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First extraction of A, B and D gravitational form factors from global DVCS analysis

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Paphos (Cyprus), 31 Oct 2019

ÉCOLE POLYTECHNIQUE

UNIVERSITÉ PARIS-SACLAY

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In classical continuum mechanics the energy-momentum tensor

$$T^{\mu\nu}(\boldsymbol{x},t) = \left(\begin{array}{c|ccc} T^{00} & T^{01} & T^{02} & T^{03} \\ \hline T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{array}\right),$$

where:
T^{00} - energy density,
T^{ii} - (not summed) represents pressure,
T^{ij} - $i \neq j$ represent shear stress,
T^{0i} - momentum flux,
T^{i0} - mass (energy) flux.
Classically $T^{\mu\nu} = T^{\nu\mu}$.

The EMT in QCD

In QCD $\hat{T}^{\mu\nu}_{\rm QCD}$ is an operator. However its expectation value on state is interpreted in the same way:

$$\mathcal{T}^{\mu\nu}(x) = \mathrm{Tr}[\hat{T}^{\mu\nu}_{\rm QCD}(x)\, \rho(\vec{0}, \vec{P})],$$

where the Wigner distribution ρ on the proton state with average momentum $\vec{P}$ and position $\vec{X}$ is

$$\rho_{R,P} = \int \frac{\mathrm{d}P^2}{2\pi} \int \frac{\mathrm{d}^4\Delta}{(2\pi)^4}\, 2\pi\, \delta(2P\cdot\Delta)\, 2\pi\, \delta(P^2 + \tfrac{\Delta^2}{4} - M^2) \left|P - \tfrac{\Delta}{2}\right\rangle\left\langle P + \tfrac{\Delta}{2}\right| e^{-i\Delta\cdot R}.$$

E. P. Wigner, Phys.Rev.40 (1932) 749

Then the expectation value of the EMT reads

$$\mathcal{T}^{\mu\nu}(x) = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{i\Delta\cdot x} \left\langle\!\left\langle T_a^{\mu\nu}(0) \right\rangle\!\right\rangle ,$$

where for unpolarized proton state we get

$$\left\langle\!\left\langle T_a^{\mu\nu}(0) \right\rangle\!\right\rangle = \frac{1}{2} \sum_{s=\uparrow,\downarrow} \frac{\langle p', s| \hat{T}^{\mu\nu}_{\mathrm{QCD}}(0) |p, s\rangle}{\sqrt{2p_0'\, 2p_0}} ,$$

where $\Delta = p' - p$, $P = \frac{1}{2}(p' + p)$ and $t = \Delta^2$.

M. V. Polyakov, Phys.Lett.B555 (2003) 57
C. Lorcé, L. Mantovani, B. Pasquini, Phys.Lett.B776 (2018) 38

Matrix elements of the general local asymmetric energy–momentum tensor for a spin-1/2 target read

$$\left\langle p', s' \right| \hat{T}^{\mu\nu}_{\mathrm{QCD}}(0) | p, s \rangle =$$

$$= \bar{u}(p', s') \left\{ \frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu} \bar{C}(t) \right.$$

$$+ \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M} \left[A(t) + B(t) + D(t) \right]$$

$$\left. + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M} \left[A(t) + B(t) - D(t) \right] \right\} u(p, s) \, .$$

X.-D. Ji, Phys.Rev.Lett.78 (1997) 610
B.L.G. Bakker, E. Leader, T.L. Trueman, Phys.Rev.D70 (2004) 114001

Mulipole model - definition

We adopt a simple multipole Ansatz for the GFFs

$$F_a(t) = \frac{F_a(0)}{\left(1 - t/\Lambda^2_{F_a}\right)^{n_F}} ,$$

which is supported by Goeke, *et.al.* calculations for $|t| < 1$ GeV^2.

We adopt a standard dipole Ansatz (i.e. $n_F = 2$) for A_a, $\bar{C}_a$ and D_a, but for B_a and C_a we choose a tripole Ansatz (i.e. $n_F = 3$) in order for the energy and pressure distributions to be realistic.

K. Goeke, *et.al.*, Phys.Rev.D75 (2007) 094021

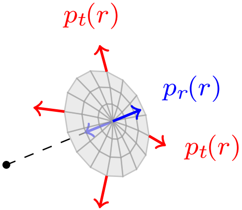
C. Lorcé, H. Moutarde, A.P. Trawiński, Eur.Phys.J. C79 (2019) 89

Parameters for the multipole model of the GFFs, in the $\overline{\text{MS}}$ scheme with renormalization scale $\mu = 2$ GeV.

F_a	n_F	$F_q(0)$	Λ_{F_q} [GeV]	$F_G(0)$	Λ_{F_G} [GeV]
A_a	2	0.55	0.91	0.45	0.91
B_a	3	−0.07	0.8	0.07	0.8
C_a	3	−0.32	0.8	−0.56	0.8
$\bar{C}_a$	2	−0.11	0.91	0.11	0.91
D_a	2	−0.33	1.74	–	–

C. Lorcé, H. Moutarde, A.P. Trawiński, Eur.Phys.J. C79 (2019) 89

This simple model allows us to identity an energy density as well as radius and tangential pressures density,



which are related to an isotropic pressure $p(r)$ and a pressure anisotropy $s(r)$ by

$$p(r) = \frac{p_r(r) + 2\,p_t(r)}{3}\,, \qquad s(r) = p_r(r) - p_t(r)\,.$$

Definitions

$$\varepsilon_a(r) = M \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4C_a(t)\right] \right\}$$

$$p_{r,a}(r) = M \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left\{ -\bar{C}_a(t) - \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{d}{dt}\left(t^{3/2}\, C_a(t)\right) \right\}$$

$$p_{t,a}(r) = M \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left\{ -\bar{C}_a(t) + \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{d}{dt}\left[t\frac{d}{dt}\left(t^{3/2}\, C_a(t)\right)\right] \right\}$$

$$p_a(r) = M \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left\{ -\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t) \right\}$$

$$s_a(r) = M \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left\{ -\frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{d^2}{dt^2}\left(t^{5/2}\, C_a(t)\right) \right\}$$

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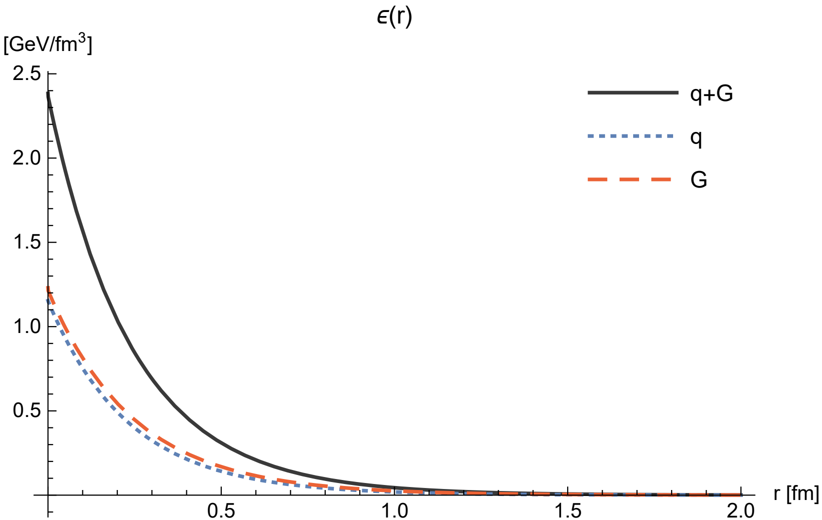
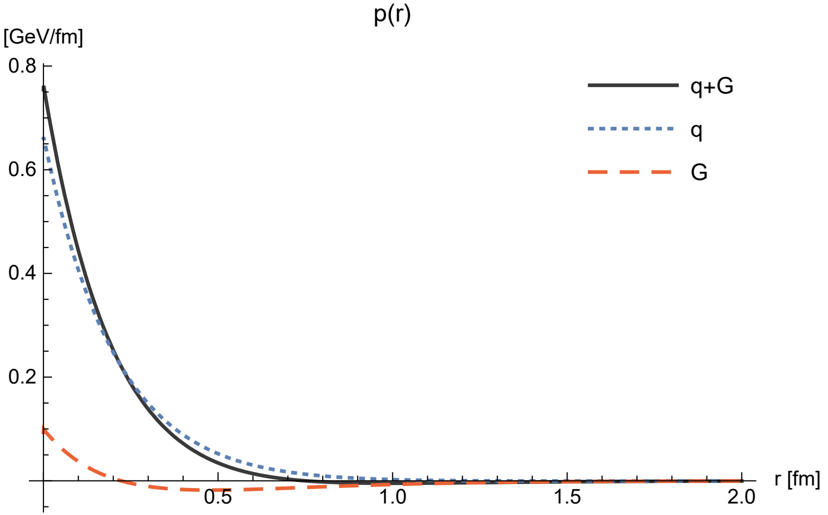
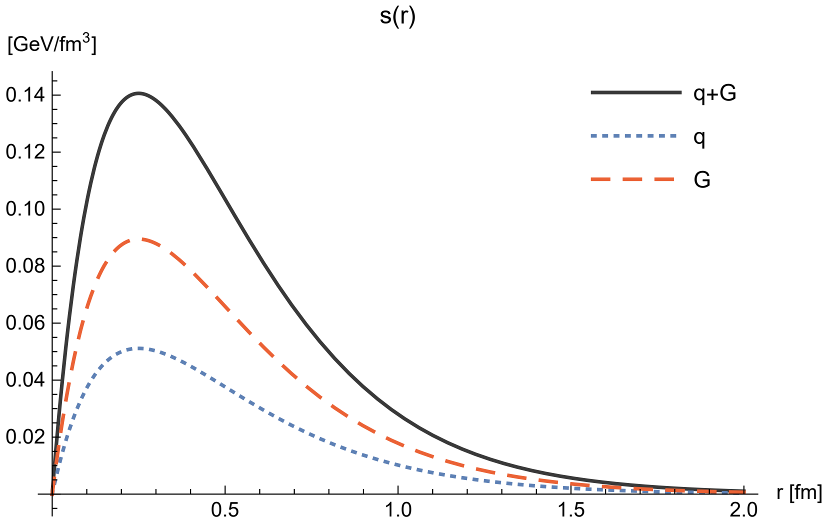
The energy density ϵ



The isotropic pressure p



The anisotropy pressure s



Subtraction constant (part 1)

The form factor $C(t)$ can be related to the so-called *subtraction constant* calculated form the GPDs (General Parton Distributions). What are GPDs will be explained in a moment, now we only point out that:

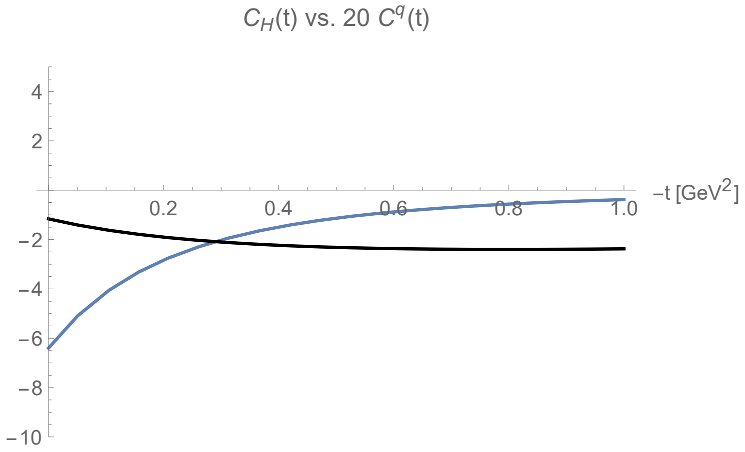
$$C_H(t) = 2\int_{-1}^{1} \frac{D(z,t)}{1-z} = 4\sum_{i=0}^{\infty} d_{2i+1}(t)\,,$$

where $D(z,t)$ is D-term, and d_i are its coefficients in the expansion in Gegenbauer polynomials. Especially interesting is $d_1(t)$, which is directly related to the GFF $C(t)$,

$$5\,C(t) = d_1(t) \approx \frac{1}{4} C_H(t)\,,$$

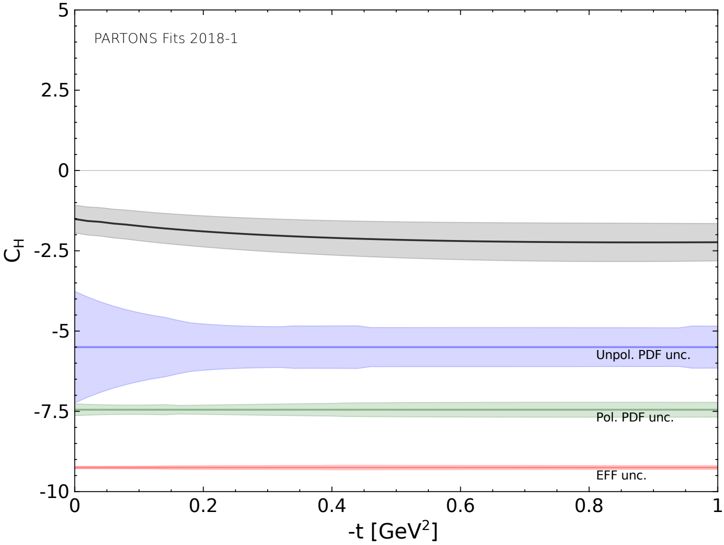
when one neglects higher orders.

$C_H(t)$ vs. $C^q(t)$ GFF



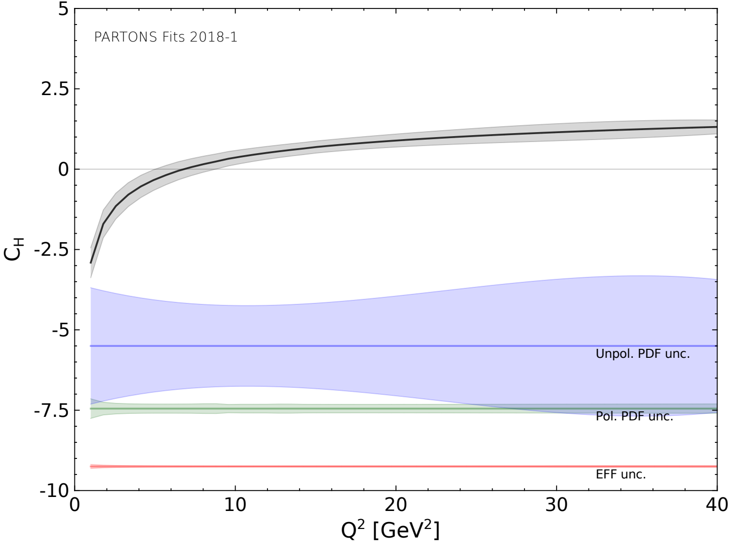
C. Lorcé, H. Moutarde, A.P. Trawiński, Eur.Phys.J. C79 (2019) 89
H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J.C78 (2018) 890
H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J. C79 (2019) 614

$C_H(t)$ – substraction constant

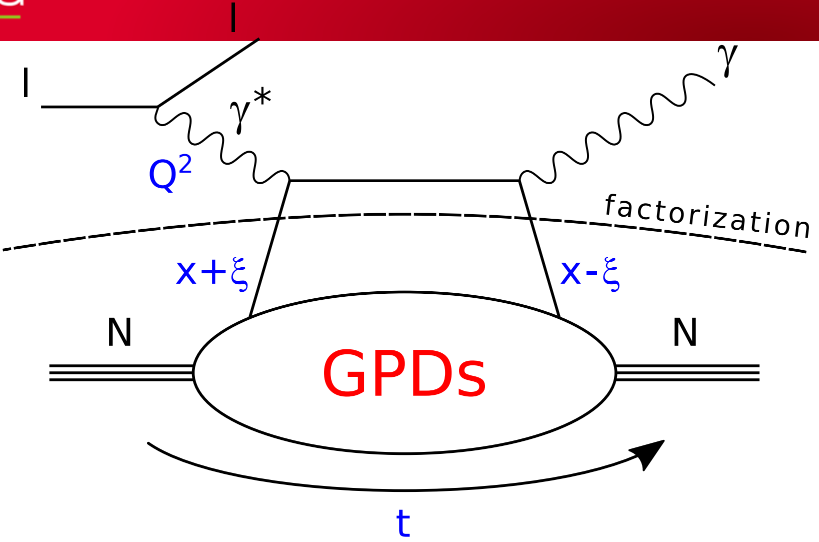


H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J.C78 (2018) 890

$C_H(t)$ – substraction constant



H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J.C78 (2018) 890



X.-D. Ji, Phys.Rev.D55 (1997) 7114

In the light cone gauge, quark GPDs for a spin-1/2 hadron are defined by the following matrix elements:

$$F^q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle\Big|_{\substack{z^+=0\\ z_\perp=0}} ,$$

$$\widetilde{F}^q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+\gamma_5 q\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle\Big|_{\substack{z^+=0\\ z_\perp=0}}$$

which can be written, with the help of the Dirac spinor bilinears, as

$$F^q(x,\xi,t) = \frac{1}{2P^+}\left[h^+\, H^q(x,\xi,t) + e^+\, E^q(x,\xi,t)\right] ,$$

$$\widetilde{F}^q(x,\xi,t) = \frac{1}{2P^+}\left[\tilde{h}^+\, \widetilde{H}^q(x,\xi,t) + \tilde{e}^+\, \widetilde{E}^q(x,\xi,t)\right] ,$$

where we identify two **unpolarized** GPDs H^q and E^q, and two **polarized** GPDs $\widetilde{H}^q$ and $\widetilde{E}^q$.

The relations with one-dimensional PDFs and EFFs are essential for the phenomenology of GPDs. In the forward limit when $\xi = t = 0$, *i.e.* both the hadron and the active quark are untouched, the unpolarized and polarized PDFs (q and Δq) read

$$H^q(x, 0, 0) = q(x) \,,$$
$$\widetilde{H}^q(x, 0, 0) = \Delta q(x) \,.$$

No similar relations exist for the GPDs E^q and $\widetilde{E}^q$.

The Dirac, Pauli, axial and pseudoscalar EFFs (F_1^q, F_2^q, g_A^q and g_P^q) can be obtained by integrating GPDs over the variable x.

The GPD model

In the Moutarde-Sznajder-Wagner model, the Ansatz that for the GPDs H^q and $\widetilde{H}^q$ at $\xi = 0$

$$\mathrm{GPD}^q(x, 0, t) = \mathrm{PDF}_G^q(x) \exp[f_G^q(x) t] ,$$

is used, what is commonly in phenomenological analyses of GPDs.

Here, $\mathrm{PDF}_G^q(x)$ is either a parameterization of the unpolarized PDF for the GPD H^q or a parameterization of the polarized PDF for the GPD $\widetilde{H}^q$. The profile function, $f_G^q(x)$, fixes the interplay between the x and t variables.

Similar Ansatz are for the GPDs E^q and $\widetilde{E}^q$.

H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J.C78 (2018) 890

Parametrization of the model

The parameterization of unpolarized and polarized PDFs read

$$\mathrm{PDF}_G(x, Q^2) = x^{-g(\delta_p,\delta_q,Q^2)}\,(1-x)^{\alpha}\sum_{i=0}^{4} g(p_i, q_i, Q^2)x^i\ ,$$

where $g(p, q, Q^2) = p + q\log\frac{Q^2}{Q_0^2}$, describes the evolution in the renormalization scale and

$$\delta_p, \delta_q, \alpha, p_i, q_i, \quad \text{where } i = 0, 1, \ldots, 4\ ,$$

are found in a fit to NNPDF.

The profile function is given by:

$$f(x) = A\log(1/x) + B(1-x)^2 + C(1-x)x\ ,$$

where A, B and C are found in a fit to experimental data.

H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J.C78 (2018) 890

Skewness and border functions

For GPDs at $\xi \neq 0$ it is used the concept of skewness function:

$$\mathrm{GPD}^q(x,\xi,t) = \mathrm{GPD}^q(x,0,t)\, g^q_G(x,\xi,t)\,,$$

which it is assumed to take following form for $\xi = x$:

$$g(x,x,t) = \frac{a}{(1-x^2)^2}\left[1 + t(1-x)(b + c\log(1+x))\right]$$

where a, b and c are "free parameters" to be constrained by experimental data, but ...

H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J.C78 (2018) 890

At the leading order, the real part of $\int_{-1}^{1} \frac{e_q^2}{x - \xi + i\epsilon} G^q(x, \xi, t)$ (called Compton form factor) can be evaluated in two different ways, using the **standard** and the **fixed-t dispersion relation**. The second one introduces the subtraction constant, which by comparison with first one can be evaluated and reads:

$$C_G^q(t) = \int_0^1 \left(G^{q(+)}(x, \xi, t) - G^{q(+)}(x, x, t) \right) \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) dx \, ,$$

where $G^{q(+)}(x, \xi, t) = G^q(x, \xi, t) \mp G^q(-x, \xi, t)$.

Unfortunately, naively setting $\xi = 0$ in the above formula results in a divergent integral.

O.V. Teryaev, Contribution to 11th EDS05 Conference
I.V. Anikin, O.V. Teryaev, Phys.Rev. D76 (2007) 056007

The analytic regularization technique

The following moments:

$$C^q_{G,j}(t) = 2\int_0^1 \left(G^{q(+)}(x,x,t) - G^{q(+)}(x,0,t)\right) x^j dx\,,$$

are well defined for odd positive j and can be analytically continued to $j = -1$ using *the analytic regularization technique*, given by the following prescription:

$$\int_{(0)}^1 \frac{f(x)}{x^{a+1}}\mathrm{d}x = \int_0^1 \frac{f(x) - f(0) - xf'(0) - \ldots}{x^{a+1}}\mathrm{d}x - \frac{f(0)}{a} - \frac{f'(0)}{a-1} + \ldots$$

K. Kumericki, D. Mueller, K. Passek-Kumericki, Nucl.Phys. B794 (2008) 244
K. Kumericki, D. Mueller, K. Passek-Kumericki, Eur.Phys.J. C58 (2008) 193
M.V. Polyakov, K.M. Semenov-Tian-Shansky, Eur.Phys.J. A40 (2009) 181
A.V. Radyushkin, Phys.Rev. D83 (2011) 076006

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PARTONS

PARtonic Tomography Of Nucleon Software

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Main Page

What is PARTONS?

PARTONS is a C++ software framework dedicated to the phenomenology of Generalized Parton Distributions (GPDs). GPDs provide a comprehensive description of the partonic structure of the nucleon and contain a wealth of new information. In particular, GPDs provide a description of the nucleon as an extended object, referred to as 3-dimensional nucleon tomography, and give an access to the orbital angular momentum of quarks.

PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments.

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Get PARTONS

Here you can learn how to get your own version of PARTONS. We offer two ways:

- You can use our provided virtual machine with an out-of-the-box PARTONS runtime and development environment. This is the easiest way to start your experience with PARTONS.
- You can also build PARTONS by your own on GNU/Linux. This is useful if you want to have PARTONS on your computer without using the virtualization technology or if you want to use PARTONS on computing farms.

Relation of EMT form-factors with GPDs

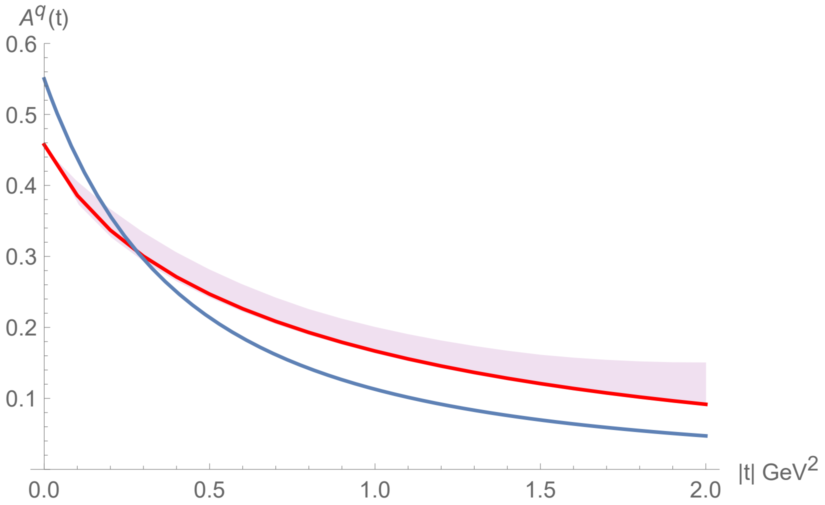
The study of the EMT became especially important after obtaining by Ji a relation between the EMT and GPDs

$$\int_{-1}^{1} dx\, x\, H(x,\xi,t) = A(t) + 4\xi^2 C(t)\,,$$
$$\int_{-1}^{1} dx\, x\, E(x,\xi,t) = B(t) - 4\xi^2 C(t)\,,$$
$$\int_{-1}^{1} dx\, \tilde{H}(x,0,t) = -D(t)\,.$$

Beside this, $\bar{C}(t)$ can be related to the scalar form factor.

X.-D. Ji, Phys.Rev.Lett.78 (1997) 610

GFF $A^q(t)$



C. Lorcé, H. Moutarde, A.P. Trawiński (in preparation)

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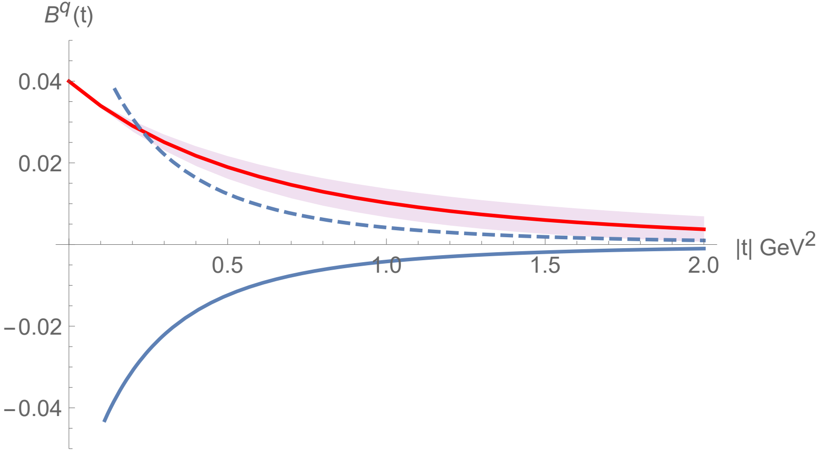
GFF $B^q(t)$



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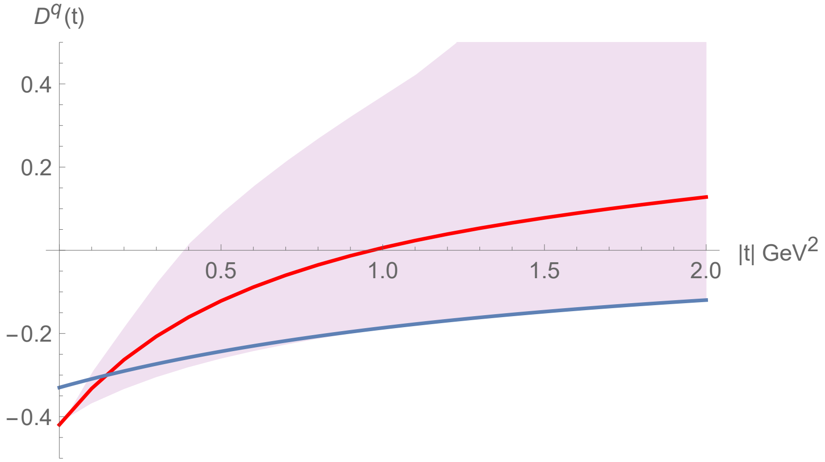
GFF $D^q(t)$ – axial form factor



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C. Lorcé, H. Moutarde, A.P. Trawiński, Eur.Phys.J. C79 (2019) 89

Summary

The study of the EMT is important because:

- $T^{\mu\nu}$ is a fundamental quantity, which allows to access for example pressures and a spin decomposition.
- DVCS gives a way to experimentally measure $T^{\mu\nu}$, *e.g.* JLab.
- Its form factors have a clear interpretation as spatial densities ($\vec{\Delta}$ is related to $\vec{r}$).
- GFFs and GPDs constrain each other.

E. Leader, C. Lorcé, Phys.Rept.541 (2014) 163

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First extraction of A, B and D gravitational form factors from global DVCS analysis

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