Spin content of the Nucleon

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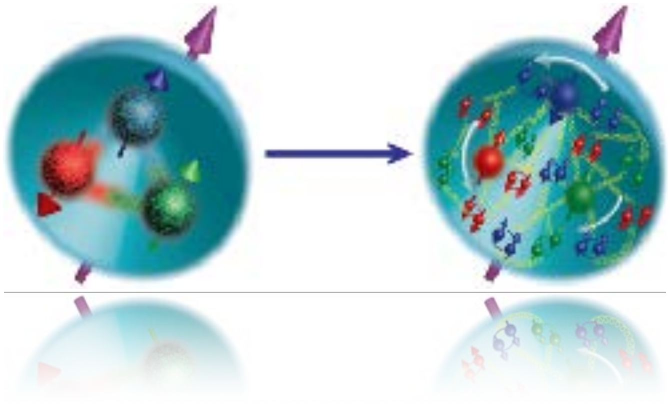
Nucleon spin

The nucleon spin decomposition has been a long standing puzzle, ever since first results from the Electron Muon Collaboration (1987) revealed a surprisingly small contribution from the quark intrinsic spins

- The Ji spin-sum rule:

$$J_N = \sum_{q=u,d,s,c\cdots} \left(\frac{1}{2}\Delta\Sigma_q + L_q\right) + J_g$$



- J_N: Nucleon spin
- $\Delta\Sigma_q$: Contribution from quark intrinsic spin
- L_q: Contribution from quark orbital motion
- J_g: Contribution from gluons (no further decomposition)

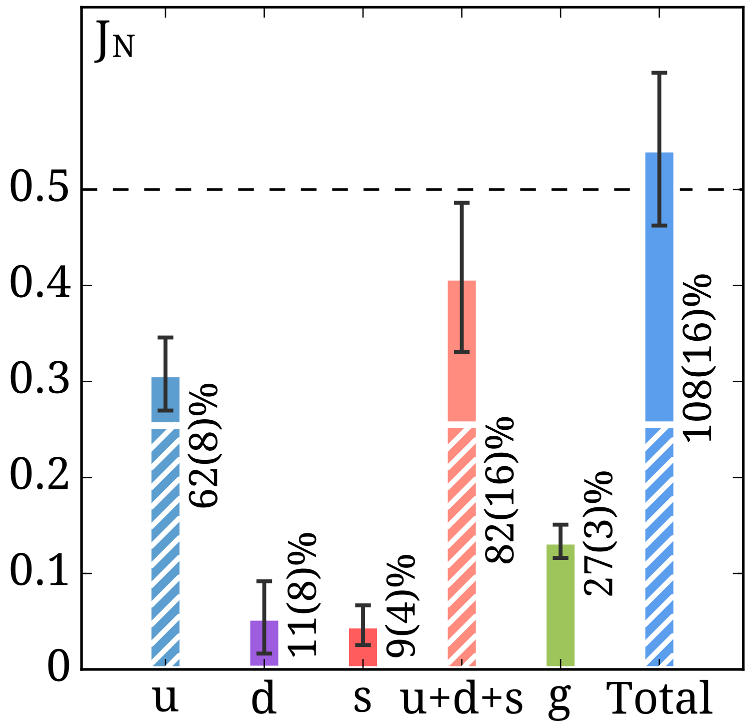
Lattice QCD: ***ab initio*** calculation of quark-intrinsic, gluon, and total spin of nucleon via nucleon matrix elements of local quark and gluon operators

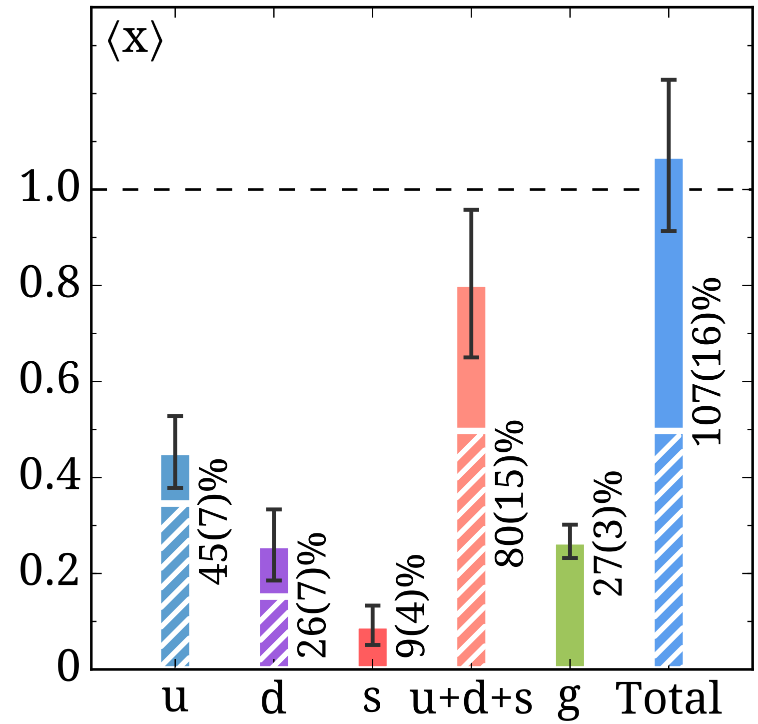
Thanks to two major breakthroughs:

- Reliable calculation of ***disconnected diagrams***
- Simulations with quark mass set to its ***physical value***

Nucleon spin

Parton spin and momentum contributions to nucleon spin





$$J^N_{u+d+s+g} = 0.541(62)(49) \qquad \langle x \rangle_{u+d+s+g} = 1.07(12)(10)$$

C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou, A. Vaquero, and C. Wiese
Phys. Rev. Lett. 119, (2017) 142002

Outline

- Methods
  - Lattice methods for nucleon structure
  - Challenges: physical point simulations, disconnected diagrams
- Nucleon spin from lattice QCD
  - Intrinsic quark spin contributions
  - Momentum fraction and total spin of nucleon
  - Gluon contribution
  - Quark orbital angular momentum contributions
- Concluding remarks

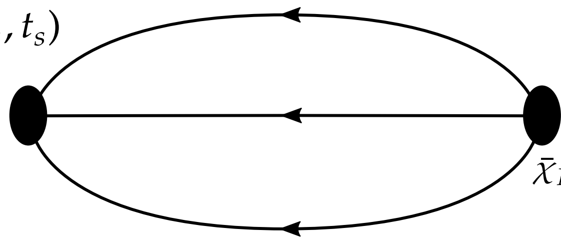
Nucleon structure on the lattice

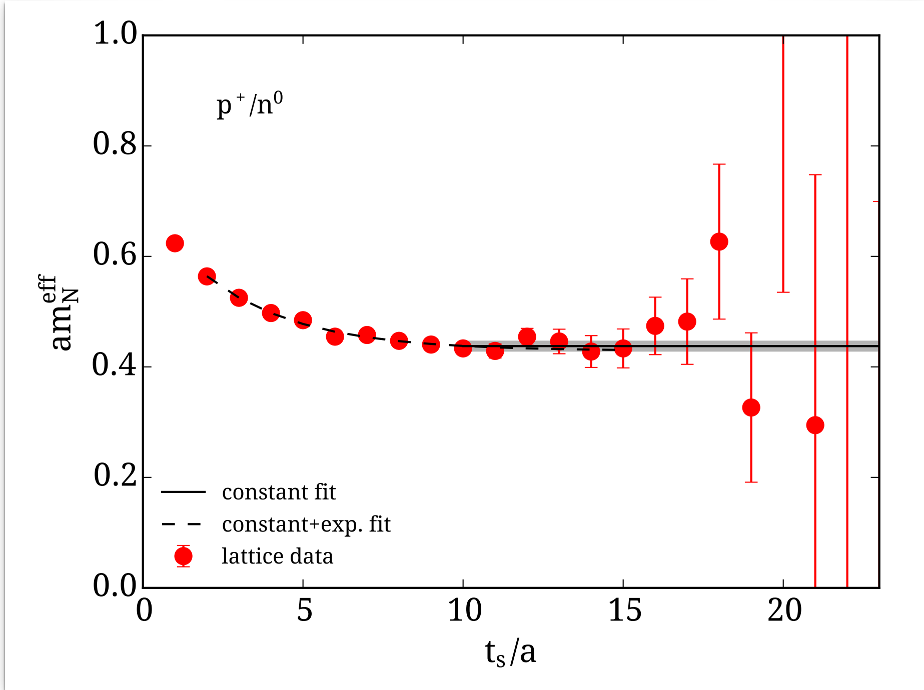
Two-point correlation functions

- Statistical error: $1/\sqrt{N}$, with MC samples
- Correlation functions: exponentially decay with time-separation

Systematic uncertainties

- Extrapolations: a, L, m_π
- Contamination from higher energy states





$$\sum_{\vec{x}_s} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(x_s) | \chi_N^{\alpha}(0) \rangle = c_0 e^{-E_0 t_s} + c_1 e^{-E_1 t_s} + \ldots$$

Nucleon structure on the lattice

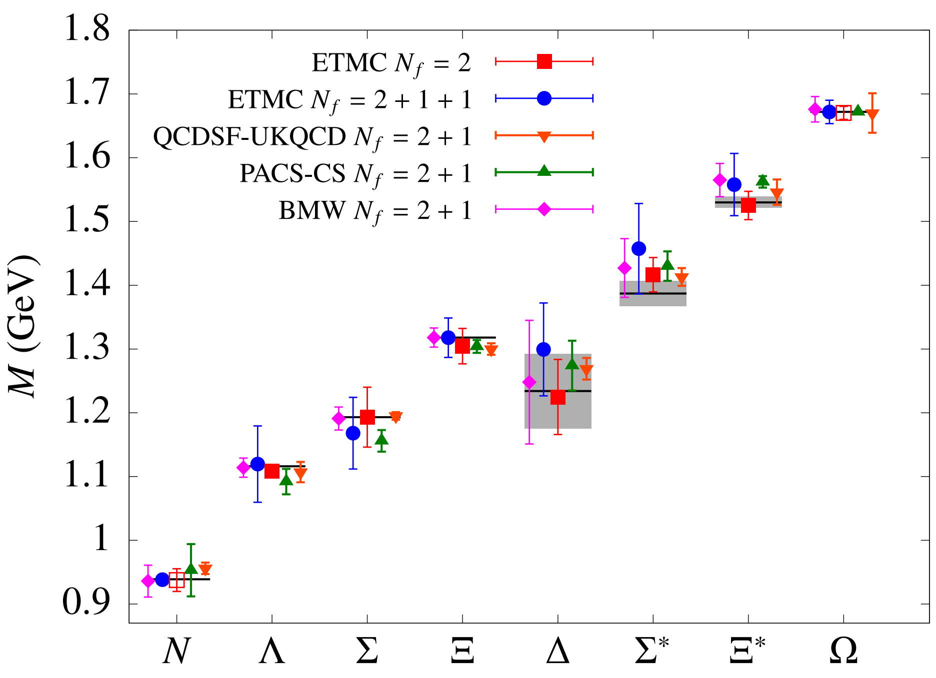
Reproduction of light baryon masses

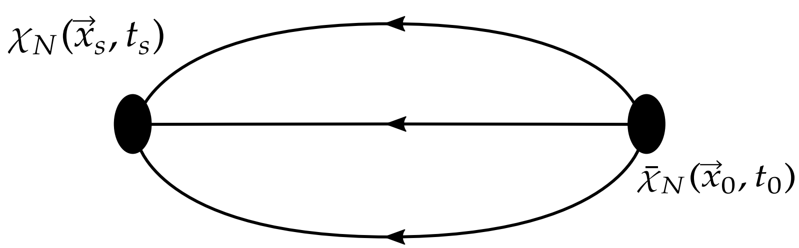
- Agreement between lattice discretisations
- Reproduction of experiment

Prediction of yet to be observed baryons

- Confidence through agreement between lattice schemes

Phys. Rev. D96 (2017) no.3, 034511 [arXiv:1704.02647]





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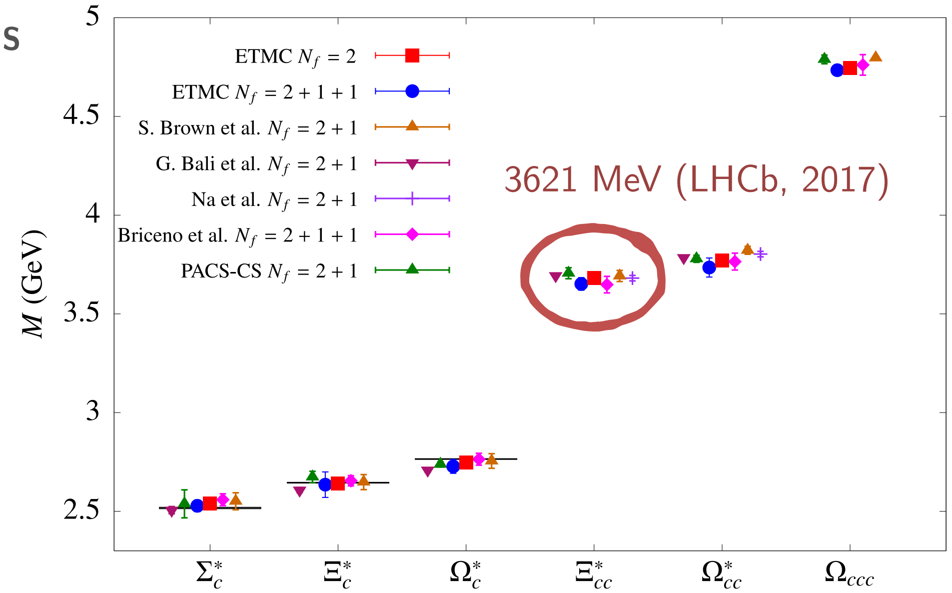

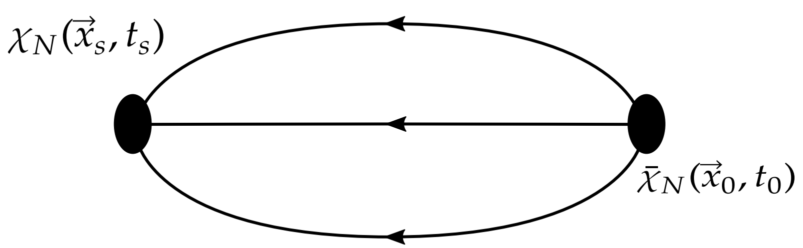

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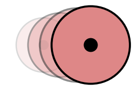
For more hadron spectrum see S. Bacchio, Wednesday 14:30 session

Nucleon structure on the lattice

– Lattice: moments are readily accessible

Unpolarised

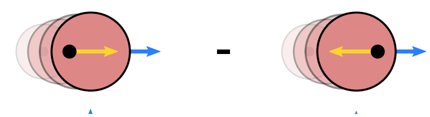
$$\mathcal{O}_{V}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{u-d} = g_V,\quad \langle x\rangle_{u-d},\quad \ldots$$

Helicity

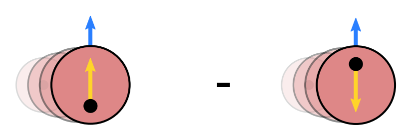
$$\mathcal{O}_{A}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{\Delta u-\Delta d} = g_A,\quad \langle x\rangle_{\Delta u-\Delta d},\quad \ldots$$

Transverse

$$\mathcal{O}_{T}^{\nu\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



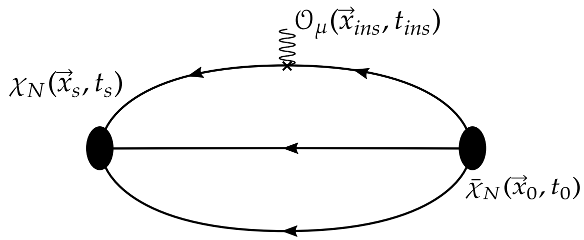
$$\langle 1\rangle_{\delta u-\delta d} = g_T,\quad \langle x\rangle_{\delta u-\delta d},\quad \ldots$$

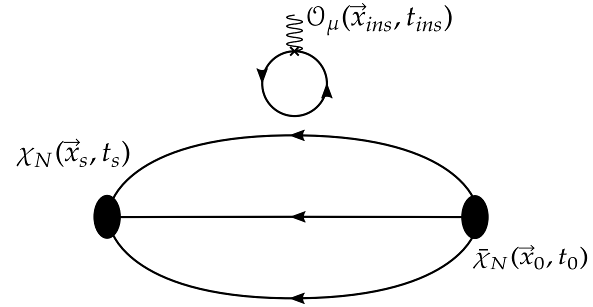
For details on charges see J. Finkenrath, next talk

Lattice evaluation of matrix elements

Three-point function:

$$G_\mu(\Gamma;\vec{q};t_s,t_{\rm ins}) = \sum_{\vec{x}_s \vec{x}_{\rm ins}} e^{-i\vec{q}.\vec{x}_{\rm ins}} \Gamma^{\alpha\beta} \langle \bar{\chi}^\beta_N(\vec{x}_s;t_s) | \mathcal{O}^\mu(\vec{x}_{\rm ins};t_{\rm ins}) | \chi^\alpha_N(\vec{0};0)\rangle$$





Analyses for identifying excited state contributions

– "Plateau":

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[t_{\rm ins}-t_0\to\infty]{t_s-t_{\rm ins}\to\infty} \mathcal{M}[1+\mathcal{O}(e^{-\Delta(t_{\rm ins}-t_0)}, e^{-\Delta'(t_s-t_{\rm ins})})]$$

fit to constant w.r.t $t_{\rm ins}$ for multiple values of t_s

– Sum over $t_{\rm ins}$ "Summation"

$$\sum_{t_{\rm ins}} R(t_s, t_{\rm ins}, t_0) \xrightarrow{t_s-t_0\to\infty} \text{Const.} + \mathcal{M}(t_s-t_0) + \mathcal{O}(t_s e^{-\Delta t_s})$$

fit to linear form, matrix element is the slope

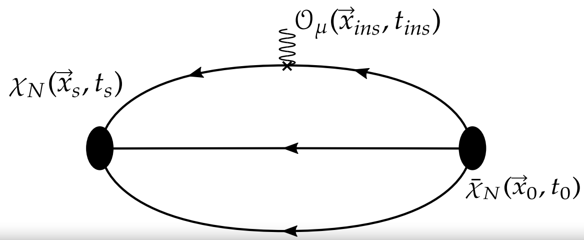
– Fit, including first excited states ("Two-state fit")

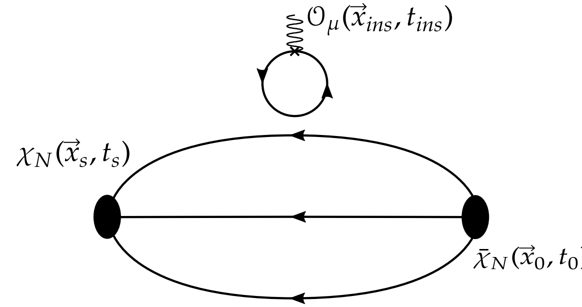
Agreement between methods signals excited state suppression

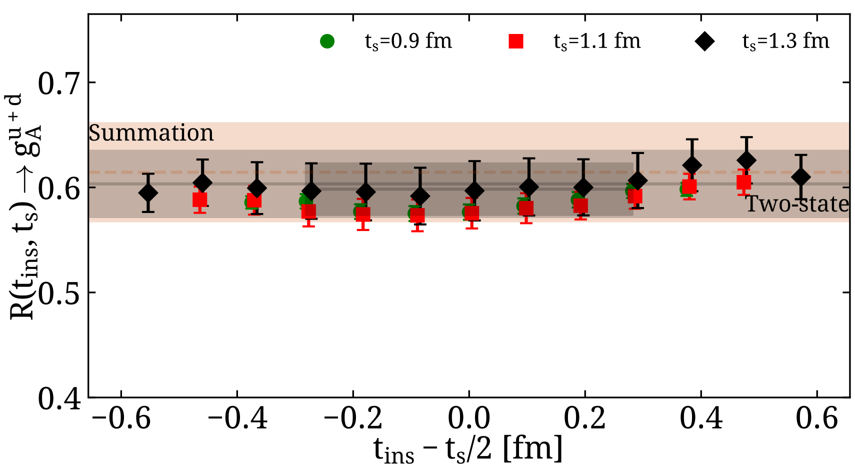
Lattice evaluation of matrix elements

Three-point function:

$$G_\mu(\Gamma; \vec{q}; t_s, t_{\text{ins}}) = \sum_{\vec{x}_s \vec{x}_{\text{ins}}} e^{-i\vec{q}.\vec{x}_{\text{ins}}} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^\beta(\vec{x}_s; t_s) | \mathcal{O}^\mu(\vec{x}_{\text{ins}}; t_{\text{ins}}) | \chi_N^\alpha(\vec{0}; 0) \rangle$$







Connected, ~9,100 statistics

tions

$+ \mathcal{O}(e^{-\Delta(t_{\text{ins}}-t_0)}, e^{-\Delta'(t_s - t_{\text{ins}})})]$

of t_s

st. $+ \mathcal{M}(t_s - t_0) + \mathcal{O}(t_s e^{-\Delta t_s})$

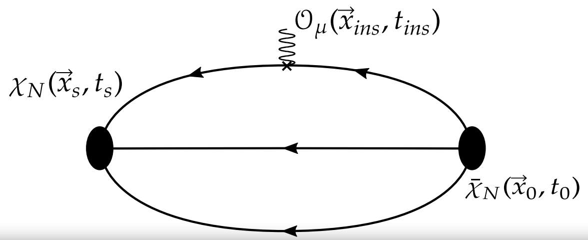
ope

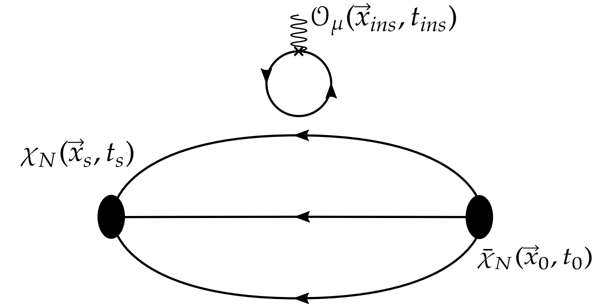
Agreement between methods signals excited state suppression

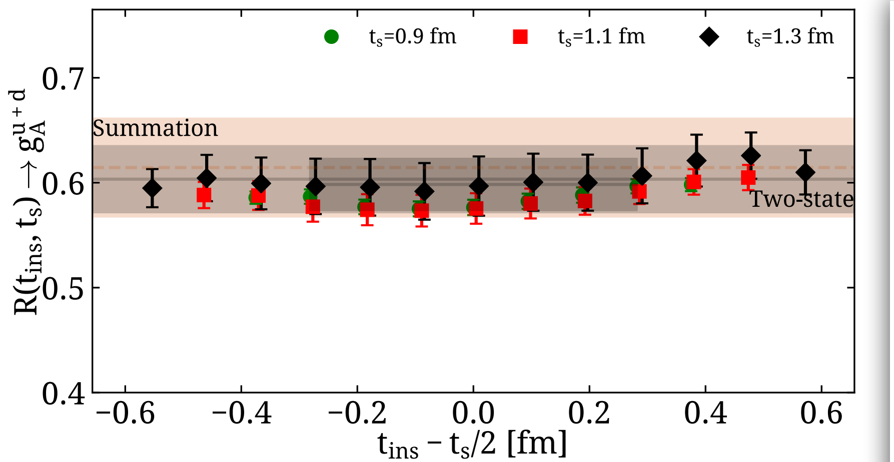
Lattice evaluation of matrix elements

Three-point function:

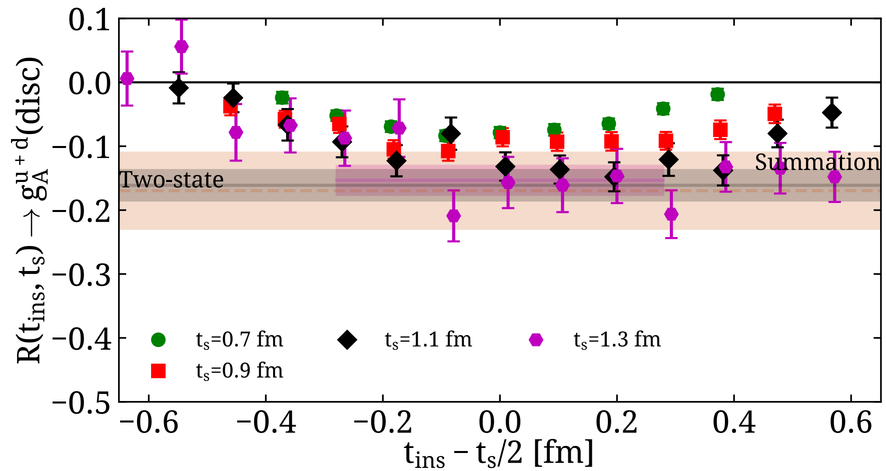
$$G_\mu(\Gamma;\vec{q};t_s,t_{\rm ins}) = \sum_{\vec{x}_s\,\vec{x}_{\rm ins}} e^{-i\vec{q}.\vec{x}_{\rm ins}}\,\Gamma^{\alpha\beta}\langle \bar{\chi}^{\beta}_N(\vec{x}_s;t_s)|\mathcal{O}^{\mu}(\vec{x}_{\rm ins};t_{\rm ins})|\chi^{\alpha}_N(\vec{0};0)\rangle$$







Connected, ~9,100 statistics



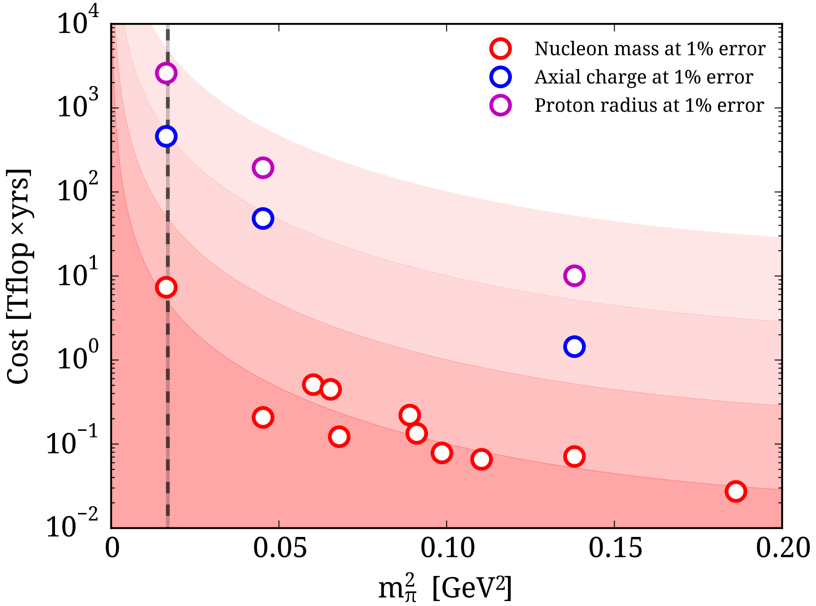
Disconnected, ~200,000 statistics

Agreement between methods signals excited state suppression

Going to the physical point

E.g. multi-grid yielding 100x improvement of computer time at physical point

- Multi-grid solvers
- Improved stochastic methods for taming noise in disconnected diagrams



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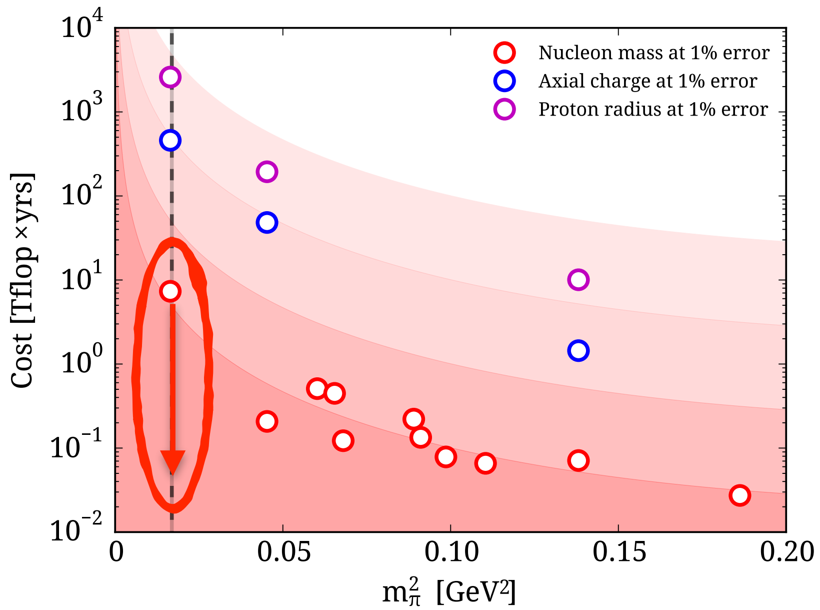

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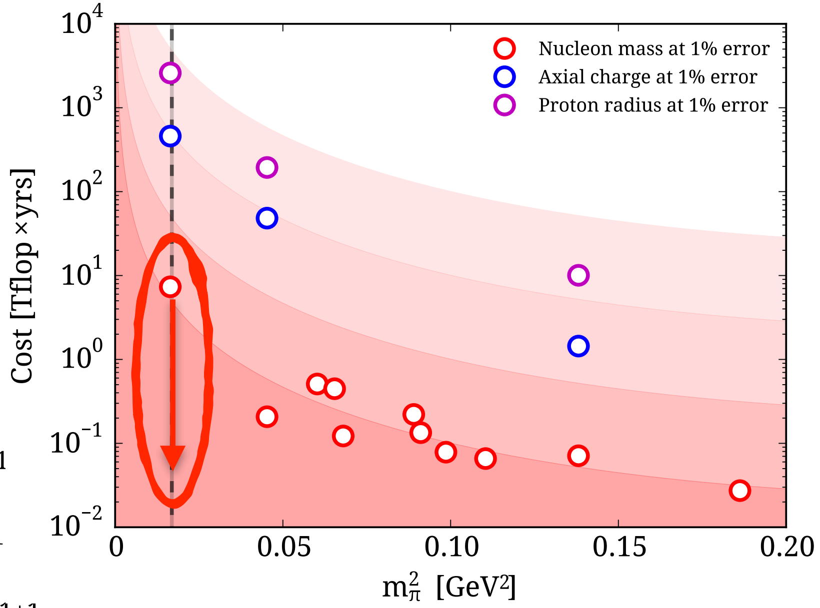
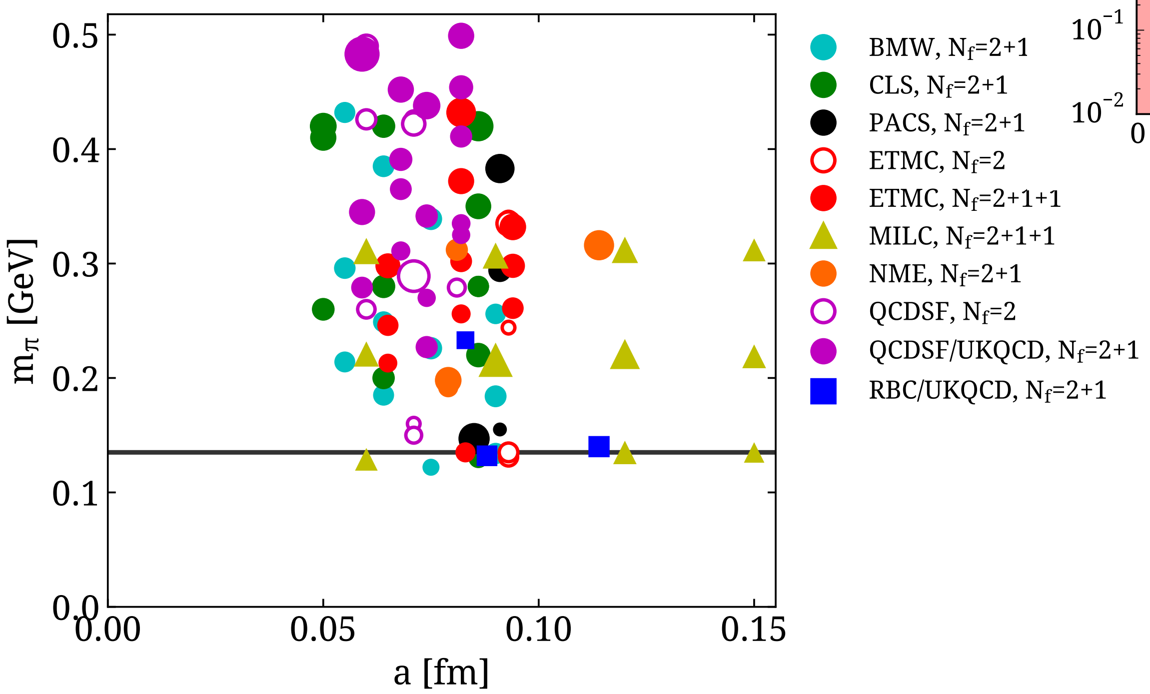




Select lattice simulation points used for hadron structure

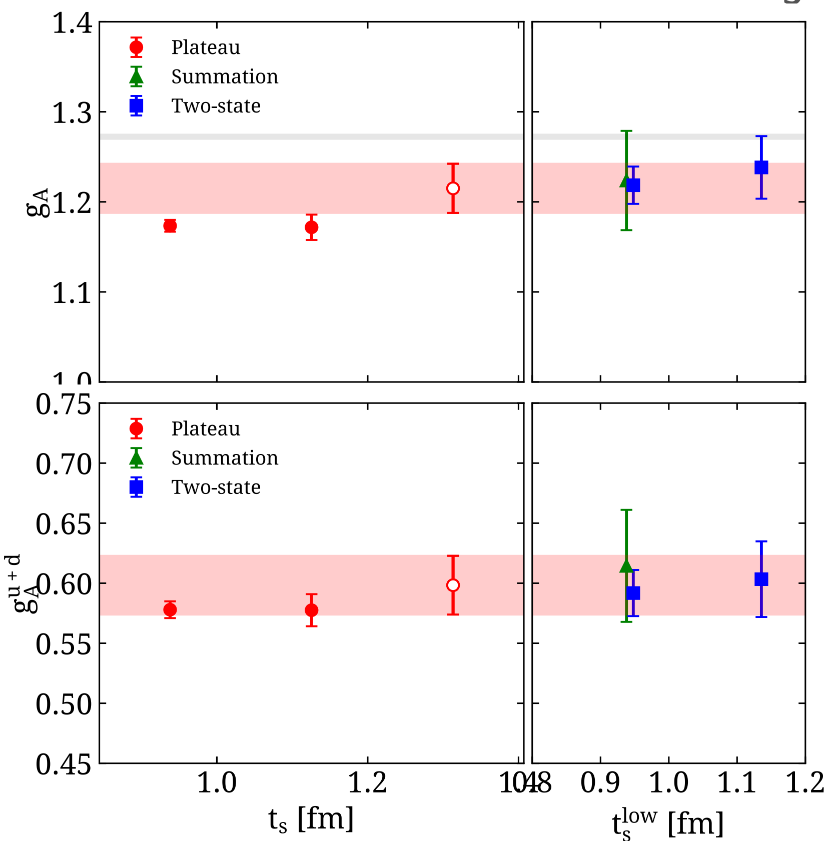
- Multiple collaborations simulating at physical pion mass

Quark intrinsic spin contributions

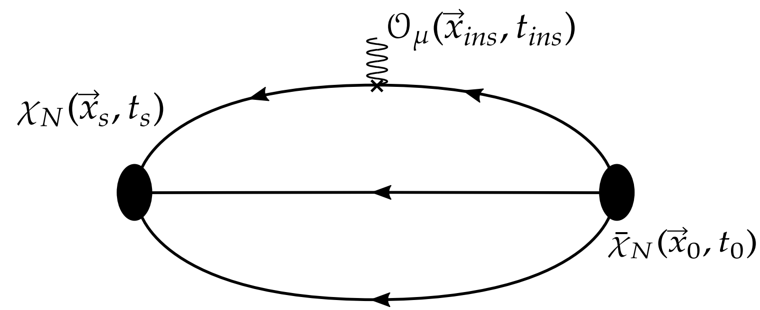
Axial matrix element of nucleon: $\langle N(\vec{p})|A^{\alpha}_{\mu}|N(\vec{p})\rangle,\ A^{\alpha}_{\mu} = \bar{\psi}\frac{\tau^{\alpha}}{2}\gamma_5\gamma_{\mu}\psi$

$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{q=u,d,s,\ldots} g_A^q$$

- Isovector contribution, $\tau^{\alpha}=\tau^3$, no disconnected contribution
- Isoscalar contribution, $\tau^{\alpha}=1$, disconnected contributions
- Strange-quark contribution, completely disconnected



- N_f=2 Twisted mass fermions, 48^3x96
- m_{π}=131 MeV, a=0.0938 fm
- Connected: statistics of 9,120
- Three sinks-source separations



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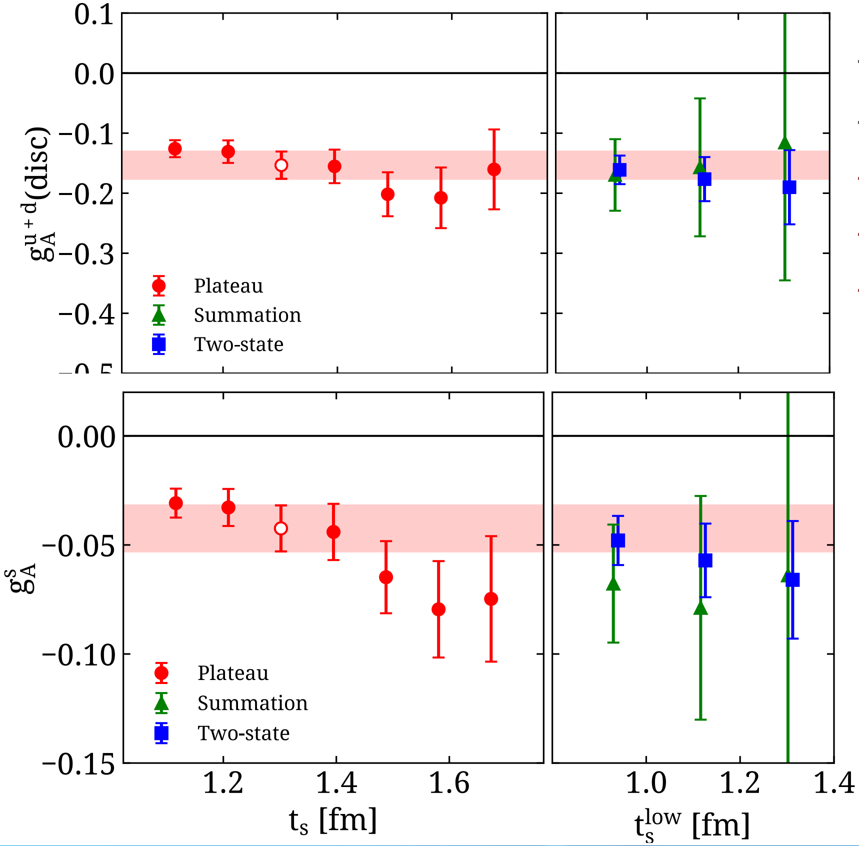
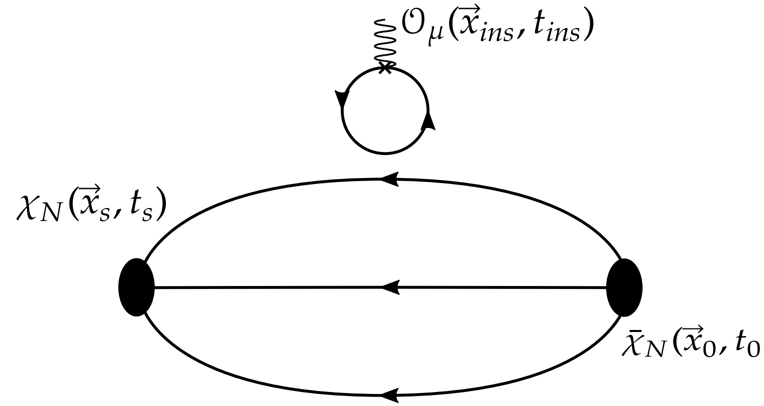


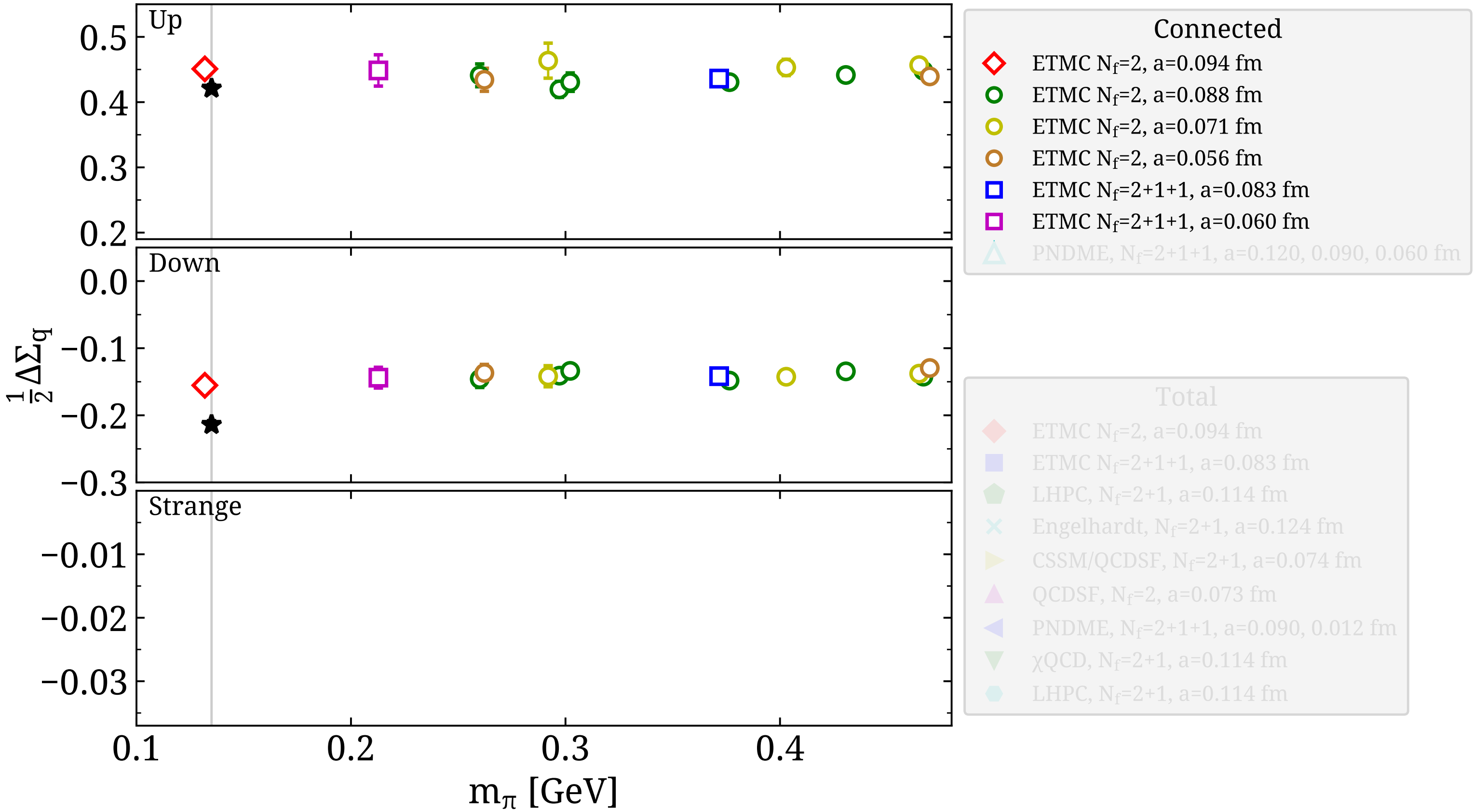
- N_f=2 Twisted mass fermions, 48^3x96
- m_{π}=131 MeV, a=0.0938 fm
- Disconnected: statistics of 2,100,000
- All time-slices (generalized one-end trick)
- Stochastic vectors
  - Light: 2250
  - Strange: TSM with (HP,LP) = (63,1024)



Quark intrinsic spin contributions

Quark intrinsic spin contributions to nucleon spin

- Mild cut-off effects
- Strange and down-quark contributions negative
- Overall agreement between formulations, and with experimental determinations



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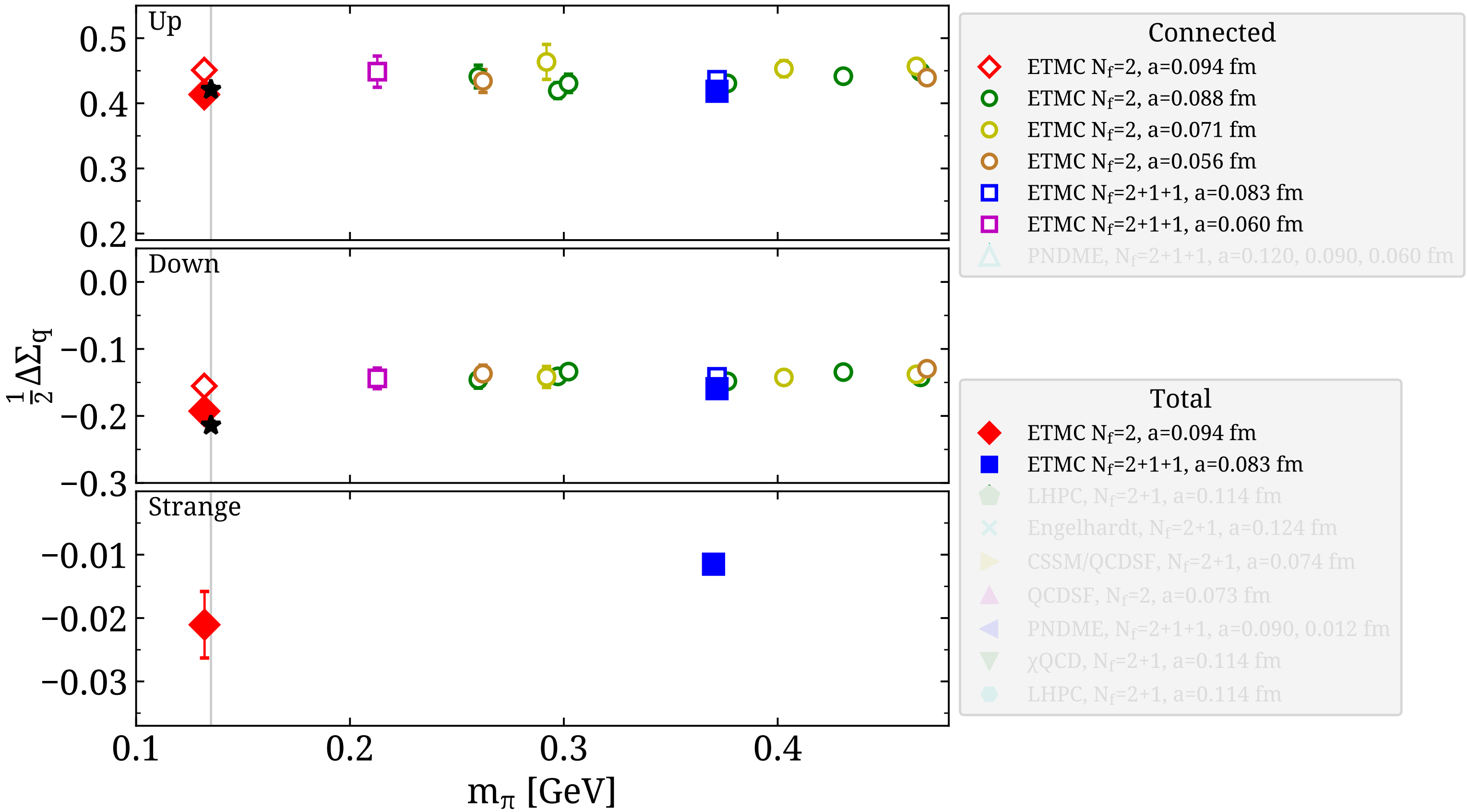

Quark intrinsic spin contributions

u, d, and s intrinsic spin contributions at 20(2)% of 1/2, at physical pion mass

Quark intrinsic spin contributions to nucleon spin

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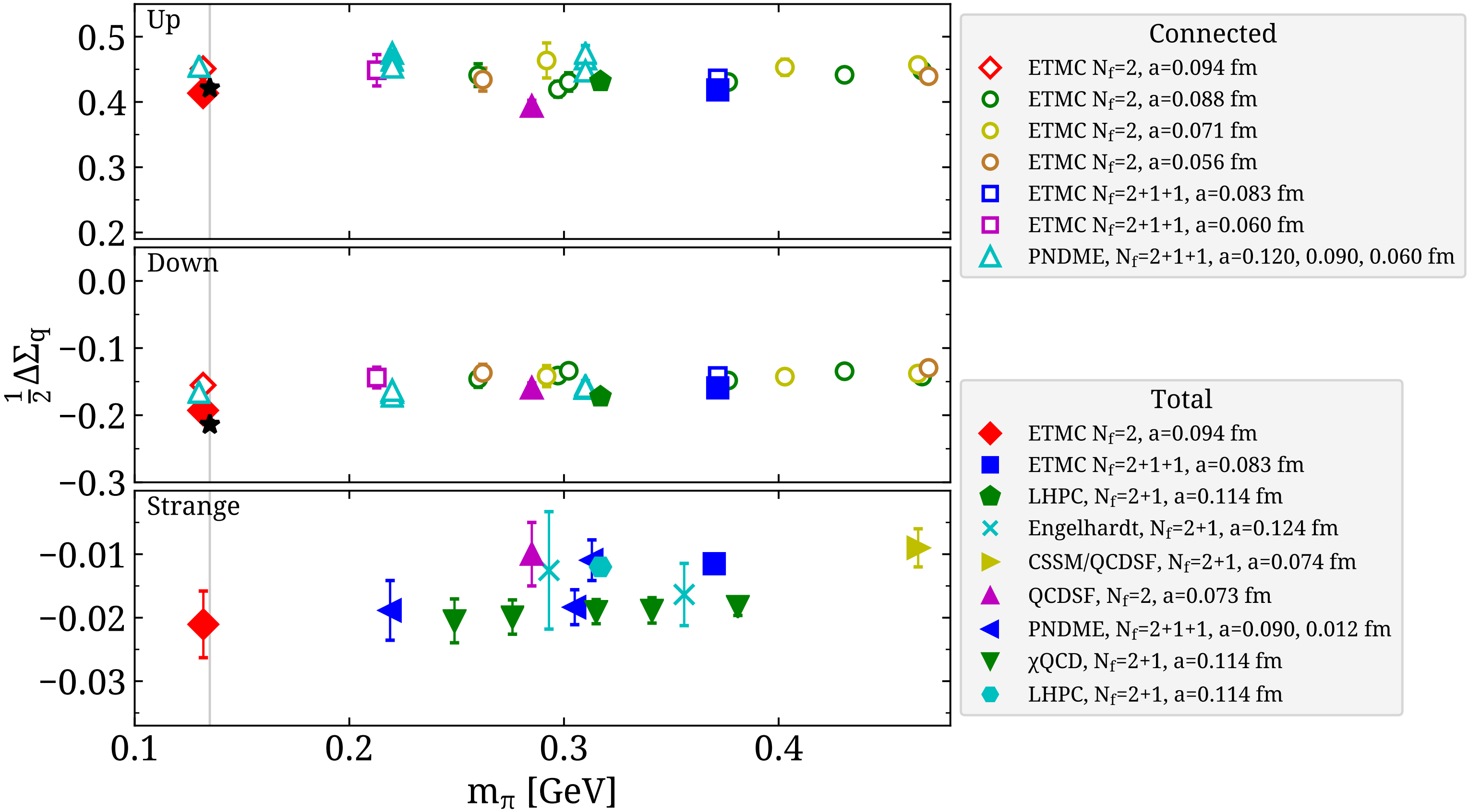


Nucleon spin

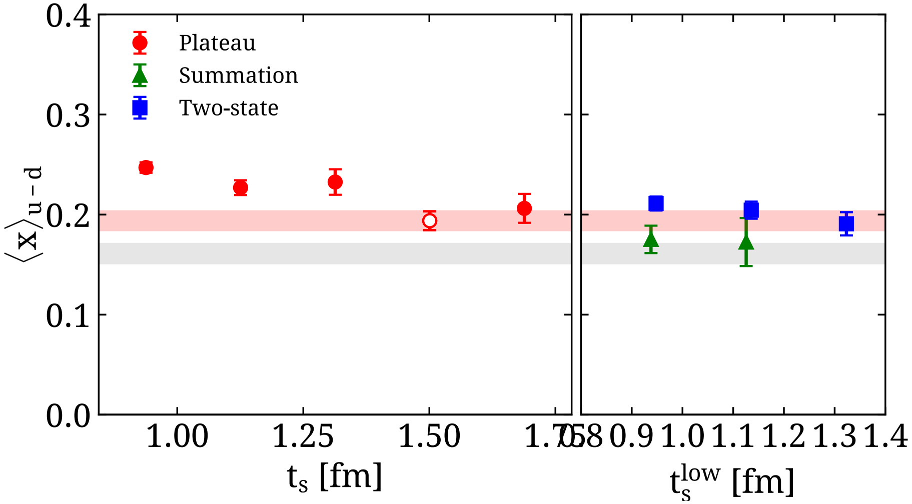
Total parton spin contributions to nucleon spin

– Ji's spin sum rule: $J_N = \sum_{q=u,d,s,c\cdots} \left(\frac{1}{2}\Delta\Sigma_q + L_q\right) + J_g$

– Quark contribution:

$$\frac{1}{2}\Delta\Sigma_q + L_q = J_q = \frac{1}{2}[A_{20}^q(0) + B_{20}^q(0)]$$

where $A_{20}^q(0)$ and $B_{20}^q(0)$ are obtained from the matrix element of the first derivative operator: $\mathcal{O}_V^{\mu\mu_1} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1\}}\psi$, i.e. $A_{20}^q(0) = \langle x \rangle_q$



– Connected contributions
– Increasing statistics with increasing sink-source separation
– 62,000 statistics at largest (~1.7 fm)

Nucleon spin

Total parton spin contributions to nucleon spin

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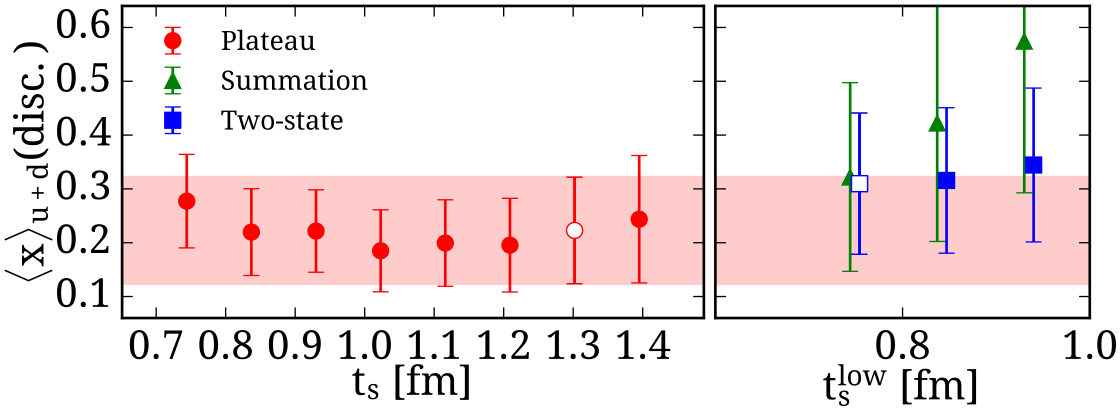

– Disconnected contributions
– 120,000 statistics
– Exact low-mode construction of loops with 500 eigenvectors
– 1,000 stochastic vectors for remaining

Nucleon spin

Total parton spin contributions to nucleon spin

- Ji's spin sum rule: $J_N = \sum_{q=u,d,s,c\cdots} \left(\frac{1}{2}\Delta\Sigma_q + L_q\right) + J_g$
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$$\frac{1}{2}\Delta\Sigma_q + L_q = J_q = \frac{1}{2}[A^q_{20}(0) + B^q_{20}(0)]$$

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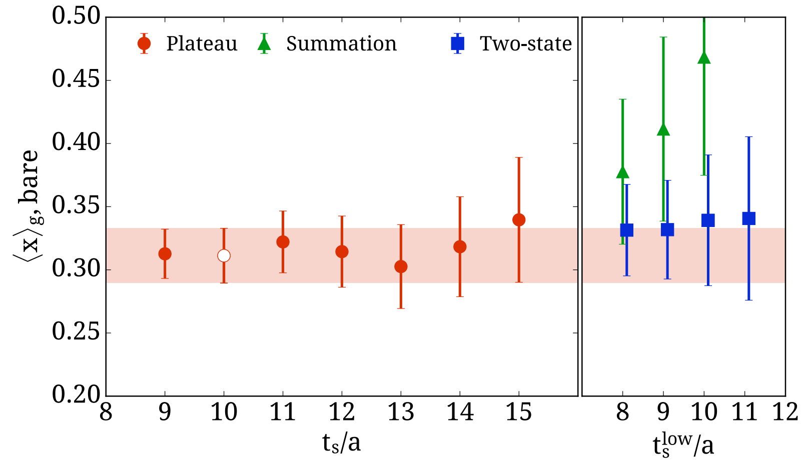
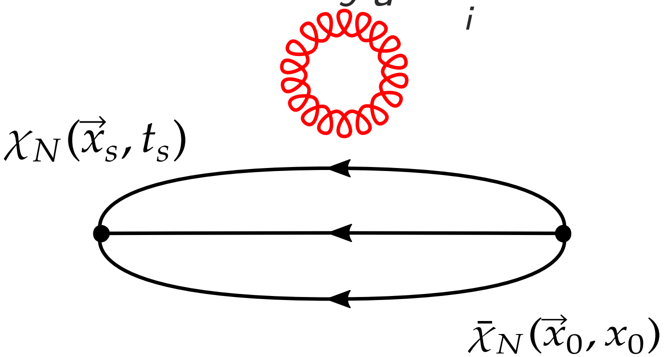
- Similarly for gluon contribution need disconnected diagram
- Statistics: 210.000. Stout smeared gluon fileds.

C. Alexandrou et al., PRD, arXiv:1611.06901

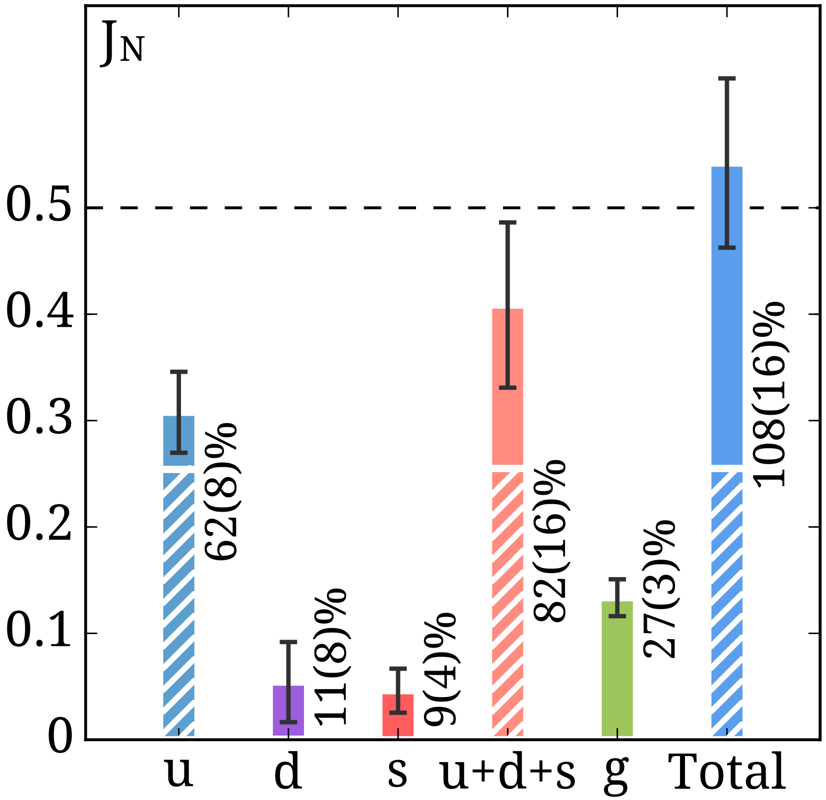


$$\mathcal{O} = \frac{2}{9}\frac{\beta}{a^4}\left[\sum_i \Re(P_{i4}) - \sum_{i<j} \Re(P_{ij})\right]$$



Nucleon spin

Parton spin and momentum contributions to nucleon spin



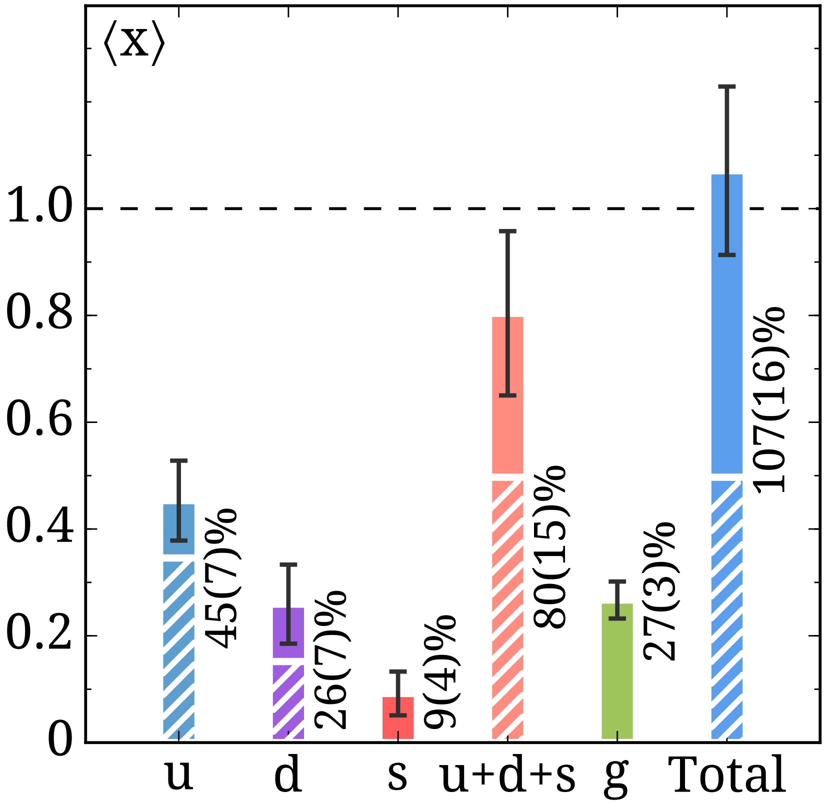
$$J^N_{u+d+s+g} = 0.541(62)(49)$$

- Includes u, d, s, and gluons simulated at physical pion mass
- Spin and momentum sums satisfied within errors
- Significant disconnected contributions (solid) compared to connected (hatched)
- About 10% uncertainties in component contributions

C. Alexandrou et al., PRL, arXiv:1706.02973

Nucleon spin

Parton spin and momentum contributions to nucleon spin



$$\langle x \rangle_{u+d+s+g} = 1.07(12)(10)$$

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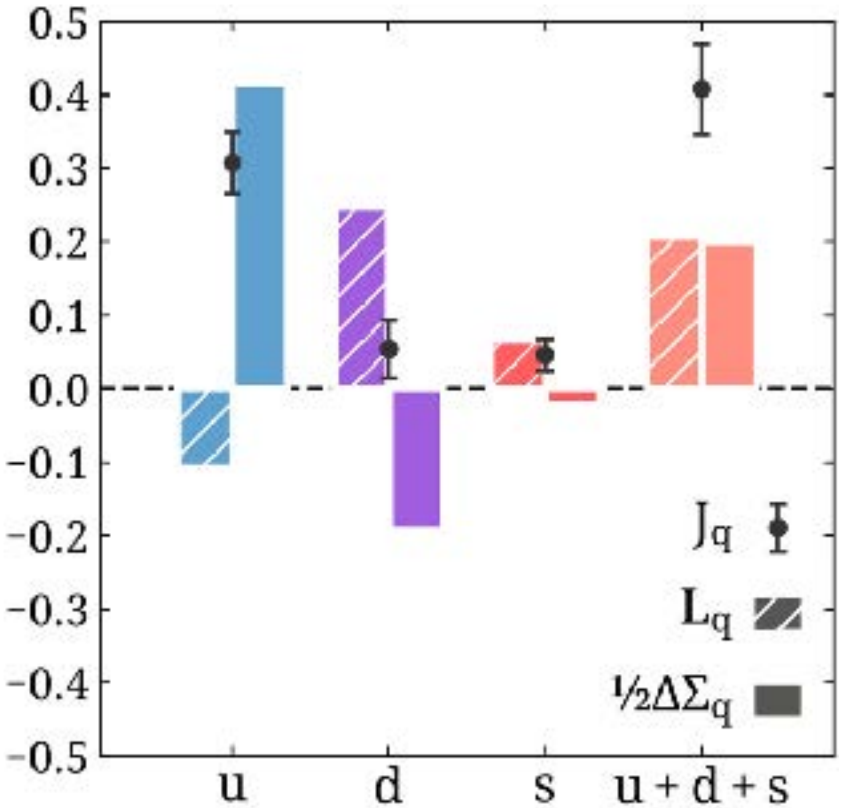

$$J^q = \frac{1}{2}\Delta\Sigma_q + L_q$$

- Angular momentum contribution deduced from difference of total and intrinsic quark spin
- Angular momentum: hatches
- Intrinsic spin: solid

	$\frac{1}{2}\Delta\Sigma$	J	L
u	0.415(13)(2)	0.308(30)(24)	-0.107(32)(24)
d	-0.193(8)(3)	0.054(29)(24)	0.247(30)(24)
s	-0.021(5)(1)	0.046(21)(0)	0.067(21)(1)
g	-	0.133(11)(14)	-
tot.	0.201(17)(5)	0.541(62)(49)	0.207(64)(45)

C. Alexandrou et al., PRL, arXiv:1706.02973

Summary

★ Lattice QCD approaching precision era for matrix elements

- Physical pion mass simulations from a number of collaborations
- Other systematic uncertainties coming under control

★ Nucleon spin decomposition from lattice QCD

- First results which include quark and gluon contributions at physical point promising
- More results coming out at physical point using other lattice actions (see e.g. arXiv:1710.09011 by the Kentucky group)
- Corroborates small contribution from quark intrinsic spins

★ETM Collaboration

European Twisted Mass Collaboration

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool), US (Temple, PA)

★Collaborators:

- C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, Ch. Kallidonis, F. Steffens, A. Vaquero, C. Wiese

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★ETM Collaboratio

(Orsay, Grenoble),
mburg, Münster),
oningen), Poland
, UK (Liverpool), US

★Collaborators:

- C. Alexandrou ... Hadjiyiannakou,
  K. Jansen, Ch...

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Deadline for applications 15th of December 2017

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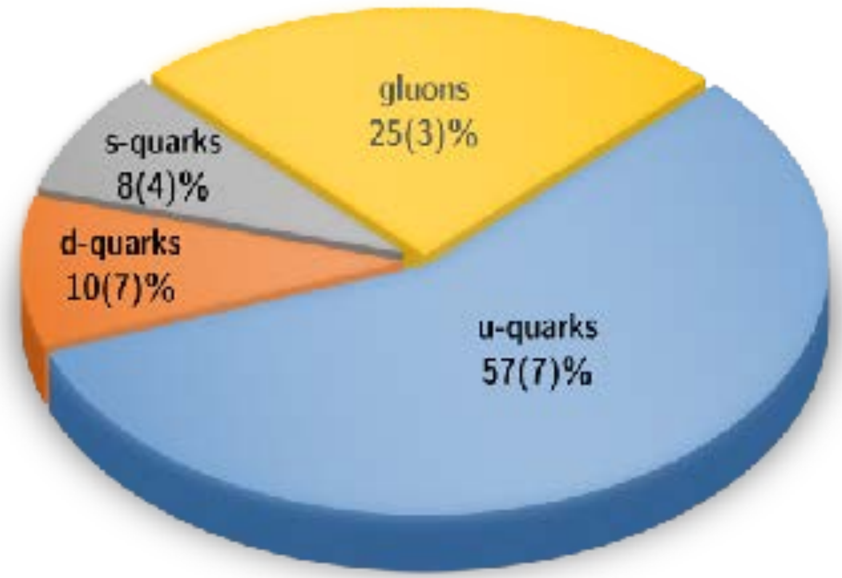
Backup

Nucleon spin

Parton spin and momentum contributions to nucleon spin

- Includes u, d, s, and gluons simulated at physical pion mass
- Spin and momentum sums satisfied within errors

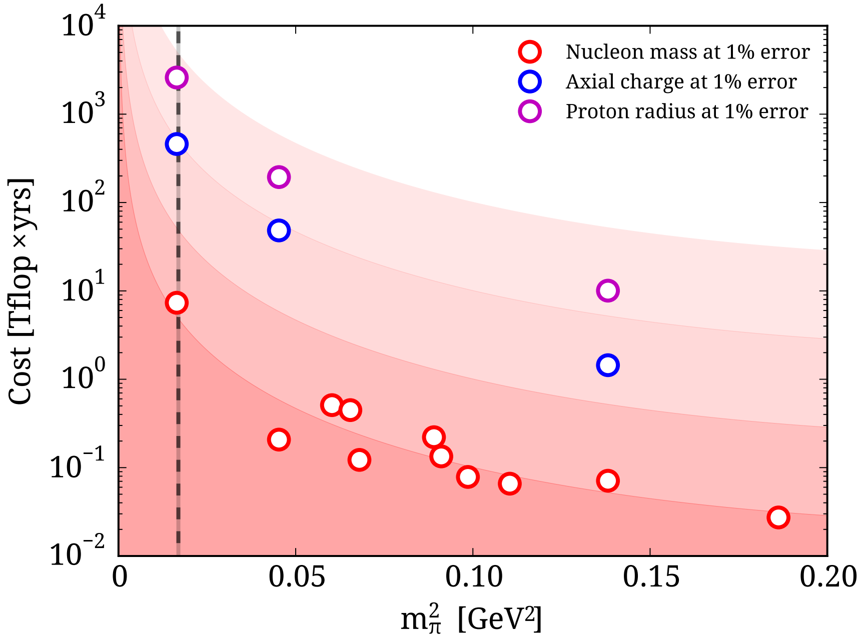
Total spin decomposition

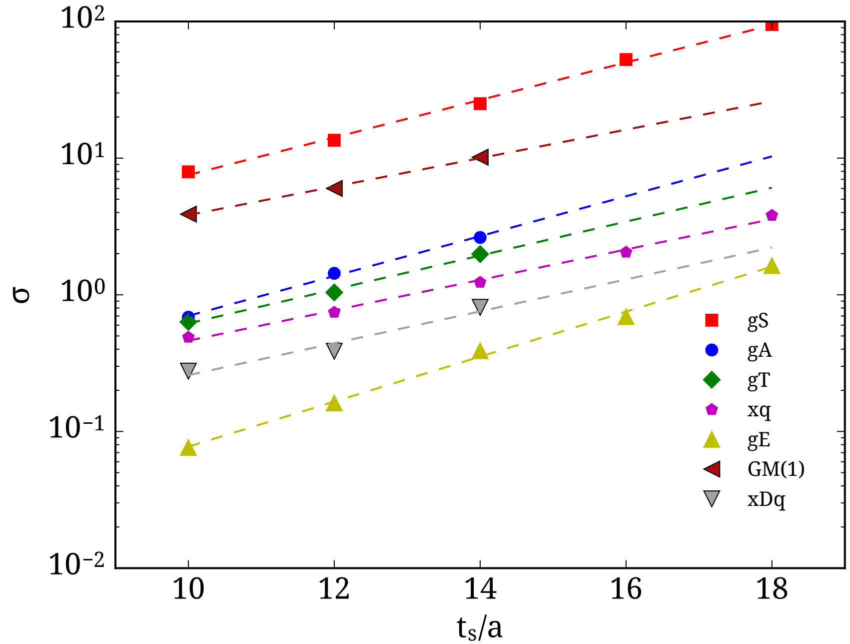


$$J^N_{u+d+s+g} = 0.541(62)(49)$$

C. Alexandrou et al., PRL, arXiv:1706.02973

Multi-petascale to exa-scale requirements





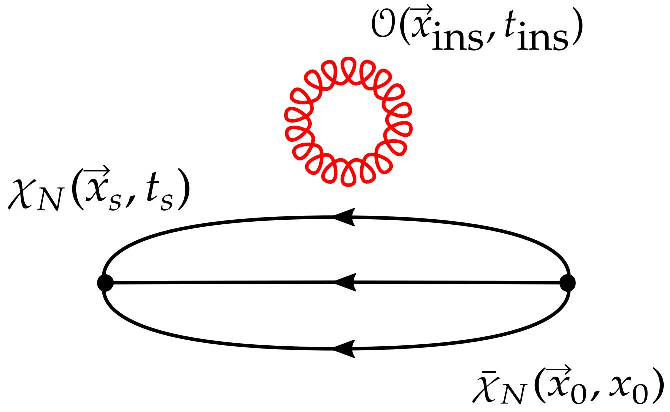
Indicative computer time requirements for nucleon structure ⟼ Increased time separations required for suppression of excited states at physical point

Gluon moment

Ji's spin sum: $$\frac{1}{2} = \sum_q (\frac{1}{2}\Delta\Sigma^q + L^q) + J^G$$

$$J^G = \frac{1}{2}[A_{20}^G(0) + B_{20}^G(0)]$$

$$\mathcal{O} = \frac{2}{9}\frac{\beta}{a^4}[\sum_i \Re(P_{i4}) - \sum_{i<j} \Re(P_{ij})]$$



Renormalisation

- Mixing with quark operator

$$\begin{pmatrix} J_q \\ J_G \end{pmatrix} = \begin{pmatrix} Z_{qq} & Z_{qG} \\ Z_{Gq} & Z_{GG} \end{pmatrix} \begin{pmatrix} J_q^{bare} \\ J_G^{bare} \end{pmatrix}$$