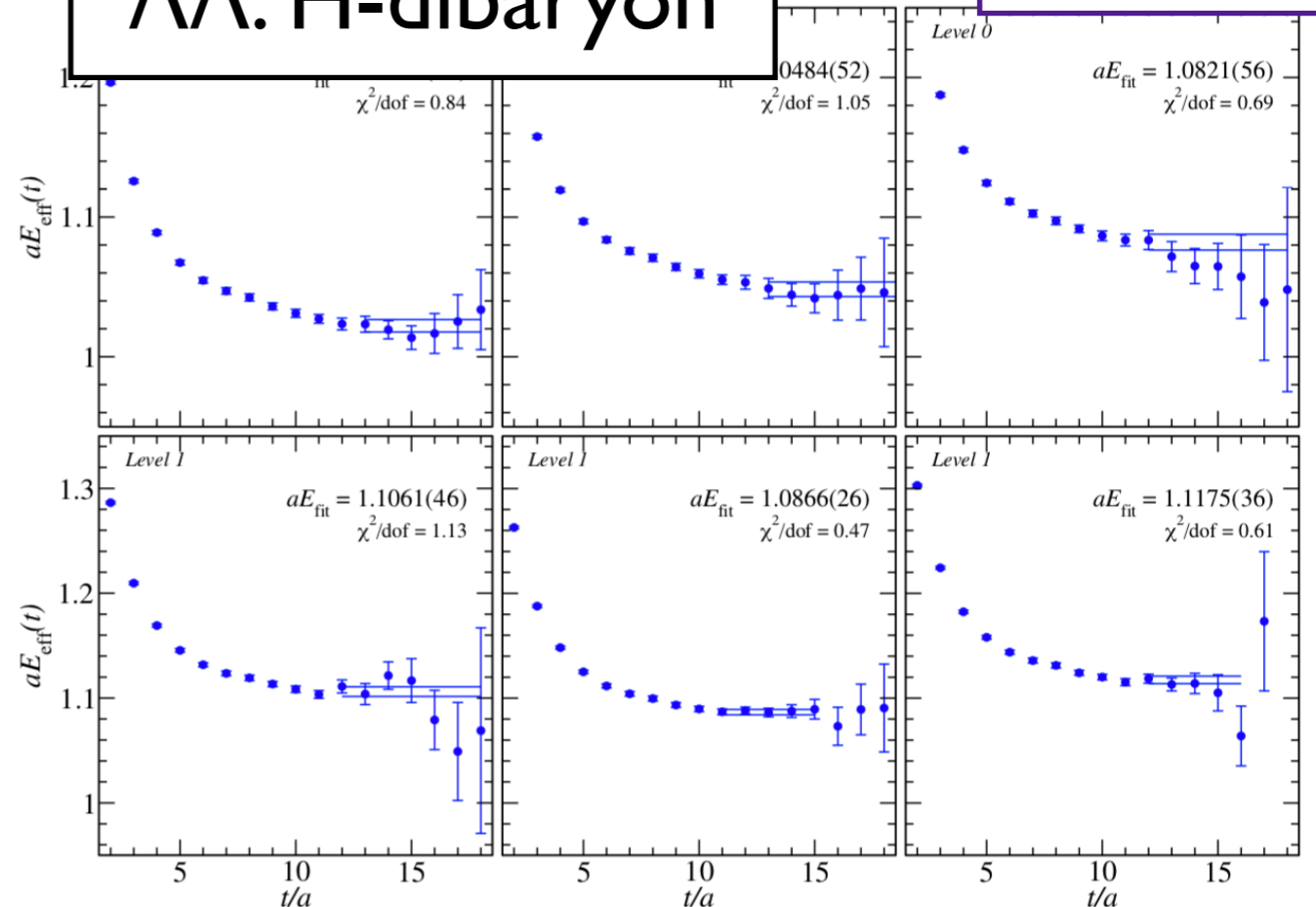
$\pi\pi$ -scattering



D. J. Wilson, R. A. Briceno,
J. J. Dudek, R. G. Edwards
and C. E. Thomas, Phys.
Rev. D 92, 094502 (2015)

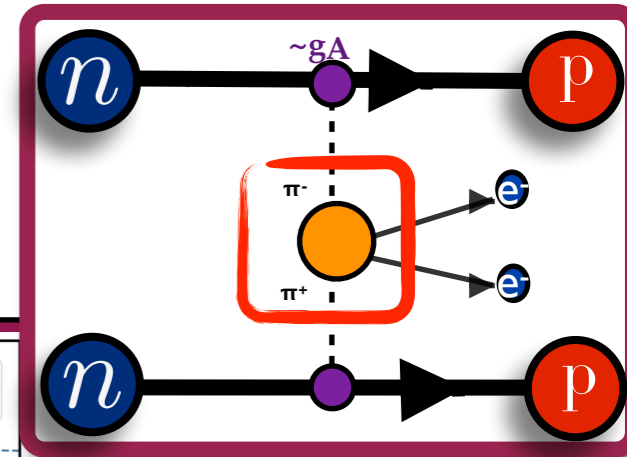
$\Lambda\Lambda: \text{H-dibaryon}$

$m_\pi \sim 800 \text{ MeV}$

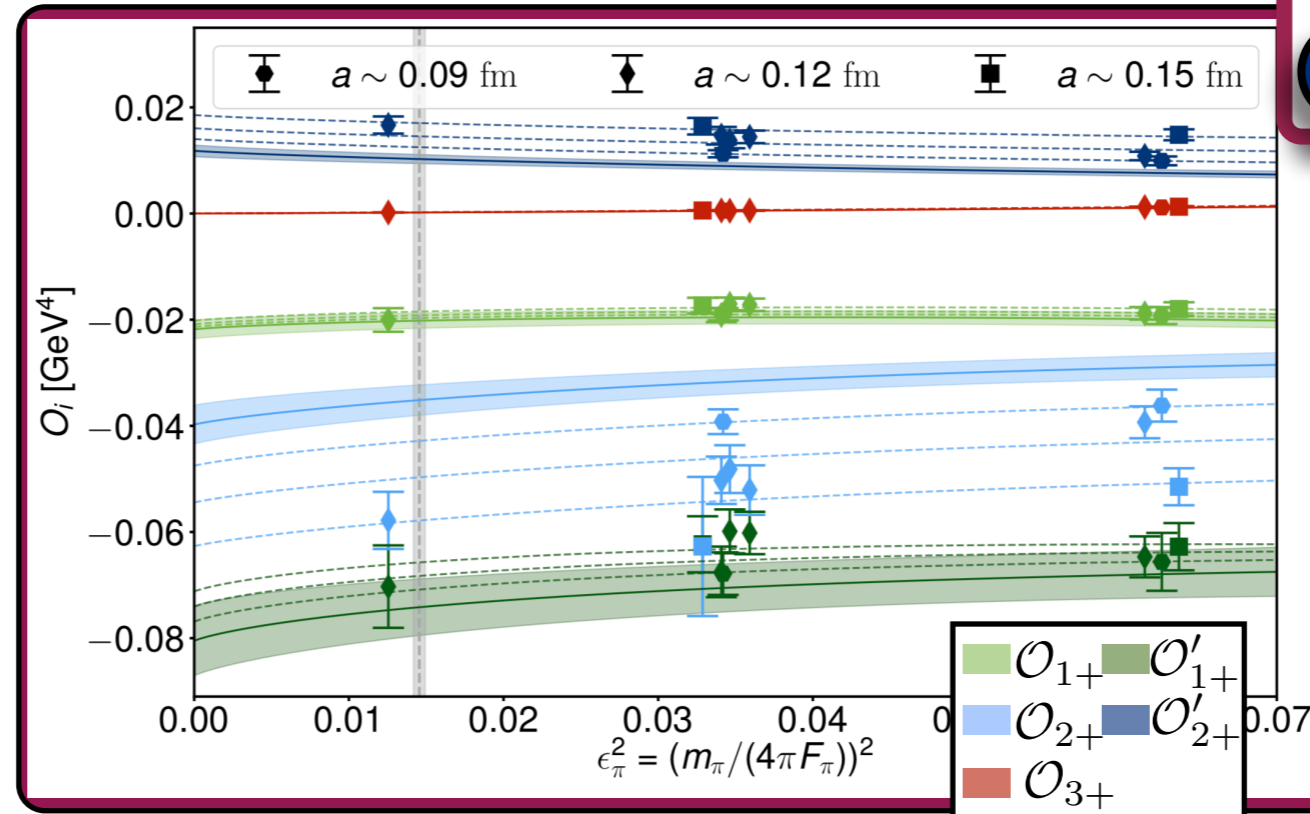
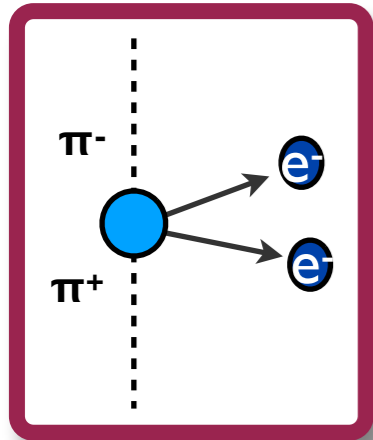


Hanlon, Francis, Green, Junnarkar, Wittig (2018)

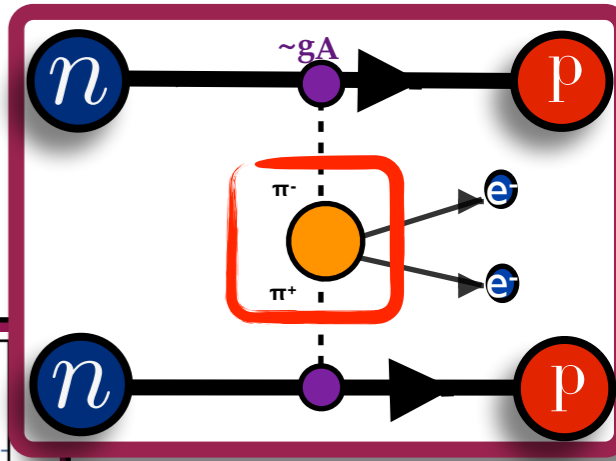
Matrix elements: Neutrinoless Double Beta Decay



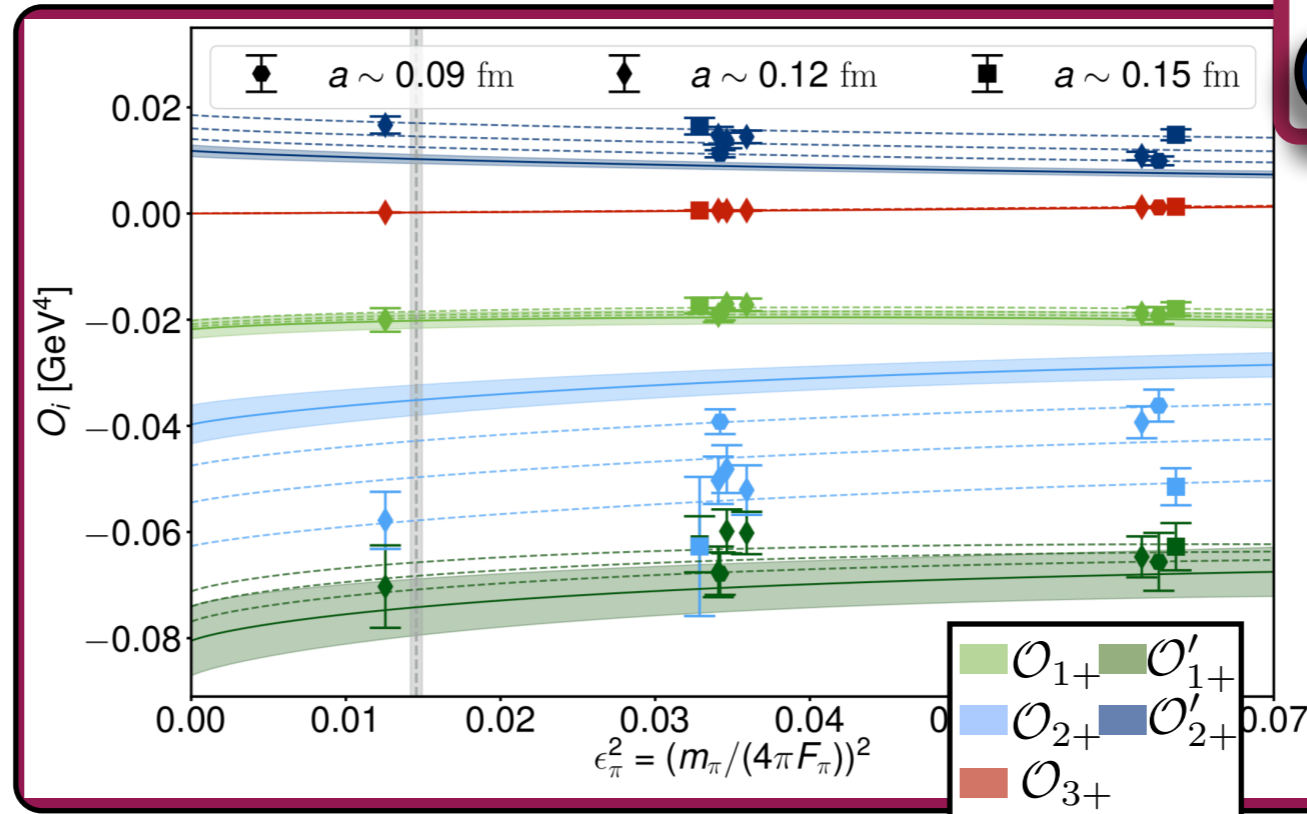
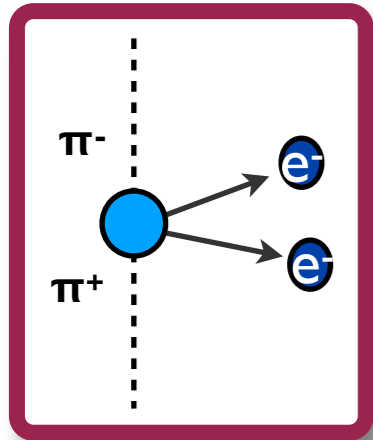
- Short-ranged contributions unconstrained from experiment



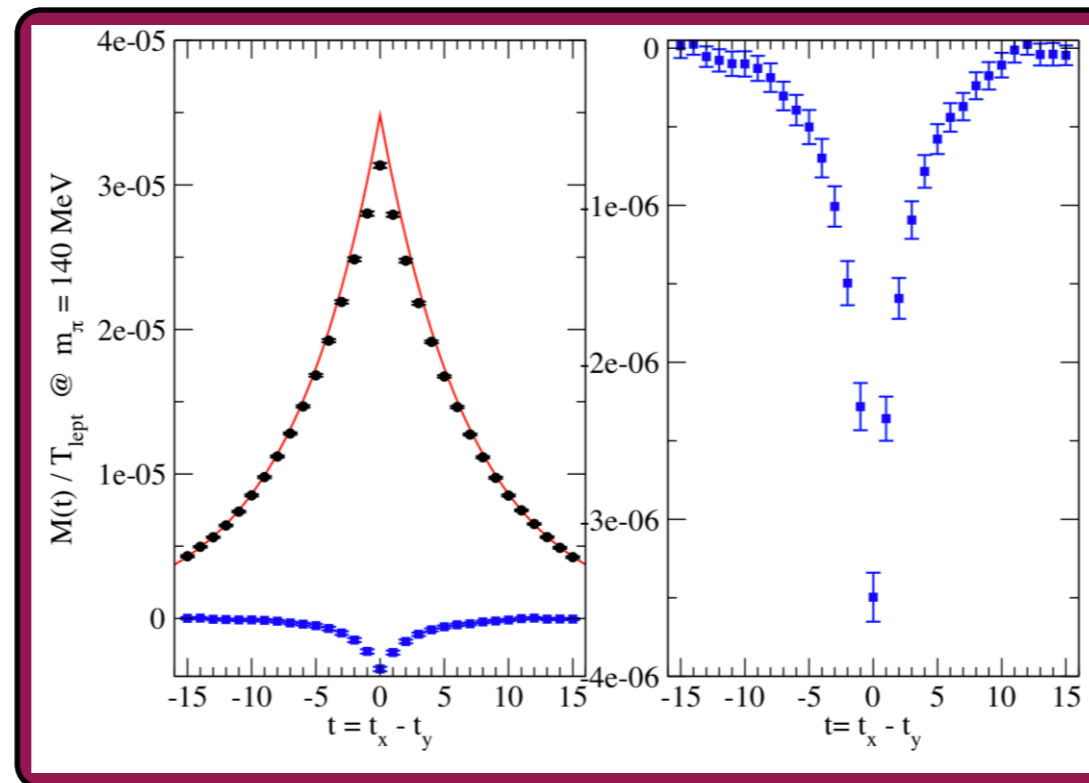
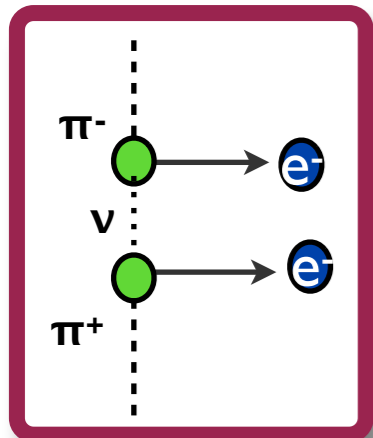
Matrix elements: Neutrinoless Double Beta Decay



- Short-ranged contributions unconstrained from experiment

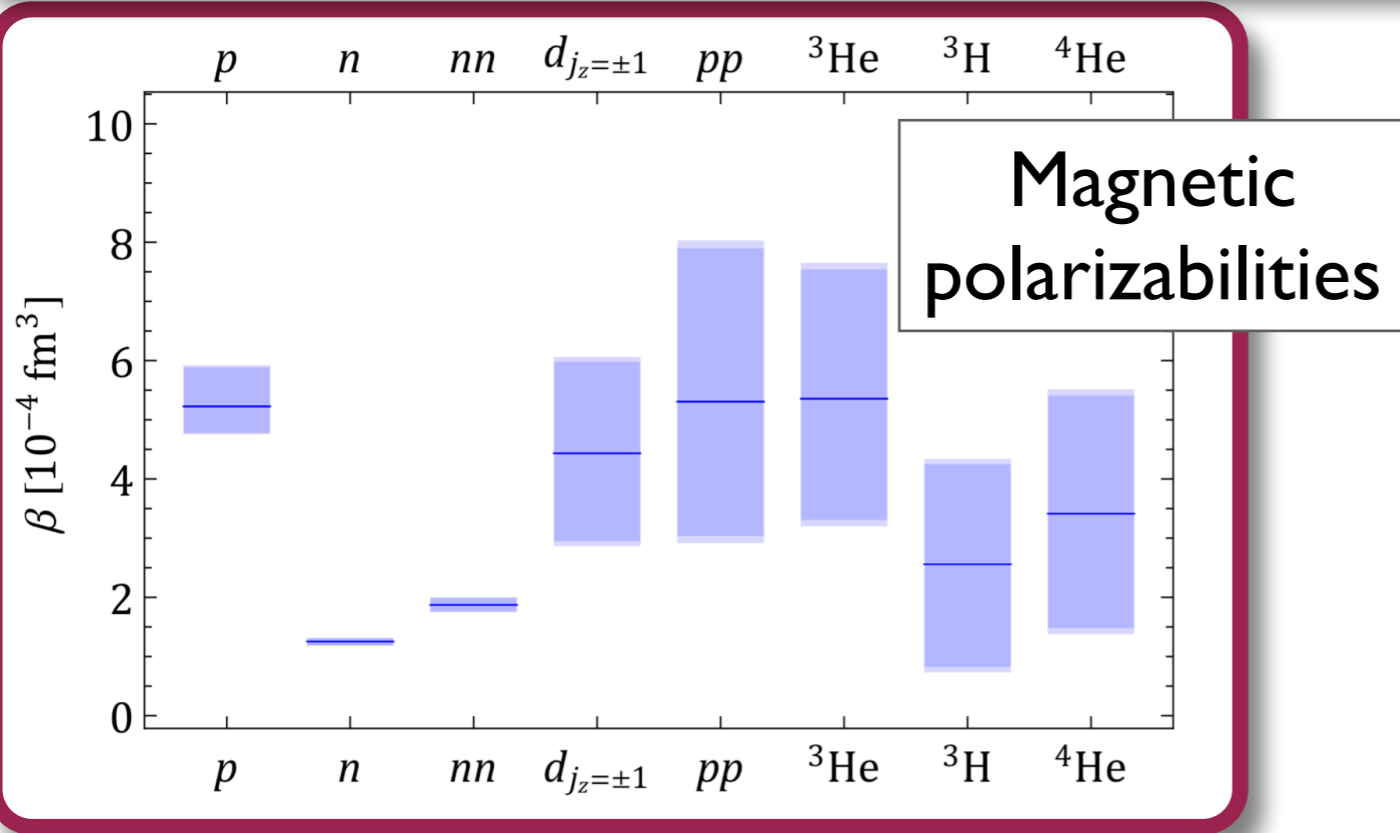


- Long-range contributions may require non-perturbative treatment even for two nucleons



Xu Feng, Lu-Chang Jin, Xin-Yu Tuo, Shi-Cheng Xia, Phys. Rev. Lett. 122, 022001 (2019)
see also D. Murphy, W. Detmold Lattice 2018

Matrix elements: Background Field



Scalar, Axial, Tensor charges

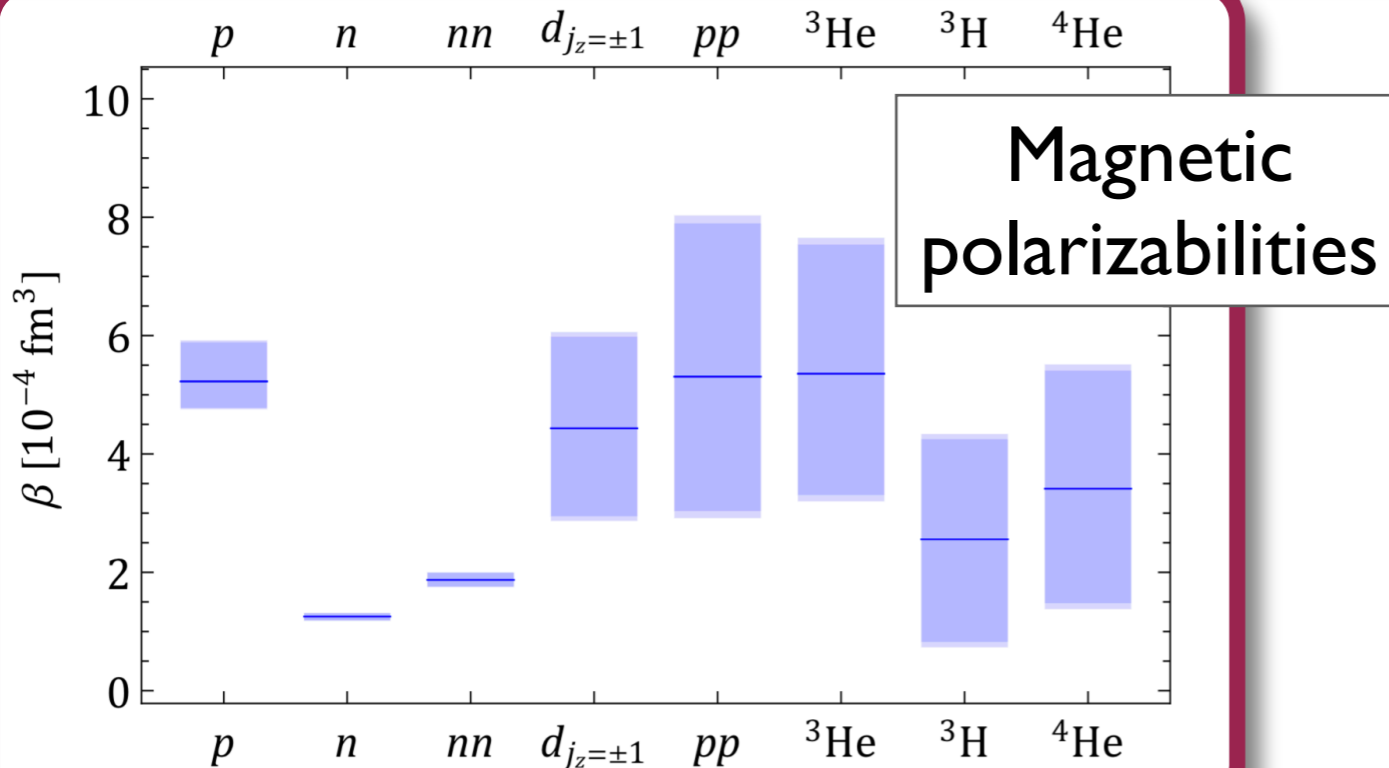
	p	d	pp	${}^3\text{He}$
$g_S^{(0)}$	3.65(7)	7.20(15)	7.22(15)	10.4(2)
$g_S^{(3)}$	0.78(2)	-	1.55(4)	0.77(2)
$g_S^{(8)}$	2.94(6)	5.84(12)	5.86(12)	8.55(18)
$g_S^{(s)}$	0.234(8)	0.45(2)	0.45(2)	0.63(3)
$g_A^{(0)}$	0.634(9)	1.26(2)	-	0.63(1)
$g_A^{(3)}$	1.14(2)	-	-	1.13(2)
$g_A^{(8)}$	0.633(9)	1.25(2)	-	0.625(9)
$g_A^{(s)}$	0.0002(6)	0.001(1)	-	0.003(2)
$g_T^{(0)}$	0.684(12)	1.36(2)	-	0.678(12)
$g_T^{(3)}$	1.12(2)	-	-	1.12(3)
$g_T^{(8)}$	0.684(12)	1.36(2)	-	0.676(12)
$g_T^{(s)}$	0.00007(13)	0.0002(2)	-	0.0004(4)

$$m_\pi \sim 800 \text{ MeV}$$



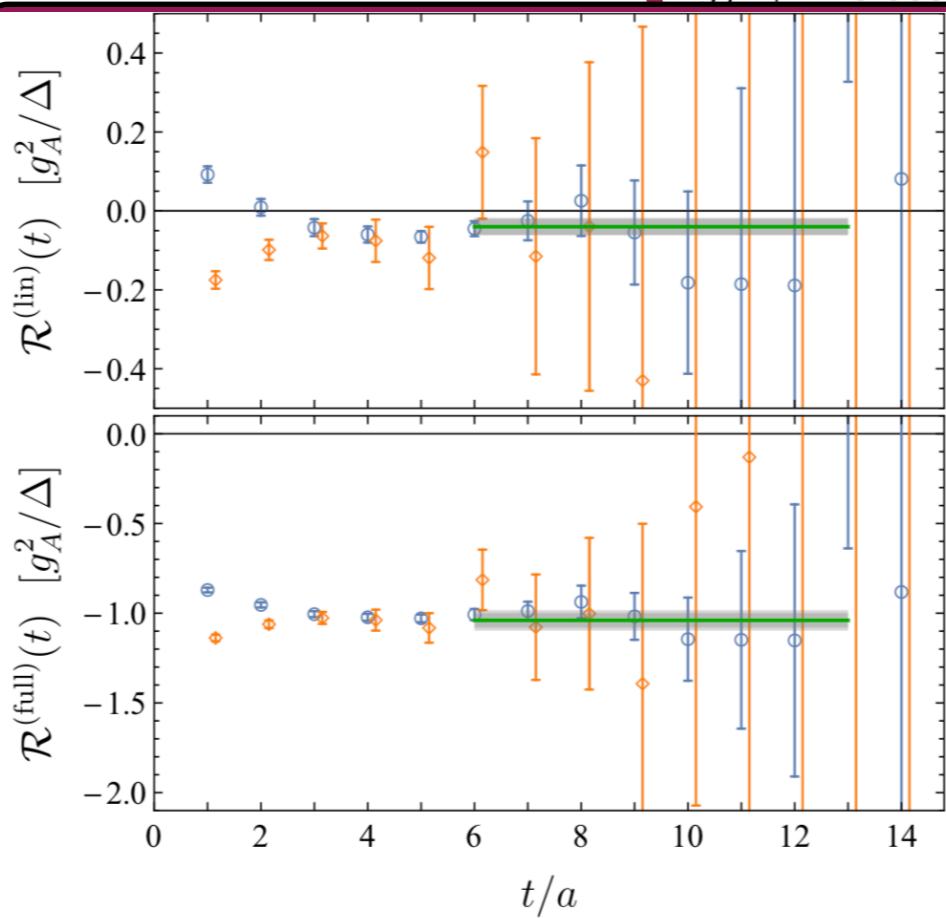
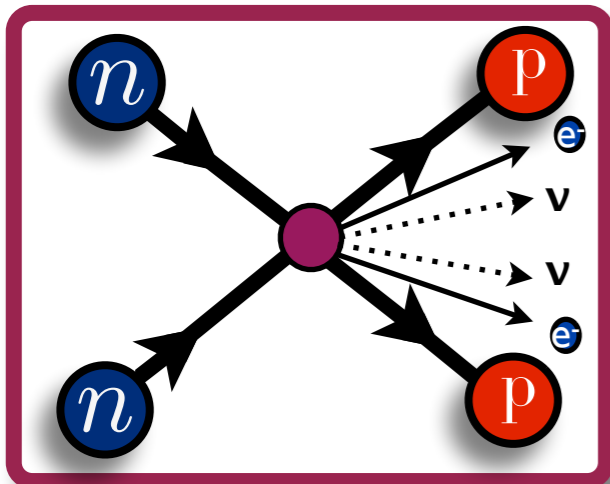
Matrix elements: Background Field

Scalar, Axial, Tensor charges



	p	d	pp	${}^3\text{He}$
$g_S^{(0)}$	3.65(7)	7.20(15)	7.22(15)	10.4(2)
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$g_T^{(8)}$	0.684(12)	1.36(2)	-	0.676(12)
$q_T^{(s)}$	0.00007(13)	0.0002(2)	-	0.0004(4)

Isotensor polarizability



$m_\pi \sim 800 \text{ MeV}$



Summary

- LQCD has entered a precision era for single nucleon observables
 - Systematics are important and must be carefully controlled
 - Convergence for single baryon HBChiPT (without Deltas) may be poor
- Multi-nucleon calculations
 - Results from several groups at heavier than physical pion mass
 - Physical pion mass will require excellent operators
 - Variational methods?
 - Francis et al. 1805.03966 (H-dibaryon)
 - Andersen, Bulava, Hörz, Morningstar, CalLat
 - Several relevant ME's now being calculated, but physical pion mass still currently only available for single hadron

- RIKEN / LBL: C.C. Chang
- RIKEN / BNL: E. Rinaldi
- NERSC: T. Kurth
- Liverpool: N. Garron
- UW / INT C. Monahan
- nVidia: M.A. Clark
- JLab: B. Joo
- WM / JLab: K. Orginos
- CCNY: B. Tiburzi
- LBL / UCB: A. Walker-Loud
- Glasgow: C. Bouchard
- LLNL: A. Gambhir, P. Vranas

- Jülich: E. Berkowitz
- WM / LBL: D. Brantley, H. Monge-Camacho

